Question 1: Mathematical Routines (MR) — 10 points

A	**(i)**	For drawing only one arrow for the momentum of Block 2 before the collision that points leftward and is 6 units long	**Point A1**
		For drawing only identical arrows for the momentum of the two-block system before and after the collision that are equal to the sum of the momentums drawn for blocks 1 and 2 before the collision	**Point A2**

Example Response

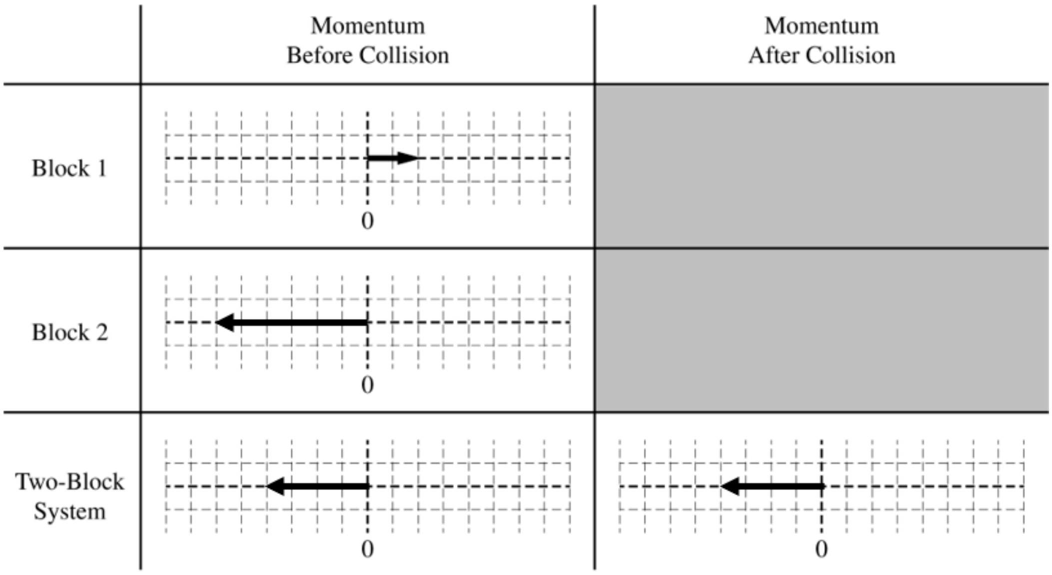


Figure 2

(ii)	For a multistep derivation that includes the integral form of impulse or the differential form of Newton's second law	**Point A3**
	For indicating **one** of the following: • The final speed of Block 1 or Block 2 is $\frac{4}{7}v_0$. • The change in the momentum of Block 2 is $+\frac{18}{7}mv_0$. • The change in the momentum of Block 1 is $-\frac{18}{7}mv_0$. • The change in the velocity of Block 2 is $+\frac{3}{7}v_0$. • The change in the velocity of Block 1 is $-\frac{18}{7}v_0$.	**Point A4**
	For substituting the given expression for $F(t)$ into an expression for the impulse exerted on either block or the differential form of Newton's second law	**Point A5**
	For an integral with appropriate limits or a constant of integration	**Point A6**
	For a correct expression for F_{max} in terms of given quantities	**Point A7**

Example Responses

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{\text{net}}(t)\, dt = \Delta\vec{p} \qquad \text{OR} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$$

$$\int \vec{F}_{\text{net}}\, dt = \Delta\vec{p}$$

$$\sum \vec{p}_0 = \sum \vec{p}_f$$

$$(m)(2v_0) + (6m)(-v_0) = (m + 6m)(v_f)$$

$$v_f = -\frac{4}{7}v_0$$

$$\Delta p_{\text{Block 2}} = -\Delta p_{\text{Block 1}} = 6m\left(-\frac{4}{7}v_0 - (-v_0)\right)$$

$$\Delta p_{\text{Block 2}} = \int_0^{t_c} F_{\max} \sin(At)\, dt$$

$$\frac{18}{7}mv_0 = \frac{F_{\max}}{A}\left[-\cos(At)\right]\Big|_0^{t_c}$$

$$\frac{18}{7}mv_0 = \frac{F_{\max}}{A}\left[1 - \cos(At_c)\right]$$

$$F_{\max} = \frac{18}{7}\left(\frac{Amv_0}{1 - \cos(At_c)}\right)$$

B	For a multistep derivation that includes conservation of momentum	**Point B1**
	For indicating that the magnitude of the final momentum of the two-block system is $7mv_0$	**Point B2**
	For a correct expression for v_1 in terms of v_0	**Point B3**

Example Response

$$\sum \vec{p}_0 = \sum \vec{p}_f$$

$$mv_1 + 6m(-v_0) = 7m(v_0)$$

$$mv_1 = 13mv_0$$

$$v_1 = 13v_0$$

Question 1: Free-Response Question — 15 points

(a)(i) For a multi-step derivation with an application of the conservation of mechanical energy that indicates that all of the energy of the system is initially U_s — **1 point**

Example Response

$$E_{\text{initial}} = E_{\text{final}}$$
$$\frac{1}{2}kx_{\text{c}}^2 = \frac{1}{2}mv^2$$

For a correct solution for v — **1 point**

Example Response

$$v = x_{\text{c}}\sqrt{\frac{k}{m}}$$

Example Solution

$$E_{\text{initial}} = E_{\text{final}}$$
$$U_s = K$$
$$\frac{1}{2}kx_{\text{c}}^2 = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{kx_{\text{c}}^2}{m}}$$
$$v = x_{\text{c}}\sqrt{\frac{k}{m}}$$

(a)(ii) For a derivation to solve for the speed at x_2 that includes **one** of the following: **1 point**

- An appropriate application of the conservation of energy
- An appropriate kinematics equation

Example Responses

$K_{\text{initial}} - \Delta E_{\text{friction}} = K_{\text{final}}$ **OR** $v^2 = v_0^2 + 2a\Delta x$

For **one** of the following that is consistent with the previous point in the response for part (a)(ii): **1 point**

- A correct expression for the energy dissipated by friction
- A correct expression for the acceleration of the block in the region with nonnegligible friction

Example Responses

$\Delta E_{\text{friction}} = \mu mgD$ **OR** $a = -\mu g$

For attempting to derive an expression for $v_{\text{A,B}}$ by using the conservation of momentum **1 point**

Example Response

$m_{\text{initial}}v_{\text{initial}} = m_{\text{A,B}}v_{\text{A,B}}$

For substituting the expression for the speed at x_2 that is consistent with the first point of the response in part (a)(ii) and substituting the correct masses into an expression for conservation of momentum **1 point**

Example Response

$$v_{\text{A,B}} = \frac{m\sqrt{\frac{kx_c^{\,2}}{m} - 2\mu gD}}{4m}$$

Example Solutions

$E_{x_1} = E_{\text{before collision}}$

$K_{x_1} - \Delta E_{\text{friction}} = K_{\text{before collision}}$

$$\frac{1}{2}m\left(\sqrt{\frac{kx_c^2}{m}}\right)^2 - \mu mgD = \frac{1}{2}mv_2^2$$

$$v_2 = \sqrt{\frac{kx_c^2}{m} - 2\mu gD}$$

$\Sigma p_{\text{before collision}} = \Sigma p_{\text{after collision}}$

$m_A v_2 = m_{A,B} v_{A,B}$

$m_A v_2 = (m + 3m) v_{A,B}$

$$v_{A,B} = \frac{m\sqrt{\frac{kx_c^2}{m} - 2\mu gD}}{4m}$$

$$v_{A,B} = \frac{1}{4}\sqrt{\frac{kx_c^2}{m} - 2\mu gD}$$

OR

$v_2^2 = v^2 + 2a\Delta x$

$$v_2^2 = \left(x_c\sqrt{\frac{k}{m}}\right)^2 + 2aD$$

$$v_2 = \sqrt{\frac{kx_c^2}{m} + 2aD}$$

$\Sigma F_x = -F_f = ma$

$-\mu mg = ma$

$a = -\mu g$

$$v_2 = \sqrt{\frac{kx_c^2}{m} - 2\mu gD}$$

$\Sigma p_{\text{before collision}} = \Sigma p_{\text{after collision}}$

$m_A v_2 = m_{A,B} v_{A,B}$

$m_A v_2 = (m + 3m) v_{A,B}$

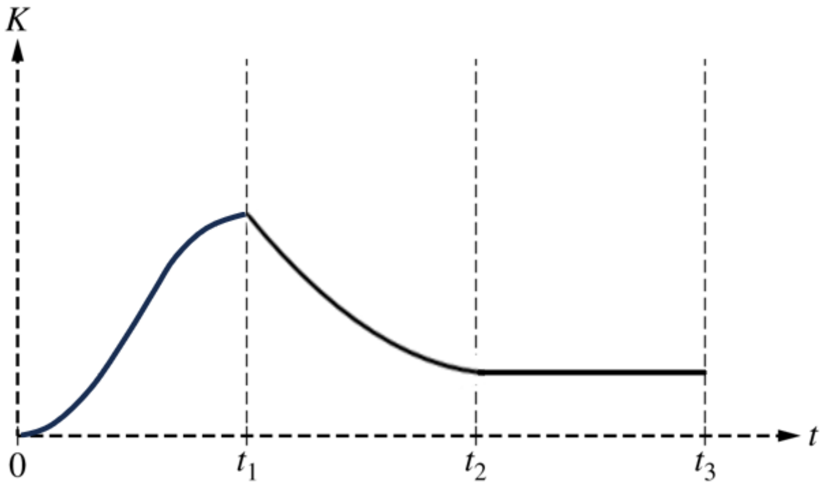
$$v_{A,B} = \frac{m\sqrt{\frac{kx_c^2}{m} - 2\mu gD}}{4m}$$

