Question 3: Free-Response Question — 15 points

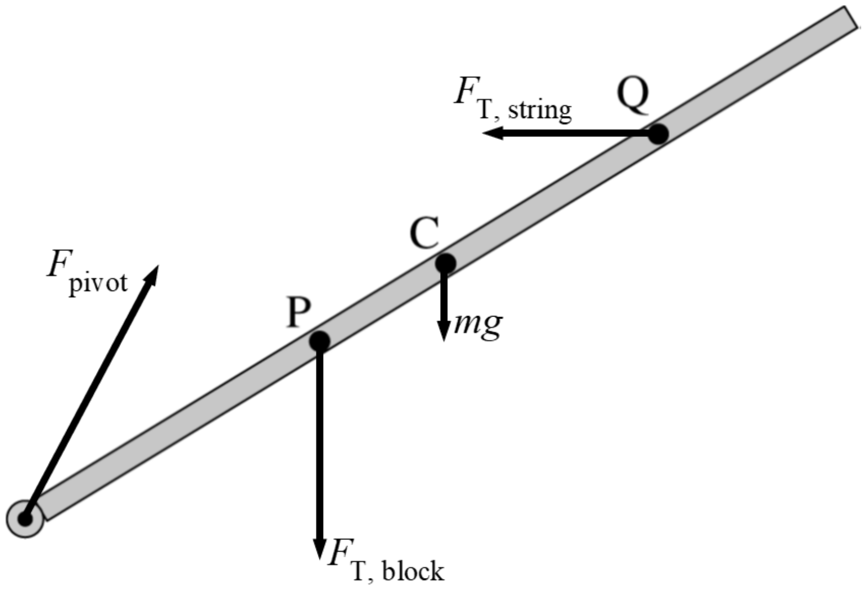
(a) For drawing and appropriately labeling the downward forces that are exerted on the rod at points P and C — **1 point**

Scoring Note: Labeling the downward force of tension as F_{block}, $3mg$, or similar, may earn this point.

For drawing and appropriately labeling a leftward force that is exerted on the rod at Point Q — **1 point**

For drawing and appropriately labeling a force that is directed up and to the right that is exerted on the rod at the pivot, and no extraneous forces are present — **1 point**

Example Response



Scoring Note: Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity.

Scoring Note: Examples of appropriate labels for the force from the pivot include: F_p, F_{pivot}, F_n, F_N, N, "normal force," "pivot force."

Scoring Note: Examples of appropriate labels for the tension force include F_t, F_T, T, F_{string}, and "Force from string."

Total for part (a) — 3 points

(b)	For indicating that the net torque exerted on the rod is equal to zero	**1 point**

Example Response

$\Sigma\tau = 0$

For a correct expression for the torque exerted on the rod by the hanging mass	**1 point**

Example Response

$$3mg\sin\theta\left(\frac{3L}{8}\right)$$

For a correct expression for the torque exerted on the rod by the gravitational force	**1 point**

Example Response

$$mg\sin\theta\left(\frac{4L}{8}\right)$$

For a correct expression for the torque exerted on the rod by the string	**1 point**

Example Responses

$F_T\sin(90° - \theta)\left(\frac{6L}{8}\right)$ **OR** $F_T\cos\theta\left(\frac{6L}{8}\right)$

Example Solution

$$\Sigma\tau = 0$$

$$3mg\sin\theta\left(\frac{3L}{8}\right) + mg\sin\theta\left(\frac{4L}{8}\right) - F_T\sin(90° - \theta)\left(\frac{6L}{8}\right) = 0$$

$$3mg\sin\theta\left(\frac{3L}{8}\right) + mg\sin\theta\left(\frac{4L}{8}\right) - F_T\cos\theta\left(\frac{6L}{8}\right) = 0$$

$$\left(\frac{9}{8}\right)mg\sin\theta + \left(\frac{4}{8}\right)mg\sin\theta = \left(\frac{6}{8}\right)F_T\cos\theta$$

$$13mg\sin\theta = 6F_T\cos\theta$$

$$F_T = \frac{13}{6}mg\tan\theta$$

Scoring Note: A maximum of three points may be earned if the trigonometric functions (sin and cos) are reversed for all three torque terms.

Total for part (b) 4 points

(c)	For indicating that the torque exerted on the rod by the string is always the same	**1 point**
	For stating that as the angle between the string and the rod increases, the force exerted on the rod by the string decreases	**1 point**

Example Response

Because the torque exerted on the rod by the string is always the same, as the angle between the string and the rod increases, the tension $F_{T,\text{ new}}$ must decrease.

Total for part (c) 2 points

(d)(i)	For indicating the total mass is the sum of differentiable masses along the length of the rod	**1 point**

Example Response

$$M = \int dm$$

For correctly writing dm in terms of x **1 point**

Example Response

$$M = \int_0^{1.2} (6 + 10x)\, dx$$

For a correct numeric answer with correct units **1 point**

Example Response

$$M = 14.4 \text{ kg}$$

Example Solution

$$M = \int dm$$

$$M = \int \lambda \, dx$$

$$M = \int_0^{1.2} (6 + 10x)\, dx$$

$$M = \left(6x + \frac{10x^2}{2}\right)\Bigg|_0^{1.2\text{ m}}$$

$$M = 6 \text{ kg/m}(1.2 \text{ m}) + \frac{10 \text{ kg/m}^2 (1.2 \text{ m})^2}{2}$$

$$M = 14.4 \text{ kg}$$

(d)(ii) For a correct substitution of λ into an integral expression of rotational inertia **1 point**

Example Response

$$I = \int_0^{1.2\text{ m}} (A + Bx)x^2 dx$$

For a correct integration **1 point**

Example Response

$$I = \left(\frac{Ax^3}{3} + \frac{Bx^4}{4}\right)\Bigg|_0^{1.2\text{ m}}$$

For a correct numeric answer with correct units **1 point**

Example Response

$$I = 8.64\text{ kg}\cdot\text{m}^2$$

Example Solution

$$I = \int r^2 dm \qquad dm = \lambda dr \text{ and } r = x$$

$$I = \int_0^{1.2\text{ m}} \lambda x^2 dx$$

$$I = \int_0^{1.2\text{ m}} (A + Bx)x^2 dx$$

$$I = \left(\frac{Ax^3}{3} + \frac{Bx^4}{4}\right)\Bigg|_0^{1.2\text{ m}}$$

$$I = \frac{(6.0\text{ kg/m})(1.2\text{ m})^3}{3} + \frac{\left(10.0\text{ kg/m}^2\right)(1.2\text{ m})^4}{4}$$

$$I = 8.64\text{ kg}\cdot\text{m}^2$$

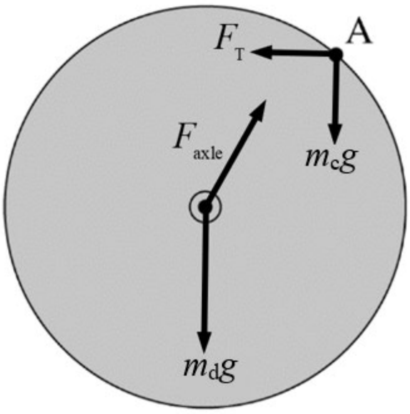
Total for part (d) 6 points

Total for question 3 15 points

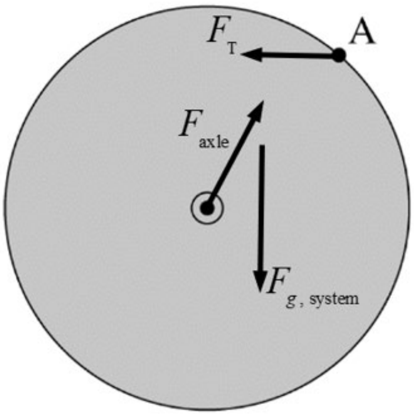
Question 3: Free-Response Question — 15 points

