Question 3: Free-Response Question — 15 points

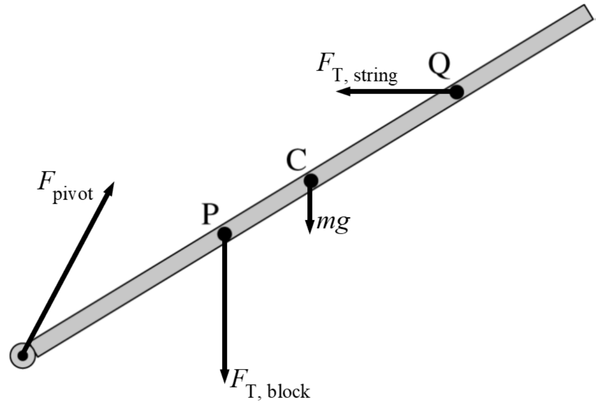
(a) For drawing and appropriately labeling the downward forces that are exerted on the rod at points P and C — **1 point**

Scoring Note: Labeling the downward force of tension as F_{block}, $3mg$, or similar, may earn this point.

For drawing and appropriately labeling a leftward force that is exerted on the rod at Point Q — **1 point**

For drawing and appropriately labeling a force that is directed up and to the right that is exerted on the rod at the pivot, and no extraneous forces are present — **1 point**

Example Response



Scoring Note: Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity.

Scoring Note: Examples of appropriate labels for the force from the pivot include: F_p, F_{pivot}, F_n, F_N, N, "normal force," "pivot force."

Scoring Note: Examples of appropriate labels for the tension force include F_t, F_T, T, F_{string}, and "Force from string."

Total for part (a) — 3 points

AP Physics C: **Mechanics: 2024 Set 1**

(b) For indicating that the net torque exerted on the rod is equal to zero — **1 point**

Example Response

$\Sigma\tau = 0$

For a correct expression for the torque exerted on the rod by the hanging mass — **1 point**

Example Response

$3mg\sin\theta\left(\frac{3L}{8}\right)$

For a correct expression for the torque exerted on the rod by the gravitational force — **1 point**

Example Response

$mg\sin\theta\left(\frac{4L}{8}\right)$

For a correct expression for the torque exerted on the rod by the string — **1 point**

Example Responses

$F_T\sin(90° - \theta)\left(\frac{6L}{8}\right)$ **OR** $F_T\cos\theta\left(\frac{6L}{8}\right)$

Example Solution

$$\Sigma\tau = 0$$

$$3mg\sin\theta\left(\frac{3L}{8}\right) + mg\sin\theta\left(\frac{4L}{8}\right) - F_T\sin(90° - \theta)\left(\frac{6L}{8}\right) = 0$$

$$3mg\sin\theta\left(\frac{3L}{8}\right) + mg\sin\theta\left(\frac{4L}{8}\right) - F_T\cos\theta\left(\frac{6L}{8}\right) = 0$$

$$\left(\frac{9}{8}\right)mg\sin\theta + \left(\frac{4}{8}\right)mg\sin\theta = \left(\frac{6}{8}\right)F_T\cos\theta$$

$$13mg\sin\theta = 6F_T\cos\theta$$

$$F_T = \frac{13}{6}mg\tan\theta$$

Scoring Note: A maximum of three points may be earned if the trigonometric functions (sin and cos) are reversed for all three torque terms.

Total for part (b) — 4 points

(c)	For indicating that the torque exerted on the rod by the string is always the same	**1 point**
	For stating that as the angle between the string and the rod increases, the force exerted on the rod by the string decreases	**1 point**

Example Response

Because the torque exerted on the rod by the string is always the same, as the angle between the string and the rod increases, the tension $F_{T,\text{new}}$ *must decrease.*

Total for part (c) 2 points

(d)(i)	For indicating the total mass is the sum of differentiable masses along the length of the rod	**1 point**

