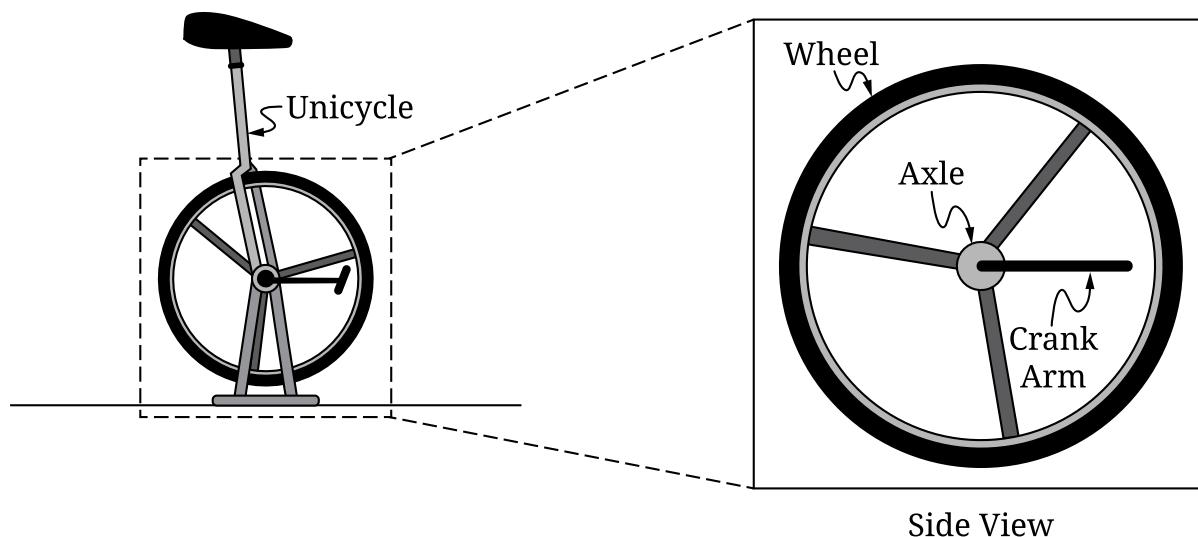


Question 4

4. A unicycle is placed on a stand such that the wheel does not touch the ground. The rotational inertia of the wheel about the axle is I_W . One end of a crank arm can be attached to the wheel, as shown in the side view.



In Scenarios 1 and 2, crank arms of different lengths are connected to the wheel. In each scenario, the wheel is initially at rest. A force of magnitude F is then exerted at the end of each crank arm. The direction of the force remains perpendicular to each crank arm at all times. The mass of each crank arm is negligible. The scenarios differ as described.

- Scenario 1: The length of the crank arm is ℓ_1 . The angular speed of the wheel immediately after the crank arm has been turned through one rotation is ω_1 .
- Scenario 2: The length of the crank arm is ℓ_2 , where $\ell_2 > \ell_1$. The angular speed of the wheel immediately after the longer crank arm has been turned through one rotation is ω_2 .

Part A

Indicate whether ω_2 is greater than, less than, or equal to ω_1 by writing one of the following.

- $\omega_2 > \omega_1$
- $\omega_2 < \omega_1$
- $\omega_2 = \omega_1$

Justify your answer. Your justification may reference equations but must include conceptual reasoning beyond algebraic solutions.

Part B

Derive an expression for the angular speed ω_1 of the wheel in Scenario 1 immediately after the crank arm has been turned through one rotation. Express your answer in terms of I_W , F , ℓ_1 , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

In Scenario 3, the unicycle is placed on the ground such that the unicycle can move forward without the wheel slipping on the ground. A force of the same magnitude F is exerted at the end of the crank arm of length ℓ_1 , as in Scenario 1. The direction of the force remains perpendicular to the crank arm at all times. The resulting angular speed of the wheel immediately after the crank arm has been turned through one rotation is ω_3 .

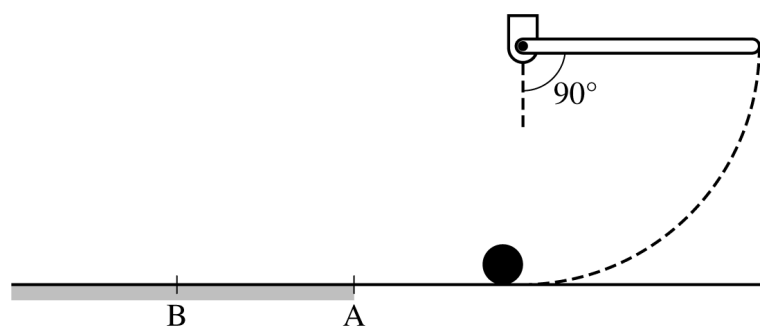
Indicate whether ω_3 is greater than, less than, or equal to ω_1 by writing one of the following.

- $\omega_3 > \omega_1$
- $\omega_3 < \omega_1$
- $\omega_3 = \omega_1$

Briefly **justify** your answer. Your justification may reference equations but must include conceptual reasoning beyond algebraic solutions.

STOP

END OF EXAM



Note: Figure not drawn to scale.

Figure 1

3. A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3} M \ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5} m R^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.
- (a) Derive an expression for the angular speed of the rod just before striking the sphere in terms of the length ℓ and physical constants as appropriate.

GO ON TO THE NEXT PAGE.

- (b) Derive an expression for the linear speed v_0 of the sphere immediately after colliding with the rod in terms of the length ℓ and physical constants as appropriate.

After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ , beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

- (c) In the following diagram, which represents the sphere while the sphere is traveling between Points A and B, draw and label the forces (not components) that act on the sphere. Each force must be represented by a distinct arrow starting on, and pointing away from, the point of application on the sphere.



- (d) Derive an expression for each of the following as the sphere is rotating and sliding between points A and B in terms of v_0 , μ , R , t , and physical constants as appropriate.

i. The linear velocity v of the center of mass of the sphere as a function of time t

ii. The angular velocity ω of the sphere as a function of time t

GO ON TO THE NEXT PAGE.

Name: _____

Continue your response to **QUESTION 3** on this page.

(e)

i. Derive an expression for the time it takes the sphere to travel from Point A to Point B in terms of v_0 , μ , and physical constants as appropriate.

ii. Derive an expression for the linear velocity of the sphere upon reaching Point B in terms of v_0 .

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STOP

END OF EXAM

Three-Blade System

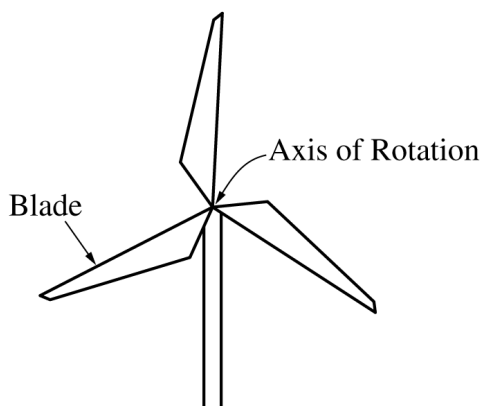


Figure 1

Single Blade

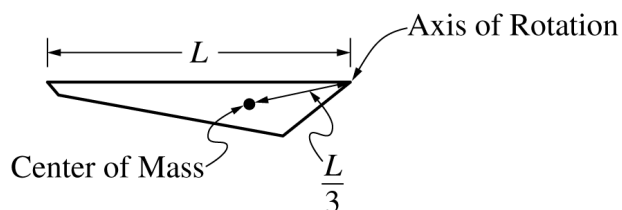


Figure 2

A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M , with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

- (a) Derive an expression for the rotational inertia of the three-blade system. Express your answer in terms of M , L , and physical constants, as appropriate. The rotational inertia of each blade about an axis through its center of mass is given by the equation $I_{\text{cm}} = \frac{1}{18}ML^2$.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

- (b) While the wind blows, the three-blade system operates at a constant angular speed $\omega_0 = 2.6 \text{ rad/s}$. The length of one blade is $L = 36 \text{ m}$. The numerical value of the rotational inertia of the system is $I_{\text{sys}} = 6.7 \times 10^6 \text{ kg}\cdot\text{m}^2$. Calculate the time T it takes the outer edge of a single blade to complete one revolution.

- (c) When the wind stops blowing, the angular speed of the system decreases. The angular speed ω of the system while slowing down is given as a function of time t by the equation $\omega = \omega_0 e^{-\beta_0 t}$, where β_0 is a constant with appropriate units, as shown on the graph in Figure 3.

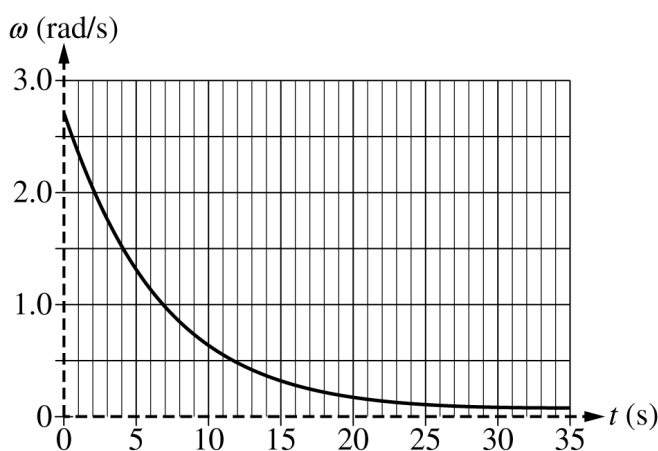


Figure 3

- i. Calculate the amount of energy dissipated from $t = 0$, when the wind stops blowing, until the system comes to rest.

GO ON TO THE NEXT PAGE.

ii. Derive an expression for the net torque exerted on the system as a function of t as the system slows down. Express your answers in terms of β_0 , ω_0 , M , L , I_{sys} , and physical constants, as appropriate.

iii. Derive an expression for the angular displacement of the system $\Delta\theta$ as a function of t . Express your answer in terms of β_0 , ω_0 , M , L , I_{sys} , and physical constants, as appropriate.

The three-blade system is now replaced with a second three-blade system identical to the first, except that the second three-blade system slows down according to the equation $\omega = \omega_0 e^{-\beta t}$, where $\omega_0 = 2.6 \text{ rad/s}$ and $\beta > \beta_0$. The original angular speed function is shown as a dashed line in Figure 4.

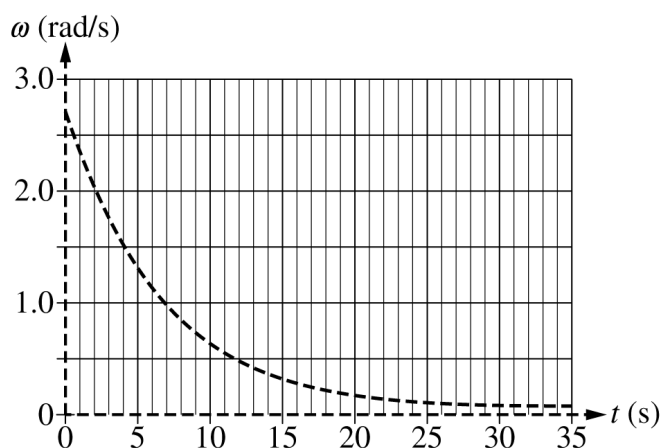


Figure 4

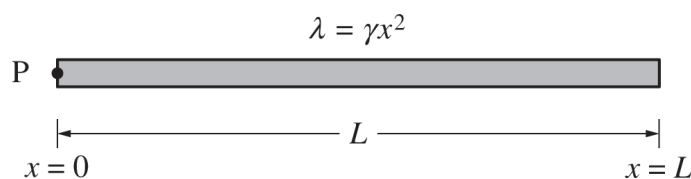
(d) On the graph in Figure 4, sketch the angular speed of the second three-blade system as a function of time t .

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STOP

END OF EXAM

Begin your response to QUESTION 3 on this page.



3. A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \frac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(a) Using integral calculus, show that the rotational inertia I of the rod about an axis perpendicular to the page and through point P is $\frac{3}{5}ML^2$.

(b) Determine the horizontal location of the center of mass of the rod relative to point P. Express your answer in terms of L .

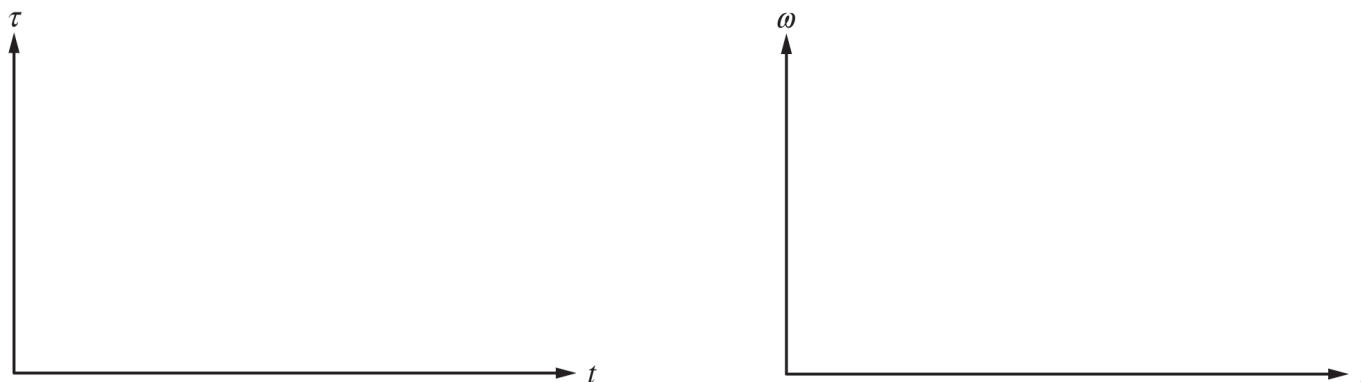
(c) For an axis perpendicular to the page, is the value of the rotational inertia of the rod around point P greater than, less than, or equal to the value of the rotational inertia of the rod around the rod's center of mass?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

The rod is released from rest in the position shown, and the rod begins to rotate about a horizontal axis perpendicular to the page and through point P.

(d) On the axes below, sketch graphs of the magnitude of the net torque τ on the rod and the angular speed ω of the rod as functions of time t from the time the rod is released until the time its center of mass reaches its lowest point.



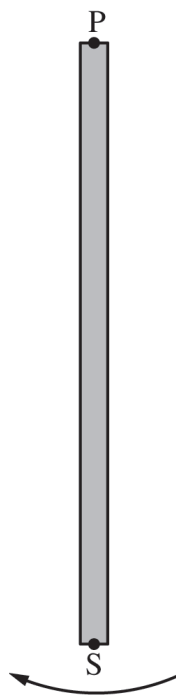
(e) As the rod rotates from the horizontal position down through vertical, is the magnitude of the angular acceleration on the rod increasing, decreasing, or not changing?

_____ Increasing _____ Decreasing _____ Not changing

Justify your answer.

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Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.



(f) The mass of the rod is 3.0 kg, and the length of the rod is 1.0 m. Calculate the linear speed v of point S as the rod swings through the vertical position shown.

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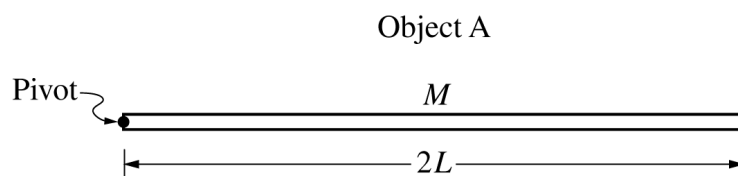
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Name:

AP Physics C: **Mechanics: 2021 Set 1**

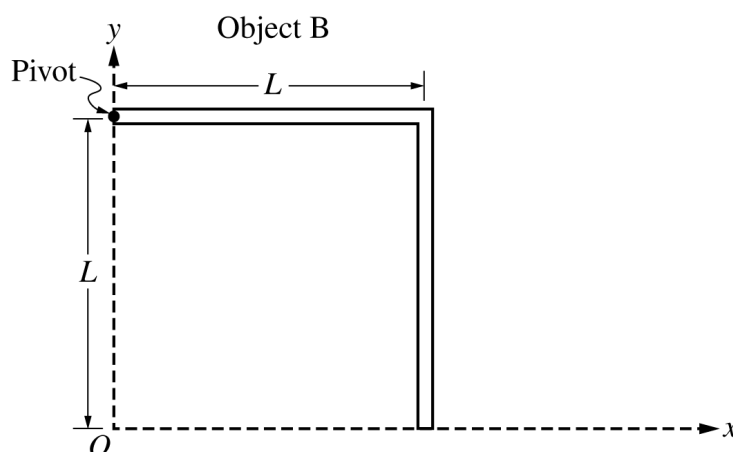
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END OF EXAM



2. Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(a) Using integral calculus, derive an expression to show that the rotational inertia I_A of object A about the pivot is given by $\frac{4}{3}ML^2$.



Object B of total mass M is formed by attaching two thin, uniform, identical rods of length L at a right angle to each other. Object B is held in place, as shown above. Express your answers in part (b) in terms of L .

(b) Determine the following for the given coordinate system shown in the figure.

i. The x -coordinate of the center of mass of object B

ii. The y -coordinate of the center of mass of object B

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Object B has a rotational inertia of I_B about its pivot.

