

Question 1: Version J

1. Two blocks, 1 and 2, slide toward each other on a horizontal surface. Block 1 has mass m and slides in the $+x$ -direction with constant speed $2v_0$. Block 2 has mass $6m$ and slides in the $-x$ -direction with constant speed v_0 , as shown in Figure 1. The blocks then collide and stick together. The collision occurs from time $t = 0$ to $t = t_c$. After the collision, where $t > t_c$, the blocks move together with the same constant speed.

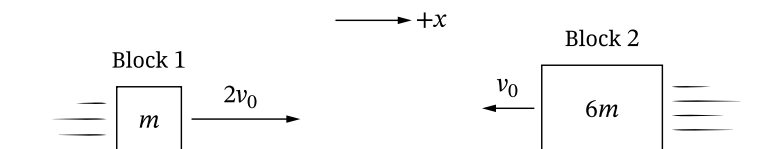


Figure 1

- A. The diagrams in Figure 2 can be used to represent the momentum of blocks 1 and 2 before and after the collision. The momentum vector diagram for Block 1 before the collision is shown.

- i. **Draw** arrows on the grids to represent the momentum vectors of Block 2 before the collision and the two-block system before and after the collision.
- Arrows should start at the zero-momentum line.
 - The length of the arrows should be proportional to the relative magnitudes of the vectors.
 - Represent an arrow of zero length by drawing a dot at zero.

	Momentum Before Collision	Momentum After Collision
Block 1		
Block 2		
Two-Block System		

Figure 2

- ii. During the time interval $0 \leq t \leq t_c$, a force F is exerted on Block 2 by Block 1 along the x -direction as a function of t that is modeled by $F(t) = F_{\max} \sin(At)$, where A is a positive constant and $F_{\max}$ is the magnitude of the maximum force exerted on Block 2 by Block 1 during the collision.

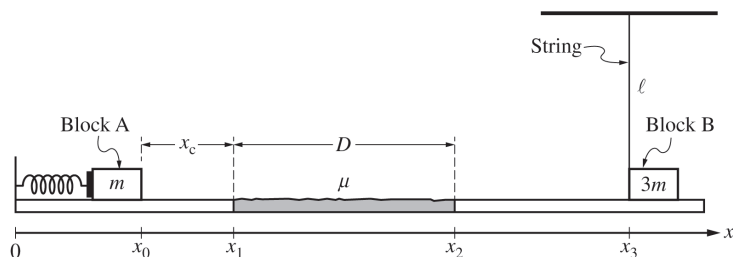
Derive an expression for $F_{\max}$. Express your answer in terms of m , v_0 , A , t_c , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

- B. Consider a new scenario where Block 1 initially slides in the $+x$ -direction with a new constant speed v_1 and Block 2 again initially slides in the $-x$ -direction with constant speed v_0 . The blocks collide and stick together. In this new scenario, the two-block system has constant speed v_0 after the collision.

Derive an expression for v_1 in terms of v_0 . Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Begin your response to **QUESTION 1** on this page.**PHYSICS C: MECHANICS****SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

Figure 1

1. Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D , where the coefficient of kinetic friction between Block A and the surface is μ . Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c .

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At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c . Block A moves with speed v , and the spring is at its equilibrium position.
- At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction.
- At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m , k , D , μ , x_c , and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1 .

ii. **Derive** an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3 .

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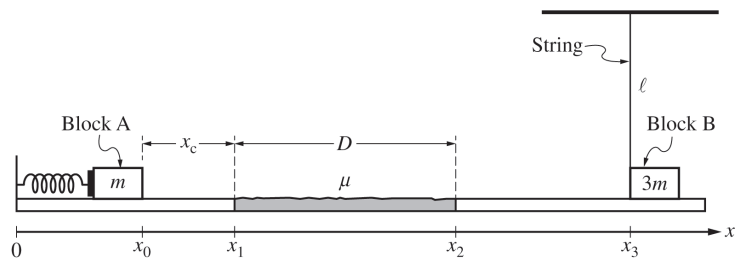
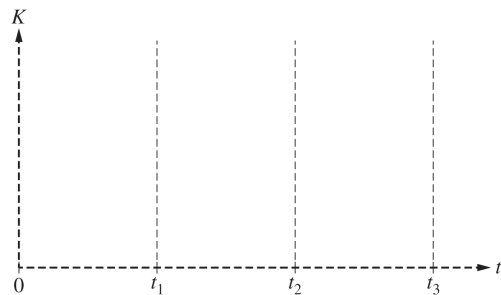
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Figure 1

(b)

- i. On the following axes, **sketch** a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to time t_3 .



- ii. Use principles of work and energy to **justify** the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

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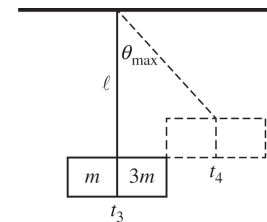
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Figure 2

After the collision, the two-block system instantaneously comes to rest at time t_4 , which occurs when the string makes a small angle $\theta_{\max}$ with the vertical, as shown in Figure 2. For times $t > t_4$, the system oscillates with frequency f_ℓ . The support holding the string is raised, and the procedure is then repeated using a new string of length 2ℓ .

