Begin your response to **QUESTION 3** on this page.

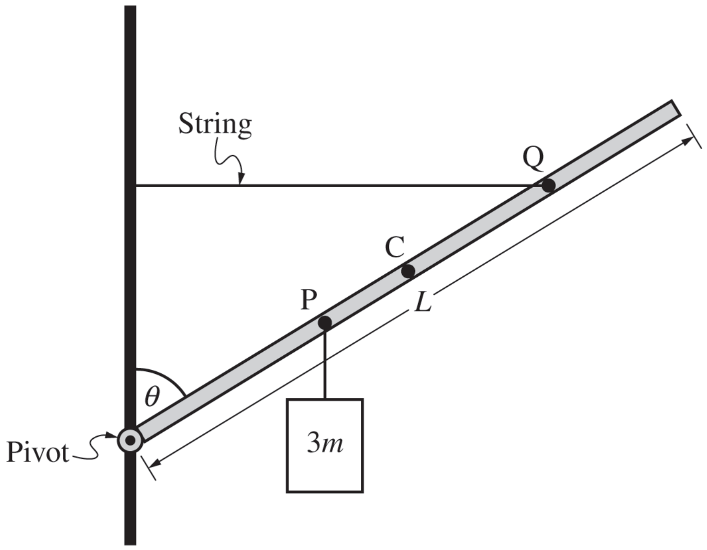
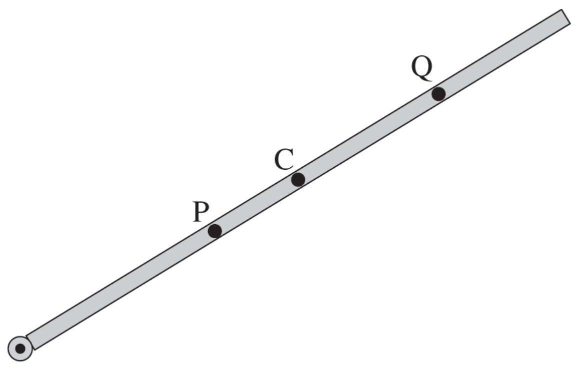


Figure 1

Note: Figure not drawn to scale.

3. A uniform rod of length L and mass m is attached to a pivot on a vertical pole, as shown in Figure 1. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

   (a) On the following representation of the rod, **draw** and **label** the forces (not components) that are exerted on the rod. Each force must be represented by a distinct arrow that starts on and points away from the point at which the force is exerted on the rod.



GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(b) In Figure 1, Point P is located $\frac{3}{8}L$ from the pivot and Point Q is located $\frac{6}{8}L$ from the pivot. **Derive** an equation for the tension F_T in the horizontal string in terms of L, m, θ, and physical constants, as appropriate.

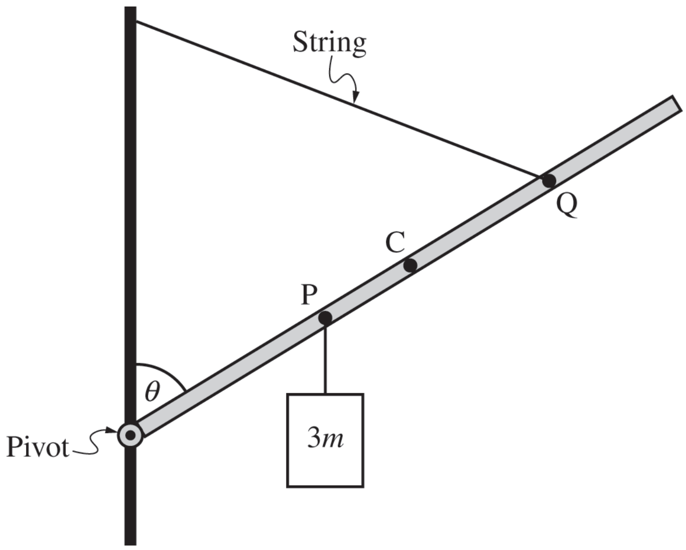


Figure 2

Note: Figure not drawn to scale.

