

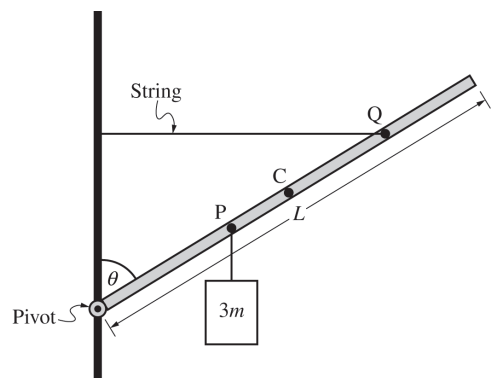
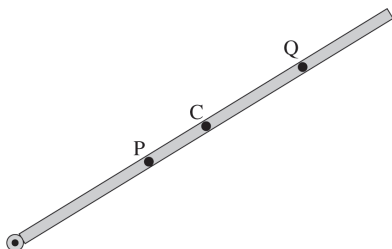
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Figure 1

Note: Figure not drawn to scale.

3. A uniform rod of length L and mass m is attached to a pivot on a vertical pole, as shown in Figure 1. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

- (a) On the following representation of the rod, **draw** and **label** the forces (not components) that are exerted on the rod. Each force must be represented by a distinct arrow that starts on and points away from the point at which the force is exerted on the rod.



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- (b) In Figure 1, Point P is located $\frac{3}{8}L$ from the pivot and Point Q is located $\frac{6}{8}L$ from the pivot. **Derive** an equation for the tension F_T in the horizontal string in terms of L , m , θ , and physical constants, as appropriate.

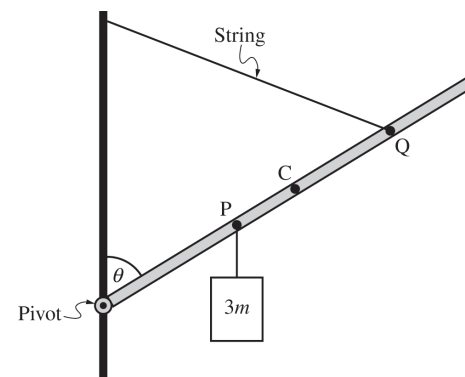


Figure 2

Note: Figure not drawn to scale.

- (c) The original string is replaced with a longer string that connects Point Q to a higher location on the vertical pole, as shown in Figure 2. The angle θ remains the same. How does the new tension $F_{T, \text{new}}$ compare with the original tension F_T from part (b)? **Justify** your reasoning.

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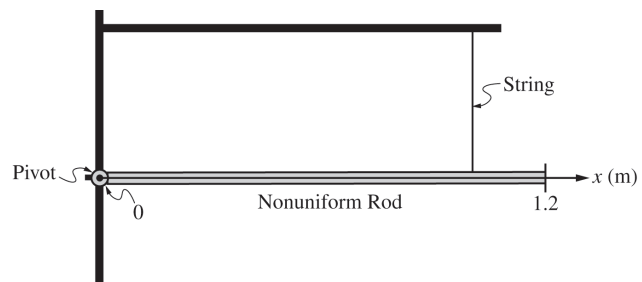


Figure 3

Note: Figure not drawn to scale.

- (d) A nonuniform rod is now attached to the pivot, as shown in Figure 3. There is negligible friction between the nonuniform rod and the pivot. The rod has a length of 1.2 m and a linear mass density $\lambda(x) = A + Bx$, where x is the distance from the pivot, $A = 6.0 \text{ kg/m}$, and $B = 10.0 \text{ kg/m}^2$.

i. **Calculate** the mass of the rod.

ii. **Calculate** the rotational inertia of the rod about the pivot.

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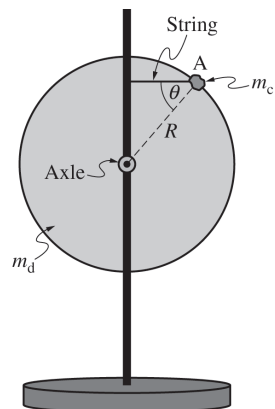
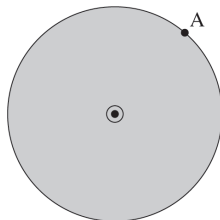
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Figure 1

Note: Figure not drawn to scale.

3. A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

- (a) On the following representation of the clay-disk system, **draw** and **label** the external forces (not components) exerted on the system. Each force must be represented by a distinct arrow that starts on, and points away from, the point at which the force is exerted on the system.

**GO ON TO THE NEXT PAGE.**Continue your response to **QUESTION 3** on this page.

- (b) **Derive** an expression for the tension F_T in the string when the clay is at Point A, as shown in Figure 1, in terms of R , m_d , m_c , θ , and physical constants, as appropriate.

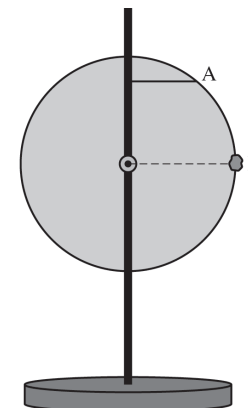


Figure 2

Note: Figure not drawn to scale.

- (c) The string remains connected to the edge of the disk at Point A. The clay is moved to Point B, which is horizontally in line with the axle, as shown in Figure 2. How does the new tension $F_{T, \text{new}}$ compare with tension F_T from part (b)? **Justify** your reasoning.

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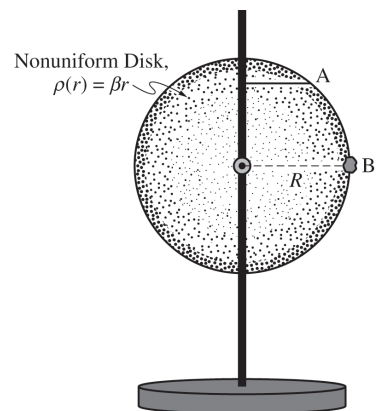


Figure 3

Note: Figure not drawn to scale.

- (d) A nonuniform disk is now attached to the axle. The lump of clay is attached to the disk at Point B, as shown in Figure 3. The clay has mass $m_c = 0.60 \text{ kg}$ and the disk has a radius $R = 0.30 \text{ m}$. The mass density of the disk varies radially and can be modeled by $\rho(r) = \beta r$, where r is the radial distance from the axle and $\beta = 4.0 \text{ kg/m}^3$.
- Calculate** the rotational inertia of the disk about the axle.

- The string connecting the disk to the pole is cut. **Calculate** the magnitude of the initial angular acceleration of the clay-disk system.

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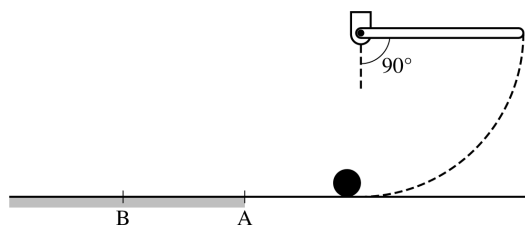
Note: Figure not drawn to scale.

Figure 1

3. A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

- (a) Derive an expression for the angular speed of the rod just before striking the sphere in terms of the length ℓ and physical constants as appropriate.

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- (b) Derive an expression for the linear speed v_0 of the sphere immediately after colliding with the rod in terms of the length ℓ and physical constants as appropriate.

After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ , beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

- (c) In the following diagram, which represents the sphere while the sphere is traveling between Points A and B, draw and label the forces (not components) that act on the sphere. Each force must be represented by a distinct arrow starting on, and pointing away from, the point of application on the sphere.



- (d) Derive an expression for each of the following as the sphere is rotating and sliding between points A and B in terms of v_0 , μ , R , t , and physical constants as appropriate.