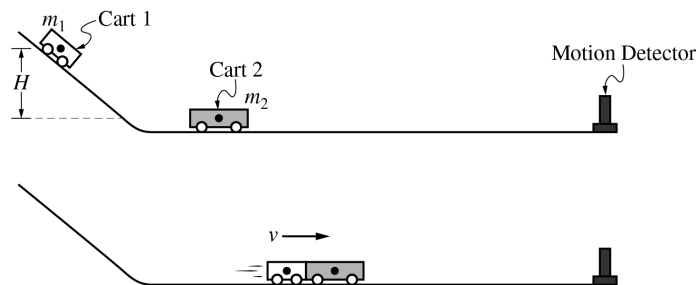
- (c) **Indicate** how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length 2ℓ will compare to the frequency of oscillation f_ℓ from the original procedure.

_____ $f_{2\ell} > f_\ell$ _____ $f_{2\ell} < f_\ell$ _____ $f_{2\ell} = f_\ell$

Briefly **justify** your answer.

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Begin your response to QUESTION 2 on this page.



Cart 1 of mass m_1 is held at rest above the bottom of an incline. Cart 2 has mass m_2 , where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and smoothly transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance of H , as shown. At time t_C , Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v . Frictional and rotational effects are negligible.

(a) During the collision, is the impulse on Cart 1 from Cart 2 greater than, less than, or equal to the magnitude of the impulse on Cart 2 from Cart 1?

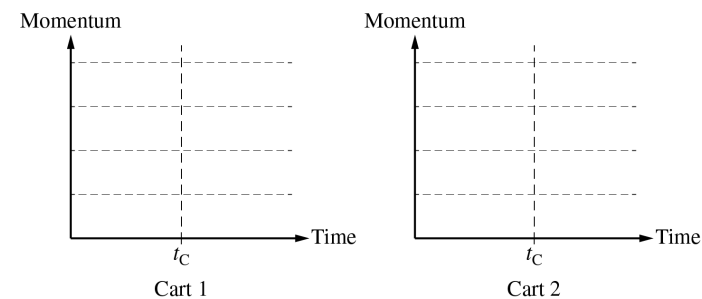
___ Greater than ___ Less than ___ Equal to

Justify your answer.

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Continue your response to **QUESTION 2** on this page.

(b) On the following axes, draw graphs of the magnitude of the momentum of each cart as a function of time t , before and after t_C . The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.



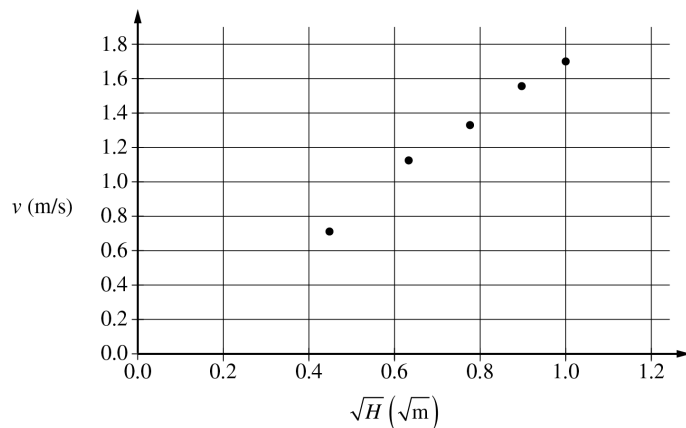
(c) Show that the velocity v of the two-cart system after the collision is given by the equation

$$v = \sqrt{2g \left(\frac{m_1}{m_1 + m_2} \right) \sqrt{H}}.$$

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Continue your response to **QUESTION 2** on this page.

- (d) A group of students use the setup to perform an experiment. They measure the mass of Cart 1 to be $m_1 = 0.250$ kg. The mass of Cart 2 is unknown. The students perform several trials and in each trial, Cart 1 is released from a different height H and the final velocity of the two-cart system is measured. The students graph v as a function of $\sqrt{H}$, as shown below.



- Draw a line that represents the best fit to the data points shown.
- Use the best-fit line to calculate the mass of Cart 2.

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Continue your response to **QUESTION 2** on this page.

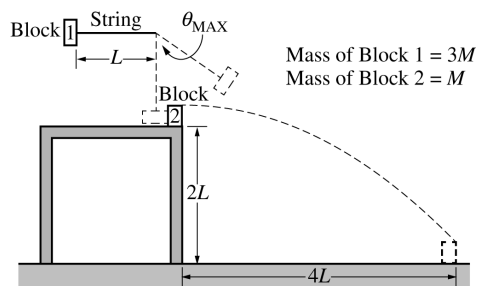
- (e) After the experiment, the students use a balance to measure the mass of Cart 2 and find it to be less than what was determined in part (d). To explain this discrepancy, one of the students proposes that the mass of Cart 1 was incorrectly measured at the beginning of the experiment. The students measure the mass of Cart 1 again and record a new value, m_1' .