$$v_{A,B} = \frac{1}{4}\sqrt{\frac{kx_c^2}{m} - 2\mu gD}$$

Total for part (a) 6 points

(b)(i)	For a nonlinear sketch that begins at zero and increases for the entire time interval $0 \le t \le t_1$	**1 point**
	For a sketch that decreases for the entire time interval $t_1 \le t \le t_2$ but does not go to zero	**1 point**
	For a sketch that is concave up for the time interval $t_1 \le t \le t_2$	**1 point**
	For a continuous function for the time interval $t_1 \le t \le t_3$ that has a horizontal line that is greater than zero for the time interval $t_2 \le t \le t_3$	**1 point**

Example Response



(b)(ii)	For a statement about the change in kinetic energy that is consistent with the graph drawn in the response for part (b)(i)	**1 point**
	For a correct explanation for why the kinetic energy is increasing, such as **one** of the following: • An increasing graph means positive work is being done on the block. • An external force is exerted on Block A, causing the velocity of the block to increase and the kinetic energy of the block to increase. • Mechanical energy is conserved and/or there is no work done for the block-spring system, and the potential energy decreases.	**1 point**
	For a correct explanation for why the graph is nonlinear, such as **one** of the following: • The rate at which the slope of the graph changes is related to the rate at which work is being done on the block. • The external force exerted on Block A is changing, which causes a nonuniform change in the velocity of Block A, which results in a nonuniform change in kinetic energy.	**1 point**

Example Response

From $0 < t < t_1$, the kinetic energy of Block A increases. The force exerted on the block by the compressed spring transfers the elastic potential energy in the block-spring system to the kinetic energy of the block. Because the force exerted by the spring is not applied at a constant rate, the kinetic energy of the block does not increase at a constant rate.

Total for part (b) **7 points**

(c)	For selecting $f_{2\ell} < f_{\ell}$ with an attempt at a relevant justification	**1 point**
	For correctly applying an equation that relates the length of a pendulum to the period or frequency of the pendulum	**1 point**

Example Response

The period of a pendulum is calculated by using $T = 2\pi\sqrt{\frac{l}{g}}$ *. Therefore, as the length is increased, the period will also increase. Because frequency and period are inversely related, an increase in period will result in a decrease in frequency.*

Total for part (c) 2 points

Total for question 1 15 points

Question 2: Free-Response Question **15 points**

(a)	For selecting "Equal to" with an attempt at a relevant justification	**1 point**
	For a correct justification	**1 point**

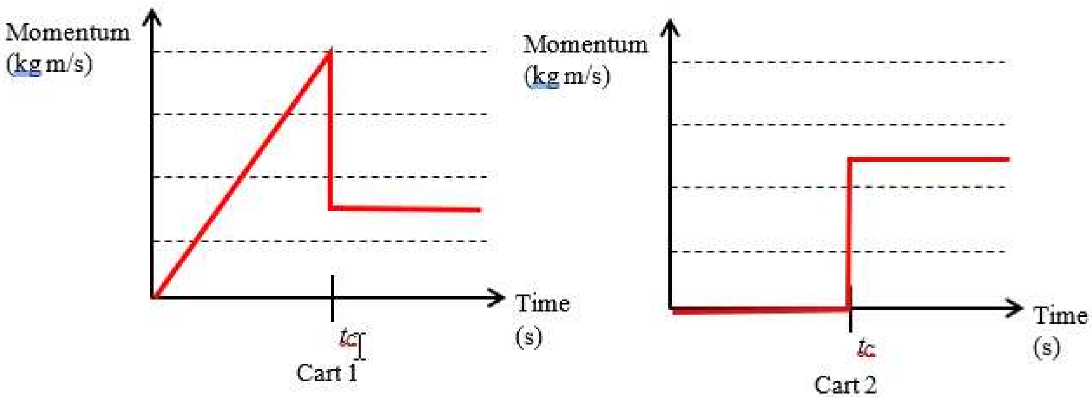
Example Response

Newton's third law says equal/opposite forces, time intervals are same during same collision, so magnitudes of impulse must be equal.

Total for part (a) 2 points

(b)	For correctly drawing the momentum for carts 1 and 2 for the time interval $0 < t < t_C$: Linear increasing momentum for Cart 1 and zero for Cart 2	**1 point**
	For drawing a horizontal line for Cart 1 when $t > t_C$ that is smaller in magnitude than the momentum of Cart 1 at time $t = t_C$	**1 point**
	For drawing a horizontal line for Cart 2 when $t > t_C$ that is greater in magnitude than the momentum of Cart 1 after time $t = t_C$	**1 point**
	For carts 1 and 2 having a change in momentum that is equal in magnitude, such that Cart 1 loses momentum and Cart 2 gains momentum or a response with changes in momentum consistent with the response in part (a)	**1 point**

Example Response



Total for part (b) 4 points

(c)	For using conservation of energy to find the speed of Cart 1 at the bottom of the incline **OR** For a correct substitution of acceleration and displacement in a kinematics equation to find the speed of Cart 1 at the bottom of the incline	**1 point**
	For using conservation of momentum to find the speed of the two-cart system after the collision	**1 point**
	For combining correct equations from above	**1 point**

Example Response

Conservation of energy:

$$m_1 gH = \frac{1}{2} m_1 v_1^2$$

$$v_1 = \sqrt{2gH}$$

OR

$$v_f^2 = v_i^2 + 2g\sin(\theta)L$$

$$H = L\sin(\theta)$$

$$v_f^2 = v_i^2 + 2g\sin(\theta)\left(\frac{H}{\sin\theta}\right)$$

$$v_f^2 = 2gH$$

Conservation of momentum:

$$m_1 v_1 = (m_1 + m_2)v_f$$

$$v_f = \frac{m_1}{m_1 + m_2} v_1$$

Combining:

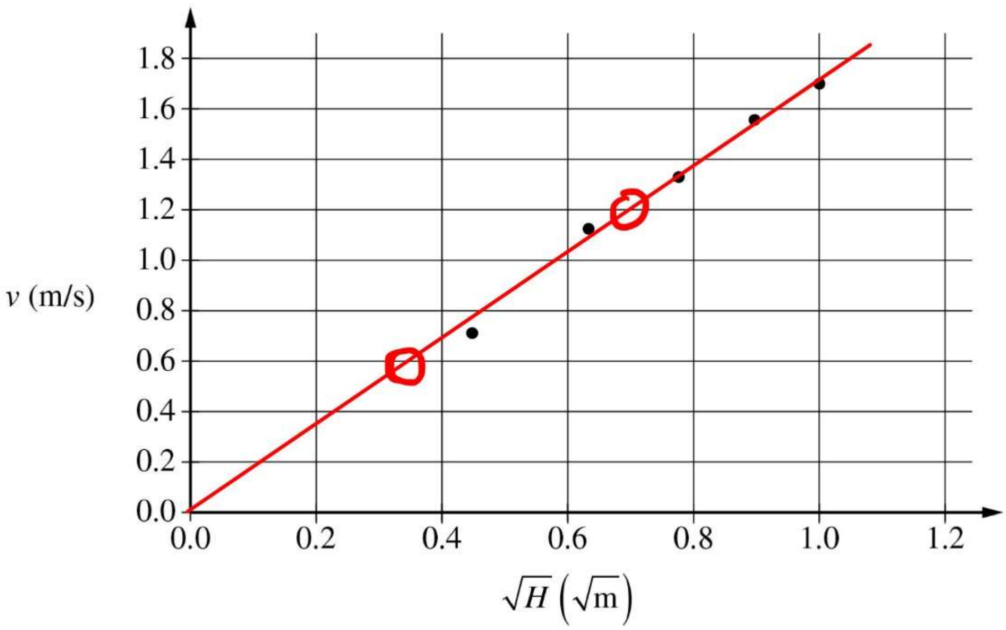
$$v_f = \frac{m_1}{m_1 + m_2}\sqrt{2gH}$$

$$v_f = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}$$

Total for part (c) 3 points

(d)(i)	For drawing an appropriate best-fit line including approximately the same number of points above and below the line	**1 point**

Example Response



(d)(ii)	For calculating the slope using two points on the best-fit line	**1 point**
	For correctly relating the slope of the best-fit line to the mass of Cart 2	**1 point**
	For a correct mass of Cart 2	**1 point**

Example Response

$$slope = \frac{(1.20 - 0.60)\text{ m/s}}{(0.70 - 0.35)\sqrt{\text{m}}} = 1.72\,\frac{\sqrt{\text{m}}}{\text{s}}$$

$$slope = \frac{m_1}{m_1 + m_2}\sqrt{2g}$$

$$m_2 = \frac{m_1\sqrt{2g}}{slope} - m_1$$

$$m_2 = \frac{(0.25\text{ kg})\sqrt{2\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)}}{1.72\,\frac{\sqrt{\text{m}}}{\text{s}}} - (0.25\text{ kg})$$

$$\therefore m_2 = 0.39\text{ kg}$$

Scoring Note: Acceptable responses for mass are 0.30 to 0.60 kg

Total for part (d) 4 points

(e)	For selecting "$m_1' < 0.250$ kg" with an attempt at a relevant justification	**1 point**
	For a correct justification	**1 point**

Example Response

A smaller m_2 indicates that the initial energy and momentum was smaller. With identical slope and height H this means that the mass m_1' must be smaller.