(c) The original string is replaced with a longer string that connects Point Q to a higher location on the vertical pole, as shown in Figure 2. The angle θ remains the same. How does the new tension $F_{T,\ new}$ compare with the original tension F_T from part (b) ? **Justify** your reasoning.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

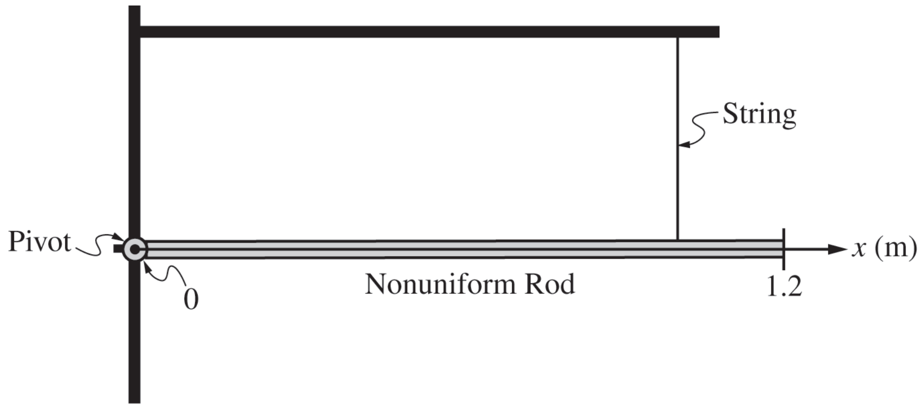


Figure 3

<u>Note</u>: Figure not drawn to scale.

(d) A nonuniform rod is now attached to the pivot, as shown in Figure 3. There is negligible friction between the nonuniform rod and the pivot. The rod has a length of 1.2 m and a linear mass density $\lambda(x) = A + Bx$, where x is the distance from the pivot, $A = 6.0$ kg/m, and $B = 10.0$ kg/m^2.

i. **Calculate** the mass of the rod.

ii. **Calculate** the rotational inertia of the rod about the pivot.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

Begin your response to **QUESTION 3** on this page.

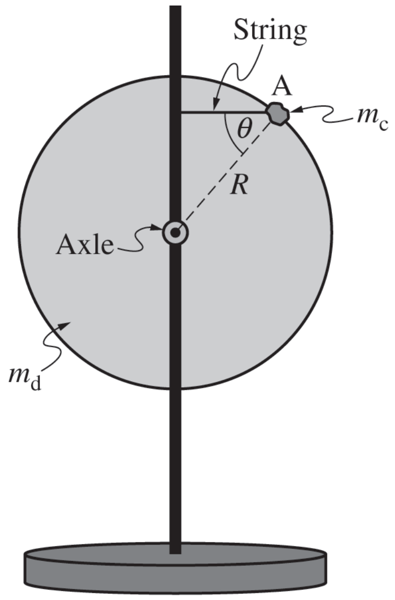
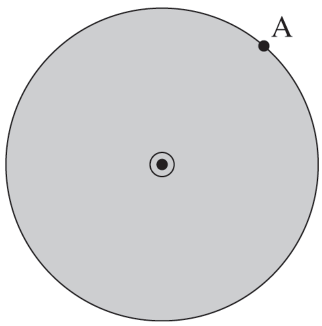


Figure 1

Note: Figure not drawn to scale.

3. A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(a) On the following representation of the clay-disk system, **draw** and **label** the external forces (not components) exerted on the system. Each force must be represented by a distinct arrow that starts on, and points away from, the point at which the force is exerted on the system.



GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(b) **Derive** an expression for the tension F_T in the string when the clay is at Point A, as shown in Figure 1, in terms of R, m_d, m_c, θ, and physical constants, as appropriate.

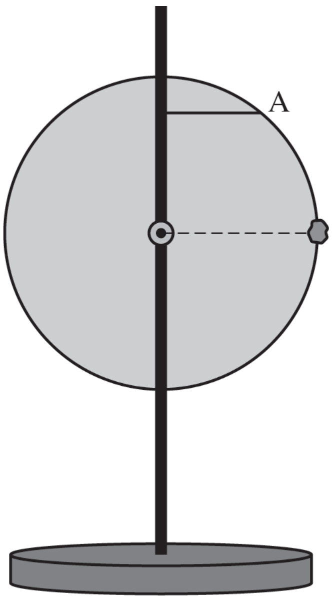


Figure 2

<u>Note:</u> Figure not drawn to scale.

