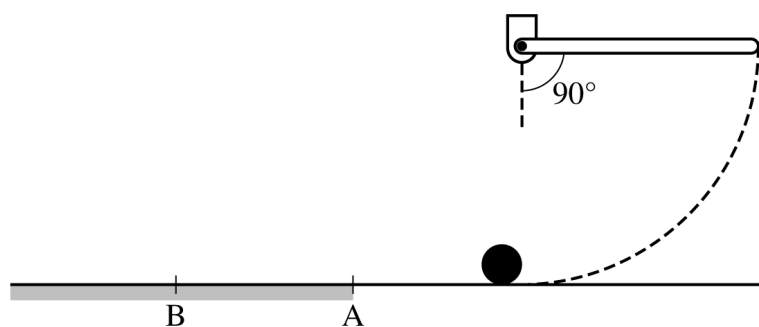




Note: Figure not drawn to scale.

Figure 1

3. A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3} M \ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5} m R^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.
- (a) Derive an expression for the angular speed of the rod just before striking the sphere in terms of the length ℓ and physical constants as appropriate.

GO ON TO THE NEXT PAGE.

- (b) Derive an expression for the linear speed v_0 of the sphere immediately after colliding with the rod in terms of the length ℓ and physical constants as appropriate.

After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ , beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

- (c) In the following diagram, which represents the sphere while the sphere is traveling between Points A and B, draw and label the forces (not components) that act on the sphere. Each force must be represented by a distinct arrow starting on, and pointing away from, the point of application on the sphere.



- (d) Derive an expression for each of the following as the sphere is rotating and sliding between points A and B in terms of v_0 , μ , R , t , and physical constants as appropriate.

i. The linear velocity v of the center of mass of the sphere as a function of time t

ii. The angular velocity ω of the sphere as a function of time t

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Name: _____

Continue your response to **QUESTION 3** on this page.

(e)

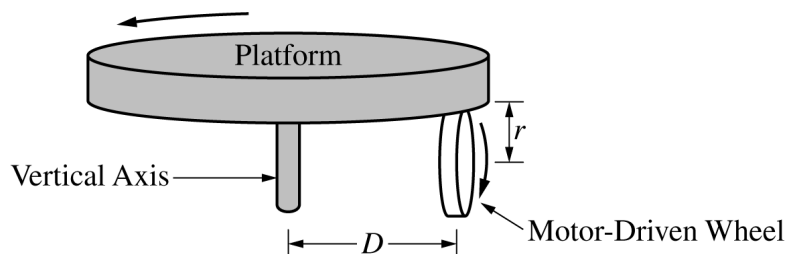
i. Derive an expression for the time it takes the sphere to travel from Point A to Point B in terms of v_0 , μ , and physical constants as appropriate.

ii. Derive an expression for the linear velocity of the sphere upon reaching Point B in terms of v_0 .

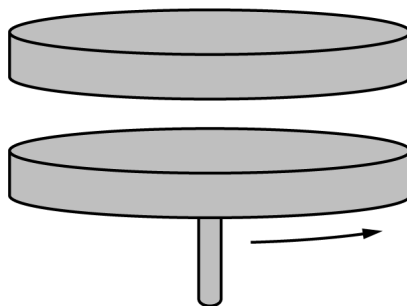
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END OF EXAM



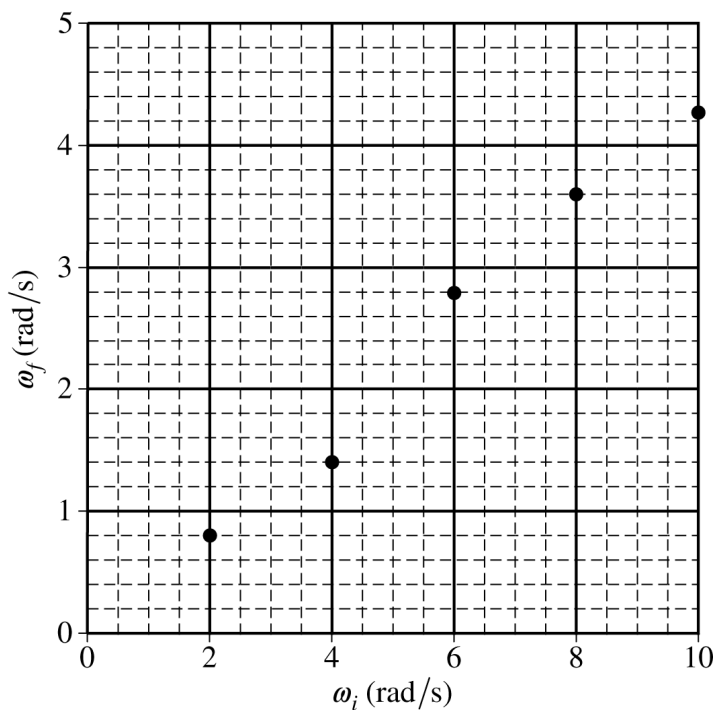
3. A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the wheel until the platform reaches an angular speed ω_P after time Δt . During time Δt , the wheel stays in contact with the platform without slipping.
- (a) Derive an expression for the angular speed ω_P of the platform. Express your answer in terms of I_P , r , D , F , Δt , and physical constants, as appropriate.
- (b) Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed ω_P . Express your answer in terms of I_P , r , D , F , Δt , and physical constants, as appropriate.
- (c) Derive an expression for the angular speed of the wheel ω_W when the platform has reached angular speed ω_P . Express your answer in terms of D , r , ω_P , and physical constants, as appropriate.