Should the students expect that m_1' will be greater than 0.250 kg, less than 0.250 kg, or equal to 0.250 kg ?

☐ $m_1' > 0.250$ kg ☐ $m_1' < 0.250$ kg ☐ $m_1' = 0.250$ kg

Justify your answer.

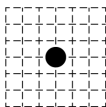
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Note: Figure not drawn to scale.

2. A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M , which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M , L , and physical constants, as appropriate.

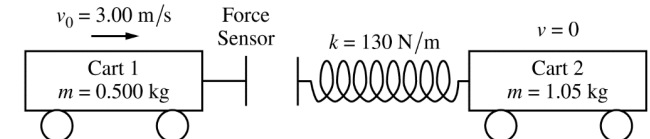
- (a) Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.
- (b) On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



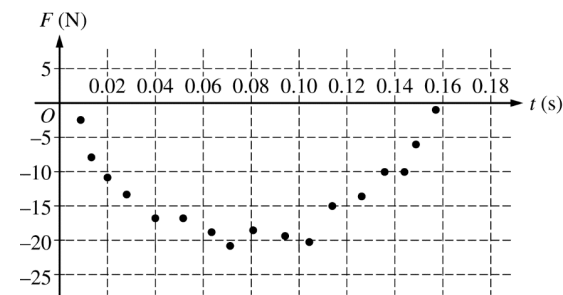
- (c) Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

- (d) Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.
- (e) Calculate the speed of block 2 as it leaves the table.
- (f) Calculate the speed of block 1 just after it collides with block 2.
- (g) Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.



2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00$ m/s toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130$ N/m. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = (3200 \text{ N/s}^2)t^2 - (500 \text{ N/s})t$.



- (a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.
- (b)
- Calculate the speed of cart 1 after the collision.
 - In which direction does cart 1 move after the collision?
☐ Left ☐ Right
☐ The direction is undefined, because the speed of cart 1 is zero after the collision.
- (c)
- Calculate the speed of cart 2 after the collision.
 - Show that the collision between the two carts is elastic.
- (d)
- Calculate the speed of the center of mass of the two-cart–spring system.
 - Calculate the maximum elastic potential energy stored in the spring.



Mech.2.

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

- (a) On the dots below, which represent the blocks of mass $2M$ and $3M$, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

Block of Mass $2M$ Block of Mass $3M$ 

The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance D , as shown above. Express your answers to parts (b), (c), and (d) in terms of M , B , v_0 , and physical constants, as appropriate.

- (b) Derive an expression for the speed of the blocks immediately after the collision.
 (c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.
 (d) Derive an expression for the maximum distance D that the spring is compressed.

(e)

- i. In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

___ Left ___ Right ___ The net force on the block of mass $2M$ is zero.

Justify your answer.

- ii. Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

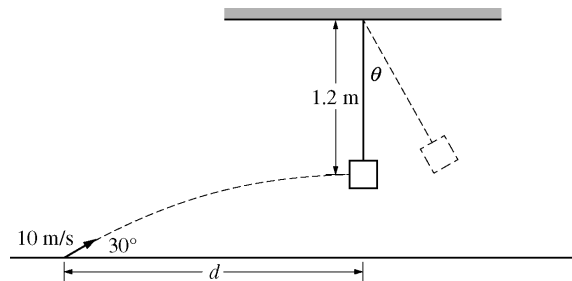
___ The magnitude of the net force is greater on the block of mass $2M$.
 ___ The magnitude of the net force is greater on the block of mass $3M$.
 ___ The magnitude of the net force on each block has the same nonzero value.
 ___ The magnitude of the net force on each block is zero.

Justify your answer.

- (f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

___ Yes ___ No

Justify your answer.



Mech.2.

A small dart of mass 0.020 kg is launched at an angle of 30° above the horizontal with an initial speed of 10 m/s. At the moment it reaches the highest point in its path and is moving horizontally, it collides with and sticks to a wooden block of mass 0.10 kg that is suspended at the end of a massless string. The center of mass of the block is 1.2 m below the pivot point of the string. The block and dart then swing up until the string makes an angle θ with the vertical, as shown above. Air resistance is negligible.

- Determine the speed of the dart just before it strikes the block.
- Calculate the horizontal distance d between the launching point of the dart and a point on the floor directly below the block.
- Calculate the speed of the block just after the dart strikes.
- Calculate the angle θ through which the dart and block on the string will rise before coming momentarily to rest.
- The block then continues to swing as a simple pendulum. Calculate the time between when the dart collides with the block and when the block first returns to its original position.
- In a second experiment, a dart with more mass is launched at the same speed and angle. The dart collides with and sticks to the same wooden block.
 - Would the angle θ that the dart and block swing to increase, decrease, or stay the same?
_____ Increase _____ Decrease _____ Stay the same
Justify your answer.
 - Would the period of oscillation after the collision increase, decrease, or stay the same?
_____ Increase _____ Decrease _____ Stay the same
Justify your answer.