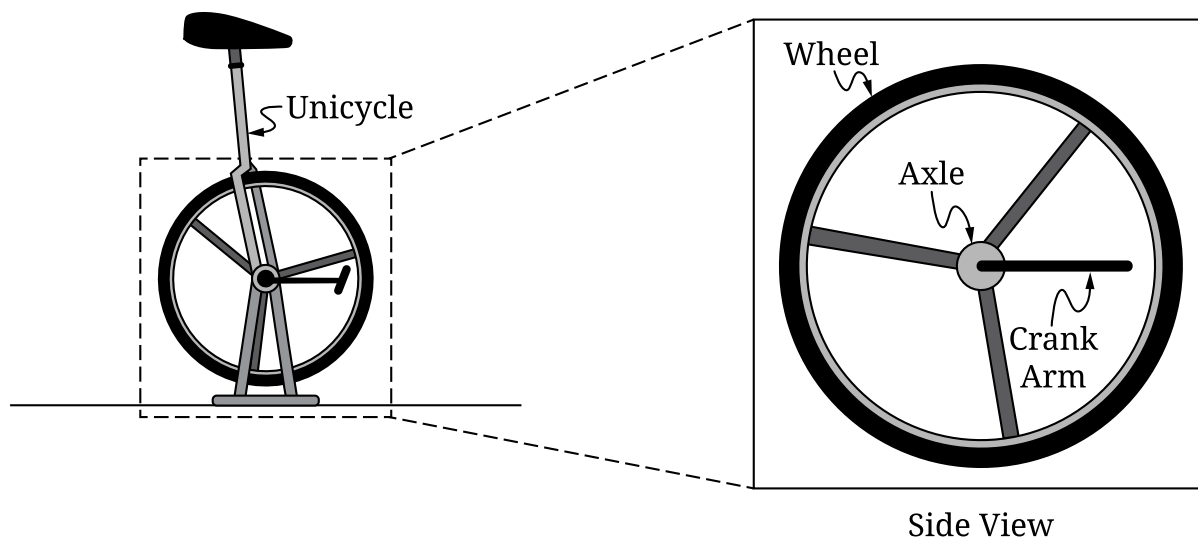


Question 4

4. A unicycle is placed on a stand such that the wheel does not touch the ground. The rotational inertia of the wheel about the axle is I_W . One end of a crank arm can be attached to the wheel, as shown in the side view.



In Scenarios 1 and 2, crank arms of different lengths are connected to the wheel. In each scenario, the wheel is initially at rest. A force of magnitude F is then exerted at the end of each crank arm. The direction of the force remains perpendicular to each crank arm at all times. The mass of each crank arm is negligible. The scenarios differ as described.

- Scenario 1: The length of the crank arm is ℓ_1 . The angular speed of the wheel immediately after the crank arm has been turned through one rotation is ω_1 .
- Scenario 2: The length of the crank arm is ℓ_2 , where $\ell_2 > \ell_1$. The angular speed of the wheel immediately after the longer crank arm has been turned through one rotation is ω_2 .

Part A

Indicate whether ω_2 is greater than, less than, or equal to ω_1 by writing one of the following.

- $\omega_2 > \omega_1$
- $\omega_2 < \omega_1$
- $\omega_2 = \omega_1$

Justify your answer. Your justification may reference equations but must include conceptual reasoning beyond algebraic solutions.

Part B

Derive an expression for the angular speed ω_1 of the wheel in Scenario 1 immediately after the crank arm has been turned through one rotation. Express your answer in terms of I_W , F , ℓ_1 , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

In Scenario 3, the unicycle is placed on the ground such that the unicycle can move forward without the wheel slipping on the ground. A force of the same magnitude F is exerted at the end of the crank arm of length ℓ_1 , as in Scenario 1. The direction of the force remains perpendicular to the crank arm at all times. The resulting angular speed of the wheel immediately after the crank arm has been turned through one rotation is ω_3 .

Indicate whether ω_3 is greater than, less than, or equal to ω_1 by writing one of the following.

- $\omega_3 > \omega_1$
- $\omega_3 < \omega_1$
- $\omega_3 = \omega_1$

Briefly **justify** your answer. Your justification may reference equations but must include conceptual reasoning beyond algebraic solutions.