When the platform is spinning at angular speed ω_P , the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia I_P as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed ω_f .

- (d) Derive an expression for ω_f . Express your answer in terms of ω_P , I_P , and physical constants, as appropriate.

A student now uses the rotating platform ($I_P = 3.1 \text{ kg}\cdot\text{m}^2$) to determine the rotational inertia I_U of an unknown object about a vertical axis that passes through the object's center of mass. The platform is rotating at an initial angular speed ω_i when the unknown object is dropped with its center of mass directly above the center of the platform. The platform and object are observed to be rotating together at angular speed ω_f . Trials are repeated for different values of ω_i . A graph of ω_f as a function of ω_i is shown on the axes below.



- (e)
- i. On the graph on the previous page, draw a best-fit line for the data.
 - ii. Using the straight line, calculate the rotational inertia of the unknown object I_U about a vertical axis passing through its center of mass.
- (f) The kinetic energy of the spinning platform before the object is dropped on it is K_i . The total kinetic energy of the platform-object system when it reaches angular speed ω_f is K_f . Which of the following expressions is true?
- _____ $K_f < K_i$ _____ $K_f = K_i$ _____ $K_f > K_i$

Justify your answer.

- (g) One of the students observes that the center of mass of the object is not actually aligned with the axis of the platform. Is the experimental value of I_U obtained in part (e) greater than, less than, or equal to the actual value of the rotational inertia of the unknown object about a vertical axis that passes through its center of mass?
- _____ Greater than _____ Less than _____ Equal to

Justify your answer.

STOP

END OF EXAM

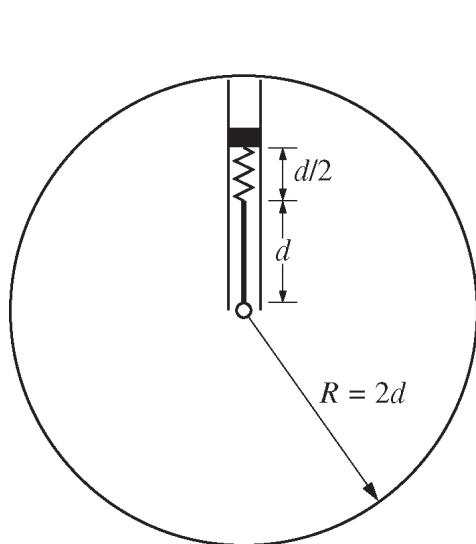


Figure 1

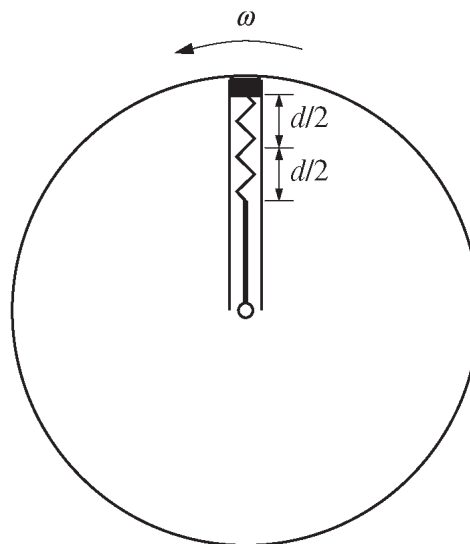


Figure 2

Mech.3.

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

	Mass	Rotational Inertia
Block	m	
Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)
Platform	$m_P = 5m$	$\frac{m_P R^2}{2}$ (about the central axis)

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω . Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω . Express all algebraic answers in terms of m , d , ω , and physical constants, as appropriate.

- (a) Derive an expression for the spring constant of the spring.
- (b)
 - i. Determine an expression for the rotational inertia of the block around the axis of the platform.
 - ii. Derive an expression for the rotational inertia of the entire system about the axis of the platform.
- (c) Determine an expression for the angular momentum of the entire system about the axis of the platform.