Total for part (e) 2 points

Total for question 2 15 points

Question 2: Free-Response Question — 15 points

(a)	For selecting "$J_S > J_2$" with an attempt at a relevant justification	**1 point**
	For a correct justification	**1 point**

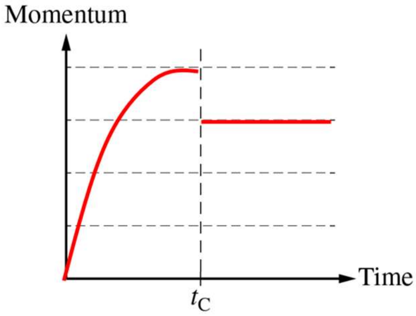
Example Response

The impulse between the two blocks is the same, so because Block 1 is moving at the end, the impulse given by the spring must be greater than the impulse given to Block 2 by Block 1.

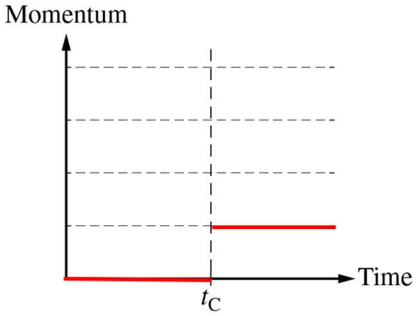
Total for part (a) 2 points

(b)	For correctly drawing the momentum for blocks 1 and 2 for the time interval $0 < t < t_C$: Block 1 shows an increasing curve with a decreasing slope (concave down) to highest value (technically a cosine curve) and Block 2 shows zero momentum up to t_C (a horizontal line along the time axis).	**1 point**
	For drawing a horizontal line for Block 1 when $t > t_C$ that is smaller in magnitude than the momentum of Block 1 at time $t = t_C$	**1 point**
	For drawing a horizontal line for Block 2 when $t > t_C$ that is smaller in magnitude than the momentum of Block 1 after time $t = t_C$	**1 point**
	For blocks 1 and 2 having a change in momentum that is equal in magnitude such that Block 1 loses momentum and Block 2 gains momentum	**1 point**

Example Response



Block 1



Block 2

Total for part (b) 4 points

(c)	For using conservation of energy to find the speed of Block 1 at $x = 0$	**1 point**
	For using conservation of momentum to find the speed of the two-block system after the collision	**1 point**
	For combining correct equations from above	**1 point**

Example Response

$$\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}m_1 v_1^2$$

$$v_1 = \sqrt{\frac{k}{m_1}}\Delta x$$

$$m_1 v_1 = (m_1 + m_2)v$$

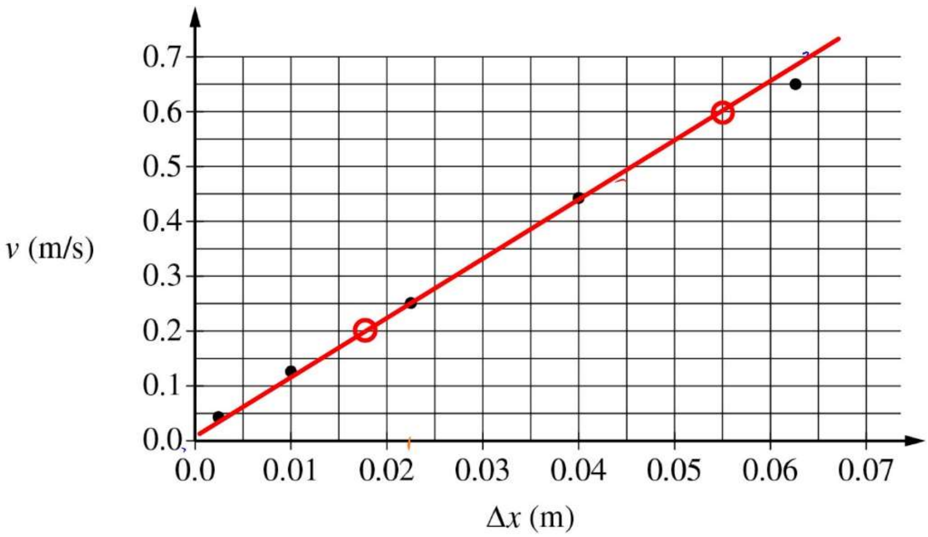
$$v = \frac{m_1 v_1}{m_1 + m_2}$$

$$\therefore\ v = \frac{\Delta x\sqrt{k m_1}}{m_1 + m_2}$$

Total for part (c) **3 points**

(d)(i)	For drawing an appropriate best-fit line including approximately the same number of points above and below the line	**1 point**

Example Response



(d)(ii)	For calculating the slope using two points on the line	**1 point**
	For correctly relating the slope of the best-fit line to the mass of Block 2	**1 point**
	For a correct mass of Block 2	**1 point**

Example Response

$$slope = \frac{(0.60 - 0.02)\ \text{m/s}}{(0.055 - 0.0175)\ \text{m}} = 10.67\ \text{s}^{-1}$$

$$slope = \frac{\sqrt{km_1}}{m_1 + m_2}$$

$$m_2 = \frac{\sqrt{km_1}}{slope} - m_1$$

$$m_2 = \frac{\sqrt{\left(150\ \frac{\text{N}}{\text{m}}\right)(0.50\ \text{kg})}}{10.67\ \text{s}^{-1}} - 0.50\ \text{kg}$$

$$m_2 = 0.31\ \text{kg}$$

Scoring Note: Acceptable responses for mass are $0.27\ \text{kg} - 0.37\ \text{kg}$.

Total for part (d) 4 points

(e)	For selecting "$k' > 150\ \text{N/m}$" with an attempt at relevant justification	**1 point**
	For a correct justification	**1 point**

Example Response

A greater m_2 indicates the spring constant k' should be greater than $150\ \text{N/m}$. The slope of the graph is the same so as m_2 increases k must be smaller.

Total for part (e) 2 points

Total for question 2 15 points

2019

AP® CollegeBoard

AP® Physics C: Mechanics

Scoring Guidelines Set 1

2019 SCORING GUIDELINES

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

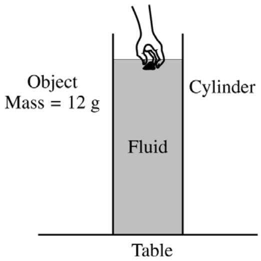
5. The scoring guidelines typically show numerical results using the value $g = 9.8\ \mathrm{m/s^2}$, but the use of $10\ \mathrm{m/s^2}$ is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

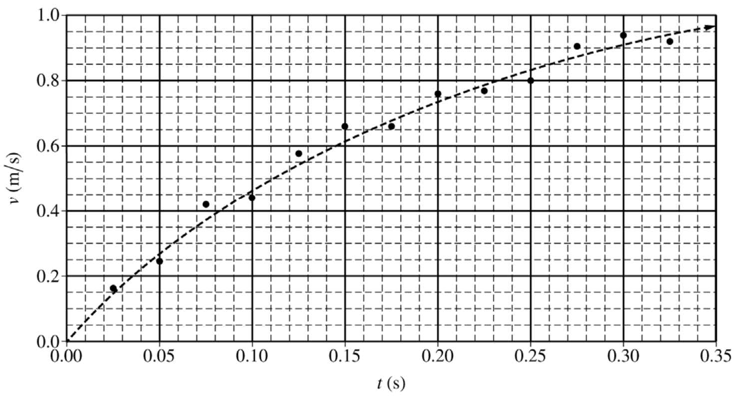
2019 SCORING GUIDELINES

Question 1

15 points



In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of $m = 12$ g is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.



(a)

i. LO CHA-1.C, SP 7.A
1 point

Does the speed of the object increase, decrease, or remain the same?

____ Increase ____ Decrease ____ Remain the same

For selecting "Increase"		1 point

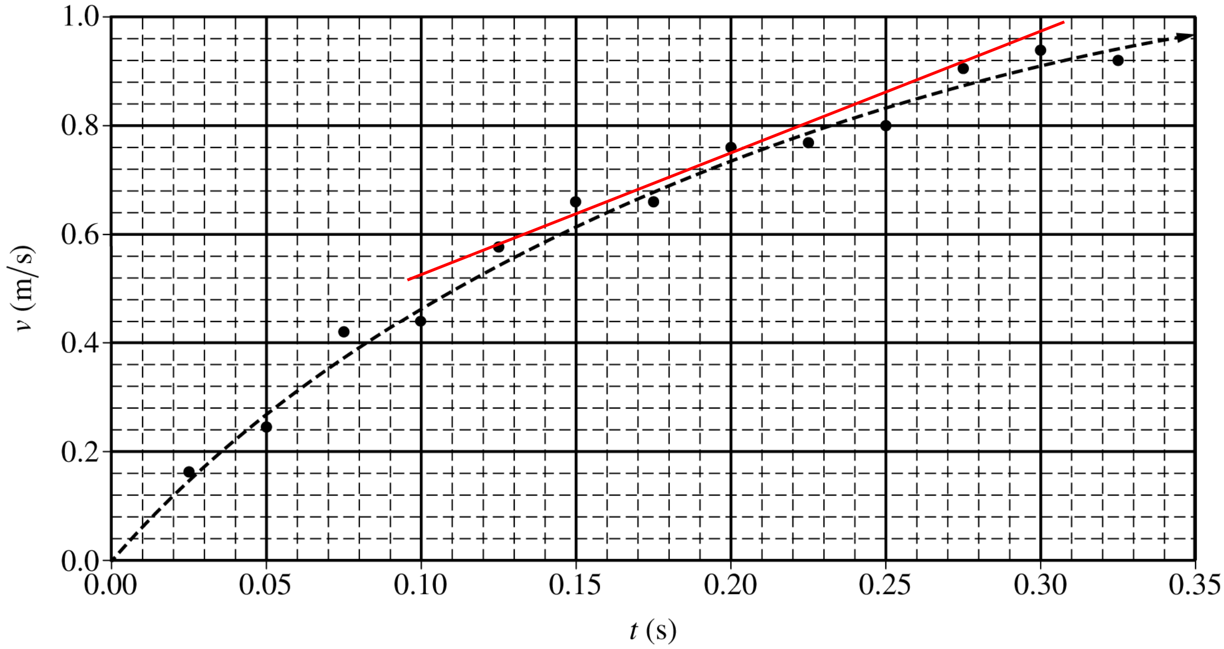
ii. LO CHA-1.C, SP 4.D
2 points

In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.

For a description that includes the direction of the acceleration as being "downwards"		1 point
For a description that includes the decrease in the magnitude of the acceleration		1 point
Example: Because the object is moving downwards and speeding up, the acceleration must be downwards. Because the slope of the graph of speed as a function of time is decreasing, the magnitude of the acceleration must be decreasing.		

Question 1 (continued)

(a) continued



iii. LO CHA-1.C, SP 4.D, 6.C
2 points

Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20$ s.

For calculating the slope of a trend line at $t = 0.20$ s		1 point
slope $= a = \dfrac{\Delta v}{\Delta t} = \dfrac{(0.8 - 0.6)\ \text{m/s}}{(0.226 - 0.136)\ \text{s}}$		
For a correct answer using points from a tangent line		1 point
$a = 2.22\ \text{m/s}^2$		

The students use the equation $v = A\left(1 - e^{-Bt}\right)$ to model the speed of the falling object and find the best-fit coefficients to be $A = 1.18\ \text{m/s}$ and $B = 5\ \text{s}^{-1}$.

2019 SCORING GUIDELINES

Question 1 (continued)

(b) Use the above equation to:

i. LO CHA-1.B, SP 6.B, 6.C
3 points

Derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time *t*.

For indicating that the vertical displacement is the integration of the velocity		1 point
$\Delta y = \int v dt = \int A\left(1 - e^{-Bt}\right)dt$		
For the equation for speed using appropriate limits or constant of integration		1 point
$\Delta y = \int_{t'=0}^{t'=t} A\left(1 - e^{-Bt'}\right)dt' = A\left[t' - \frac{1}{-B}e^{-Bt'}\right]_{t'=0}^{t'=t} A\left[\left(t + \frac{1}{B}e^{-Bt}\right) - \left(0 + \frac{1}{B}e^{0}\right)\right]$		
For a correct answer		1 point
$\Delta y = A\left(t + \frac{1}{B}\left(e^{-Bt} - 1\right)\right) = (1.18)\left(t + \frac{1}{5}\left(e^{-5t} - 1\right)\right)$		
Note: Credit given for using *A* and *B* or plugging in the given values		

ii. LO INT-1.C.d, SP 6.B, 6.C
3 points

Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time *t*.

For attempting the derivative of the equation for speed		1 point
$a = \frac{dv}{dt} = \frac{d}{dt}\left[A\left(1 - e^{-Bt}\right)\right]$		
For a correct equation for the acceleration		1 point
$a = ABe^{-Bt}$		
For multiplying the equation above by the mass of the object		1 point
$F = ma = m\left(ABe^{-Bt}\right) = mABe^{-Bt}$		
$F = (.012)(1.18)(5)e^{-5t} = 0.071e^{-5t}$		
Note: Credit given for using *A*, *B*, and *m* or plugging in the given values		

2019 SCORING GUIDELINES

Question 1 (continued)

(c)

The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed.

i. LO INT-1.I, SP 7.A, 7.C
2 points

Determine the constant speed of the object.
Justify your answer.

For stating the constant speed is $v = A$		1 point
For indicating that the constant speed can be determined by setting the time equal to infinity		1 point
Example: After a long time, the falling object will reach a terminal constant speed in the fluid. This can be determined by setting the time t in the equation for speed equal to infinity. By doing this, the constant speed is determined to be $v = A$.		
Note: Credit given for solving mathematically		

ii. LO INT-1.H.b, SP 7.A, 7.C
2 points

Determine the force exerted by the fluid on the object at this time. Justify your answer.

For indicating that the net force is equal to zero when the object moves with constant speed		1 point
For indicating the resistive force is equal to the weight of the object at this time		1 point
Example: When the falling object reaches a constant speed in the fluid, the net force must be zero. Because the only vertical forces acting on the object are Earth's gravitational pull and the resistive force of the fluid, these two forces must be equal. So, the resistive force must be equal to the weight of the object or 0.12 N.		
Note: Credit given for solving mathematically		

Question 1 (continued)

Learning Objectives

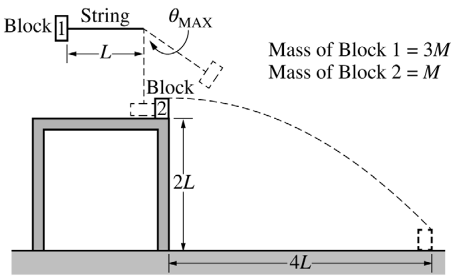
CHA-1.B: Determine functions of position, velocity, and acceleration that are consistent with each other, for the motion of an object with a nonuniform acceleration.
CHA-1.C: Describe the motion of an object in terms of the consistency that exists between position and time, velocity and time, and acceleration and time.
INT-1.C.d: Derive an expression for the net force on an object in translational motion.
INT-1.H.b: Describe the acceleration, velocity, or position in relation to time for an object subject to a resistive force (with different initial conditions, i.e., falling from rest or projected vertically).
INT-1.I: Calculate the terminal velocity of an object moving vertically under the influence of a resistive force of a given relationship.

Science Practices

4.D: Select relevant features of a graph to describe a physical situation or solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

Question 2

15 points



Note: Figure not drawn to scale.

A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

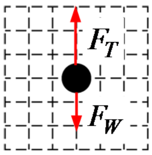
(a) LO CON-2.C.a, SP 5.E
1 point

Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

For correctly calculating the speed of block 1		1 point
$U_{g1} = K_2 \therefore mgh = \frac{1}{2}mv^2 \therefore gL = \frac{1}{2}v^2$		
$v = \sqrt{2gL}$		
Note: Credit is earned even if no work is shown.		

(b) LO INT-2.B.b, SP 3.D
2 points

On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



For correctly drawing and labeling the weight of the block and the tension in the string		1 point
For drawing the tension longer than the weight of the block		1 point
Note: A maximum of one point can be earned if there are any extraneous vectors.		