(a)	For drawing and appropriately labeling separate gravitational forces that are exerted on the disk and the lump of clay and exerted at the correct locations **Scoring Note:** Drawing a downward gravitational force exerted on the clay-disk system at the correct location can earn this point.	**1 point**
	For drawing and appropriately labeling a leftward force exerted on the system at Point A	**1 point**
	For drawing and appropriately labeling a force directed up and right that is exerted on the system at the axle, and no extraneous forces are present	**1 point**

Example Responses



OR



Scoring Note: Examples of appropriate labels for the gravitational force include F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on disk," "F on disk by Earth," $F_{\text{Earth on Disk}}$, $F_{\text{E,Disk}}$, and $F_{\text{Disk,E}}$. The labels G or g are not appropriate labels for the gravitational force.

Scoring Note: Examples of appropriate labels for the normal force from the axle include F_N, F_{axle}, N, "normal force," and "axle force."

Scoring Note: Examples of appropriate labels for the tension force include F_t, F_T, T, F_{string}, and "Force from string."

Total for part (a) — 3 points

(b) For a multi-step derivation that indicates the net torque is zero **1 point**

For indicating only the weight of the clay and the tension in the string exert torques on the system about the axle **1 point**

Example Response

$\tau_{\text{net}} = \tau_{\text{clay}} - \tau_{\text{string}}$

For a correct expression for the torque exerted on the system by the weight of the clay **1 point**

Example Response

$\tau_{\text{clay}} = Rm_{\text{c}}g\cos\theta$

For a correct expression for the torque exerted on the system by the tension in the string **1 point**

Example Response

$\tau_{\text{string}} = RF_{\text{T}}\sin\theta$

Example Solution

$$\Sigma\tau = 0$$
$$\tau_{\text{clay}} - \tau_{\text{string}} = 0$$
$$F_{g,\,\text{clay}}R\cos\theta - RF_{\text{T}}\sin\theta = 0$$
$$Rm_{\text{c}}g\cos\theta = RF_{\text{T}}\sin\theta$$
$$F_{\text{T}} = \frac{Rm_{\text{c}}g\cos\theta}{R\sin\theta}$$
$$F_{\text{T}} = m_{\text{c}}g\cot\theta$$

Scoring Note: A maximum of three points can be earned if the trigonometric functions ($\sin$ and $\cos$) are reversed for both torque expressions.

Total for part (b) 4 points

(c)	For indicating a direct relationship between the torque exerted by the clay and the tension in the string	**1 point**
	For correctly relating a greater torque exerted by the clay from at least **one** of the following: • An increase in the angle between the radial direction and the weight of the clay • A greater perpendicular component of the weight of the clay • A greater lever arm	**1 point**
	Example Response *The torque caused by the weight of the clay at Point* B *is greater than when the clay is at Point* A *because the component of the weight that is perpendicular to the lever arm is larger. To maintain equilibrium, the net torque on the system is still zero, therefore the tension* $F_{\text{T, new}}$ *must be greater.*	
	Total for part (c)	**2 points**

(d)(i)	For using integration to calculate rotational inertia	**1 point**
	For **one** of the following: • Substituting $\rho(2\pi r)dr$ for dm • Indicating the correct limits of integration	**1 point**
	Example Response $I = \int_0^{0.3\text{ m}} r^2 \rho(2\pi r)dr$	
	For a correct answer of $I = 0.012\text{ kg}\cdot\text{m}^2$, including units	**1 point**
	Example Response $I = 0.012\text{ kg}\cdot\text{m}^2$	

Example Solution

$$I = \int r^2 dm \qquad dm = \rho dA \quad \text{and} \quad dA = 2\pi r\, dr$$

$$I = \int_0^R r^2 \rho dA$$

$$I = \int_0^{0.3\text{ m}} 2\pi\beta r^4 dr$$

$$I = \left.\frac{2\pi\beta R^5}{5}\right|_0^{0.3\text{ m}}$$

$$I = \frac{2\pi(4.0\,\text{kg/m}^3)(0.3\text{ m})^5}{5}$$

$$I = 0.012\text{ kg}\cdot\text{m}^2$$

(d)(ii)	For using the rotational form of Newton's second law	**1 point**
	For a correct expression for the net torque exerted on the clay-disk system	**1 point**

Example Response

$$\tau_{\text{net}} = Rm_c g$$

For indicating that the rotational inertia of the clay-disk system is the sum of the rotational inertia of the disk and the rotational inertia of the lump of clay — **1 point**

Example Response

$$I_{\text{system}} = I_{\text{disk}} + I_{\text{clay}}$$

Example Solution

$$\Sigma\tau = I\alpha$$

$$\alpha = \frac{\tau_{\text{net}}}{I}$$

$$\alpha = \frac{Rm_c g}{I_{\text{disk}} + I_{\text{clay}}}$$

$$\alpha = \frac{(0.3\text{ m})(0.60\text{ kg})\left(9.8\text{ m/s}^2\right)}{\left(\left(0.012\text{ kg}\cdot\text{m}^2\right) + (0.60\text{ kg})(0.3\text{m})^2\right)}$$

$$\alpha = 26.7\ \text{rad/s}^2$$

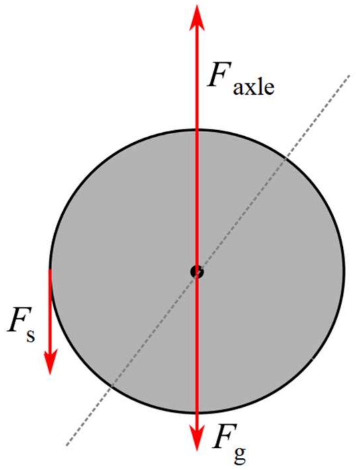
Total for part (d) 6 points

Total for question 3 15 points

Question 3: Free-Response Question — 15 points

(a)	For indicating that the sum of the torques on the disk equals zero $\Sigma\tau_{\text{on disk}} = 0$ $\tau_g = \tau_s$ **OR** For indicating that the sum of the forces equals zero $\Sigma F = 0$ $F_g = F_s$	**1 point**
	For correctly substituting the expressions for the forces $F_g R = F_s R$ $m_B g R = k\Delta x R$ $m_B g = k\Delta x$ **OR** $F_g = F_s$ $m_B g = k\Delta x$	**1 point**
	For correctly substituting for Δx $m_B g = kR\theta$ $m_B = \dfrac{kR\theta}{g}$	**1 point**
	Total for part (a)	**3 points**
(b)	For drawing and labeling the force of the tension exerted on the disk anywhere between point P and the left edge of the disk, including point P and the left edge of the disk, tangent to the disk	**1 point**
	For correct location and label of the force due to gravity exerted on the disk, directed straight down	**1 point**
	For correct location and label of the force exerted on the disk by the axle, directed such that the disk remains in translational equilibrium (i.e., $\Sigma F = 0$)	**1 point**

Example Response



Scoring Notes:

- Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force, which can be used instead of F_{axle}. F_{spring}, F_s, T_{spring}, T, "spring force," or similar labels may be used for the tension force exerted by the spring.
- A response with extraneous forces or vectors can earn a maximum of two points.