- i. The linear velocity v of the center of mass of the sphere as a function of time t

- ii. The angular velocity ω of the sphere as a function of time t

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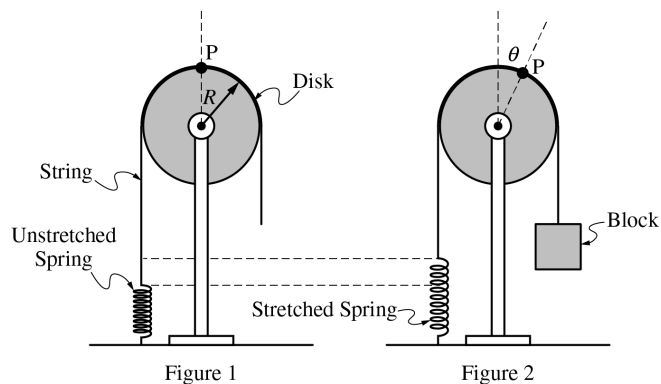
- (e)
- i. Derive an expression for the time it takes the sphere to travel from Point A to Point B in terms of v_0 , μ , and physical constants as appropriate.
- ii. Derive an expression for the linear velocity of the sphere upon reaching Point B in terms of v_0 .

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Note: Figures not drawn to scale.

A solid uniform disk is supported by a vertical stand. The disk is able to rotate with negligible friction about an axle that passes through the center of the disk. The mass and radius of the disk are given by M_d and R , respectively. The rotational inertia of the disk is $I_d = \frac{1}{2} M_d R^2$. A string of negligible mass is draped over the disk and attached to the top of the disk at point P. One end of the string is connected to an unstretched ideal spring of spring constant k , which is fixed to the ground as shown in Figure 1.

A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ .

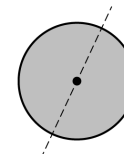
Give all algebraic answers in terms of M_d , R , k , θ , and physical constants, as appropriate.

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(a) Derive an expression for the mass m_B of the block.

(b) At time $t = 0$, the string on the right side of the disk is cut and the block falls to the ground. On the circle below, which represents the disk, draw and label the forces (not components) that act on the disk immediately after the string is cut and the block is falling to the ground. Each force should be represented by an arrow that starts on and is directed away from the point of application.

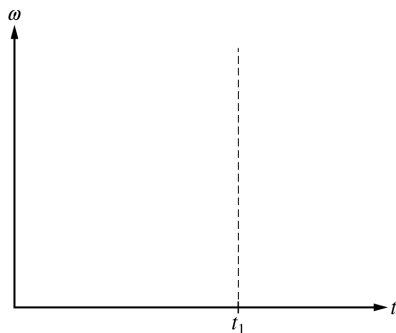


(c) Derive an expression for the angular acceleration α of the disk immediately after the string is cut.

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- (d) At $t = t_1$, the disk has rotated and point P is again directly above the axle. Sketch a graph of the magnitude of the angular velocity ω of the disk as a function of time t from $t = 0$ to $t = t_1$.



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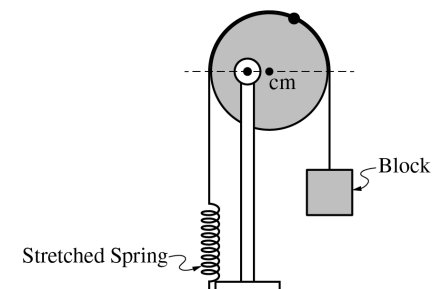
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Figure 3

Note: Figure not drawn to scale.

- (e) The disk is adjusted on the support so that the axle does not pass through the center of mass of the disk. The block is again hung on the right side of the disk and the spring-disk-block system comes to equilibrium, as shown in Figure 3. The axle does not exert a torque on the disk. For each force on the disk, indicate whether the magnitude of the torque about the axle caused by that force increases, decreases, or stays the same relative to part (b).