2019 SCORING GUIDELINES

Question 2 (continued)

(c) LO INT-2.B.b, SP 5.A, 5.E
2 points

Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

For using an appropriate equation to calculate the tension in the string		1 point
$F_T = mg + ma_C = mg + \dfrac{mv^2}{r} = 3Mg + \dfrac{(3M)\left(\sqrt{2gL}\right)^2}{L}$		
For a correct answer		1 point
$F_T = 9Mg$		

For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

(d) LO CHA-2.C, SP 6.A, 6.C
2 points

Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

For using a correct kinematic equation to calculate the time		1 point
$\Delta y = v_1 t + \frac{1}{2}at^2 = \frac{1}{2}at^2 \therefore t = \sqrt{\dfrac{2\Delta y}{a}} = \sqrt{\dfrac{2(2L)}{g}} = \sqrt{\dfrac{(4)(0.75\ \text{m})}{\left(9.8\ \text{m/s}^2\right)}}$		
For a correct answer		1 point
$t = 0.55$ s		

(e) LO CHA-2.C, SP 6.A, 6.C
2 points

Calculate the speed of block 2 as it leaves the table.

For using a correct kinematic equation to calculate the speed		1 point
$\Delta x = vt \therefore v = \dfrac{\Delta x}{t} = \dfrac{4L}{t}$		
For substituting the time from part (d) into equation above		1 point
$v = \dfrac{(4)(0.75\ \text{m})}{(0.55\ \text{s})} = 5.45\ \text{m/s}$		

Question 2 (continued)

(f) LO CON-4.E.a, SP 6.B, 6.C
3 points

Calculate the speed of block 1 just after it collides with block 2.

For indicating the use of the correct conservation of momentum to calculate the speed		1 point
$p_i = p_f \therefore m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$		
For correctly substituting the mass into equation above		1 point
$(3M)v_{1i} + 0 = (3M)v_{1f} + (M)v_{2f}$		
For substituting the speeds from parts (a) and (e) into equation above		1 point
$v_{1f} = \dfrac{(3M)v_{1i} - (M)v_{2f}}{(3M)} = \dfrac{(3M)\sqrt{2gL} - (M)v_{2f}}{(3M)}$		
$v_{1f} = \dfrac{(3)\sqrt{(2)(9.8\ \text{m/s}^2)(0.75\ \text{m})} - (1)(5.45\ \text{m/s})}{(3)} = 2.02\ \text{m/s}$		
($v_{1f} = 2.06\ \text{m/s}$ when using $g = 10\ \text{m/s}^2$)		

(g) LO CON-2.C.a, SP 6.B, 6.C
3 points

Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.

For using conservation of energy to calculate the angle		1 point
$K_1 = U_{g2}$		
$\frac{1}{2}mv^2 = mgh$		
For correctly substituting $h = L(1 - \cos\theta)$ into the equation above		1 point
For correctly substituting the speed from part (f) into the equation above		1 point
$\frac{1}{2}mv^2 = mgL(1 - \cos\theta) \therefore \dfrac{v^2}{2gL} = 1 - \cos\theta$		
$\theta = \cos^{-1}\left(1 - \dfrac{v^2}{2gL}\right) = \cos^{-1}\left(1 - \dfrac{(2.02\ \text{m/s})^2}{(2)(9.8\ \text{m/s}^2)(0.75\ \text{m})}\right) = 43.5°$		
($\theta = 44.1°$ when using $g = 10\ \text{m/s}^2$)		

2019 SCORING GUIDELINES

Question 2 (continued)

Learning Objectives

CHA-2.C: Calculate kinematic quantities of an object in projectile motion, such as displacement, velocity, speed, acceleration, and time, given initial conditions of various launch angles, including a horizontal launch at some point in its trajectory.
INT-2.B.b: Describe forces that are exerted on objects undergoing horizontal circular motion, vertical circular motion, or horizontal circular motion on a banked curve.
CON-2.C.a: Calculate unknown quantities (e.g., speed or positions of an object) that are in a conservative system of connected objects, such as the masses in an Atwood machine, masses connected with pulley/string combinations, or the masses in a modified Atwood machine.
CON-4.E.a: Calculate the changes in speeds, changes in velocities, changes in kinetic energy, or changes in momenta of objects in all types of collisions (elastic or inelastic) in one dimension, given the initial conditions of the objects.

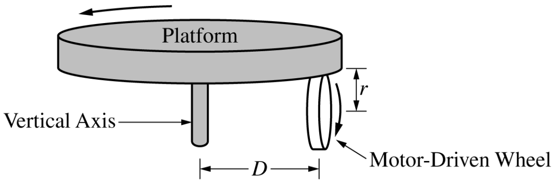
Science Practices

3.D: Create appropriate diagrams to represent physical situations.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

2019 SCORING GUIDELINES

Question 3

15 points



A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the wheel until the platform reaches an angular speed ω_P after time Δt . During time Δt , the wheel stays in contact with the platform without slipping.

(a) LO INT-7.A.b, CHA-4.A.b, SP 5.A, 5.E
2 points

Derive an expression for the angular speed ω_P of the platform. Express your answer in terms of I_P , r, D, F, Δt , and physical constants, as appropriate.

For correctly substituting into the rotational form of Newton's second law			1 point
$\tau = I\alpha \therefore FD = I_P\alpha$			
$\alpha = \dfrac{FD}{I_P}$			
For correctly substituting into a rotational kinematic equation to calculate the angular speed			1 point
$\omega_2 = \omega_1 + \alpha\Delta t = 0 + \left(\dfrac{FD}{I_P}\right)\Delta t$			
$\omega_P = \dfrac{FD\Delta t}{I_P}$			

(b) LO INT-7.D.a, SP 5.A, 5.E
2 points

Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed ω_P . Express your answer in terms of I_P , r, D, F, Δt , and physical constants, as appropriate.

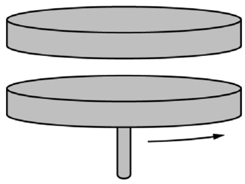
For using the equation for rotational kinetic energy		1 point
$K = \dfrac{1}{2}I\omega_P^2 = \left(\dfrac{1}{2}\right)(I)\left(\dfrac{FD\Delta t}{I}\right)^2$		
For an answer consistent with part (a)		1 point
$K = \dfrac{(FD\Delta t)^2}{2I}$		

Question 3 (continued)

(c) LO INT-7.C, SP 5.A, 5.E
2 points

Derive an expression for the angular speed of the wheel ω_W when the platform has reached angular speed ω_P. Express your answer in terms of D, r, ω_P, and physical constants, as appropriate.

For indicating that the linear speed of the platform is equal to the linear speed of the wheel		1 point
$v_P = v_W$ OR $(r\omega)_P = (r\omega)_W$		
For correctly relating the linear speeds to the angular speeds in the above equation		1 point
$D\omega_P = r\omega_W$		
$\omega_W = \dfrac{D\omega_P}{r}$		



When the platform is spinning at angular speed ω_P, the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia I_P as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed ω_f.

(d) LO CON-5.D.c, SP 5.A, 5.E
2 points

Derive an expression for ω_f. Express your answer in terms of ω_P, I_P, and physical constants, as appropriate.

For using an expression for the conservation of angular momentum		1 point
$L_1 = L_2 \therefore I_1\omega_1 = I_2\omega_2$		
For correctly substituting into the above equation		1 point
$I\omega_P = (2I)\omega_F$		
$\omega_F = \frac{1}{2}\omega_P$		

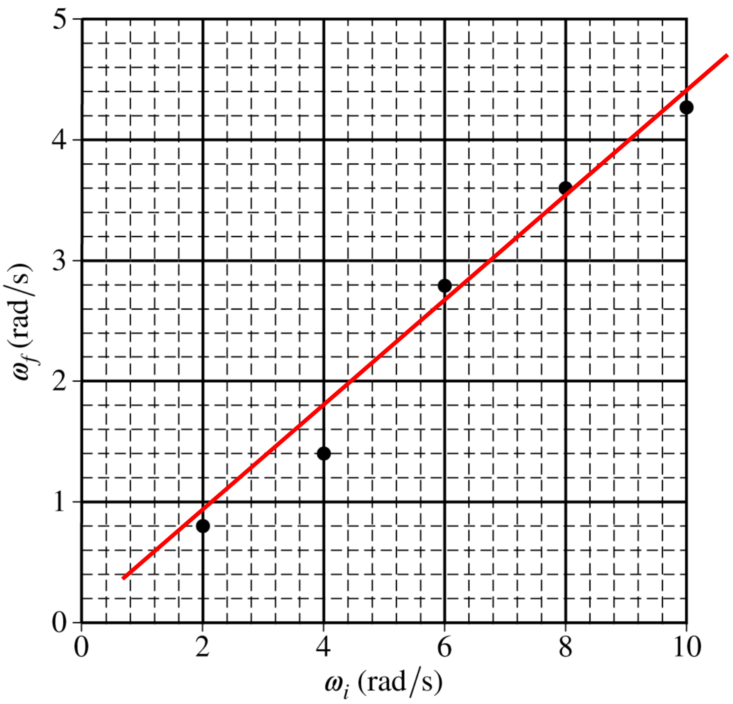
Question 3 (continued)

A student now uses the rotating platform $\left(I_P = 3.1\ \text{kg} \bullet \text{m}^2\right)$ to determine the rotational inertia I_U of an unknown object about a vertical axis that passes through the object's center of mass. The platform is rotating at an initial angular speed ω_i when the unknown object is dropped with its center of mass directly above the center of the platform. The platform and object are observed to be rotating together at angular speed ω_f. Trials are repeated for different values of ω_i. A graph of ω_f as a function of ω_i is shown on the axes below.

(e)

i. LO CON-5.D.c, SP 4.C
1 points

On the graph on the previous page, draw a best-fit line for the data.



For an appropriate best-fit line for the graph above		1 point

Question 3 (continued)

(e) continued
ii. LO CON-5.D.c, SP 4.D, 6.C
2 points

Using the straight line, calculate the rotational inertia of the unknown object I_U about a vertical axis passing through its center of mass.

For using conservation of angular momentum to derive an expression that includes I_U		1 point
$L_i = L_f \therefore I_P\omega_i = (I_P + I_U)\omega_f$		
$I_U = I_P\left(\dfrac{\omega_i}{\omega_f}\right) - I_P$		
For substituting points from the best-fit line into the expression above		1 point
$I_U = \left(3.1\ \text{kg}\bullet\text{m}^2\right)\left(\dfrac{(4.0-1.0)\ \text{rad/s}}{(9.3-2.4)\ \text{rad/s}}\right) - \left(3.1\ \text{kg}\bullet\text{m}^2\right) = 4.1\ \text{kg}\bullet\text{m}^2$		
Note: The point (0, 0) can be used implicitly if the best-fit line goes through the origin.		

(f) LO CON-5.D.c, SP 7.A, 7.C
2 points

The kinetic energy of the spinning platform before the object is dropped on it is K_i. The total kinetic energy of the platform-object system when it reaches angular speed ω_f is K_f. Which of the following expressions is true?

____ $K_f < K_i$ ____ $K_f = K_i$ ____ $K_f > K_i$

Justify your answer.

For selecting $K_f < K_i$ with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because the two disks will be rotating with the same final angular speed, this is an inelastic collision, and kinetic energy will be lost during the collision.		

Question 3 (continued)

(g) LO INT-6.E, SP 7.A, 7.C
2 points

One of the students observes that the center of mass of the object is not actually aligned with the axis of the platform. Is the experimental value of I_U obtained in part (e) greater than, less than, or equal to the actual value of the rotational inertia of the unknown object about a vertical axis that passes through its center of mass?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

For selecting "Greater than" with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because the center of mass of the object is off the axis of the platform, the parallel axis theorem would be used to calculate the total rotational inertia of the platform-object system. Using $I = I_{CM} + Mh^2$, the experimental value will be increased by Mh^2.		

Learning Objectives

INT-6.E: Derive the moments of inertia of an extended rigid body for different rotational axes (parallel to an axis that goes through the object's center of mass) if the moment of inertia is known about an axis through the object's center of mass.
CHA-4.A.b: Calculate unknown quantities such as angular positions, displacement, angular speeds, or angular acceleration of a rigid body in uniformly accelerated motion, given initial conditions.
INT-7.A.b: Calculate unknown quantities such as net torque, angular acceleration, or moment of inertia for a rigid body undergoing rotational acceleration.
INT-7.C: Derive expressions for physical systems such as Atwood Machines, pulleys with rotational inertia, or strings connecting discs or strings connecting multiple pulleys that relate linear or translational motion characteristics to the angular motion characteristics of rigid bodies in the system that are: **(a)** rolling (or rotating on a fixed axis) without slipping. **(b)** rotating and sliding simultaneously.
INT-7.D.a: Calculate the rotational kinetic energy of a rotating rigid body.
CON-5.D.c: Calculate the changes of angular momentum of each disc in a rotating system of two rotating discs that collide with each other inelastically about a common rotational axis.

Science Practices

4.C: Linearize data and/or determine a best-fit line or curve.
4.D: Select relevant features of a graph to describe a physical situation or solve problems.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

2018

AP® CollegeBoard

AP Physics C: Mechanics

Scoring Guidelines

2018 SCORING GUIDELINES

General Notes About 2018 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

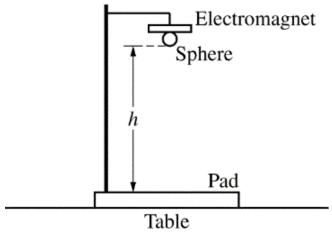
5. The scoring guidelines typically show numerical results using the value $g = 9.8\ \mathrm{m/s^2}$, but the use of $10\ \mathrm{m/s^2}$ is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

AP® PHYSICS C: MECHANICS
2018 SCORING GUIDELINES

Question 1

15 points total | **Distribution of points**



A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

(a) 2 points

While taking the first data point, the student notices that the electromagnet actually releases the sphere after the timer begins. Would the value of g calculated from this one measurement be greater than, less than, or equal to the actual value of g at the student's location?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

For selecting "Less than" with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because the measured time to fall the same distance will be larger, the acceleration must be less.		
Example: Because the time is larger and $a \propto 1/t^2$ *, then g must decrease.*		

2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

The electromagnet is replaced so that the timer begins when the sphere is released. The student varies the distance h. The student measures and records the time Δt of the fall for each particular height, resulting in the following data table.

h (m)	0.10	0.20	0.60	0.80	1.00
Δt (s)	0.105	0.213	0.342	0.401	0.451

(b) 1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for g.

Vertical axis: ____________

Horizontal axis: ____________

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given in the table. Label each row you use and include units.

For correctly indicating two variables that will yield a straight line that could be used to determine a value for g		1 point
Example: Vertical Axis: h *Horizontal Axis:* $(\Delta t)^2$		
Note: Student earns full credit if axes are reversed or if they use another acceptable combination.		

AP® PHYSICS C: MECHANICS

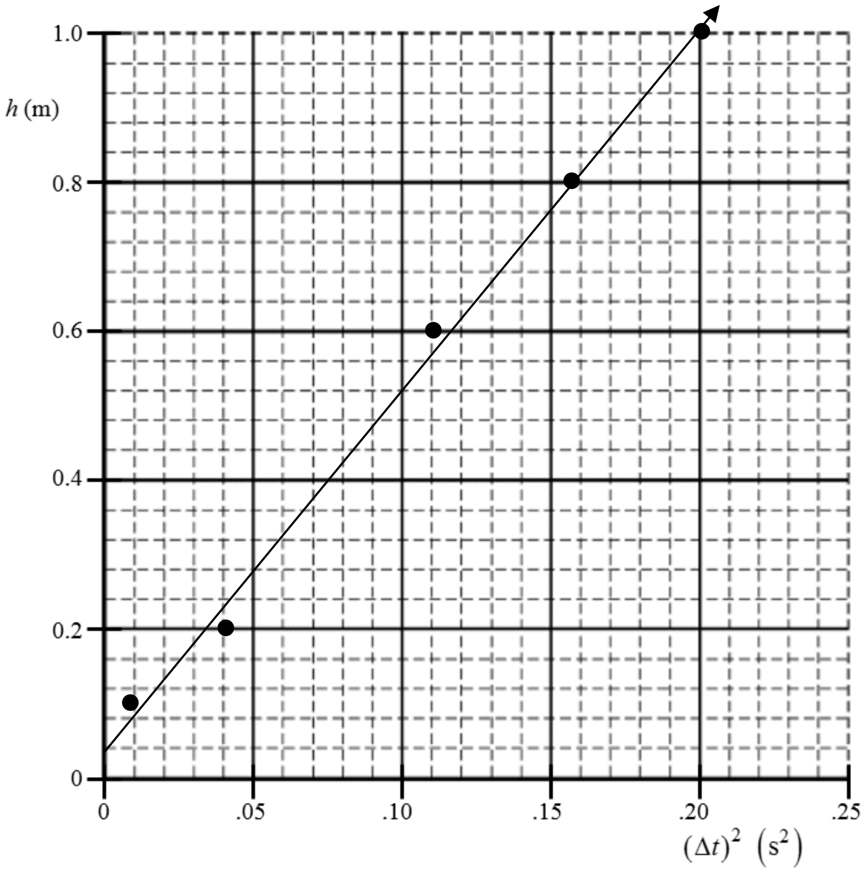
2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(c) 4 points

Plot the data points for the quantities indicated in part (b) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



For a correct scale that uses more than half the grid		1 point
For correctly labeling the axis with variables and units consistent with part (b)		1 point
For correctly plotting data consistent with part (b)		1 point
For drawing a straight line consistent with the plotted data		1 point

2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(d) 2 points

Using the straight line, calculate an experimental value for g.

For using points on the line rather than the data to calculate the slope		1 point
slope $= \dfrac{\Delta h}{\Delta(\Delta t)^2} = \dfrac{(0.80 - 0.20)\ \text{m}}{(0.160 - 0.039)\ \text{s}^2} = 4.96\ \text{m/s}^2$ (Linear regression = $4.83\ \text{m/s}^2$)		
For correctly relating the slope to the acceleration due to gravity		1 point
slope $= \dfrac{1}{2}g \therefore g = 2 \times \text{slope} = 2 \times \left(4.96\ \text{m/s}^2\right)$		
$g = 9.92\ \text{m/s}^2$ (Linear regression = $9.66\ \text{m/s}^2$)		

Another student fits the data in the table to a quadratic equation. The student's equation for the distance fallen y as a function of time t is $y = At^2 + Bt + C$, where $A = 5.75\ \text{m/s}^2$, $B = -0.524\ \text{m/s}$, and $C = +0.080\ \text{m}$. Vertically down is the positive direction.

(e) Using the student's equation above, do the following.

i. 1 point

Derive an expression for the velocity of the sphere as a function of time.

For correctly taking the time derivative of the given equation		1 point
$y = y_0 + v_1 t + \dfrac{1}{2}at^2 = At^2 + Bt + C$		
$v(t) = \dfrac{dy}{dt} = 2At + B$		
Note: Credit is earned for substituting numbers: $v(t) = \left(11.5\ \text{m/s}^2\right)t - 0.524\ \text{m/s}$.		

ii. 2 points

Calculate the new experimental value for g.

For correctly relating the given equation to a correct kinematic equation		1 point
$\Delta y = y - y_0 = v_1 t + \dfrac{1}{2}at^2$		
$y = y_0 + v_1 t + \dfrac{1}{2}at^2 = At^2 + Bt + C$		
For correctly relating the equation to the value of g		1 point
$A = \dfrac{1}{2}a = \dfrac{1}{2}g$		
$g = 2A = (2)\left(5.75\ \text{m/s}^2\right) = 11.5\ \text{m/s}^2$		

AP® PHYSICS C: MECHANICS

2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

iii. 1 point

Using 9.81 m/s^2 as the accepted value for g at this location, calculate the percent error for the value found in part (e)(ii).

For correctly calculating the percent error		1 point
$\%\text{error} = \dfrac{\lvert \text{acc} - \text{exp} \rvert}{\text{acc}} \times 100\% = \dfrac{\left\lvert \left(11.5 \text{ m/s}^2\right) - \left(9.81 \text{ m/s}^2\right) \right\rvert}{\left(9.81 \text{ m/s}^2\right)} \times 100\%$		
$\%\text{error} = 17.2\%$		
Note: Credit is earned if percent error is expressed as positive or negative.		

iv. 2 points

Assuming the sphere is at a height of 1.40 m at $t = 0$, calculate the velocity of the sphere just before it strikes the pad.

For relating the coefficients of the equation to the kinematic variables		1 point
$y = At^2 + Bt + C \therefore v_1 = B = -0.524 \text{ m/s}$		
$a = 2A = 2 \times \left(5.75 \text{ m/s}^2\right) = 11.5 \text{ m/s}^2$		
$y_0 = C = 0.080 \text{ m}$		
For correctly using an appropriate kinematics equation to determine the velocity of the sphere		1 point
$v_2^2 = v_1^2 + 2a\Delta y$		
$v = \sqrt{v_1^2 + 2a\Delta y} = \sqrt{(-0.524 \text{ m/s})^2 + (2)\left(11.5 \text{ m/s}^2\right)(1.40 \text{ m} - 0.080 \text{ m})}$		
$v = 5.54 \text{ m/s}$		
Alternate solution:		*Alternate Points*
For correctly determining the time of fall for the sphere		*1 point*
$y = At^2 + Bt + C$		
$1.40 = \left(5.75 \text{ m/s}^2\right)t^2 + (-0.524 \text{ m/s})t + (0.080 \text{ m})$		
$5.75t^2 - 0.524t - 1.32 = 0$		
$t = 0.53 \text{ s or } -0.44 \text{ s} \therefore t = 0.53 \text{ s}$		
For correctly using the equation from part (e)(i)		*1 point*
$v(t) = \left(11.5 \text{ m/s}^2\right)(0.53 \text{ s}) - 0.524 \text{ m/s}$		
$v = 5.54 \text{ m/s}$		

2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(e)

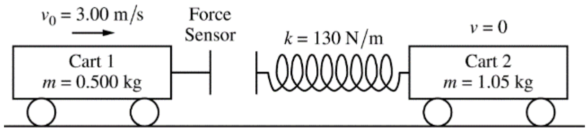
iv. (continued)

Alternate Solution — Conservation of Energy		
For relating the coefficients of the equation to the initial height and speed		1 point
$y = At^2 + Bt + C \ \therefore v_1 = B = -0.524 \text{ m/s}$		
$y_0 = C = 0.080 \text{ m}$		
For correctly using conservation of energy to determine the velocity of the sphere		1 point
$U_i + K_i = U_f + K_f$		
$mgh + \frac{1}{2}mv_1^2 = \frac{1}{2}mv^2$		
$v = \sqrt{2(11.5)(1.40 - 0.080) + (-0.524)^2} = 5.54 \text{ m/s}$		

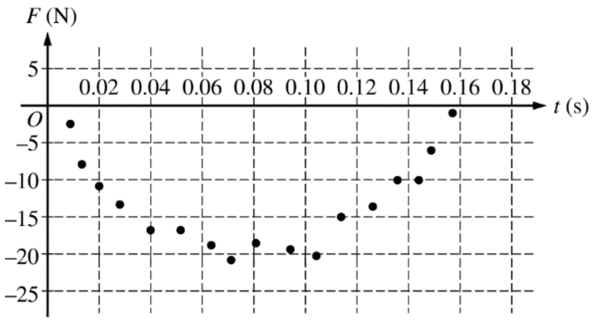
2018 SCORING GUIDELINES

Question 2

15 points total | **Distribution of points**



Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00\ \text{m/s}$ toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130\ \text{N/m}$. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200\ \text{N/s}^2\right)t^2 - \left(500\ \text{N/s}\right)t$.



(a) 3 points

Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

For using the given force equation in the integral to determine the impulse delivered to the cart		1 point
$J = \int F \bullet dt = \int \left(3200t^2 - 500t\right) dt$		
For integrating the force using the correct limits or constant of integration		1 point
$J = \int_{t=0}^{t=0.16\ \text{s}} \left(3200t^2 - 500t\right)dt = \left[\frac{3200}{3}t^3 - 250t^2\right]_{t=0}^{t=0.16}$		
$J = \left(\left(\frac{3200}{3}\right)(0.16)^3 - (250)(0.16)^2\right) - 0$		
For a correct answer		1 point
$J = -2.03\ \text{N}\bullet\text{s}$		

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Question 2 (continued)

Distribution of points

(b)

i. 1 point

Calculate the speed of cart 1 after the collision.

For correct substitution into the impulse-momentum equation of the answer from part (a) to determine the speed of cart 1		1 point
$J = m_1\left(v_{1f} - v_{1i}\right) \therefore v_{1f} = \dfrac{J}{m_1} + v_{1i} = \dfrac{(-2.03\ \text{N}\bullet\text{s})}{(0.500\ \text{kg})} + (3.00\ \text{m/s})$		
$v_{1f} = -1.06\ \text{m/s}$		

ii. 1 point

In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

For correctly selecting "Left"		1 point

(c)

i. 2 points

Calculate the speed of cart 2 after the collision.

For using a correct equation to determine the speed of cart 2		1 point
$-J = -m_1\left(v_{1f} - v_{1i}\right) = m_2 v_{2f} \therefore v_{2f} = \dfrac{-J}{m_2}$		
For correct substitution of the answer from part (a)		1 point
$v_{2f} = \dfrac{(2.03\ \text{N}\bullet\text{s})}{(1.05\ \text{kg})}$		
$v_{2f} = 1.93\ \text{m/s}$		

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Question 2 (continued)

Distribution of points

(c)

i. (continued)

Alternate solution		*Alternate Points*
For using the conservation of momentum to calculate the speed of cart 2 after the collision		*1 point*
$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \therefore v_{2f} = \dfrac{m_1 \left(v_{1i} - v_{1f} \right)}{m_2}$		
For correct substitution of the answer from part (b)(i)		*1 point*
$v_{2f} = \dfrac{(0.500\ \text{kg})((3.00\ \text{m/s}) - (-1.06\ \text{m/s}))}{(1.05\ \text{kg})} = 1.93\ \text{m/s}$		

ii. 2 points

Show that the collision between the two carts is elastic.

For indicating that the initial and final kinetic energies must be equal		1 point
$K_{1i} = K_{1f} + K_{2f}$		
For correct substitutions of answers from parts (b)(i) and (c)(i) into the calculations of the initial and final kinetic energies		1 point
$\left(\dfrac{1}{2}\right)(0.500\ \text{kg})(3.00\ \text{m/s})^2 = \left(\dfrac{1}{2}\right)(0.500\ \text{kg})(-1.06\ \text{m/s})^2 + \left(\dfrac{1}{2}\right)(1.05\ \text{kg})(1.93\ \text{m/s})^2$		
$2.25\ \text{J} \approx 2.24\ \text{J}$		

(d)

i. 2 points

Calculate the speed of the center of mass of the two-cart-spring system.