(c) The string remains connected to the edge of the disk at Point A. The clay is moved to Point B, which is horizontally in line with the axle, as shown in Figure 2. How does the new tension $F_{T,\ new}$ compare with tension F_T from part (b)? **Justify** your reasoning.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

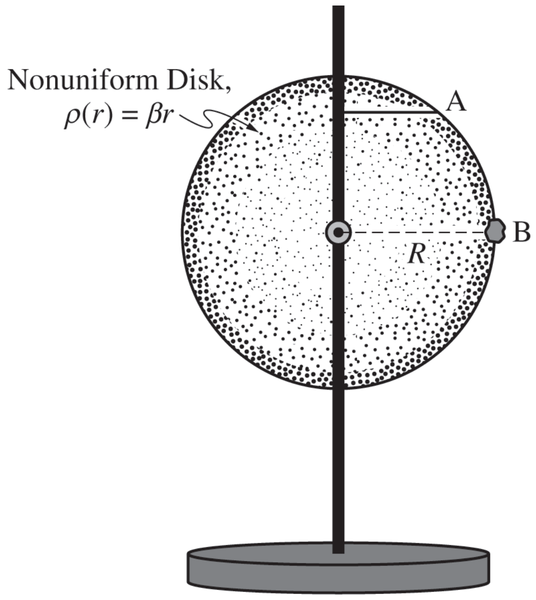


Figure 3

<u>Note:</u> Figure not drawn to scale.

(d) A nonuniform disk is now attached to the axle. The lump of clay is attached to the disk at Point B, as shown in Figure 3. The clay has mass $m_c = 0.60\ \text{kg}$ and the disk has a radius $R = 0.30\ \text{m}$. The mass density of the disk varies radially and can be modeled by $\rho(r) = \beta r$, where r is the radial distance from the axle and $\beta = 4.0\ \text{kg/m}^3$.

i. **Calculate** the rotational inertia of the disk about the axle.

ii. The string connecting the disk to the pole is cut. **Calculate** the magnitude of the initial angular acceleration of the clay-disk system.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

Begin your response to **QUESTION 3** on this page.

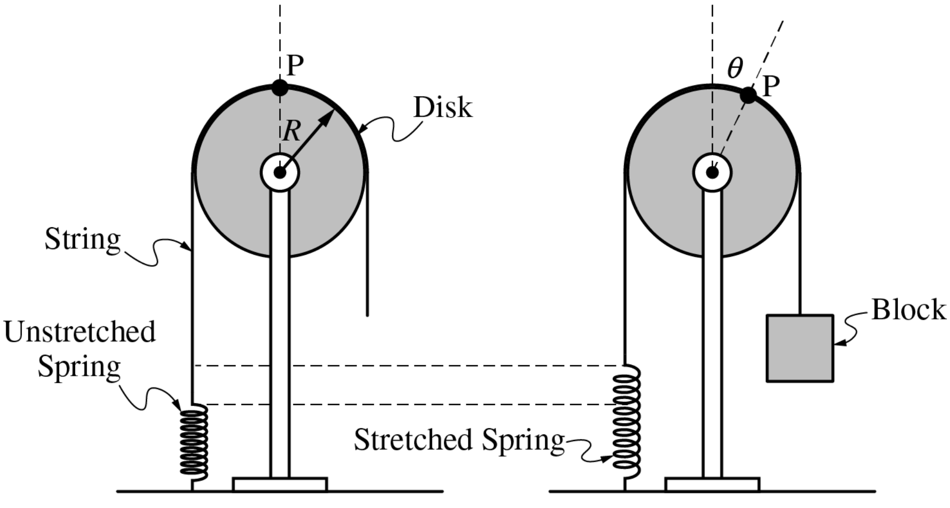


Figure 1 Figure 2

Note: Figures not drawn to scale.

A solid uniform disk is supported by a vertical stand. The disk is able to rotate with negligible friction about an axle that passes through the center of the disk. The mass and radius of the disk are given by M_d and R, respectively. The rotational inertia of the disk is $I_d = \frac{1}{2}M_d R^2$. A string of negligible mass is draped over the disk and attached to the top of the disk at point P. One end of the string is connected to an unstretched ideal spring of spring constant k, which is fixed to the ground as shown in Figure 1.

