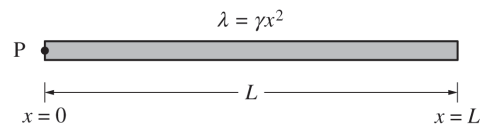


Begin your response to **QUESTION 3** on this page.

3. A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \frac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(a) Using integral calculus, show that the rotational inertia I of the rod about an axis perpendicular to the page and through point P is $\frac{3}{5}ML^2$.

(b) Determine the horizontal location of the center of mass of the rod relative to point P. Express your answer in terms of L .

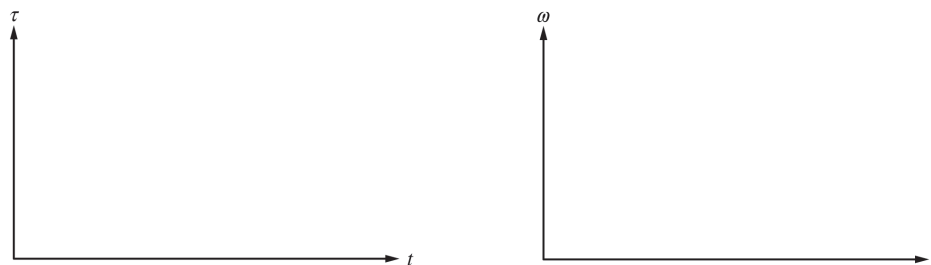
(c) For an axis perpendicular to the page, is the value of the rotational inertia of the rod around point P greater than, less than, or equal to the value of the rotational inertia of the rod around the rod's center of mass?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

The rod is released from rest in the position shown, and the rod begins to rotate about a horizontal axis perpendicular to the page and through point P.

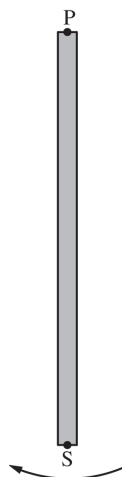
(d) On the axes below, sketch graphs of the magnitude of the net torque τ on the rod and the angular speed ω of the rod as functions of time t from the time the rod is released until the time its center of mass reaches its lowest point.



(e) As the rod rotates from the horizontal position down through vertical, is the magnitude of the angular acceleration on the rod increasing, decreasing, or not changing?

_____ Increasing _____ Decreasing _____ Not changing

Justify your answer.

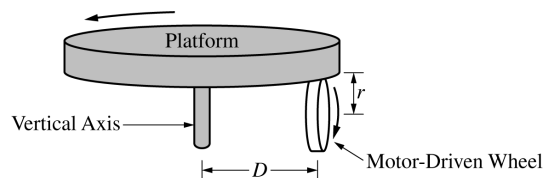


(f) The mass of the rod is 3.0 kg, and the length of the rod is 1.0 m. Calculate the linear speed v of point S as the rod swings through the vertical position shown.

GO ON TO THE NEXT PAGE.

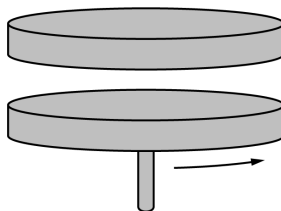
Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.

STOP
END OF EXAM



3. A horizontal circular platform with rotational inertia I_p rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the wheel until the platform reaches an angular speed ω_p after time Δt . During time Δt , the wheel stays in contact with the platform without slipping.

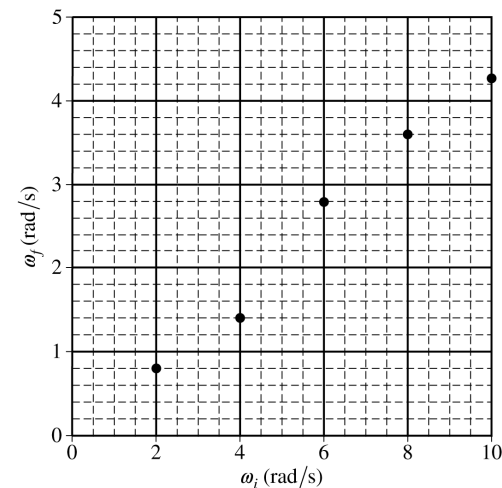
- Derive an expression for the angular speed ω_p of the platform. Express your answer in terms of I_p , r , D , F , Δt , and physical constants, as appropriate.
- Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed ω_p . Express your answer in terms of I_p , r , D , F , Δt , and physical constants, as appropriate.
- Derive an expression for the angular speed of the wheel ω_w when the platform has reached angular speed ω_p . Express your answer in terms of D , r , ω_p , and physical constants, as appropriate.



When the platform is spinning at angular speed ω_p , the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia I_p as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed ω_f .

- Derive an expression for ω_f . Express your answer in terms of ω_p , I_p , and physical constants, as appropriate.

A student now uses the rotating platform ($I_p = 3.1 \text{ kg}\cdot\text{m}^2$) to determine the rotational inertia I_U of an unknown object about a vertical axis that passes through the object's center of mass. The platform is rotating at an initial angular speed ω_i when the unknown object is dropped with its center of mass directly above the center of the platform. The platform and object are observed to be rotating together at angular speed ω_f . Trials are repeated for different values of ω_i . A graph of ω_f as a function of ω_i is shown on the axes below.



(e)

- i. On the graph on the previous page, draw a best-fit line for the data.
- ii. Using the straight line, calculate the rotational inertia of the unknown object I_U about a vertical axis passing through its center of mass.

- (f) The kinetic energy of the spinning platform before the object is dropped on it is K_i . The total kinetic energy of the platform-object system when it reaches angular speed ω_f is K_f . Which of the following expressions is true?

_____ $K_f < K_i$ _____ $K_f = K_i$ _____ $K_f > K_i$

Justify your answer.

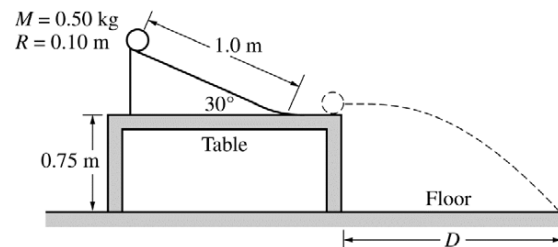
- (g) One of the students observes that the center of mass of the object is not actually aligned with the axis of the platform. Is the experimental value of I_U obtained in part (e) greater than, less than, or equal to the actual value of the rotational inertia of the unknown object about a vertical axis that passes through its center of mass?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

STOP

END OF EXAM



3. A uniform solid cylinder of mass $M = 0.50 \text{ kg}$ and radius $R = 0.10 \text{ m}$ is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m . The inclined plane makes an angle of 30° with the horizontal. The cylinder lands on the floor a distance D away from the edge of the table, as shown in the figure above. There is a smooth transition from the inclined plane to the horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

- (a) Calculate the total kinetic energy of the cylinder as it reaches the horizontal table.
- (b) Calculate the angular velocity of the cylinder around its axis at the moment it reaches the floor.
- (c) Calculate the ratio of the rotational kinetic energy to the total kinetic energy for the cylinder at the moment it reaches the floor.
- (d) Calculate the horizontal distance D .

A sphere of the same mass and radius is now rolled down the same inclined plane. The rotational inertia of a sphere around its center is $\frac{2}{5}MR^2$.

(e)

- i. Is the total kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the total kinetic energy of the cylinder at the moment it reaches the floor?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

- ii. Is the rotational kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the rotational kinetic energy of the cylinder at the moment it reaches the floor?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

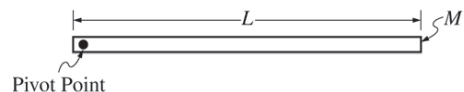
- iii. Is the horizontal distance the sphere travels from the table to where it hits the floor greater than, less than, or equal to the horizontal distance the cylinder travels from the table to where it hits the floor?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

STOP

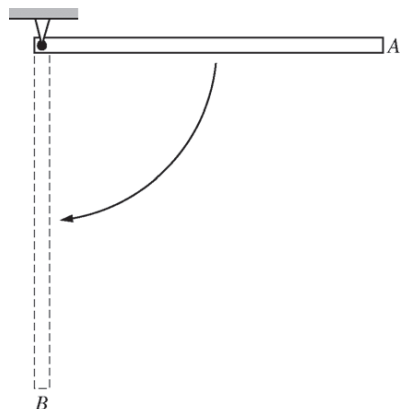
END OF EXAM



Mech.3.

A uniform, thin rod of length L and mass M is allowed to pivot about its end, as shown in the figure above.

- (a) Using integral calculus, derive the rotational inertia for the rod around its end to show that it is $ML^2/3$.



The rod is fixed at one end and allowed to fall from the horizontal position A through the vertical position B .

- (b) Derive an expression for the velocity of the free end of the rod at position B . Express your answer in terms of M , L , and physical constants, as appropriate.

An experiment is designed to test the validity of the expression found in part (b). A student uses rods of various lengths that all have a uniform mass distribution. The student releases each of the rods from the horizontal position A and uses photogates to measure the velocity of the free end at position B . The data are recorded below.

Length (m)	0.25	0.50	0.75	1.00	1.25	1.50
Velocity (m/s)	2.7	3.8	4.6	5.2	5.8	6.3

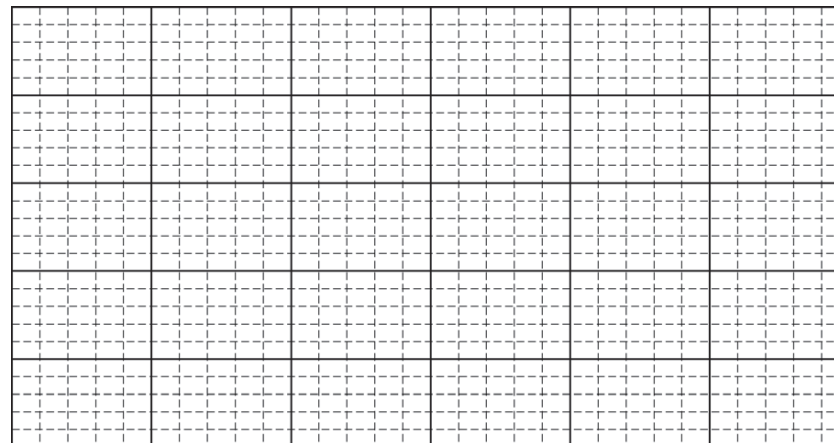
- (c) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the acceleration due to gravity g .

Horizontal axis: _____

Vertical axis: _____

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given. Label each row you use and include units.

- (d) Plot the straight line data points on the grid below. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



- (e)

- Using your straight line, determine an experimental value for g .
- Describe two ways in which the effects of air resistance could be reduced.

STOP

END OF EXAM