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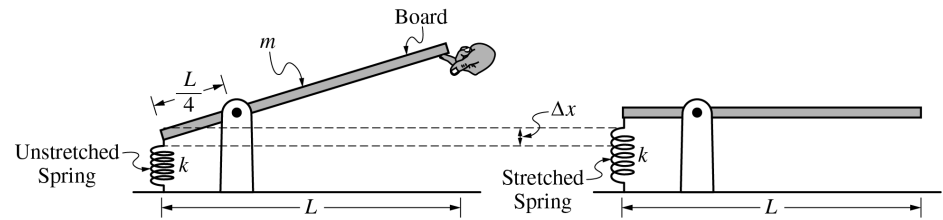


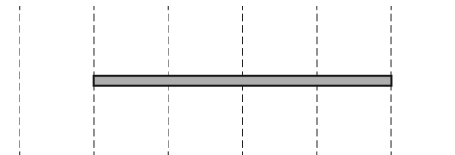
Figure 1

Figure 2

Note: Figures not drawn to scale.

A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The right end of the board is initially held by a student so that the spring is unstretched, as shown in Figure 1. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring stretched, as shown in Figure 2. The rotational inertia of the board about the pivot is I .

- (a) On the rectangle below, which represents the board, draw and label the forces (not components) that act on the board while the board-spring system is in equilibrium. Each force should be represented by an arrow that starts on, and points away from, the board, and should represent the location at which that force acts.

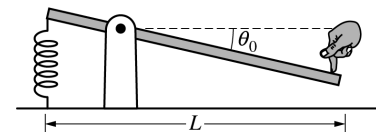


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- (b) Derive an expression for the distance the spring stretches, Δx , when the board is in equilibrium. Express your answer in terms of k , L , m , and physical constants, as appropriate.

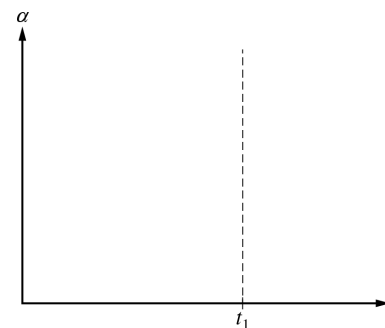
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Note: Figure not drawn to scale.

- (c) A student pushes the board down on the right side, stretching the spring a new distance Δx_2 from the unstretched position. The board is held at a small angle θ_0 with the horizontal, as shown. The student then releases the board from rest.

- i. At time $t = 0$, the board is released. At $t = t_1$, the board first crosses the horizontal. Sketch a graph of the magnitude of the angular acceleration α of the board as a function of time t from $t = 0$ to $t = t_1$.



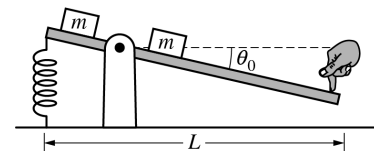
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- ii. Derive an expression for the angular acceleration α_0 of the board immediately after the board is released. Express your answer in terms of k , L , m , I , Δx_2 , θ_0 , and physical constants, as appropriate.

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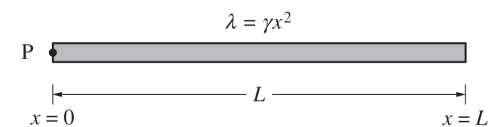
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- (d) Two blocks of equal mass m are attached to the board equal distances from the pivot point, as shown. The board is again pushed down on the right side so that the spring stretches the same distance Δx_2 as in part (c). The board is then released. How does the new angular acceleration α' when the blocks are attached compare to the angular acceleration α_0 from part (c) ?

___ $\alpha' > \alpha_0$ ___ $\alpha' < \alpha_0$ ___ $\alpha' = \alpha_0$

Justify your answer.

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3. A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \frac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(a) Using integral calculus, show that the rotational inertia I of the rod about an axis perpendicular to the page and through point P is $\frac{3}{5}ML^2$.

(b) Determine the horizontal location of the center of mass of the rod relative to point P . Express your answer in terms of L .