A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

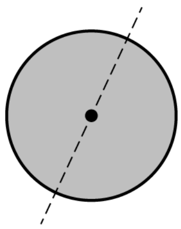
Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(a) Derive an expression for the mass m_B of the block.

(b) At time $t = 0$, the string on the right side of the disk is cut and the block falls to the ground. On the circle below, which represents the disk, draw and label the forces (not components) that act on the disk immediately after the string is cut and the block is falling to the ground. Each force should be represented by an arrow that starts on and is directed away from the point of application.

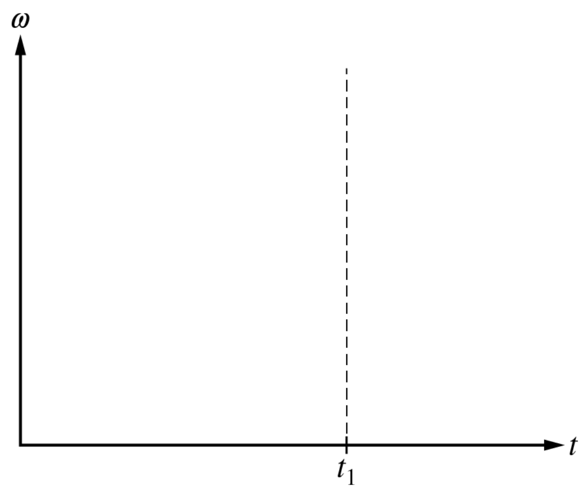


(c) Derive an expression for the angular acceleration α of the disk immediately after the string is cut.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(d) At $t = t_1$, the disk has rotated and point P is again directly above the axle. Sketch a graph of the magnitude of the angular velocity ω of the disk as a function of time t from $t = 0$ to $t = t_1$.



GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

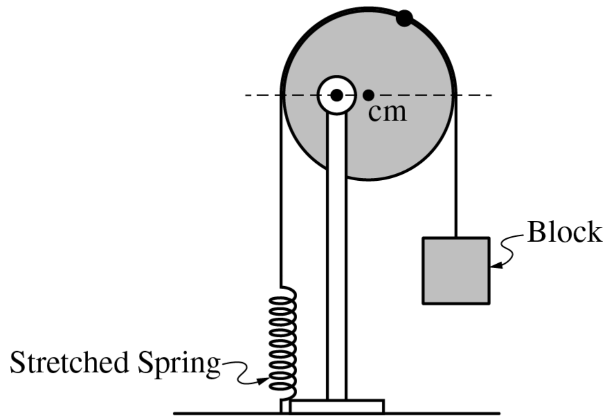


Figure 3

Note: Figure not drawn to scale.

(e) The disk is adjusted on the support so that the axle does not pass through the center of mass of the disk. The block is again hung on the right side of the disk and the spring-disk-block system comes to equilibrium, as shown in Figure 3. The axle does not exert a torque on the disk. For each force on the disk, indicate whether the magnitude of the torque about the axle caused by that force increases, decreases, or stays the same relative to part (b).

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

Begin your response to **QUESTION** [illegible] on this page.

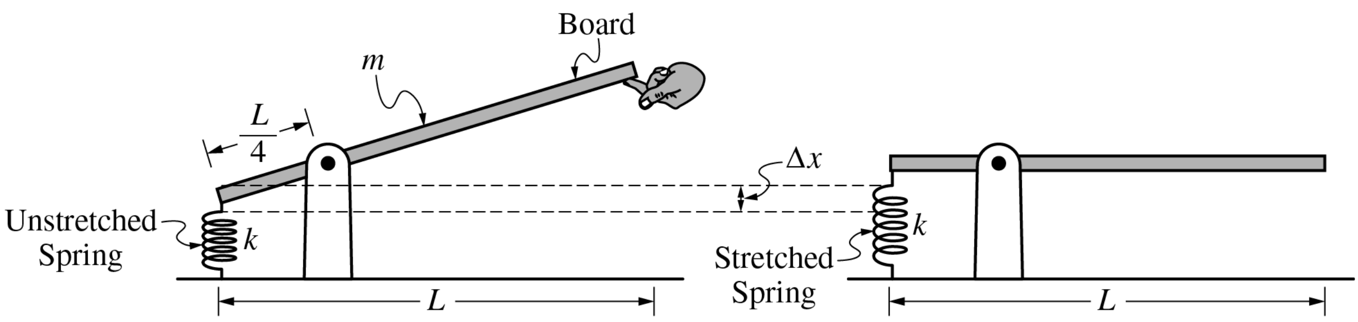
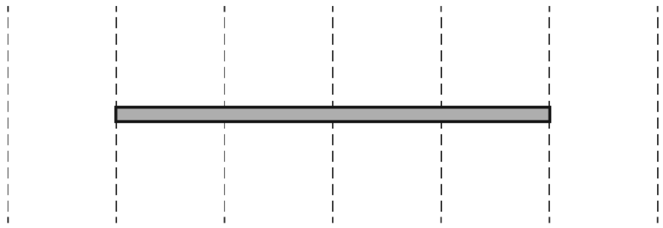