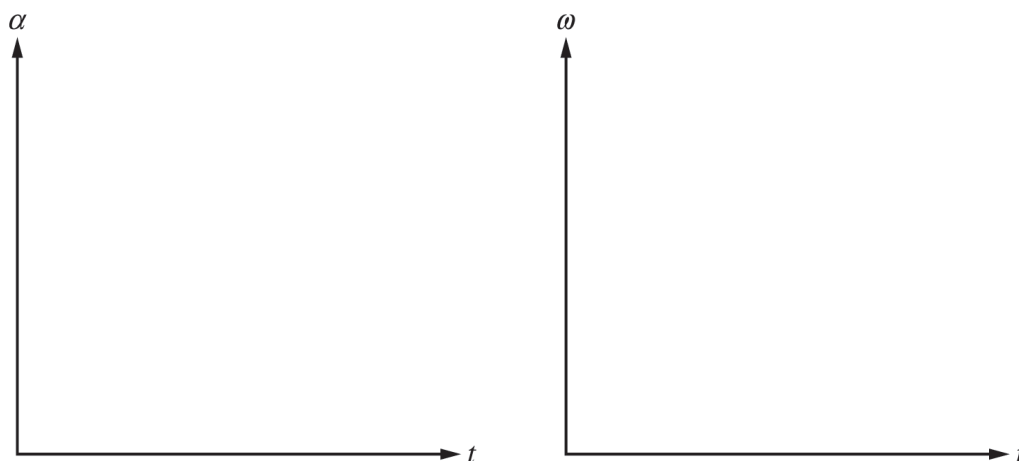
(c) Is the value of I_B greater than, less than, or equal to I_A ?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

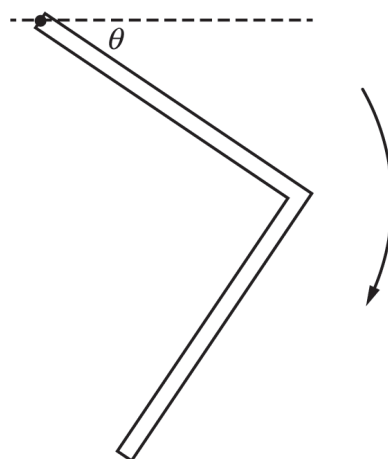
Object B is released from rest and begins to rotate about its pivot.

(d) On the axes below, sketch graphs of the magnitude of the angular acceleration α and the angular speed ω of object B as functions of time t from the time it is released to the time its center of mass reaches its lowest point.



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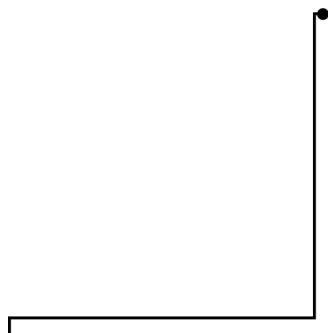
(e) While object B rotates from the horizontal position down through the angle θ shown above, is the magnitude of its angular acceleration increasing, decreasing, or not changing?

_____ Increasing

_____ Decreasing

_____ Not changing

Justify your answer.

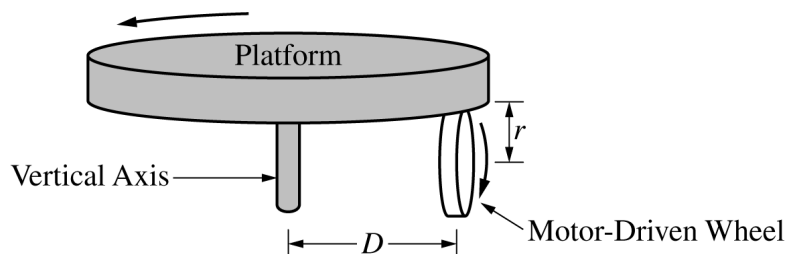


Object B rotates through the position shown above.

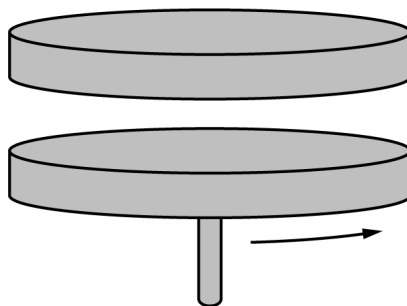
(f) Derive an expression for the angular speed of object B when it is in the position shown above. Express your answer in terms of M , L , I_B , and physical constants, as appropriate.