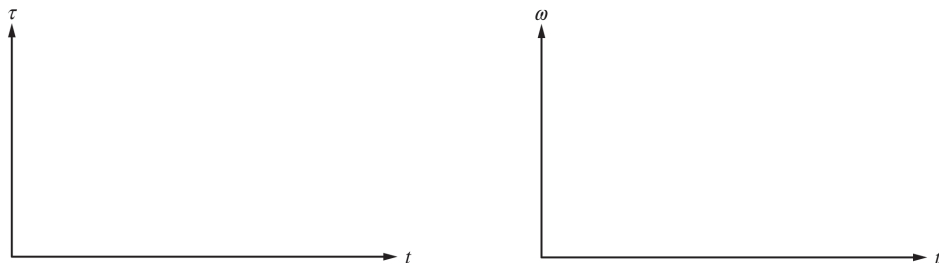
(c) For an axis perpendicular to the page, is the value of the rotational inertia of the rod around point P greater than, less than, or equal to the value of the rotational inertia of the rod around the rod's center of mass?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

The rod is released from rest in the position shown, and the rod begins to rotate about a horizontal axis perpendicular to the page and through point P.

(d) On the axes below, sketch graphs of the magnitude of the net torque τ on the rod and the angular speed ω of the rod as functions of time t from the time the rod is released until the time its center of mass reaches its lowest point.



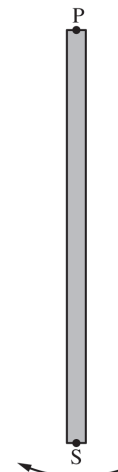
(e) As the rod rotates from the horizontal position down through vertical, is the magnitude of the angular acceleration on the rod increasing, decreasing, or not changing?

_____ Increasing _____ Decreasing _____ Not changing

Justify your answer.

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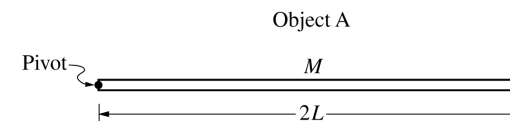
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(f) The mass of the rod is 3.0 kg, and the length of the rod is 1.0 m. Calculate the linear speed v of point S as the rod swings through the vertical position shown.

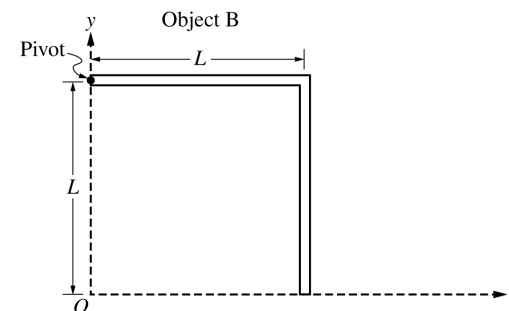
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2. Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(a) Using integral calculus, derive an expression to show that the rotational inertia I_A of object A about the pivot is given by $\frac{4}{3}ML^2$.



Object B of total mass M is formed by attaching two thin, uniform, identical rods of length L at a right angle to each other. Object B is held in place, as shown above. Express your answers in part (b) in terms of L .

(b) Determine the following for the given coordinate system shown in the figure.

- The x -coordinate of the center of mass of object B
- The y -coordinate of the center of mass of object B

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Object B has a rotational inertia of I_B about its pivot.

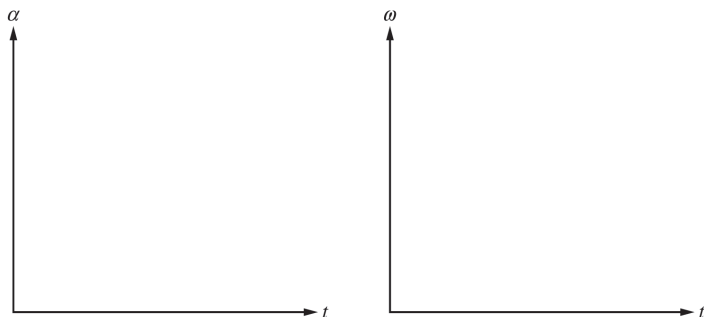
(c) Is the value of I_B greater than, less than, or equal to I_A ?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

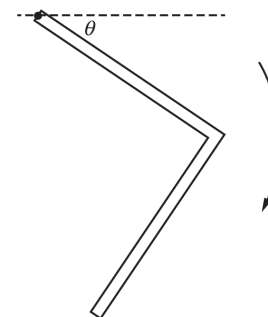
Object B is released from rest and begins to rotate about its pivot.

