

3. A triangular rod, shown above, has length L , mass M , and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(a) Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

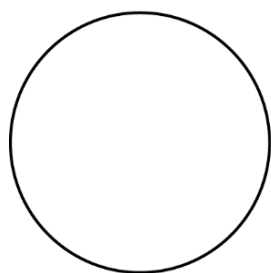


Figure 1

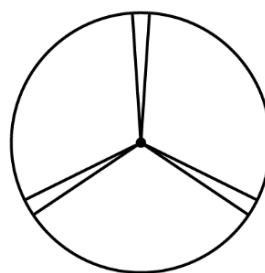


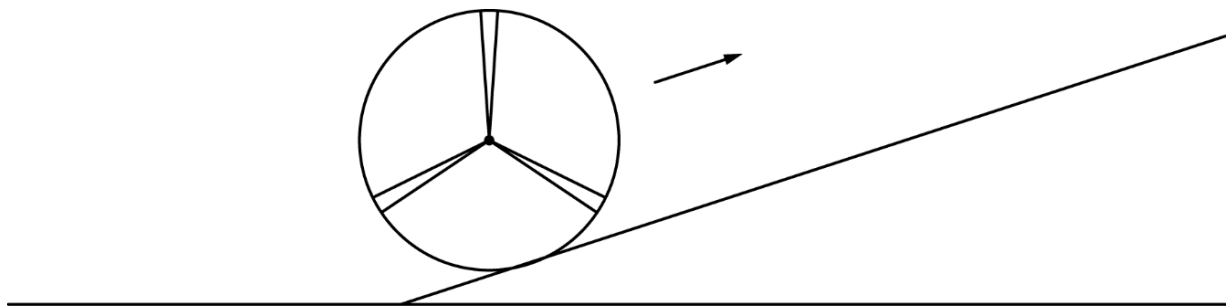
Figure 2

The thin hoop shown above in Figure 1 has a mass M , radius L , and a rotational inertia around its center of ML^2 . Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

- (b) Derive an expression for the rotational inertia I_{tot} of the hoop-rods system about the center of the hoop. Express your answer in terms of M , L , and physical constants, as appropriate.

The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt .

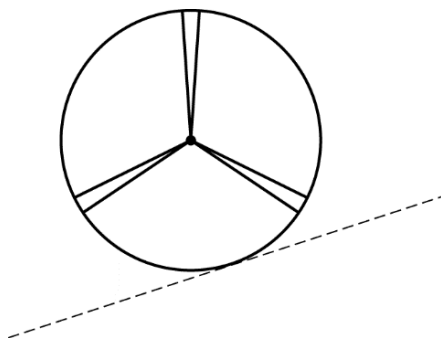
- (c) Derive an expression for the final angular speed ω of the hoop-rods system. Express your answer in terms of M , L , F , Δt , and physical constants, as appropriate.



The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed ω found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

(d)

- i. On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.



- ii. Justify your choice for the direction of each of the forces drawn in part (d)i.

- (e) Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of M , L , I_{tot} , ω , and physical constants, as appropriate.

STOP

END OF EXAM

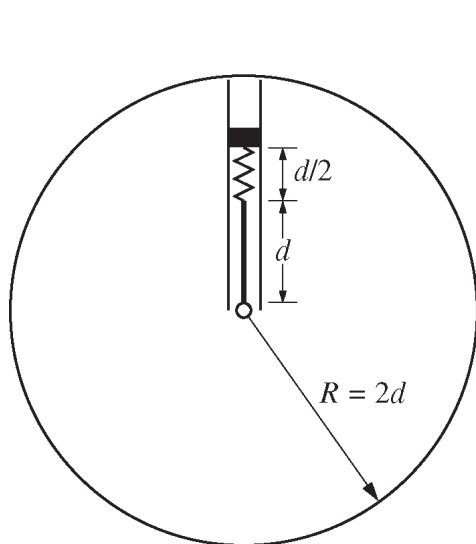


Figure 1

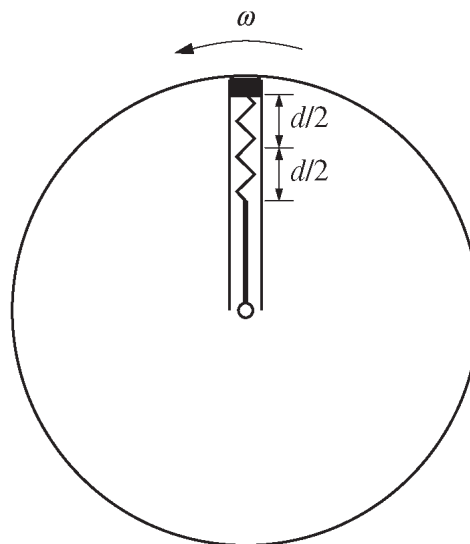


Figure 2

Mech.3.

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

	Mass	Rotational Inertia
Block	m	
Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)
Platform	$m_P = 5m$	$\frac{m_P R^2}{2}$ (about the central axis)

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω . Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω . Express all algebraic answers in terms of m , d , ω , and physical constants, as appropriate.

- (a) Derive an expression for the spring constant of the spring.
- (b)
 - i. Determine an expression for the rotational inertia of the block around the axis of the platform.
 - ii. Derive an expression for the rotational inertia of the entire system about the axis of the platform.
- (c) Determine an expression for the angular momentum of the entire system about the axis of the platform.

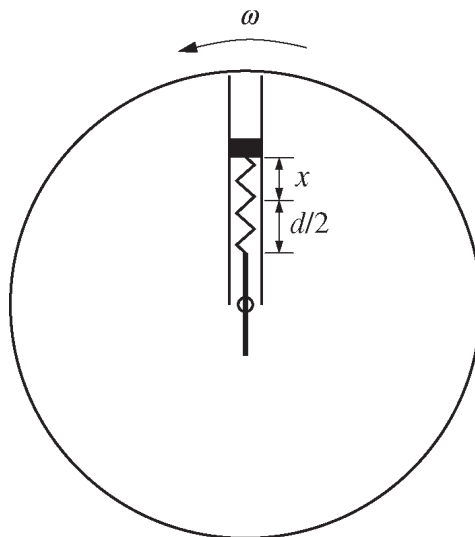


Figure 3

While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed ω is maintained by the motor driving the platform.

- (d) Derive an expression for the distance x that the spring is stretched when the rod reaches the position shown in Figure 3 above.

For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

- (e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

_____ Increasing _____ Decreasing _____ Staying the same

Justify your answer.

- (f) In order to keep the system rotating with constant angular speed ω , is the motor doing positive work, negative work, or no work on the rotating system?

_____ Positive _____ Negative _____ No work

Justify your answer.

- (g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.

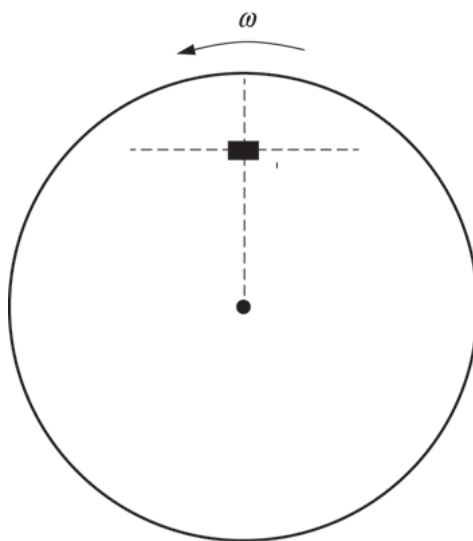


Figure 4

STOP

END OF EXAM