Total for part (b) 3 points

(c) For indicating that the net torque is due only to the force exerted on the disk by the tension in the rotational form of Newton's second law — **1 point**

$\tau_s = I_d \alpha$

For correctly expressing the torque on the disk by the tension in terms of the spring force, which is equal to the tension, and the lever (moment) arm — **1 point**

$F_s R = I_d \alpha$

For correctly substituting for F_s — **1 point**

$-k\Delta x R = I_d \alpha$

For correctly substituting I_d and Δx, or an expression for Δx consistent with part (a) — **1 point**

$$-k(R\theta)R = \frac{1}{2}M_d R^2 \alpha$$

$$\alpha = -\frac{2k\theta}{M_d}$$

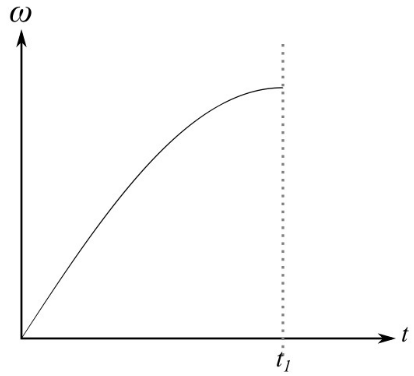
Scoring Note: The negative sign is not necessary to earn this point.

Total for part (c) 4 points

(d) For a sketch that starts at zero and monotonically increases until time $t = t_1$ — **1 point**

For a sketch that is concave down between time $t = 0$ and $t = t_1$ — **1 point**

Example Response



Scoring Note: Any part of the graph beyond t_1 is not considered in scoring.

Total for part (d) — 2 points

(e) For indicating that the torque exerted by the force due to gravity on the disk increased — **1 point**

Example Response

The force due to gravity on the disk now has a non-zero lever arm and hence it exerts a larger torque on the disk.

For indicating that the torque exerted by the tension caused by the force due to gravity on the block increased — **1 point**

Example Response

The force exerted by the right side of the string (from the block) on the disk has a longer lever arm, hence the torque it exerts is larger.

For indicating that the torque exerted by the tension increased — **1 point**

Example Response

The counterclockwise torque due to the tension caused by the spring must increase to counteract the increase in clockwise torques due to the force due to gravity of the disk and tension caused by the force of gravity due to block to keep the disk in equilibrium.

Scoring Notes:

- A response that references the torque due to the force at the axle staying the same can earn all 3 points.
- A response that references the torque due to the force on the axle changing, or any additional torques can earn a maximum of 2 points.

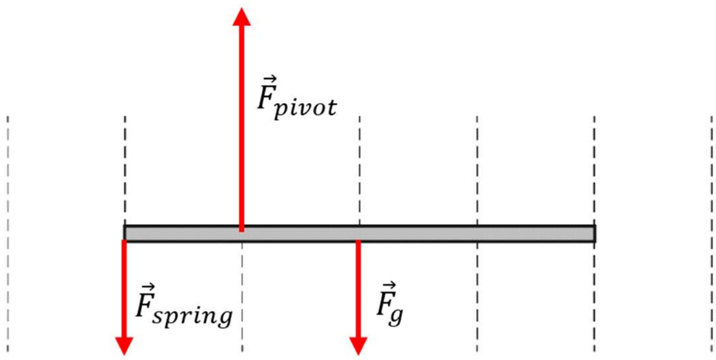
Total for part (e) — 3 points

Total for question 3 — 15 points

Question 3: Free-Response Question — 15 points

(a)	For correct location and label of the force exerted on the board by the spring	**1 point**
	For correct location and label of the force exerted on the board by gravity	**1 point**
	For correct location and label of the force exerted on the board by the pivot	**1 point**

Example Response



Scoring Notes:

- Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force, which can be used instead of F_{pivot}. F_{spring}, F_S, "spring force," or similar labels may be used for the force exerted by the spring.
- A response with extraneous forces or vectors can earn a maximum of two points.

Total for part (a) — 3 points

(b)	For indicating that the sum of the torques equals zero $\Sigma\tau = 0$	**1 point**
	For substituting mg for the force of gravity and $k\Delta x$ for the spring force into a torque equation $\frac{L}{4}k\Delta x - \frac{L}{4}mg = 0$	**1 point**
	For a correct expression for Δx $\Delta x = \frac{mg}{k}$	**1 point**

Example Response

$\Sigma \tau = 0$

$F_{\text{spring}} d_{\text{spring}} - F_{\text{weight}} d_{\text{weight}} = 0$

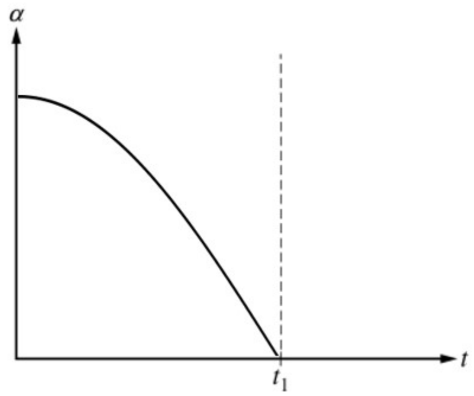
$\frac{L}{4} k \Delta x - \frac{L}{4} mg = 0$

$\Delta x = \frac{mg}{k}$

Scoring Note: This point can be earned regardless of whether a negative sign is present.

	Total for part (b)	**3 points**
(c)(i)	For a sketch that begins at a maximum value and monotonically decreases to a minimum of zero at $t = t_1$	**1 point**
	For a sketch that is concave down between $t = 0$ and $t = t_1$	**1 point**

Example Response



Scoring Note: Any part of the graph beyond t_1 is not considered in scoring.