STOP

END OF EXAM

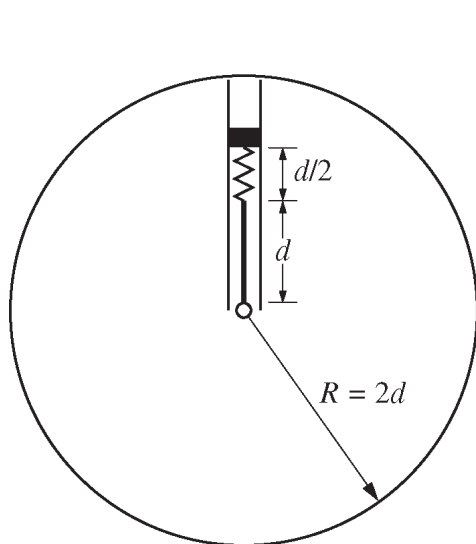


Figure 1

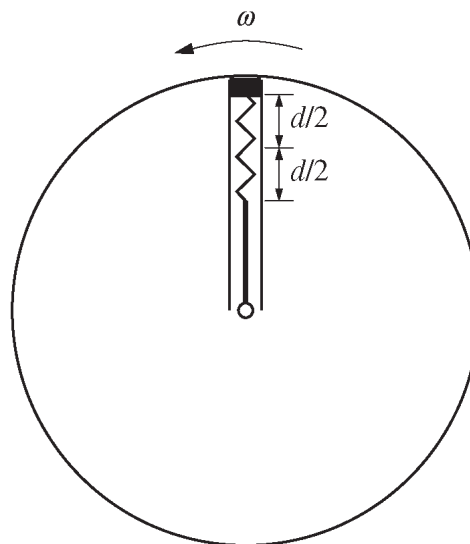


Figure 2

Mech.3.

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

	Mass	Rotational Inertia
Block	m	
Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)
Platform	$m_P = 5m$	$\frac{m_P R^2}{2}$ (about the central axis)

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω . Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω . Express all algebraic answers in terms of m , d , ω , and physical constants, as appropriate.

- (a) Derive an expression for the spring constant of the spring.
- (b)
 - i. Determine an expression for the rotational inertia of the block around the axis of the platform.
 - ii. Derive an expression for the rotational inertia of the entire system about the axis of the platform.
- (c) Determine an expression for the angular momentum of the entire system about the axis of the platform.

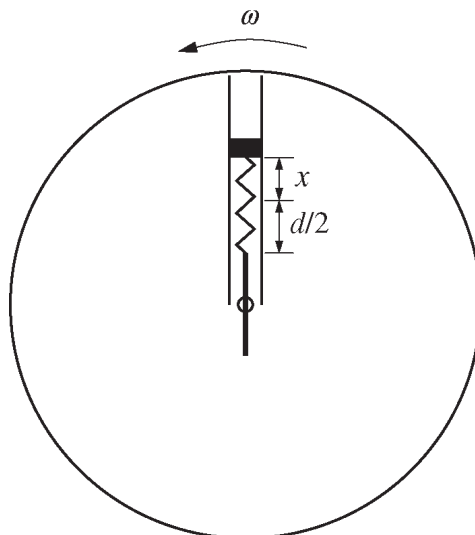


Figure 3

While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed ω is maintained by the motor driving the platform.

- (d) Derive an expression for the distance x that the spring is stretched when the rod reaches the position shown in Figure 3 above.

For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

- (e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

_____ Increasing _____ Decreasing _____ Staying the same

Justify your answer.

- (f) In order to keep the system rotating with constant angular speed ω , is the motor doing positive work, negative work, or no work on the rotating system?

_____ Positive _____ Negative _____ No work

Justify your answer.

- (g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.

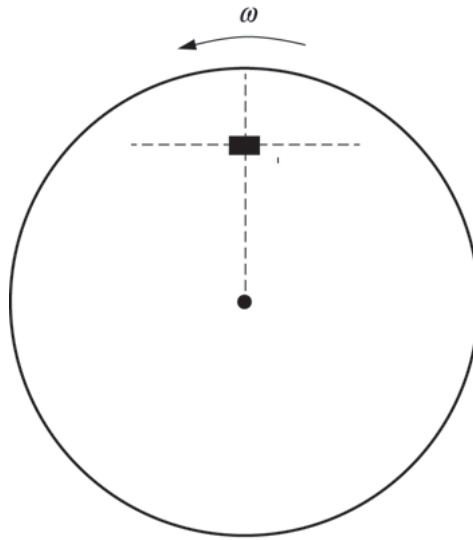


Figure 4

STOP

END OF EXAM