(d) On the axes below, sketch graphs of the magnitude of the angular acceleration α and the angular speed ω of object B as functions of time t from the time it is released to the time its center of mass reaches its lowest point.



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(e) While object B rotates from the horizontal position down through the angle θ shown above, is the magnitude of its angular acceleration increasing, decreasing, or not changing?

_____ Increasing _____ Decreasing _____ Not changing

Justify your answer.

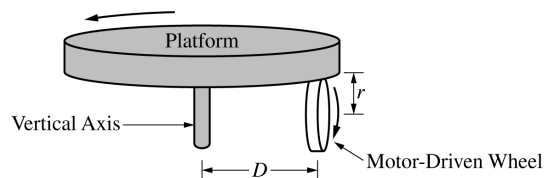


Object B rotates through the position shown above.

(f) Derive an expression for the angular speed of object B when it is in the position shown above. Express your answer in terms of M , L , I_B , and physical constants, as appropriate.

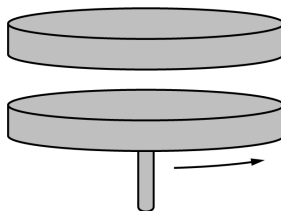
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3. A horizontal circular platform with rotational inertia I_p rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the wheel until the platform reaches an angular speed ω_p after time Δt . During time Δt , the wheel stays in contact with the platform without slipping.

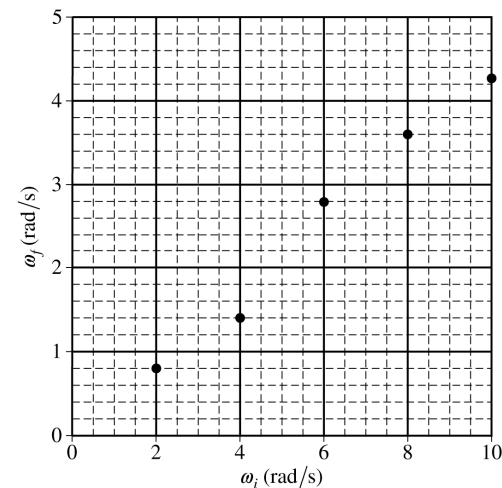
- Derive an expression for the angular speed ω_p of the platform. Express your answer in terms of I_p , r , D , F , Δt , and physical constants, as appropriate.
- Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed ω_p . Express your answer in terms of I_p , r , D , F , Δt , and physical constants, as appropriate.
- Derive an expression for the angular speed of the wheel ω_w when the platform has reached angular speed ω_p . Express your answer in terms of D , r , ω_p , and physical constants, as appropriate.



When the platform is spinning at angular speed ω_p , the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia I_p as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed ω_f .

- Derive an expression for ω_f . Express your answer in terms of ω_p , I_p , and physical constants, as appropriate.

A student now uses the rotating platform ($I_p = 3.1 \text{ kg}\cdot\text{m}^2$) to determine the rotational inertia I_U of an unknown object about a vertical axis that passes through the object's center of mass. The platform is rotating at an initial angular speed ω_i when the unknown object is dropped with its center of mass directly above the center of the platform. The platform and object are observed to be rotating together at angular speed ω_f . Trials are repeated for different values of ω_i . A graph of ω_f as a function of ω_i is shown on the axes below.



(e)

- i. On the graph on the previous page, draw a best-fit line for the data.
 - ii. Using the straight line, calculate the rotational inertia of the unknown object I_U about a vertical axis passing through its center of mass.
- (f) The kinetic energy of the spinning platform before the object is dropped on it is K_i . The total kinetic energy of the platform-object system when it reaches angular speed ω_f is K_f . Which of the following expressions is true?

_____ $K_f < K_i$ _____ $K_f = K_i$ _____ $K_f > K_i$

Justify your answer.

- (g) One of the students observes that the center of mass of the object is not actually aligned with the axis of the platform. Is the experimental value of I_U obtained in part (e) greater than, less than, or equal to the actual value of the rotational inertia of the unknown object about a vertical axis that passes through its center of mass?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

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