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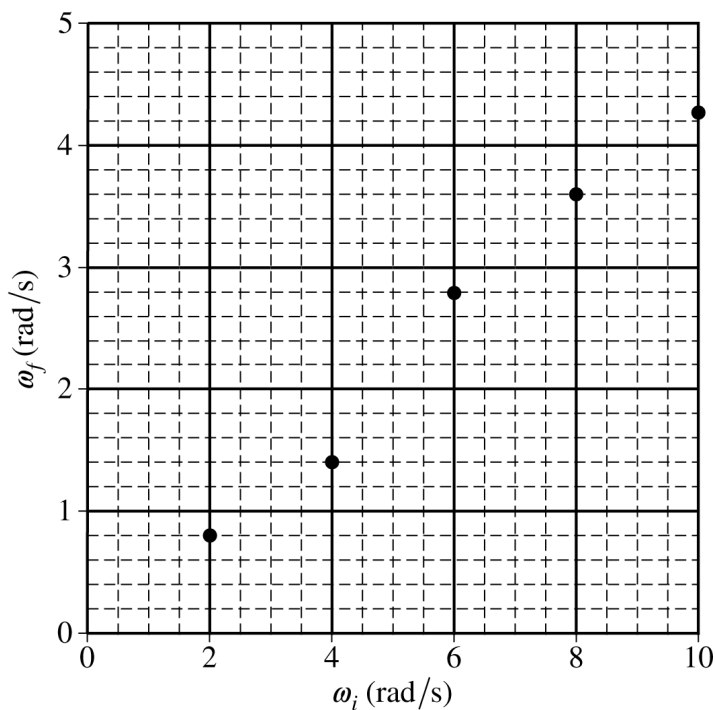
3. A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the wheel until the platform reaches an angular speed ω_P after time Δt . During time Δt , the wheel stays in contact with the platform without slipping.
- (a) Derive an expression for the angular speed ω_P of the platform. Express your answer in terms of I_P , r , D , F , Δt , and physical constants, as appropriate.
- (b) Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed ω_P . Express your answer in terms of I_P , r , D , F , Δt , and physical constants, as appropriate.
- (c) Derive an expression for the angular speed of the wheel ω_W when the platform has reached angular speed ω_P . Express your answer in terms of D , r , ω_P , and physical constants, as appropriate.



When the platform is spinning at angular speed ω_P , the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia I_P as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed ω_f .

- (d) Derive an expression for ω_f . Express your answer in terms of ω_P , I_P , and physical constants, as appropriate.

A student now uses the rotating platform ($I_P = 3.1 \text{ kg}\cdot\text{m}^2$) to determine the rotational inertia I_U of an unknown object about a vertical axis that passes through the object's center of mass. The platform is rotating at an initial angular speed ω_i when the unknown object is dropped with its center of mass directly above the center of the platform. The platform and object are observed to be rotating together at angular speed ω_f . Trials are repeated for different values of ω_i . A graph of ω_f as a function of ω_i is shown on the axes below.



- (e)
- i. On the graph on the previous page, draw a best-fit line for the data.
 - ii. Using the straight line, calculate the rotational inertia of the unknown object I_U about a vertical axis passing through its center of mass.
- (f) The kinetic energy of the spinning platform before the object is dropped on it is K_i . The total kinetic energy of the platform-object system when it reaches angular speed ω_f is K_f . Which of the following expressions is true?
- _____ $K_f < K_i$ _____ $K_f = K_i$ _____ $K_f > K_i$

Justify your answer.

- (g) One of the students observes that the center of mass of the object is not actually aligned with the axis of the platform. Is the experimental value of I_U obtained in part (e) greater than, less than, or equal to the actual value of the rotational inertia of the unknown object about a vertical axis that passes through its center of mass?
- _____ Greater than _____ Less than _____ Equal to

Justify your answer.

STOP

END OF EXAM

3. A triangular rod, shown above, has length L , mass M , and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(a) Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

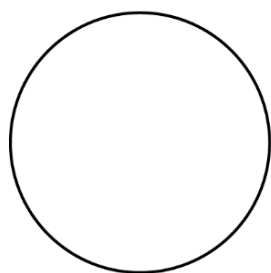


Figure 1

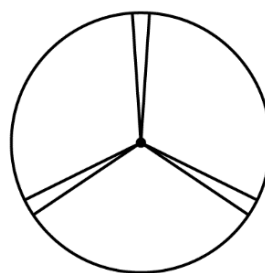


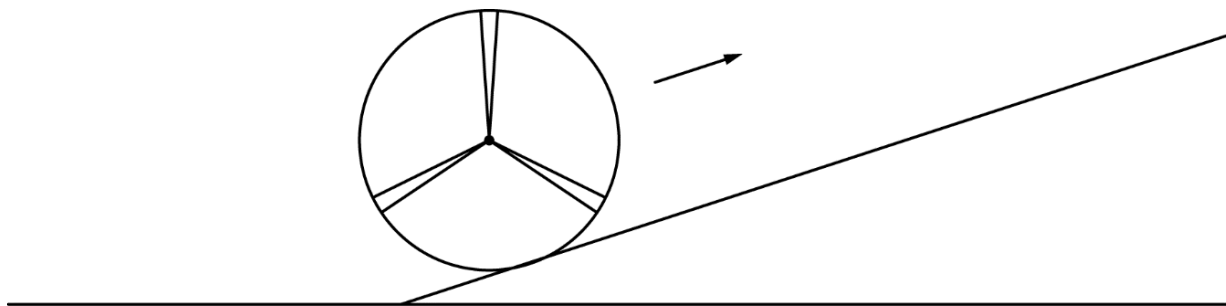
Figure 2

The thin hoop shown above in Figure 1 has a mass M , radius L , and a rotational inertia around its center of ML^2 . Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

- (b) Derive an expression for the rotational inertia I_{tot} of the hoop-rods system about the center of the hoop. Express your answer in terms of M , L , and physical constants, as appropriate.

