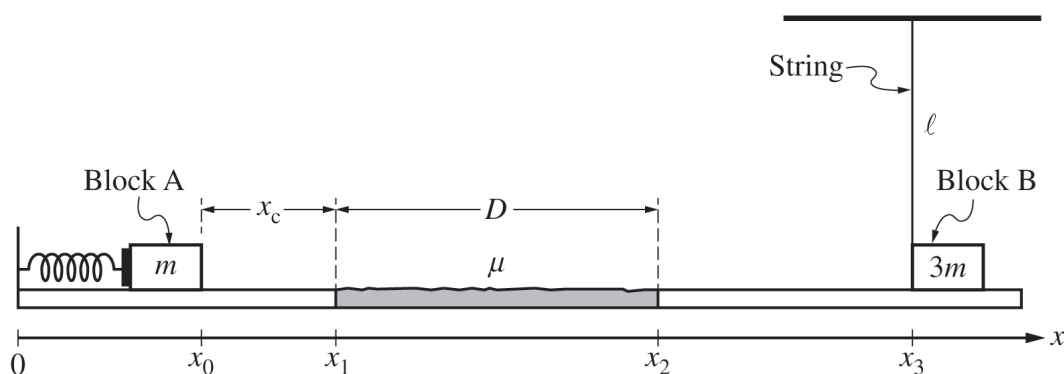


Begin your response to **QUESTION 1** on this page.**PHYSICS C: MECHANICS****SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

Figure 1

1. Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D , where the coefficient of kinetic friction between Block A and the surface is μ . Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c .

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Continue your response to **QUESTION 1** on this page.

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c . Block A moves with speed v , and the spring is at its equilibrium position.
- At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction.
- At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m , k , D , μ , x_c , and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1 .

ii. **Derive** an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3 .

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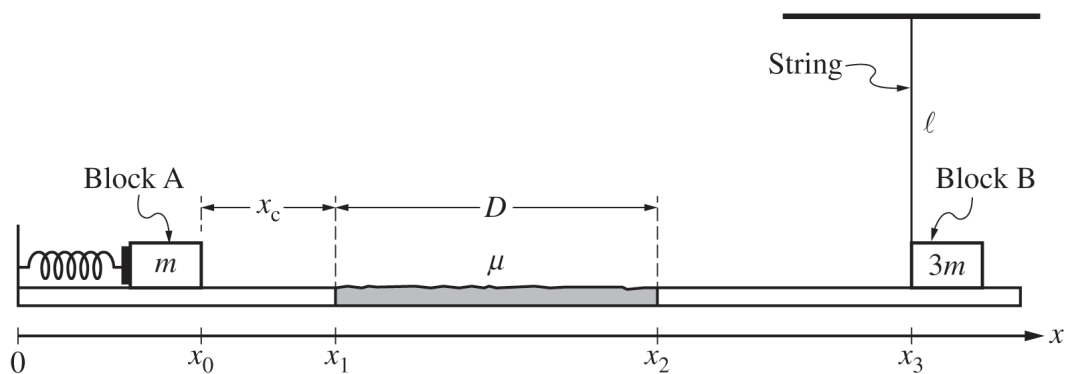
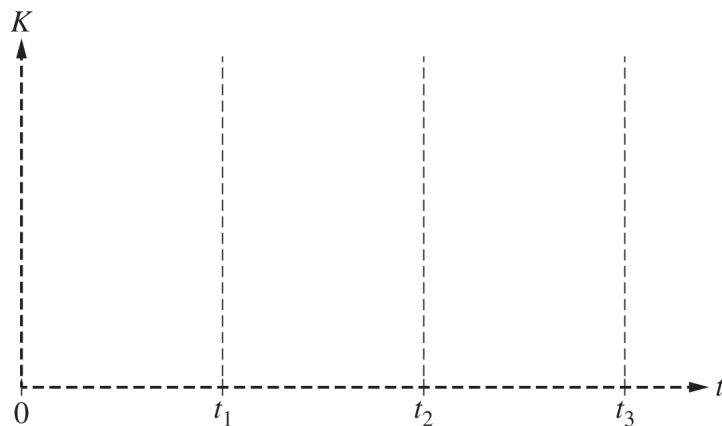
Continue your response to **QUESTION 1** on this page.Note: Figure not drawn to scale.