(c)(ii)	For indicating that the torques are in opposite directions in the rotational form of Newton's second law $\tau_{\text{spring}} - \tau_g = I\alpha$	**1 point**
	For including $\sin(90 - \theta_0)$, $\sin(90 + \theta_0)$, or $\cos\theta_0$ in an expression for the net torque exerted on the rod $F_{spring} d_{spring} (\cos\theta) - F_g d_g (\cos\theta) = I_B \alpha_0$	**1 point**
	For substituting mg for the force due to gravity and substituting $k\Delta x_2$ for the spring force in an expression for the net torque exerted on the rod $d_{spring} k \Delta x_2 (\cos\theta_0) - d_g mg (\cos\theta_0) = I_B \alpha_0$	**1 point**
	For substituting correct lever arms in an expression for the net torque exerted on the rod $\frac{L}{4}(\cos\theta_0) k \Delta x_2 - \frac{L}{4}(\cos\theta_0) mg = I_B \alpha_0$	**1 point**

Example Response

$\tau_{\text{spring}} - \tau_g = I\alpha$

$F_{spring} d_{spring} (\cos\theta_0) - F_g d_g (\cos\theta_0) = I_B \alpha_0$

$l_{spring} (\cos\theta_0) k\Delta x_2 - l_g (\cos\theta_0) mg = I_B \alpha_0$

$\frac{L}{4}(\cos\theta_0) k\Delta x_2 - \frac{L}{4}(\cos\theta_0) mg = I_B \alpha_0$

$\alpha_0 = \frac{L}{4I_B}(k\Delta x_2 \cos\theta_0 - mg\cos\theta_0)$

	Total for part (c)	**6 points**
(d)	For selecting "$\alpha' < \alpha_0$" with an attempt at a relevant justification	**1 point**
	For indicating that the net torque does not change	**1 point**
	For indicating the rotational inertia increases	**1 point**

Example Response

The net torque on the system is the same (the spring and the person exert the same force and the additional torques from the gravitational forces on the masses cancel) but the rotational inertia of the system is now larger. This means that the angular acceleration of the system is now smaller than it was before.

Total for part (d) 3 points

Total for question 3 15 points

Question 3: Free-Response Question — 15 points

(a) For using integral calculus to calculate the rotational inertia of the rod — **1 point**

$$I = \int r^2 dm$$

$$dm = \lambda dr = \gamma x^2 dx$$

For correctly substituting γx^2 into the above equation — **1 point**

$$I = \int x^2 \left(\gamma x^2 dx\right) = \int_{x=0}^{x=L} \gamma x^4 dx = \gamma \left[\frac{x^5}{5}\right]_{x=0}^{x=L} = \left(\frac{3M}{L^3}\right)\left(\frac{L^5}{5} - 0\right) = \frac{3}{5}ML^2$$

Total for part (a) — 2 points

(b) For using integral calculus to determine the center of mass of the rod — **1 point**

$$X_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{\int x dm}{\int dm}$$

For correctly substituting γx^2 into the numerator of the above equation — **1 point**

For correctly substituting M into the denominator of the above equation OR evaluating the integral $\int dm$ to find the mass of the rod — **1 point**

$$X_{CM} = \frac{\int x\lambda dx}{M} = \frac{\int x\left(\gamma x^2\right)dx}{M} = \frac{\int_{x=0}^{x=L} \gamma x^3 dx}{M} = \frac{\left[\frac{\gamma x^4}{4}\right]_{x=0}^{x=L}}{M} = \frac{\left(\frac{3M}{L^3}\right)\frac{L^4}{4}}{M} = \frac{3}{4}L$$

OR

$$X_{CM} = \frac{\int x\lambda dx}{\int \lambda dx} = \frac{\int x\left(\gamma x^2\right)dx}{\int_{x=0}^{x=L} \gamma x^2 dx} = \frac{\int_{x=0}^{x=L} \gamma x^3 dx}{\left[\frac{\gamma x^2}{3}\right]_{x=0}^{x=L}} = \frac{\left[\frac{\gamma x^4}{4}\right]_{x=0}^{x=L}}{\frac{\gamma L^2}{3}} = \frac{\frac{\gamma L^4}{4}}{\frac{\gamma L^2}{3}} = \frac{3}{4}L$$

Total for part (b) — 3 points

(c)	For selecting "Greater than" with an attempted justification	**1 point**
	For a correct justification	**1 point**

Example responses for part (c)

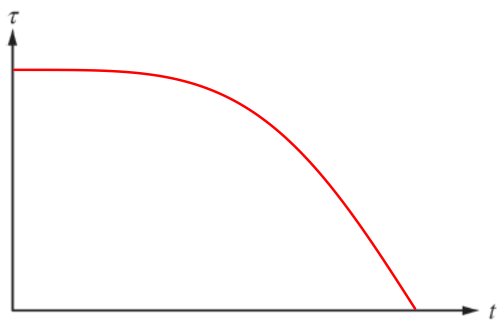
Because more of the mass of the rod is at the end of the rod opposite point P, more mass is concentrated away from the axis of rotation; thus, the rotational inertia of the rod would be greater around point P than around its center of mass.

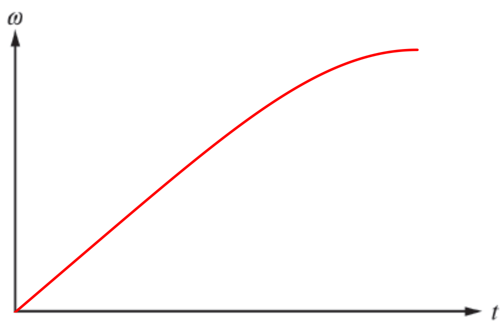
OR

According to the parallel axis theorem, $I = I_{cm} + md^2$*, if the axis is at a position away from the center of mass, the rotational inertia is larger than if the axis were at the center of mass.*

Total for part (c) 2 points

(d)	For a concave down curve that decreases to zero for the graph of τ as a function of t	**1 point**
	For a concave down curve that approaches horizontal for the graph of ω as a function of t	**1 point**
	For consistency between the two graphs	**1 point**





Total for part (d) 3 points

(e)	For selecting "Decreases" with an attempted justification	**1 point**
	For a correct justification	**1 point**

Example responses for part (e)

As the rod rotates downward, the angle θ *in the torque equation* $\tau = rF\sin\theta$ *decreases. Thus, the torque on the rod decreases.*

OR

As the rod rotates downward, the lever arm between point P and the rod's center of mass continues to decrease; thus, the torque on the rod decreases.