Example Response

$$M = \int dm$$

For correctly writing dm in terms of x **1 point**

Example Response

$$M = \int_0^{1.2} (6 + 10x)\, dx$$

For a correct numeric answer with correct units **1 point**

Example Response

$$M = 14.4 \text{ kg}$$

Example Solution

$$M = \int dm$$
$$M = \int \lambda\, dx$$
$$M = \int_0^{1.2} (6 + 10x) dx$$
$$M = \left(6x + \frac{10x^2}{2}\right)\Bigg|_0^{1.2\text{ m}}$$
$$M = 6\text{ kg/m}(1.2\text{ m}) + \frac{10\text{ kg/m}^2(1.2\text{ m})^2}{2}$$
$$M = 14.4\text{ kg}$$

AP Physics C: **Mechanics: 2024 Set 1**

(d)(ii)	For a correct substitution of λ into an integral expression of rotational inertia	**1 point**

Example Response

$$I = \int_0^{1.2\text{ m}} (A + Bx)x^2 dx$$

For a correct integration **1 point**

Example Response

$$I = \left(\frac{Ax^3}{3} + \frac{Bx^4}{4}\right)\Bigg|_0^{1.2\text{ m}}$$

For a correct numeric answer with correct units **1 point**

Example Response

$$I = 8.64\text{ kg}\cdot\text{m}^2$$

Example Solution

$$I = \int r^2 dm \qquad dm = \lambda dr \text{ and } r = x$$
$$I = \int_0^{1.2\text{ m}} \lambda x^2 dx$$
$$I = \int_0^{1.2\text{ m}} (A + Bx)x^2 dx$$
$$I = \left(\frac{Ax^3}{3} + \frac{Bx^4}{4}\right)\Bigg|_0^{1.2\text{ m}}$$
$$I = \frac{(6.0\text{ kg/m})(1.2\text{ m})^3}{3} + \frac{\left(10.0\text{ kg/m}^2\right)(1.2\text{ m})^4}{4}$$
$$I = 8.64\text{ kg}\cdot\text{m}^2$$

Total for part (d) 6 points

Total for question 3 15 points

Question 3: Free-Response Question — 15 points

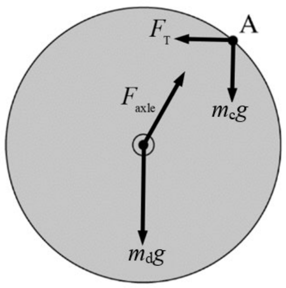
(a) For drawing and appropriately labeling separate gravitational forces that are exerted on the disk and the lump of clay and exerted at the correct locations — **1 point**

Scoring Note: Drawing a downward gravitational force exerted on the clay-disk system at the correct location can earn this point.

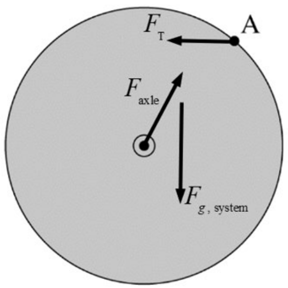
For drawing and appropriately labeling a leftward force exerted on the system at Point A — **1 point**

For drawing and appropriately labeling a force directed up and right that is exerted on the system at the axle, and no extraneous forces are present — **1 point**

Example Responses



OR



Scoring Note: Examples of appropriate labels for the gravitational force include F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on disk," "F on disk by Earth," $F_{\text{Earth on Disk}}$, $F_{\text{E,Disk}}$, and $F_{\text{Disk,E}}$. The labels G or g are not appropriate labels for the gravitational force.

Scoring Note: Examples of appropriate labels for the normal force from the axle include F_N, F_{axle}, N, "normal force," and "axle force."

Scoring Note: Examples of appropriate labels for the tension force include F_t, F_T, T, F_{string}, and "Force from string."

Total for part (a) — 3 points

(b) For a multi-step derivation that indicates the net torque is zero — **1 point**

For indicating only the weight of the clay and the tension in the string exert torques on the system about the axle — **1 point**

Example Response

$\tau_{net} = \tau_{clay} - \tau_{string}$

For a correct expression for the torque exerted on the system by the weight of the clay — **1 point**

Example Response

$\tau_{clay} = Rm_c g\cos\theta$

For a correct expression for the torque exerted on the system by the tension in the string — **1 point**

Example Response

$\tau_{string} = RF_T\sin\theta$

Example Solution

$\Sigma\tau = 0$

$\tau_{clay} - \tau_{string} = 0$

$F_{g,\text{ clay}}R\cos\theta - RF_T\sin\theta = 0$

$Rm_c g\cos\theta = RF_T\sin\theta$

$$F_T = \frac{Rm_c g\cos\theta}{R\sin\theta}$$

$F_T = m_c g\cot\theta$

Scoring Note: A maximum of three points can be earned if the trigonometric functions ($\sin$ and $\cos$) are reversed for both torque expressions.

Total for part (b) — 4 points

(c)	For indicating a direct relationship between the torque exerted by the clay and the tension in the string	**1 point**

For correctly relating a greater torque exerted by the clay from at least **one** of the following: **1 point**

- An increase in the angle between the radial direction and the weight of the clay
- A greater perpendicular component of the weight of the clay
- A greater lever arm

Example Response

The torque caused by the weight of the clay at Point B *is greater than when the clay is at Point* A *because the component of the weight that is perpendicular to the lever arm is larger. To maintain equilibrium, the net torque on the system is still zero, therefore the tension* $F_{\text{T, new}}$ *must be greater.*

Total for part (c) 2 points

(d)(i) For using integration to calculate rotational inertia **1 point**

For **one** of the following: **1 point**

- Substituting $\rho(2\pi r)dr$ for dm
- Indicating the correct limits of integration

Example Response

$$I = \int_0^{0.3\text{ m}} r^2 \rho(2\pi r)dr$$

For a correct answer of $I = 0.012\text{ kg}\cdot\text{m}^2$, including units **1 point**

Example Response

$$I = 0.012\text{ kg}\cdot\text{m}^2$$

Example Solution

$$I = \int r^2 dm \qquad dm = \rho dA \quad \text{and} \quad dA = 2\pi r\,dr$$

$$I = \int_0^R r^2 \rho dA$$

$$I = \int_0^{0.3\text{ m}} 2\pi\beta r^4 dr$$

$$I = \left.\frac{2\pi\beta R^5}{5}\right|_0^{0.3\text{ m}}$$

$$I = \frac{2\pi(4.0\,\text{kg/m}^3)(0.3\text{ m})^5}{5}$$

$$I = 0.012\text{ kg}\cdot\text{m}^2$$

(d)(ii) For using the rotational form of Newton's second law **1 point**

For a correct expression for the net torque exerted on the clay-disk system **1 point**

Example Response

$$\tau_{\text{net}} = Rm_c g$$

For indicating that the rotational inertia of the clay-disk system is the sum of the rotational inertia of the disk and the rotational inertia of the lump of clay **1 point**

Example Response

$$I_{\text{system}} = I_{\text{disk}} + I_{\text{clay}}$$

Example Solution

$$\Sigma\tau = I\alpha$$

$$\alpha = \frac{\tau_{\text{net}}}{I}$$

$$\alpha = \frac{Rm_c g}{I_{\text{disk}} + I_{\text{clay}}}$$

$$\alpha = \frac{(0.3\text{ m})(0.60\text{ kg})\left(9.8\text{ m/s}^2\right)}{\left(\left(0.012\text{ kg}\cdot\text{m}^2\right) + (0.60\text{ kg})(0.3\text{m})^2\right)}$$

$$\alpha = 26.7\ \text{rad/s}^2$$

Total for part (d) 6 points

Total for question 3 15 points

Question 3: Free-Response Question — 15 points

(a) For indicating that the sum of the torques on the disk equals zero — **1 point**

$\Sigma\tau_{\text{on disk}} = 0$

$\tau_g = \tau_s$

OR

For indicating that the sum of the forces equals zero

$\Sigma F = 0$

$F_g = F_s$

For correctly substituting the expressions for the forces — **1 point**

$F_g R = F_s R$

$m_B g R = k\Delta x R$

$m_B g = k\Delta x$

OR

$F_g = F_s$

$m_B g = k\Delta x$

For correctly substituting for Δx — **1 point**

$m_B g = kR\theta$

$m_B = \dfrac{kR\theta}{g}$

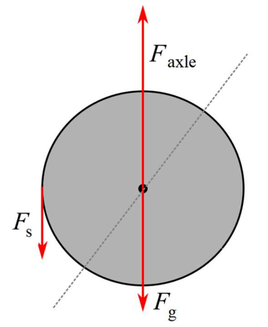
Total for part (a) — 3 points

(b) For drawing and labeling the force of the tension exerted on the disk anywhere between point P and the left edge of the disk, including point P and the left edge of the disk, tangent to the disk — **1 point**

For correct location and label of the force due to gravity exerted on the disk, directed straight down — **1 point**

For correct location and label of the force exerted on the disk by the axle, directed such that the disk remains in translational equilibrium (i.e., $\Sigma F = 0$) — **1 point**

Example Response



Scoring Notes:

- Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force, which can be used instead of F_{axle}. F_{spring}, F_s, T_{spring}, T, "spring force," or similar labels may be used for the tension force exerted by the spring.
- A response with extraneous forces or vectors can earn a maximum of two points.

Total for part (b) — 3 points

(c) For indicating that the net torque is due only to the force exerted on the disk by the tension in the rotational form of Newton's second law — **1 point**

$\tau_s = I_d\alpha$

For correctly expressing the torque on the disk by the tension in terms of the spring force, which is equal to the tension, and the lever (moment) arm — **1 point**

$F_s R = I_d\alpha$

For correctly substituting for F_s — **1 point**

$-k\Delta x R = I_d\alpha$

For correctly substituting I_d and Δx, or an expression for Δx consistent with part (a) — **1 point**

$-k(R\theta)R = \frac{1}{2}M_d R^2\alpha$

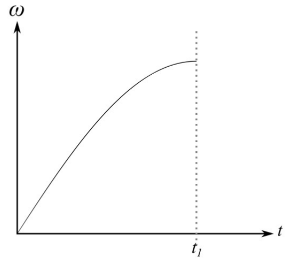
$\alpha = -\dfrac{2k\theta}{M_d}$

Scoring Note: The negative sign is not necessary to earn this point.

Total for part (c) — 4 points

(d)	For a sketch that starts at zero and monotonically increases until time $t = t_1$	**1 point**
	For a sketch that is concave down between time $t = 0$ and $t = t_1$	**1 point**

Example Response



Scoring Note: Any part of the graph beyond t_1 is not considered in scoring.

Total for part (d) **2 points**

(e)	For indicating that the torque exerted by the force due to gravity on the disk increased	**1 point**

Example Response

The force due to gravity on the disk now has a non-zero lever arm and hence it exerts a larger torque on the disk.

For indicating that the torque exerted by the tension caused by the force due to gravity on the block increased	**1 point**

Example Response

The force exerted by the right side of the string (from the block) on the disk has a longer lever arm, hence the torque it exerts is larger.

For indicating that the torque exerted by the tension increased	**1 point**

Example Response

The counterclockwise torque due to the tension caused by the spring must increase to counteract the increase in clockwise torques due to the force due to gravity of the disk and tension caused by the force of gravity due to block to keep the disk in equilibrium.

Scoring Notes:

- A response that references the torque due to the force at the axle staying the same can earn all 3 points.
- A response that references the torque due to the force on the axle changing, or any additional torques can earn a maximum of 2 points.

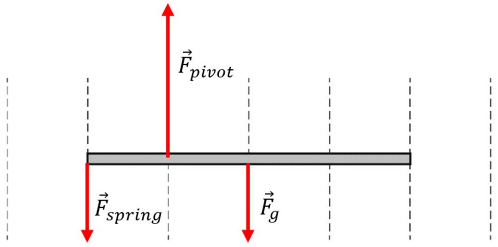
Total for part (e) **3 points**

Total for question 3 **15 points**

Question 3: Free-Response Question **15 points**

(a)	For correct location and label of the force exerted on the board by the spring	**1 point**
	For correct location and label of the force exerted on the board by gravity	**1 point**
	For correct location and label of the force exerted on the board by the pivot	**1 point**

Example Response



Scoring Notes:

- Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force, which can be used instead of F_{pivot}. F_{spring}, F_S, "spring force," or similar labels may be used for the force exerted by the spring.
- A response with extraneous forces or vectors can earn a maximum of two points.

Total for part (a) **3 points**

(b)	For indicating that the sum of the torques equals zero	**1 point**

$\Sigma\tau = 0$

For substituting mg for the force of gravity and $k\Delta x$ for the spring force into a torque equation	**1 point**

$$\frac{L}{4}k\Delta x - \frac{L}{4}mg = 0$$

For a correct expression for Δx	**1 point**

$$\Delta x = \frac{mg}{k}$$

Example Response

$\Sigma\tau = 0$

$F_{\text{spring}}d_{\text{spring}} - F_{\text{weight}}d_{\text{weight}} = 0$

$\frac{L}{4}k\Delta x - \frac{L}{4}mg = 0$

$\Delta x = \frac{mg}{k}$

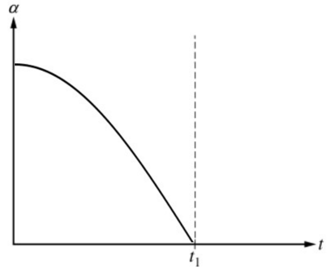
Scoring Note: This point can be earned regardless of whether a negative sign is present.

Total for part (b) 3 points

(c)(i) For a sketch that begins at a maximum value and monotonically decreases to a minimum of zero at $t = t_1$ — **1 point**

For a sketch that is concave down between $t = 0$ and $t = t_1$ — **1 point**

Example Response



Scoring Note: Any part of the graph beyond t_1 is not considered in scoring.

(c)(ii) For indicating that the torques are in opposite directions in the rotational form of Newton's second law — **1 point**

$\tau_{\text{spring}} - \tau_g = I\alpha$

For including $\sin(90 - \theta_0)$, $\sin(90 + \theta_0)$, or $\cos\theta_0$ in an expression for the net torque exerted on the rod — **1 point**

$F_{spring}d_{spring}(\cos\theta) - F_g d_g(\cos\theta) = I_B\alpha_0$

For substituting mg for the force due to gravity and substituting $k\Delta x_2$ for the spring force in an expression for the net torque exerted on the rod — **1 point**

$d_{spring}k\Delta x_2(\cos\theta_0) - d_g mg(\cos\theta_0) = I_B\alpha_0$

For substituting correct lever arms in an expression for the net torque exerted on the rod — **1 point**

$\frac{L}{4}(\cos\theta_0)k\Delta x_2 - \frac{L}{4}(\cos\theta_0)mg = I_B\alpha_0$

Example Response

$\tau_{\text{spring}} - \tau_g = I\alpha$

$F_{spring}d_{spring}(\cos\theta_0) - F_g d_g(\cos\theta_0) = I_B\alpha_0$

$l_{spring}(\cos\theta_0)k\Delta x_2 - l_g(\cos\theta_0)mg = I_B\alpha_0$

$\frac{L}{4}(\cos\theta_0)k\Delta x_2 - \frac{L}{4}(\cos\theta_0)mg = I_B\alpha_0$

$\alpha_0 = \frac{L}{4I_B}(k\Delta x_2\cos\theta_0 - mg\cos\theta_0)$

Total for part (c) 6 points

(d) For selecting "$\alpha' < \alpha_0$" with an attempt at a relevant justification — **1 point**

For indicating that the net torque does not change — **1 point**

For indicating the rotational inertia increases — **1 point**

Example Response

The net torque on the system is the same (the spring and the person exert the same force and the additional torques from the gravitational forces on the masses cancel) but the rotational inertia of the system is now larger. This means that the angular acceleration of the system is now smaller than it was before.

Total for part (d) 3 points

Total for question 3 15 points