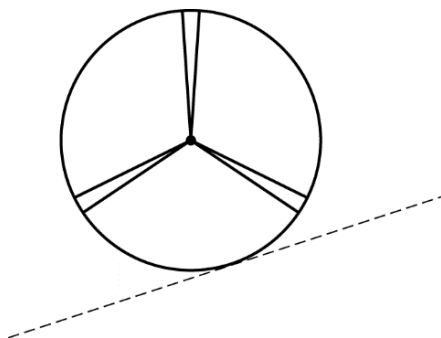
The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt .

- (c) Derive an expression for the final angular speed ω of the hoop-rods system. Express your answer in terms of M , L , F , Δt , and physical constants, as appropriate.



The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed ω found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

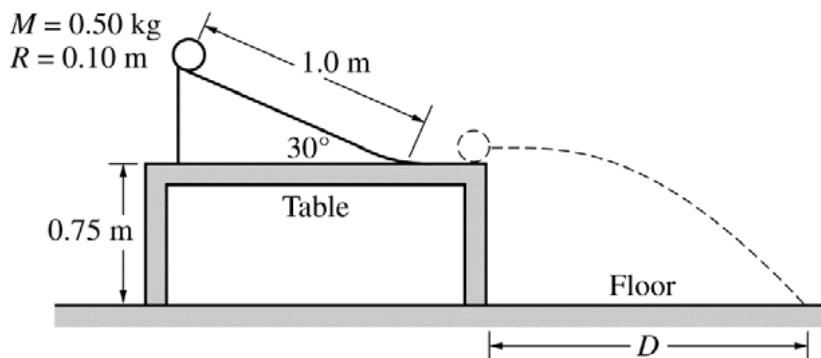
- (d)
- i. On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.



- ii. Justify your choice for the direction of each of the forces drawn in part (d)i.
- (e) Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of M , L , I_{tot} , ω , and physical constants, as appropriate.

STOP

END OF EXAM



3. A uniform solid cylinder of mass $M = 0.50 \text{ kg}$ and radius $R = 0.10 \text{ m}$ is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m . The inclined plane makes an angle of 30° with the horizontal. The cylinder lands on the floor a distance D away from the edge of the table, as shown in the figure above. There is a smooth transition from the inclined plane to the horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

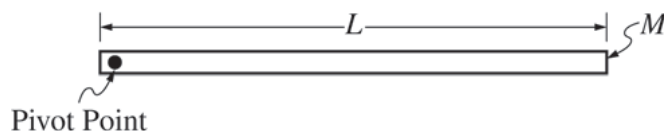
- (a) Calculate the total kinetic energy of the cylinder as it reaches the horizontal table.
- (b) Calculate the angular velocity of the cylinder around its axis at the moment it reaches the floor.
- (c) Calculate the ratio of the rotational kinetic energy to the total kinetic energy for the cylinder at the moment it reaches the floor.
- (d) Calculate the horizontal distance D .

A sphere of the same mass and radius is now rolled down the same inclined plane. The rotational inertia of a sphere around its center is $\frac{2}{5}MR^2$.

- (e)
 - i. Is the total kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the total kinetic energy of the cylinder at the moment it reaches the floor?
____ Greater than ____ Less than ____ Equal to
Justify your answer.
 - ii. Is the rotational kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the rotational kinetic energy of the cylinder at the moment it reaches the floor?
____ Greater than ____ Less than ____ Equal to
Justify your answer.
 - iii. Is the horizontal distance the sphere travels from the table to where it hits the floor greater than, less than, or equal to the horizontal distance the cylinder travels from the table to where it hits the floor?
____ Greater than ____ Less than ____ Equal to
Justify your answer.

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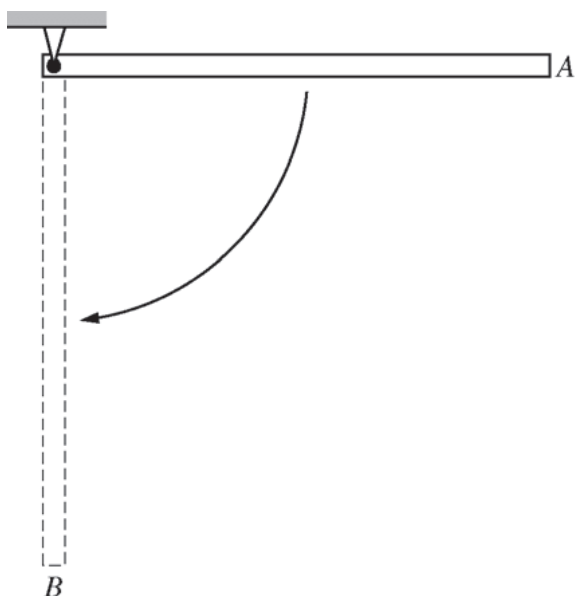
END OF EXAM



Mech.3.

A uniform, thin rod of length L and mass M is allowed to pivot about its end, as shown in the figure above.

- (a) Using integral calculus, derive the rotational inertia for the rod around its end to show that it is $ML^2/3$.



The rod is fixed at one end and allowed to fall from the horizontal position A through the vertical position B .

- (b) Derive an expression for the velocity of the free end of the rod at position B . Express your answer in terms of M , L , and physical constants, as appropriate.

An experiment is designed to test the validity of the expression found in part (b). A student uses rods of various lengths that all have a uniform mass distribution. The student releases each of the rods from the horizontal position A and uses photogates to measure the velocity of the free end at position B . The data are recorded below.

Length (m)	0.25	0.50	0.75	1.00	1.25	1.50
Velocity (m/s)	2.7	3.8	4.6	5.2	5.8	6.3

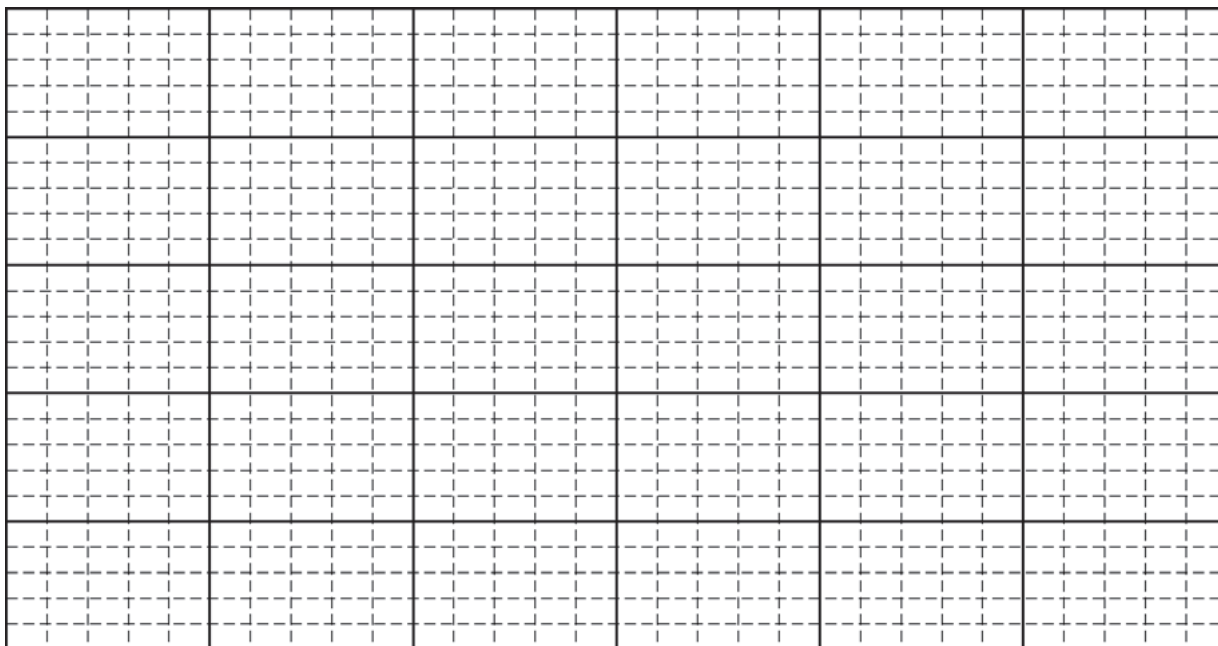
- (c) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the acceleration due to gravity g .

Horizontal axis: _____

Vertical axis: _____

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given. Label each row you use and include units.

- (d) Plot the straight line data points on the grid below. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



- (e)
- i. Using your straight line, determine an experimental value for g .
 - ii. Describe two ways in which the effects of air resistance could be reduced.

STOP

END OF EXAM