Total for part (e) 2 points

(f) For using conservation of energy to calculate the speed of the rotating rod **1 point**

$$U_i + K_i = U_f + K_f$$
$$U_i + 0 = 0 + K_f$$
$$U_i = K_f$$

For correctly substituting into the above equation **1 point**

$$mgh_i = \frac{1}{2} I \omega_f^2$$

For correctly solving for the linear speed of point S **1 point**

$$Mg\left(\frac{3}{4}L\right) = \frac{1}{2}\left(\frac{3}{5}ML^2\right)\left(\frac{v}{L}\right)^2$$

$$\frac{3}{4}MgL = \frac{3}{10}Mv^2 \therefore v = \sqrt{\tfrac{5}{2}gL} = \sqrt{\tfrac{5}{2}\left(9.8 \text{ m/s}^2\right)(1.0 \text{ m})} = 4.9 \text{ m/s}$$

Total for part (f) 3 points

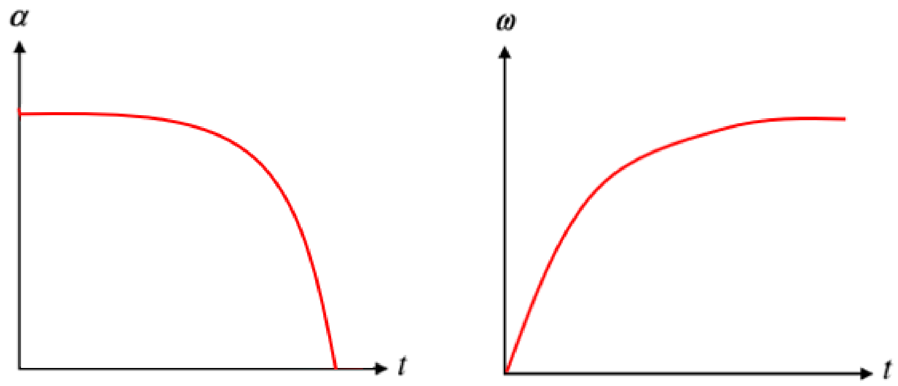
Total for question 3 15 points

Question 2: Free-Response Question — 15 points

(a)	For integrating using the correct limits or constant of integration	**1 point**
	$I = \int_{r=0}^{r=2L} \lambda r^2 dr = \lambda \left[\dfrac{r^3}{3} \right]_{r=0}^{r=2L} = \dfrac{\lambda}{3}\left((2L)^3 - 0 \right)$	
	For correctly relating λ to M and L	**1 point**
	$\lambda = \dfrac{m}{\ell} = \dfrac{M}{2L} \therefore I = \left(\dfrac{1}{3}\right)\left(\dfrac{M}{2L}\right)\left(8L^3\right) = \dfrac{4}{3}ML^2$	
	Total for part (a)	**2 points**
(b) i.	For correctly substituting into an equation for the center of mass of an object in the horizontal direction	**1 point**
	$X_{CM} = \dfrac{\sum m_i x_i}{\sum m_i} = \dfrac{\left[\left(\frac{M}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{M}{2}\right)(L)\right]}{\left(\frac{M}{2} + \frac{M}{2}\right)} = \dfrac{\left(\frac{ML}{4} + \frac{ML}{2}\right)}{M} = \dfrac{3}{4}L$	
ii.	For correctly substituting into an equation for the center of mass of an object in the vertical direction	**1 point**
	$Y_{CM} = \dfrac{\sum m_i y_i}{\sum m_i} = \dfrac{\left[\left(\frac{M}{2}\right)(L) + \left(\frac{M}{2}\right)\left(\frac{L}{2}\right)\right]}{\left(\frac{M}{2} + \frac{M}{2}\right)} = \dfrac{\left(\frac{ML}{2} + \frac{ML}{4}\right)}{M} = \dfrac{3}{4}L$	
	Total for part (b)	**2 points**
(c)	For selecting "Less than" and attempting a relevant justification	**1 point**
	For a correct justification	**1 point**
	Example response for part (c) *Because object B has more of its mass closer to the pivot than object A, the rotational inertia of object B must be less than that of object A.*	
	Total for part (c)	**2 points**

(d)	For an acceleration graph that is concave down and begins horizontally	**1 point**
	For an angular speed graph that is concave down and ends horizontally	**1 point**
	For consistency between the angular acceleration and angular speed graphs	**1 point**

Example responses for part (d)



Total for part (d) 3 points

(e)	For selecting "Decreasing" and attempting a relevant justification	**1 point**
	For a justification that indicates the lever arm for the torque is decreasing	**1 point**

Example response for part (e)

Because the horizontal position of the center of mass for the object is moving closer to the pivot, the lever arm for the force of gravity is decreasing so the angular acceleration decreases.

Total for part (e) 2 points

(f) For using conservation of energy **1 point**

$U_{g1} = K_2$

For correctly relating the change in rotational kinetic energy to the change in gravitational potential energy **1 point**

$mgh = \frac{1}{2}I\omega^2$

For correctly substituting for h into the equation above **1 point**

$mg\left(\frac{L}{2}\right) = \frac{1}{2}I\omega^2$

For an expression for ω that uses only the allowed symbols and is algebraically consistent with the previous steps **1 point**

$\omega = \sqrt{\frac{2mgh}{I}} = \sqrt{\frac{2Mg(L/2)}{I_B}}$

$\omega = \sqrt{\frac{MgL}{I_B}}$

Total for part (f) 4 points

Total for question 2 15 points

2019

AP CollegeBoard

AP® Physics C: Mechanics

Scoring Guidelines

Set 1

2019 SCORING GUIDELINES

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

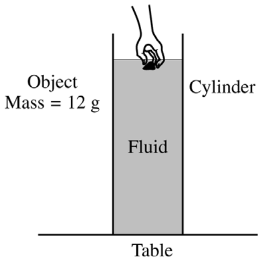
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

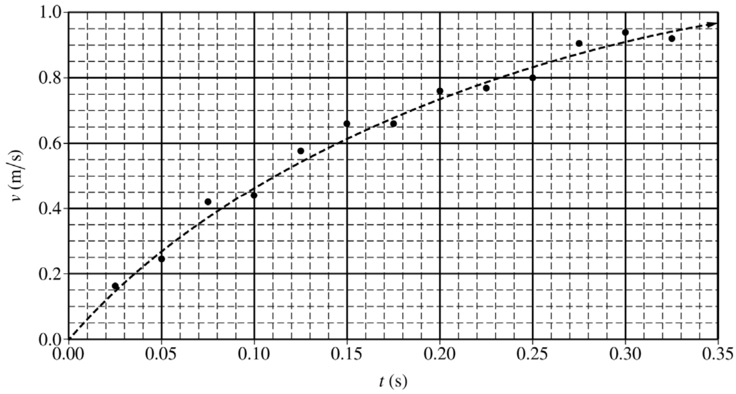
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 1

15 points



In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of $m = 12$ g is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.



(a)

i. LO CHA-1.C, SP 7.A
1 point

Does the speed of the object increase, decrease, or remain the same?

____ Increase ____ Decrease ____ Remain the same

For selecting "Increase"		1 point

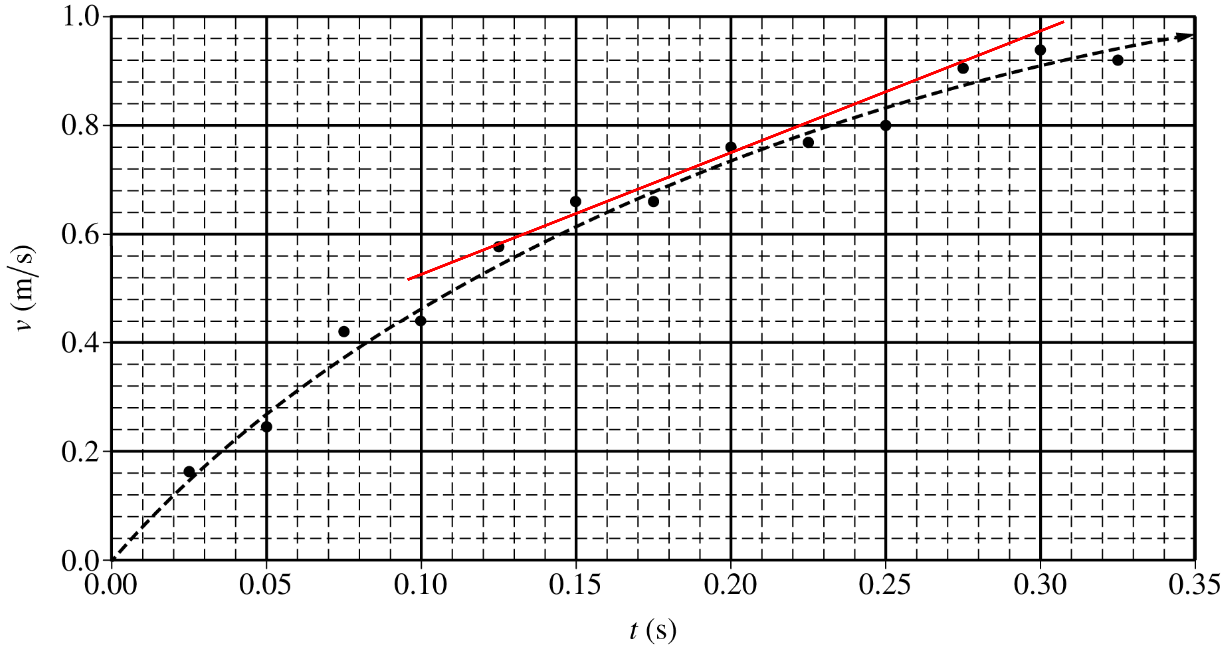
ii. LO CHA-1.C, SP 4.D
2 points

In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.

For a description that includes the direction of the acceleration as being "downwards"		1 point
For a description that includes the decrease in the magnitude of the acceleration		1 point
Example: Because the object is moving downwards and speeding up, the acceleration must be downwards. Because the slope of the graph of speed as a function of time is decreasing, the magnitude of the acceleration must be decreasing.		

Question 1 (continued)

(a) continued



iii. LO CHA-1.C, SP 4.D, 6.C
2 points

Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20$ s.

For calculating the slope of a trend line at $t = 0.20$ s		1 point
$\text{slope} = a = \dfrac{\Delta v}{\Delta t} = \dfrac{(0.8 - 0.6)\ \text{m/s}}{(0.226 - 0.136)\ \text{s}}$		
For a correct answer using points from a tangent line		1 point
$a = 2.22\ \text{m/s}^2$		

The students use the equation $v = A\left(1 - e^{-Bt}\right)$ to model the speed of the falling object and find the best-fit coefficients to be $A = 1.18\ \text{m/s}$ and $B = 5\ \text{s}^{-1}$.

2019 SCORING GUIDELINES

Question 1 (continued)

(b) Use the above equation to:

i. LO CHA-1.B, SP 6.B, 6.C
3 points

Derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time *t*.

For indicating that the vertical displacement is the integration of the velocity		1 point
$\Delta y = \int v dt = \int A\left(1 - e^{-Bt}\right)dt$		
For the equation for speed using appropriate limits or constant of integration		1 point
$\Delta y = \int_{t'=0}^{t'=t} A\left(1 - e^{-Bt'}\right)dt' = A\left[t' - \frac{1}{-B}e^{-Bt'}\right]_{t'=0}^{t'=t} A\left[\left(t + \frac{1}{B}e^{-Bt}\right) - \left(0 + \frac{1}{B}e^{0}\right)\right]$		
For a correct answer		1 point
$\Delta y = A\left(t + \frac{1}{B}\left(e^{-Bt} - 1\right)\right) = (1.18)\left(t + \frac{1}{5}\left(e^{-5t} - 1\right)\right)$		
<u>Note:</u> Credit given for using *A* and *B* or plugging in the given values		

ii. LO INT-1.C.d, SP 6.B, 6.C
3 points

Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time *t*.

For attempting the derivative of the equation for speed		1 point
$a = \frac{dv}{dt} = \frac{d}{dt}\left[A\left(1 - e^{-Bt}\right)\right]$		
For a correct equation for the acceleration		1 point
$a = ABe^{-Bt}$		
For multiplying the equation above by the mass of the object		1 point
$F = ma = m\left(ABe^{-Bt}\right) = mABe^{-Bt}$		
$F = (.012)(1.18)(5)e^{-5t} = 0.071e^{-5t}$		
<u>Note:</u> Credit given for using *A*, *B*, and *m* or plugging in the given values		

Question 1 (continued)

(c)

The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed.

i. LO INT-1.I, SP 7.A, 7.C
2 points

Determine the constant speed of the object.
Justify your answer.

For stating the constant speed is $v = A$		1 point
For indicating that the constant speed can be determined by setting the time equal to infinity		1 point
Example: After a long time, the falling object will reach a terminal constant speed in the fluid. This can be determined by setting the time t in the equation for speed equal to infinity. By doing this, the constant speed is determined to be $v = A$.		
Note: Credit given for solving mathematically		

ii. LO INT-1.H.b, SP 7.A, 7.C
2 points

Determine the force exerted by the fluid on the object at this time. Justify your answer.

For indicating that the net force is equal to zero when the object moves with constant speed		1 point
For indicating the resistive force is equal to the weight of the object at this time		1 point
Example: When the falling object reaches a constant speed in the fluid, the net force must be zero. Because the only vertical forces acting on the object are Earth's gravitational pull and the resistive force of the fluid, these two forces must be equal. So, the resistive force must be equal to the weight of the object or 0.12 N.		
Note: Credit given for solving mathematically		

2019 SCORING GUIDELINES

Question 1 (continued)

Learning Objectives

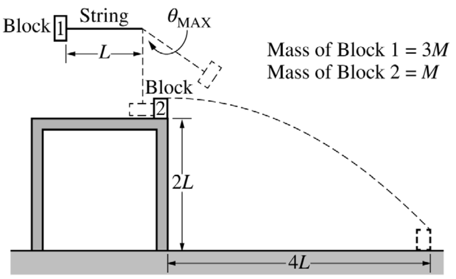
CHA-1.B: Determine functions of position, velocity, and acceleration that are consistent with each other, for the motion of an object with a nonuniform acceleration.
CHA-1.C: Describe the motion of an object in terms of the consistency that exists between position and time, velocity and time, and acceleration and time.
INT-1.C.d: Derive an expression for the net force on an object in translational motion.
INT-1.H.b: Describe the acceleration, velocity, or position in relation to time for an object subject to a resistive force (with different initial conditions, i.e., falling from rest or projected vertically).
INT-1.I: Calculate the terminal velocity of an object moving vertically under the influence of a resistive force of a given relationship.

Science Practices

4.D: Select relevant features of a graph to describe a physical situation or solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

Question 2

15 points



Note: Figure not drawn to scale.

A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

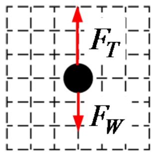
(a) LO CON-2.C.a, SP 5.E
1 point

Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

For correctly calculating the speed of block 1		1 point
$U_{g1} = K_2 \therefore mgh = \frac{1}{2}mv^2 \therefore gL = \frac{1}{2}v^2$		
$v = \sqrt{2gL}$		
Note: Credit is earned even if no work is shown.		

(b) LO INT-2.B.b, SP 3.D
2 points

On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



For correctly drawing and labeling the weight of the block and the tension in the string		1 point
For drawing the tension longer than the weight of the block		1 point
Note: A maximum of one point can be earned if there are any extraneous vectors.		

Question 2 (continued)

(c) LO INT-2.B.b, SP 5.A, 5.E
2 points

Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

For using an appropriate equation to calculate the tension in the string		1 point
$F_T = mg + ma_C = mg + \dfrac{mv^2}{r} = 3Mg + \dfrac{(3M)\left(\sqrt{2gL}\right)^2}{L}$		
For a correct answer		1 point
$F_T = 9Mg$		

For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

(d) LO CHA-2.C, SP 6.A, 6.C
2 points

Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

For using a correct kinematic equation to calculate the time		1 point
$\Delta y = v_1 t + \frac{1}{2}at^2 = \frac{1}{2}at^2 \therefore t = \sqrt{\dfrac{2\Delta y}{a}} = \sqrt{\dfrac{2(2L)}{g}} = \sqrt{\dfrac{(4)(0.75\text{ m})}{\left(9.8\text{ m/s}^2\right)}}$		
For a correct answer		1 point
$t = 0.55$ s		

(e) LO CHA-2.C, SP 6.A, 6.C
2 points

Calculate the speed of block 2 as it leaves the table.

For using a correct kinematic equation to calculate the speed		1 point
$\Delta x = vt \therefore v = \dfrac{\Delta x}{t} = \dfrac{4L}{t}$		
For substituting the time from part (d) into equation above		1 point
$v = \dfrac{(4)(0.75\text{ m})}{(0.55\text{ s})} = 5.45\text{ m/s}$		

Question 2 (continued)

(f) LO CON-4.E.a, SP 6.B, 6.C
3 points

Calculate the speed of block 1 just after it collides with block 2.

For indicating the use of the correct conservation of momentum to calculate the speed		1 point
$p_i = p_f \therefore m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$		
For correctly substituting the mass into equation above		1 point
$(3M)v_{1i} + 0 = (3M)v_{1f} + (M)v_{2f}$		
For substituting the speeds from parts (a) and (e) into equation above		1 point
$v_{1f} = \dfrac{(3M)v_{1i} - (M)v_{2f}}{(3M)} = \dfrac{(3M)\sqrt{2gL} - (M)v_{2f}}{(3M)}$		
$v_{1f} = \dfrac{(3)\sqrt{(2)(9.8\ \text{m/s}^2)(0.75\ \text{m})} - (1)(5.45\ \text{m/s})}{(3)} = 2.02\ \text{m/s}$		
($v_{1f} = 2.06\ \text{m/s}$ when using $g = 10\ \text{m/s}^2$)		

(g) LO CON-2.C.a, SP 6.B, 6.C
3 points

Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.

For using conservation of energy to calculate the angle		1 point
$K_1 = U_{g2}$		
$\frac{1}{2}mv^2 = mgh$		
For correctly substituting $h = L(1 - \cos\theta)$ into the equation above		1 point
For correctly substituting the speed from part (f) into the equation above		1 point
$\frac{1}{2}mv^2 = mgL(1 - \cos\theta) \therefore \dfrac{v^2}{2gL} = 1 - \cos\theta$		
$\theta = \cos^{-1}\left(1 - \dfrac{v^2}{2gL}\right) = \cos^{-1}\left(1 - \dfrac{(2.02\ \text{m/s})^2}{(2)(9.8\ \text{m/s}^2)(0.75\ \text{m})}\right) = 43.5°$		
($\theta = 44.1°$ when using $g = 10\ \text{m/s}^2$)		

Question 2 (continued)

Learning Objectives

CHA-2.C: Calculate kinematic quantities of an object in projectile motion, such as displacement, velocity, speed, acceleration, and time, given initial conditions of various launch angles, including a horizontal launch at some point in its trajectory.
INT-2.B.b: Describe forces that are exerted on objects undergoing horizontal circular motion, vertical circular motion, or horizontal circular motion on a banked curve.
CON-2.C.a: Calculate unknown quantities (e.g., speed or positions of an object) that are in a conservative system of connected objects, such as the masses in an Atwood machine, masses connected with pulley/string combinations, or the masses in a modified Atwood machine.
CON-4.E.a: Calculate the changes in speeds, changes in velocities, changes in kinetic energy, or changes in momenta of objects in all types of collisions (elastic or inelastic) in one dimension, given the initial conditions of the objects.

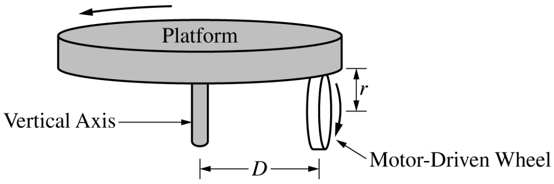
Science Practices

3.D: Create appropriate diagrams to represent physical situations.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

2019 SCORING GUIDELINES

Question 3

15 points



A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the wheel until the platform reaches an angular speed ω_P after time Δt . During time Δt , the wheel stays in contact with the platform without slipping.

(a) LO INT-7.A.b, CHA-4.A.b, SP 5.A, 5.E
2 points

Derive an expression for the angular speed ω_P of the platform. Express your answer in terms of I_P , r, D, F, Δt , and physical constants, as appropriate.

For correctly substituting into the rotational form of Newton's second law			1 point
$\tau = I\alpha \therefore FD = I_P\alpha$			
$\alpha = \dfrac{FD}{I_P}$			
For correctly substituting into a rotational kinematic equation to calculate the angular speed			1 point
$\omega_2 = \omega_1 + \alpha\Delta t = 0 + \left(\dfrac{FD}{I_P}\right)\Delta t$			
$\omega_P = \dfrac{FD\Delta t}{I_P}$			

(b) LO INT-7.D.a, SP 5.A, 5.E
2 points

Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed ω_P . Express your answer in terms of I_P , r, D, F, Δt , and physical constants, as appropriate.

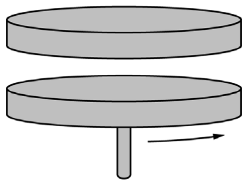
For using the equation for rotational kinetic energy		1 point
$K = \dfrac{1}{2}I\omega_P^2 = \left(\dfrac{1}{2}\right)(I)\left(\dfrac{FD\Delta t}{I}\right)^2$		
For an answer consistent with part (a)		1 point
$K = \dfrac{(FD\Delta t)^2}{2I}$		

Question 3 (continued)

(c) LO INT-7.C, SP 5.A, 5.E
2 points

Derive an expression for the angular speed of the wheel ω_W when the platform has reached angular speed ω_P. Express your answer in terms of D, r, ω_P, and physical constants, as appropriate.