Figure 1

(b)

- i. On the following axes, **sketch** a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to time t_3 .



- ii. Use principles of work and energy to **justify** the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

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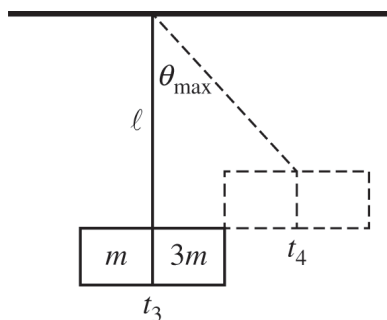
Continue your response to **QUESTION 1** on this page.Note: Figure not drawn to scale.

Figure 2

After the collision, the two-block system instantaneously comes to rest at time t_4 , which occurs when the string makes a small angle $\theta_{\max}$ with the vertical, as shown in Figure 2. For times $t > t_4$, the system oscillates with frequency f_ℓ . The support holding the string is raised, and the procedure is then repeated using a new string of length 2ℓ .

(c) **Indicate** how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length 2ℓ will compare to the frequency of oscillation f_ℓ from the original procedure.

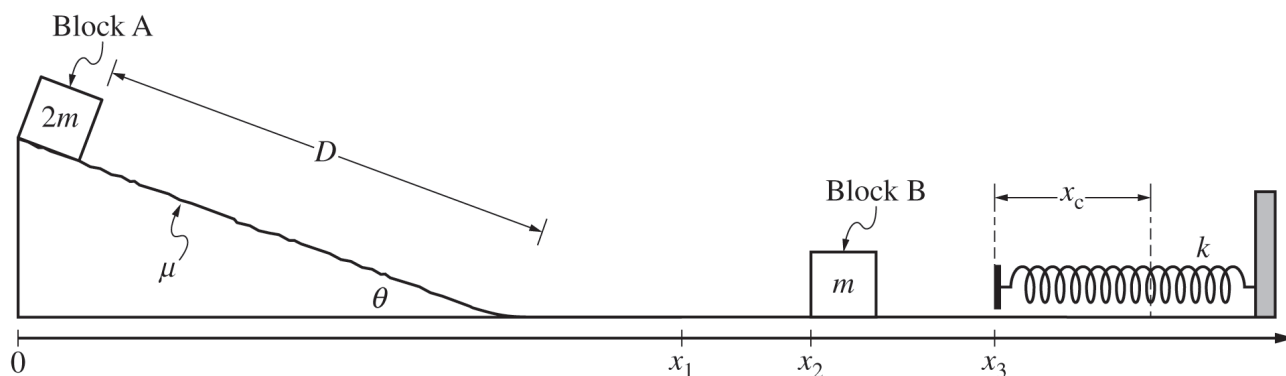
_____ $f_{2\ell} > f_\ell$ _____ $f_{2\ell} < f_\ell$ _____ $f_{2\ell} = f_\ell$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 1** on this page.**PHYSICS C: MECHANICS****SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

1. Blocks A and B of masses $2m$ and m , respectively, are arranged in a setup consisting of a ramp that makes an angle θ with a smooth horizontal table and an ideal spring of spring constant k fixed to a wall, as shown. Block A is held at rest a distance D up the ramp, and Block B is at rest on the horizontal table. The coefficient of kinetic friction between Block A and the rough ramp is μ in the region of length D , and there is negligible friction between the blocks and the smooth table.

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Continue your response to **QUESTION 1** on this page.

At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.

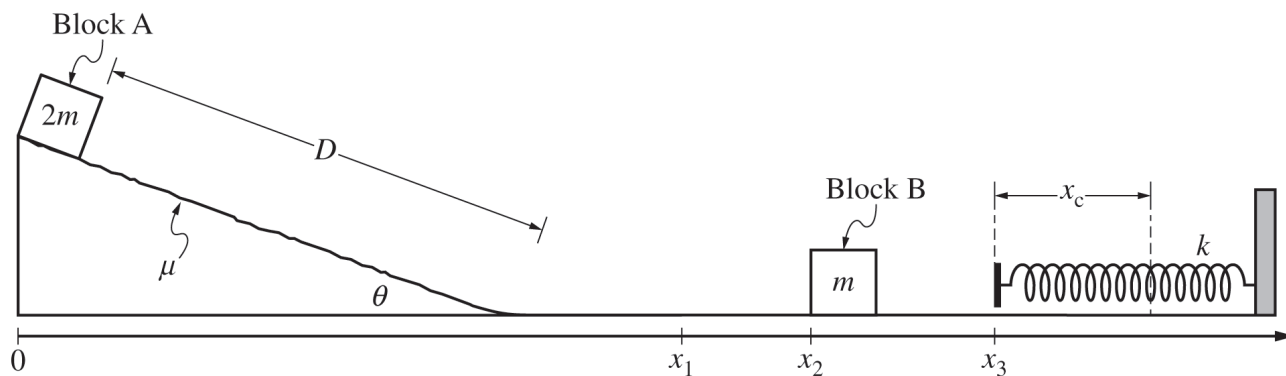
- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$.
- At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B.
- At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position.
- At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m , θ , D , μ , x_c , and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1 .

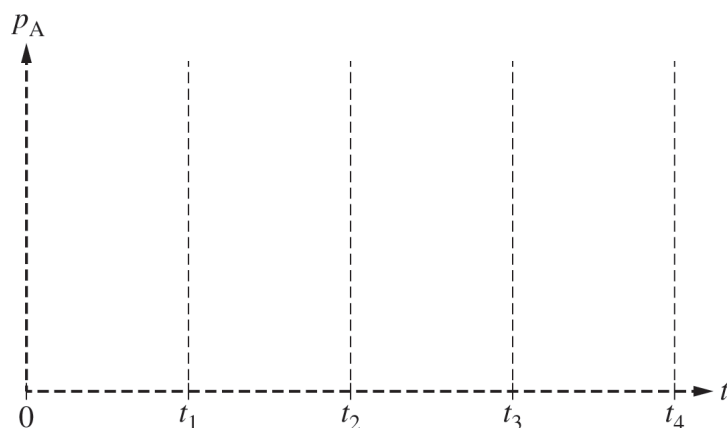
ii. **Derive** an expression for the spring constant k of the spring.

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Continue your response to **QUESTION 1** on this page.Note: Figure not drawn to scale.

(b)

- i. On the following axes, **sketch** a graph of the magnitude of the momentum p_A of Block A as a function of time t from $t = 0$ to t_4 .



- ii. Use principles of forces to **justify** the graph drawn in part (b)(i) for the time interval $t = t_3$ to $t = t_4$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

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Continue your response to **QUESTION 1** on this page.

For times $t > t_4$, the two-block-spring system oscillates with period T_O . The procedure is then repeated using a new ramp, where there is negligible friction between Block A and the ramp.

(c) **Indicate** how the new period of oscillation T_N in the procedure that uses the new ramp compares with the period of oscillation T_O from the original procedure.

_____ $T_N > T_O$ _____ $T_N < T_O$ _____ $T_N = T_O$

Briefly **justify** your answer.

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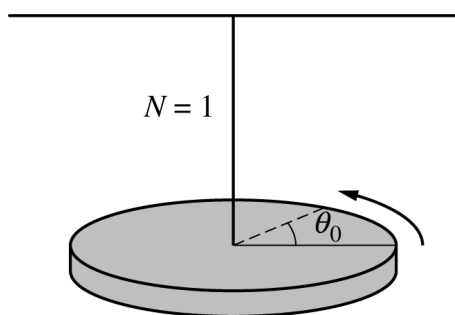


Figure 1

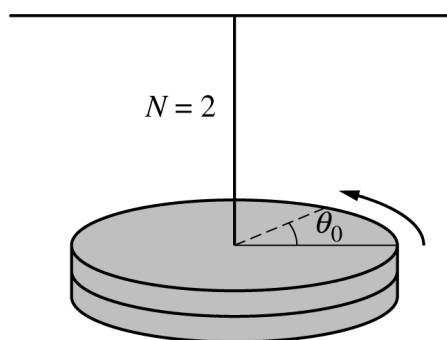


Figure 2

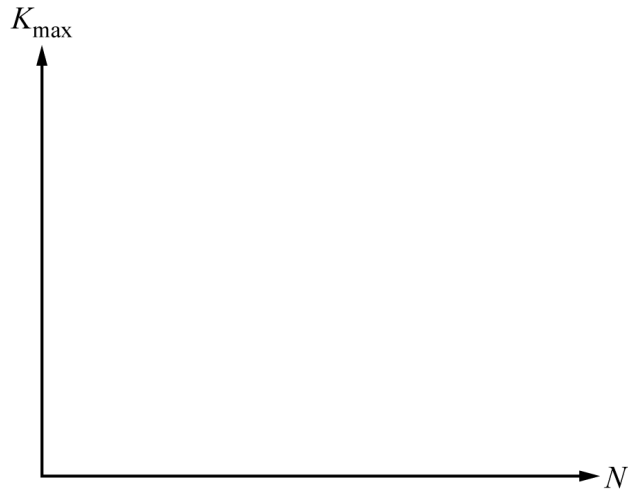
2. A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

- (a) Using $T = 2\pi\sqrt{\frac{I}{\kappa}}$, derive an expression for T as a function of N . Express your answer in terms of M , R , κ , N , and physical constants, as appropriate.

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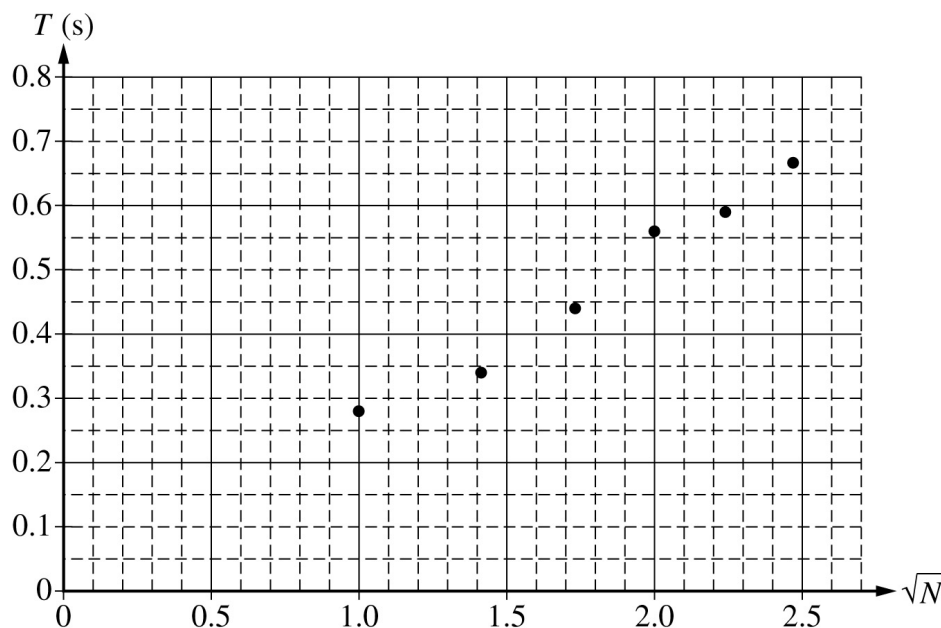
- (b) The potential energy stored in the torsional pendulum when the disks are displaced is $U = \frac{1}{2} \kappa (\Delta\theta)^2$. On the following axes, sketch a graph of the maximum kinetic energy $K_{\max}$ of the torsional pendulum as a function of N for $N \geq 1$.



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Continue your response to **QUESTION 2** on this page.

- (c) The student plots the data for T as a function of $\sqrt{N}$, as shown.



- Draw the best-fit line for the data.
- The student previously determined that the radius of a disk is $R = 0.2$ m and found that $\kappa = 1.6$ N·m. Using the graph, calculate the mass M of a single disk.
- The student finds that the value given by the manufacturer for the mass of the disk is less than the value determined experimentally in part (c)(ii). Determine a single source of experimental error that could result in the observed difference in the value of M . Justify your answer.

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(d) The student repeats the experiment, but now the disks have a density that varies as a function of the radius of the disk according to $\rho = 0.3r$.

i. Would the slope of the best-fit line for this new data be greater than, less than, or the same as the slope of the best-fit line in part (c)(i) ?

___ greater than ___ less than ___ the same as

Justify your answer.

ii. When $N = 1$, the maximum angular speed of the torsional pendulum with a uniform disk is found to be ω_U . When $N = 1$, the maximum angular speed of the torsional pendulum with a nonuniform disk is $\omega_{\text{non-U}}$. Which of the following correctly compares ω_U and $\omega_{\text{non-U}}$?

___ $\omega_U > \omega_{\text{non-U}}$ ___ $\omega_U < \omega_{\text{non-U}}$ ___ $\omega_U = \omega_{\text{non-U}}$

Briefly justify your answer.

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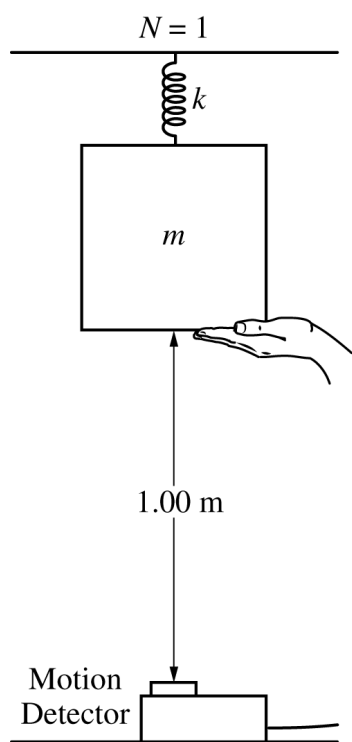


Figure 1

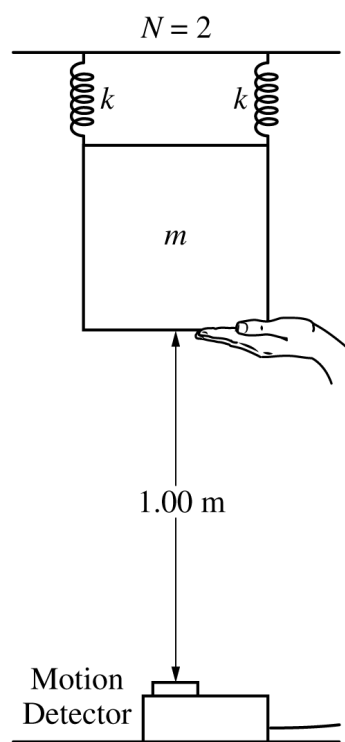


Figure 2

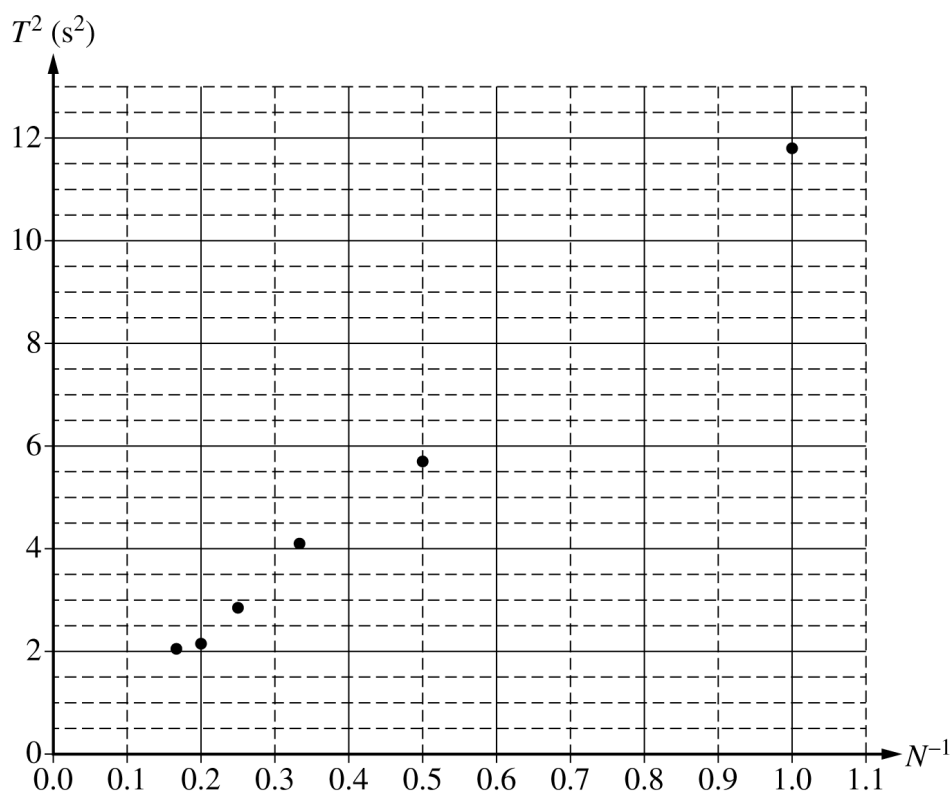
A student conducts an investigation to determine the relationship between the period of oscillation T of a system consisting of a block and N attached springs. The student starts with a block of mass m attached to a single ideal spring of spring constant k , as shown in Figure 1. The student holds the block so that the spring is neither stretched nor compressed at a vertical height 1.00 m above a motion detector. The student releases the block from rest and records the period of oscillation for the system consisting of the single spring and block. An additional identical spring is attached in parallel, as shown in Figure 2, and the procedure is repeated for $N = 2$ springs. This procedure is repeated through $N = 10$ springs.

- (a) Derive an expression for T as a function of N . Express your answer in terms of m , k , N , and physical constants as appropriate.

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Continue your response to **QUESTION 2** on this page.(b) On the following axes, sketch a graph of T as a function of N for $N \geq 1$.**GO ON TO THE NEXT PAGE.**

(c) The student plots the data for T^2 as a function of N^{-1} , as shown.



i. Draw the best-fit line for the data.

ii. The mass of the block is measured to be $m = 1.5$ kg. Using the graph, calculate an experimental value for the spring constant k for a single spring.

iii. The student finds that the value given by the manufacturer for the spring constant is larger than the value determined experimentally in part (c)(ii). Determine a single source of experimental error that could result in the observed difference in the value for k . Briefly justify your answer.

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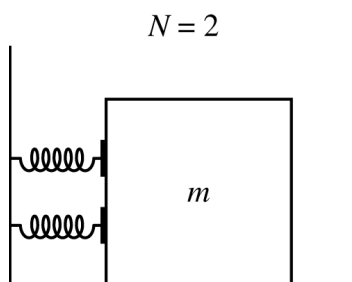
Continue your response to **QUESTION 2** on this page.

Figure 3

- (d) The student conducts a similar investigation to determine the relationship between the period of oscillation T of a system consisting of a block and a number N of identical springs but arranges the block-spring system horizontally on a table, as shown in Figure 3. Frictional forces between the table and the block are negligible. In each trial, the block is displaced the same horizontal distance from equilibrium and released from rest.

i. The student plots T^2 as a function of N^{-1} for this new data. Would the slope of the best-fit line from this new investigation be greater than, less than, or the same as the slope of the best-fit line in part (c)(i) ?

____ greater than ____ less than ____ the same as

Briefly justify your answer.

ii. When $N = 1$, the maximum speed of the block is found to be $v_{\max}$. When N increases, will $v_{\max}$ increase, decrease, or stay the same?

____ increase ____ decrease ____ stay the same

Justify your answer.

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Note: Figure not drawn to scale.

2. A block of mass m starts at rest at the top of an inclined plane of height h , as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k . There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m , h , k , and physical constants, as appropriate.

- (a)
- Derive an expression for the speed of the block just before it collides with the spring.
 - Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

- (b) Derive an expression for the maximum compression of the spring.
- (c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0 . Now suppose the block experiences a resistive force as it slides on the horizontal surface.

The magnitude of the resistive force F is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of kg/m .

- (d)
- Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m , h , k , β , v , and physical constants, as appropriate.
 - Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time t can be written in the form $\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$, where v_0 is the speed at $t = 0$.

- (e) Sketch graphs of position x as a function of time t , velocity v as a function of time t , and acceleration a as a function of time t for the block as it is moving on the horizontal surface before it reaches the spring.