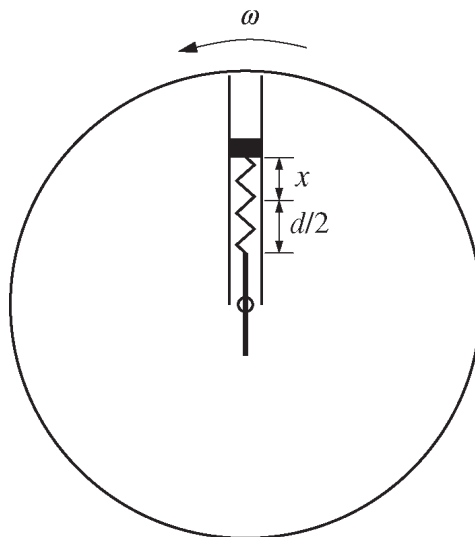


Figure 3

While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed ω is maintained by the motor driving the platform.

- (d) Derive an expression for the distance x that the spring is stretched when the rod reaches the position shown in Figure 3 above.

For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

- (e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

_____ Increasing _____ Decreasing _____ Staying the same

Justify your answer.

- (f) In order to keep the system rotating with constant angular speed ω , is the motor doing positive work, negative work, or no work on the rotating system?

_____ Positive _____ Negative _____ No work

Justify your answer.

- (g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.

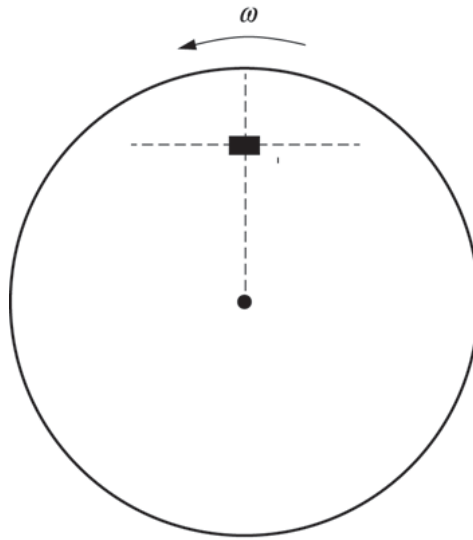
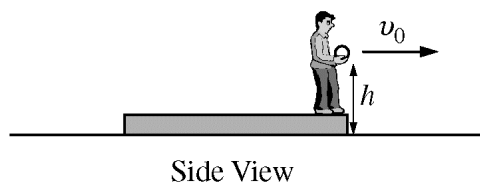
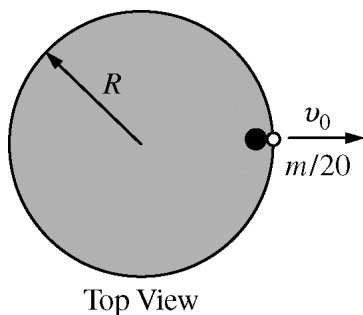


Figure 4

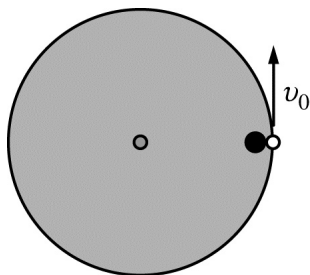
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END OF EXAM

**Mech. 3.**

A large circular disk of mass m and radius R is initially stationary on a horizontal icy surface. A person of mass $m/2$ stands on the edge of the disk. Without slipping on the disk, the person throws a large stone of mass $m/20$ horizontally at initial speed v_0 from a height h above the ice in a radial direction, as shown in the figures above. The coefficient of friction between the disk and the ice is μ . All velocities are measured relative to the ground. The time it takes to throw the stone is negligible. Express all algebraic answers in terms of m , R , v_0 , h , μ , and fundamental constants, as appropriate.

- Derive an expression for the length of time it will take the stone to strike the ice.
- Assuming that the disk is free to slide on the ice, derive an expression for the speed of the disk and person immediately after the stone is thrown.
- Derive an expression for the time it will take the disk to stop sliding.



Top View

The person now stands on a similar disk of mass m and radius R that has a fixed pole through its center so that it can only rotate on the ice. The person throws the same stone horizontally in a tangential direction at initial speed v_0 , as shown in the figure above. The rotational inertia of the disk is $mR^2/2$.

- (d) Derive an expression for the angular speed ω of the disk immediately after the stone is thrown.
- (e) The person now stands on the disk at rest $R/2$ from the center of the disk. The person now throws the stone horizontally with a speed v_0 in the same direction as in part (d). Is the angular speed of the disk immediately after throwing the stone from this new position greater than, less than, or equal to the angular speed found in part (d) ?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

STOP

END OF EXAM