For indicating that the linear speed of the platform is equal to the linear speed of the wheel		1 point
$v_P = v_W$ OR $(r\omega)_P = (r\omega)_W$		
For correctly relating the linear speeds to the angular speeds in the above equation		1 point
$D\omega_P = r\omega_W$		
$\omega_W = \dfrac{D\omega_P}{r}$		



When the platform is spinning at angular speed ω_P, the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia I_P as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed ω_f.

(d) LO CON-5.D.c, SP 5.A, 5.E
2 points

Derive an expression for ω_f. Express your answer in terms of ω_P, I_P, and physical constants, as appropriate.

For using an expression for the conservation of angular momentum		1 point
$L_1 = L_2 \therefore I_1\omega_1 = I_2\omega_2$		
For correctly substituting into the above equation		1 point
$I\omega_P = (2I)\omega_F$		
$\omega_F = \dfrac{1}{2}\omega_P$		

2019 SCORING GUIDELINES

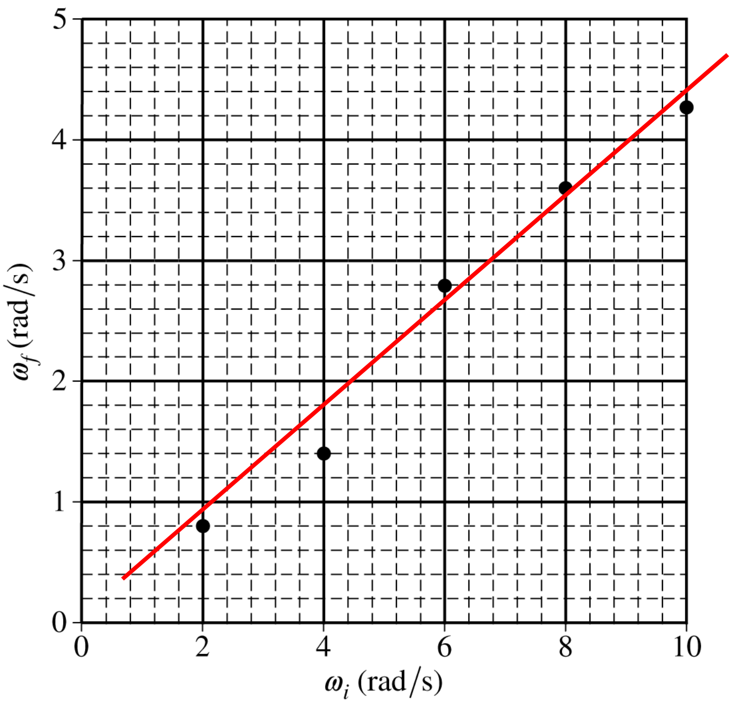
Question 3 (continued)

A student now uses the rotating platform $\left(I_P = 3.1 \text{ kg} \bullet \text{m}^2\right)$ to determine the rotational inertia I_U of an unknown object about a vertical axis that passes through the object's center of mass. The platform is rotating at an initial angular speed ω_i when the unknown object is dropped with its center of mass directly above the center of the platform. The platform and object are observed to be rotating together at angular speed ω_f. Trials are repeated for different values of ω_i. A graph of ω_f as a function of ω_i is shown on the axes below.

(e)

i. LO CON-5.D.c, SP 4.C
1 points

On the graph on the previous page, draw a best-fit line for the data.



For an appropriate best-fit line for the graph above		1 point

Question 3 (continued)

(e) continued

ii. LO CON-5.D.c, SP 4.D, 6.C
2 points

Using the straight line, calculate the rotational inertia of the unknown object I_U about a vertical axis passing through its center of mass.

For using conservation of angular momentum to derive an expression that includes I_U		1 point
$L_i = L_f \therefore I_P\omega_i = (I_P + I_U)\omega_f$		
$I_U = I_P\left(\dfrac{\omega_i}{\omega_f}\right) - I_P$		
For substituting points from the best-fit line into the expression above		1 point
$I_U = \left(3.1 \text{ kg}\bullet\text{m}^2\right)\left(\dfrac{(4.0 - 1.0) \text{ rad/s}}{(9.3 - 2.4) \text{ rad/s}}\right) - \left(3.1 \text{ kg}\bullet\text{m}^2\right) = 4.1 \text{ kg}\bullet\text{m}^2$		
<u>Note:</u> The point (0, 0) can be used implicitly if the best-fit line goes through the origin.		

(f) LO CON-5.D.c, SP 7.A, 7.C
2 points

The kinetic energy of the spinning platform before the object is dropped on it is K_i. The total kinetic energy of the platform-object system when it reaches angular speed ω_f is K_f. Which of the following expressions is true?

____ $K_f < K_i$ ____ $K_f = K_i$ ____ $K_f > K_i$

Justify your answer.

For selecting $K_f < K_i$ with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because the two disks will be rotating with the same final angular speed, this is an inelastic collision, and kinetic energy will be lost during the collision.		

2019 SCORING GUIDELINES

Question 3 (continued)

(g) LO INT-6.E, SP 7.A, 7.C
2 points

One of the students observes that the center of mass of the object is not actually aligned with the axis of the platform. Is the experimental value of I_U obtained in part (e) greater than, less than, or equal to the actual value of the rotational inertia of the unknown object about a vertical axis that passes through its center of mass?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

For selecting "Greater than" with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because the center of mass of the object is off the axis of the platform, the parallel axis theorem would be used to calculate the total rotational inertia of the platform-object system. Using $I = I_{CM} + Mh^2$, the experimental value will be increased by Mh^2.		

Learning Objectives

INT-6.E: Derive the moments of inertia of an extended rigid body for different rotational axes (parallel to an axis that goes through the object's center of mass) if the moment of inertia is known about an axis through the object's center of mass.
CHA-4.A.b: Calculate unknown quantities such as angular positions, displacement, angular speeds, or angular acceleration of a rigid body in uniformly accelerated motion, given initial conditions.
INT-7.A.b: Calculate unknown quantities such as net torque, angular acceleration, or moment of inertia for a rigid body undergoing rotational acceleration.
INT-7.C: Derive expressions for physical systems such as Atwood Machines, pulleys with rotational inertia, or strings connecting discs or strings connecting multiple pulleys that relate linear or translational motion characteristics to the angular motion characteristics of rigid bodies in the system that are: **(a)** rolling (or rotating on a fixed axis) without slipping. **(b)** rotating and sliding simultaneously.
INT-7.D.a: Calculate the rotational kinetic energy of a rotating rigid body.
CON-5.D.c: Calculate the changes of angular momentum of each disc in a rotating system of two rotating discs that collide with each other inelastically about a common rotational axis.

Science Practices

4.C: Linearize data and/or determine a best-fit line or curve.
4.D: Select relevant features of a graph to describe a physical situation or solve problems.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.