Figure 1 Figure 2

Note: Figures not drawn to scale.

. A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The right end of the board is initially held by a student so that the spring is unstretched, as shown in Figure 1. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring stretched, as shown in Figure 2. The rotational inertia of the board about the pivot is I.

(a) On the rectangle below, which represents the board, draw and label the forces (not components) that act on the board while the board-spring system is in equilibrium. Each force should be represented by an arrow that starts on, and points away from, the board, and should represent the location at which that force acts.



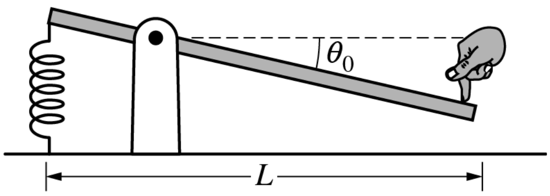
GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(b) Derive an expression for the distance the spring stretches, Δx, when the board is in equilibrium. Express your answer in terms of k, L, m, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

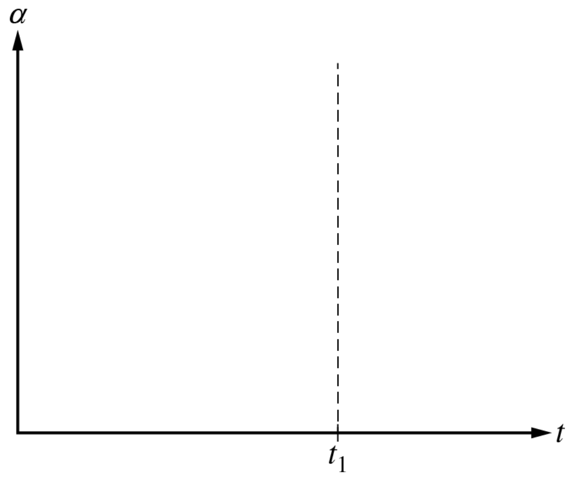
Continue your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

(c) A student pushes the board down on the right side, stretching the spring a new distance Δx_2 from the unstretched position. The board is held at a small angle θ_0 with the horizontal, as shown. The student then releases the board from rest.

i. At time $t = 0$, the board is released. At $t = t_1$, the board first crosses the horizontal. Sketch a graph of the magnitude of the angular acceleration α of the board as a function of time t from $t = 0$ to $t = t_1$.



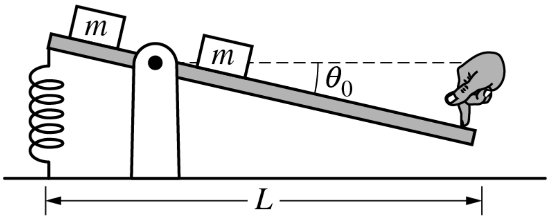
GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

ii. Derive an expression for the angular acceleration α_0 of the board immediately after the board is released. Express your answer in terms of k, L, m, I, Δx_2, θ_0, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

(d) Two blocks of equal mass m are attached to the board equal distances from the pivot point, as shown. The board is again pushed down on the right side so that the spring stretches the same distance Δx_2 as in part (c). The board is then released. How does the new angular acceleration α' when the blocks are attached compare to the angular acceleration α_0 from part (c) ?

___$\alpha' > \alpha_0$ ___$\alpha' < \alpha_0$ ___ $\alpha' = \alpha_0$

Justify your answer.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM