Question 3: Free-Response Question — 15 points

(a)

For indicating that the sum of the torques on the disk equals zero — **1 point**

$\Sigma\tau_{\text{on disk}} = 0$

$\tau_g = \tau_s$

OR

For indicating that the sum of the forces equals zero

$\Sigma F = 0$

$F_g = F_s$

For correctly substituting the expressions for the forces — **1 point**

$F_g R = F_s R$

$m_B g R = k\Delta x R$

$m_B g = k\Delta x$

OR

$F_g = F_s$

$m_B g = k\Delta x$

For correctly substituting for Δx — **1 point**

$m_B g = kR\theta$

$m_B = \dfrac{kR\theta}{g}$

Total for part (a) 3 points

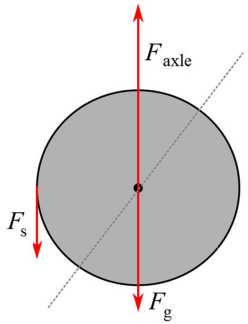
(b)

For drawing and labeling the force of the tension exerted on the disk anywhere between point P and the left edge of the disk, including point P and the left edge of the disk, tangent to the disk — **1 point**

For correct location and label of the force due to gravity exerted on the disk, directed straight down — **1 point**

For correct location and label of the force exerted on the disk by the axle, directed such that the disk remains in translational equilibrium (i.e., $\Sigma F = 0$) — **1 point**

Example Response



Scoring Notes:

- Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force, which can be used instead of F_{axle}. F_{spring}, F_s, T_{spring}, T, "spring force," or similar labels may be used for the tension force exerted by the spring.
- A response with extraneous forces or vectors can earn a maximum of two points.

Total for part (b) 3 points

(c)

For indicating that the net torque is due only to the force exerted on the disk by the tension in the rotational form of Newton's second law — **1 point**

$\tau_s = I_d\alpha$

For correctly expressing the torque on the disk by the tension in terms of the spring force, which is equal to the tension, and the lever (moment) arm — **1 point**

$F_s R = I_d\alpha$

For correctly substituting for F_s — **1 point**

$-k\Delta x R = I_d\alpha$

For correctly substituting I_d and Δx, or an expression for Δx consistent with part (a) — **1 point**

$-k(R\theta)R = \dfrac{1}{2}M_d R^2\alpha$

$\alpha = -\dfrac{2k\theta}{M_d}$

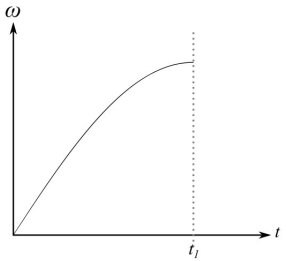
Scoring Note: The negative sign is not necessary to earn this point.

Total for part (c) 4 points

(d) For a sketch that starts at zero and monotonically increases until time $t = t_1$ — **1 point**

For a sketch that is concave down between time $t = 0$ and $t = t_1$ — **1 point**

Example Response



Scoring Note: Any part of the graph beyond t_1 is not considered in scoring.

Total for part (d) — **2 points**

(e) For indicating that the torque exerted by the force due to gravity on the disk increased — **1 point**

Example Response

The force due to gravity on the disk now has a non-zero lever arm and hence it exerts a larger torque on the disk.

For indicating that the torque exerted by the tension caused by the force due to gravity on the block increased — **1 point**

Example Response

The force exerted by the right side of the string (from the block) on the disk has a longer lever arm, hence the torque it exerts is larger.

For indicating that the torque exerted by the tension increased — **1 point**

Example Response

The counterclockwise torque due to the tension caused by the spring must increase to counteract the increase in clockwise torques due to the force due to gravity of the disk and tension caused by the force of gravity due to block to keep the disk in equilibrium.

Scoring Notes:

- A response that references the torque due to the force at the axle staying the same can earn all 3 points.
- A response that references the torque due to the force on the axle changing, or any additional torques can earn a maximum of 2 points.

Total for part (e) — **3 points**

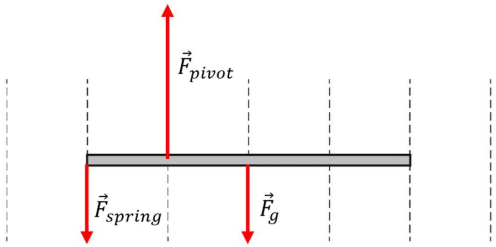
Question 3: Free-Response Question — 15 points

(a) For correct location and label of the force exerted on the board by the spring — **1 point**

For correct location and label of the force exerted on the board by gravity — **1 point**

For correct location and label of the force exerted on the board by the pivot — **1 point**

Example Response



Scoring Notes:

- Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{E,Block}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force, which can be used instead of F_{pivot}. F_{spring}, F_S, "spring force," or similar labels may be used for the force exerted by the spring.
- A response with extraneous forces or vectors can earn a maximum of two points.

Total for part (a) — **3 points**

(b) For indicating that the sum of the torques equals zero — **1 point**

$\Sigma\tau = 0$

For substituting mg for the force of gravity and $k\Delta x$ for the spring force into a torque equation — **1 point**

$$\frac{L}{4}k\Delta x - \frac{L}{4}mg = 0$$

For a correct expression for Δx — **1 point**

$$\Delta x = \frac{mg}{k}$$

Example Response

$\Sigma\tau = 0$

$F_{\text{spring}}d_{\text{spring}} - F_{\text{weight}}d_{\text{weight}} = 0$

$\frac{L}{4}k\Delta x - \frac{L}{4}mg = 0$

$\Delta x = \frac{mg}{k}$

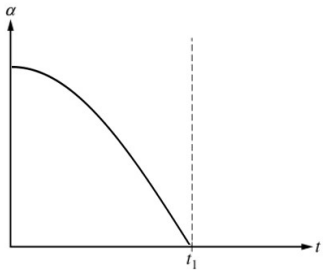
Scoring Note: This point can be earned regardless of whether a negative sign is present.

Total for part (b) 3 points

(c)(i) For a sketch that begins at a maximum value and monotonically decreases to a minimum of zero at $t = t_1$ **1 point**

For a sketch that is concave down between $t = 0$ and $t = t_1$ **1 point**

Example Response



Scoring Note: Any part of the graph beyond t_1 is not considered in scoring.

(c)(ii) For indicating that the torques are in opposite directions in the rotational form of Newton's second law **1 point**

$\tau_{\text{spring}} - \tau_g = I\alpha$

For including $\sin(90 - \theta_0)$, $\sin(90 + \theta_0)$, or $\cos\theta_0$ in an expression for the net torque exerted on the rod **1 point**

$F_{spring}d_{spring}(\cos\theta) - F_g d_g(\cos\theta) = I_B\alpha_0$

For substituting mg for the force due to gravity and substituting $k\Delta x_2$ for the spring force in an expression for the net torque exerted on the rod **1 point**

$d_{spring}k\Delta x_2(\cos\theta_0) - d_g mg(\cos\theta_0) = I_B\alpha_0$

For substituting correct lever arms in an expression for the net torque exerted on the rod **1 point**

$\frac{L}{4}(\cos\theta_0)k\Delta x_2 - \frac{L}{4}(\cos\theta_0)mg = I_B\alpha_0$

Example Response

$\tau_{\text{spring}} - \tau_g = I\alpha$

$F_{spring}d_{spring}(\cos\theta_0) - F_g d_g(\cos\theta_0) = I_B\alpha_0$

$l_{spring}(\cos\theta_0)k\Delta x_2 - l_g(\cos\theta_0)mg = I_B\alpha_0$

$\frac{L}{4}(\cos\theta_0)k\Delta x_2 - \frac{L}{4}(\cos\theta_0)mg = I_B\alpha_0$

$\alpha_0 = \frac{L}{4I_B}(k\Delta x_2\cos\theta_0 - mg\cos\theta_0)$

Total for part (c) 6 points

(d) For selecting "$\alpha' < \alpha_0$" with an attempt at a relevant justification **1 point**

For indicating that the net torque does not change **1 point**

For indicating the rotational inertia increases **1 point**

Example Response

The net torque on the system is the same (the spring and the person exert the same force and the additional torques from the gravitational forces on the masses cancel) but the rotational inertia of the system is now larger. This means that the angular acceleration of the system is now smaller than it was before.

Total for part (d) 3 points

Total for question 3 15 points

Question 2: Free-Response Question — 15 points

(a) For integrating using the correct limits or constant of integration — **1 point**

$$I = \int_{r=0}^{r=2L} \lambda r^2 dr = \lambda\left[\frac{r^3}{3}\right]_{r=0}^{r=2L} = \frac{\lambda}{3}\left((2L)^3 - 0\right)$$

For correctly relating λ to M and L — **1 point**

$$\lambda = \frac{m}{\ell} = \frac{M}{2L} \therefore I = \left(\frac{1}{3}\right)\left(\frac{M}{2L}\right)\left(8L^3\right) = \frac{4}{3}ML^2$$

Total for part (a) 2 points

(b) i. For correctly substituting into an equation for the center of mass of an object in the horizontal direction — **1 point**

$$X_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\left[\left(\frac{M}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{M}{2}\right)(L)\right]}{\left(\frac{M}{2} + \frac{M}{2}\right)} = \frac{\left(\frac{ML}{4} + \frac{ML}{2}\right)}{M} = \frac{3}{4}L$$

ii. For correctly substituting into an equation for the center of mass of an object in the vertical direction — **1 point**

$$Y_{CM} = \frac{\sum m_i y_i}{\sum m_i} = \frac{\left[\left(\frac{M}{2}\right)(L) + \left(\frac{M}{2}\right)\left(\frac{L}{2}\right)\right]}{\left(\frac{M}{2} + \frac{M}{2}\right)} = \frac{\left(\frac{ML}{2} + \frac{ML}{4}\right)}{M} = \frac{3}{4}L$$

Total for part (b) 2 points

(c) For selecting "Less than" and attempting a relevant justification — **1 point**

For a correct justification — **1 point**

Example response for part (c)

Because object B has more of its mass closer to the pivot than object A, the rotational inertia of object B must be less than that of object A.

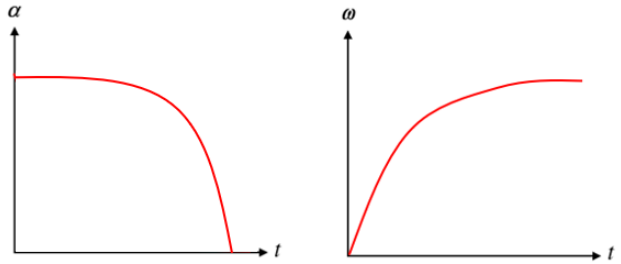
Total for part (c) 2 points

(d) For an acceleration graph that is concave down and begins horizontally — **1 point**

For an angular speed graph that is concave down and ends horizontally — **1 point**

For consistency between the angular acceleration and angular speed graphs — **1 point**

Example responses for part (d)



Total for part (d) 3 points

(e) For selecting "Decreasing" and attempting a relevant justification — **1 point**

For a justification that indicates the lever arm for the torque is decreasing — **1 point**

Example response for part (e)

Because the horizontal position of the center of mass for the object is moving closer to the pivot, the lever arm for the force of gravity is decreasing so the angular acceleration decreases.

Total for part (e) 2 points

(f) For using conservation of energy — **1 point**

$$U_{g1} = K_2$$

For correctly relating the change in rotational kinetic energy to the change in gravitational potential energy — **1 point**

$$mgh = \frac{1}{2}I\omega^2$$

For correctly substituting for h into the equation above — **1 point**

$$mg\left(\frac{L}{2}\right) = \frac{1}{2}I\omega^2$$

For an expression for ω that uses only the allowed symbols and is algebraically consistent with the previous steps — **1 point**

$$\omega = \sqrt{\frac{2mgh}{I}} = \sqrt{\frac{2Mg(L/2)}{I_B}}$$

$$\omega = \sqrt{\frac{MgL}{I_B}}$$

Total for part (f) 4 points

Total for question 2 15 points