For using the equation for the conservation of momentum to calculate the speed for the center of mass of the system		1 point
$m_1 v_{1i} = (m_1 + m_2) v_{cm} \therefore v_{cm} = \dfrac{m_1 v_{1i}}{(m_1 + m_2)}$		
For correct substitution into the equation above		1 point
$v_{cm} = \dfrac{(0.500\ \text{kg})(3.00\ \text{m/s})}{(0.500\ \text{kg} + 1.05\ \text{kg})} = 0.97\ \text{m/s}$		

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Question 2 (continued)

Distribution of points

(d) (continued)

ii. 3 points

Calculate the maximum elastic potential energy stored in the spring.

For using conservation of energy to calculate the maximum elastic potential energy stored in the spring		1 point
$K_i = K_f + U_{Sf}$		
For using the speed of the center of mass of the system for kinetic energy		1 point
$\frac{1}{2}m_1v_{1i}^2 = \frac{1}{2}(m_1 + m_2)v_{cm}{}^2 + U_{Sf}$		
$U_{Sf} = \frac{1}{2}m_1v_{1i}^2 - \frac{1}{2}(m_1 + m_2)v_{cm}{}^2$		
For correct substitution into above equation		1 point
$U_S = \left(\frac{1}{2}\right)(0.500\text{ kg})(3.00\text{ m/s})^2 - \frac{1}{2}(0.500\text{ kg} + 1.05\text{ kg})(0.97\text{ m/s})^2$		
$U_S = 1.52\text{ J}$		
Alternate Solution:		*Alternate Points*
For correctly determining the magnitude of the maximum force exerted between the carts		1 point
Set $\frac{dF}{dt} = 0 = 6400t - 500 \therefore 6400t = 500 \therefore t = 0.078\text{ s}$		
$F_{MAX} = 3200(0.078)^2 - 500(0.078) = -19.5$		
$\lvert F_{MAX} \rvert = 19.5\text{ N}$		
Note: Can estimate from the graph, and accept the range of 20 N to 21 N.		
For calculating the maximum compression of the spring		1 point
$F_{MAX} = -kx_{MAX}$		
$x_{MAX} = -\frac{F_{MAX}}{k} = -\frac{(-19.5\text{ N})}{(130\text{ N/m})} = 0.15\text{ m}$		
Note: If estimating from the graph, accept the range from 0.15 m to 0.16 m.		
For substituting into an equation for the elastic potential energy at maximum spring compression		1 point
$U_S = \frac{1}{2}kx_{MAX}^2 = \left(\frac{1}{2}\right)(130\text{ N/m})(0.15\text{ m})^2$		
$U_S = 1.46\text{ J}$		
Note: If estimating from the graph, accept range from 1.46 J to 1.70 J.		

Units point: 1 point for correct units on all calculated answers

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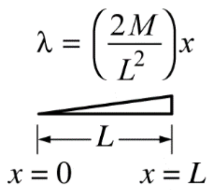
Question 3

15 points total **Distribution of points**



A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \dfrac{2M}{L^2}x$, where x is the distance from one end of the rod.

(a) 3 points

Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

For relating x to r properly in an integral to calculate the moment of inertia		1 point
$I = \int r^2 dm = \int x^2 dm$		
For correctly using the linear mass density to substitute into the equation above		1 point
$m = \int \lambda dx = \int \left(2M/L^2\right) x dx \therefore dm = \left(2M/L^2\right) x dx$		
$I = \int \left(2M/L^2\right) x^3 dx$		
For integrating using the correct limits or constant of integration		1 point
$I = \int_{x=0}^{x=L} \left(2M/L^2\right) x^3 dx = \left[\dfrac{\left(2M/L^2\right)x^4}{4}\right]_{x=0}^{x=L} = \dfrac{2M}{L^2}\left(\dfrac{L^4}{4} - 0\right) = ML^2/2$		

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Question 3 (continued)

Distribution of points

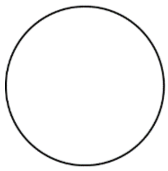

Figure 1

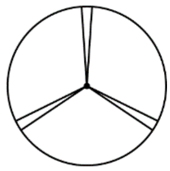

Figure 2

The thin hoop shown above in Figure 1 has a mass M, radius L, and a rotational inertia around its center of ML^2. Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

(b) 2 points

Derive an expression for the rotational inertia I_{tot} of the hoop-rods system about the center of the hoop. Express your answer in terms of M, L, and physical constants, as appropriate.

For setting the total rotational inertia for the hoop-rod system equal to the sum of the rotational inertias of the hoop and the three rods		1 point
$I = 3I_{rod} + I_{hoop}$		
$I = 3\left(ML^2/2\right) + ML^2$		
For a correct answer with work		1 point
$I = \frac{5}{2}ML^2$		

The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt.

(c) 3 points

Derive an expression for the final angular speed ω of the hoop-rods system. Express your answer in terms of M, L, F, Δt, and physical constants, as appropriate.

For using an appropriate equation to determine the final angular speed of the hoop		1 point
$\tau\Delta t = I(\omega_2 - \omega_1)$		
$Fr\Delta t = I\omega$		
$\omega = \frac{Fr\Delta t}{I}$		
For relating the lever arm to the length of the rod		1 point
$\omega = \frac{FL\Delta t}{I}$		
For correct substitution from part (b)		1 point
$\omega = \frac{FL\Delta t}{\left(\frac{5}{2}\right)ML^2} = \frac{2F\Delta t}{5ML}$		

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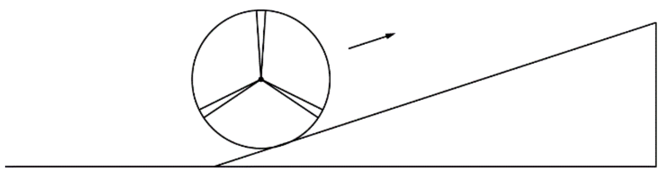
Question 3 (continued)

Distribution of points

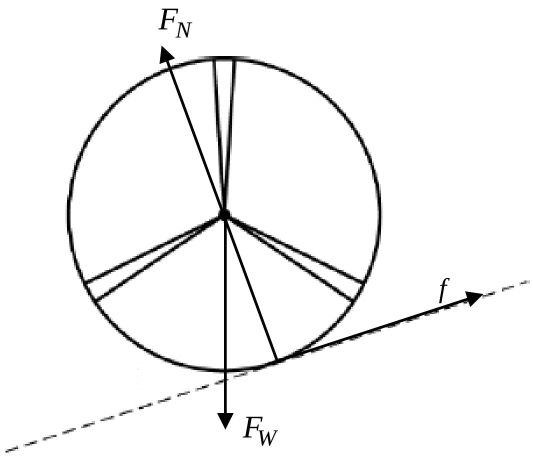


The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed ω found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

(d)

i. 3 points

On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.



For drawing the weight of the hoop-rod system in the correct direction		1 point
For drawing the normal force in the correct direction		1 point
For drawing the frictional force in the correct direction		1 point
Note: A maximum of two points can be earned if there are any extraneous vectors or any vector has an incorrect point of exertion.		

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Question 3 (continued)

Distribution of points

ii. 1 point

Justify your choice for the direction of each of the forces drawn in part (d)(i).

For a correct justification of the direction of the forces in part (d)(i)		1 point
Example: The normal force is always perpendicular to the surface, so it will be directed up and to the left. The gravitational force is always vertically downward. The friction will be opposite of the direction of rotation; therefore, it is directed up the incline.		

(e) 3 points

Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of M, L, I_{tot}, ω, and physical constants, as appropriate.

For including both linear and rotational kinetic energy in an equation for the conservation of energy to determine the final height of the hoop		1 point
$K_1 = U_{g2}$		
$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgH$		
$H = \dfrac{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2}{mg}$		
For correct substitution of $v = L\omega$		1 point
$H = \dfrac{\frac{1}{2}m(L\omega)^2 + \frac{1}{2}I_{tot}\omega^2}{mg}$		
$H = \dfrac{\frac{1}{2}\left(m_{tot}L^2 + I_{tot}\right)\omega^2}{m_{tot}g}$		
For correct substitution of inertias into energy equation		1 point
$H = \dfrac{\frac{1}{2}\left((3M + M)L^2 + I_{tot}\right)\omega^2}{(3M + M)g} = \dfrac{\left(4ML^2 + I_{tot}\right)\omega^2}{8Mg}$		

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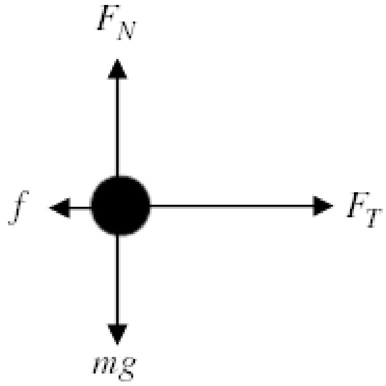
2016 SCORING GUIDELINES

Question

15 points total **Distribution of points**

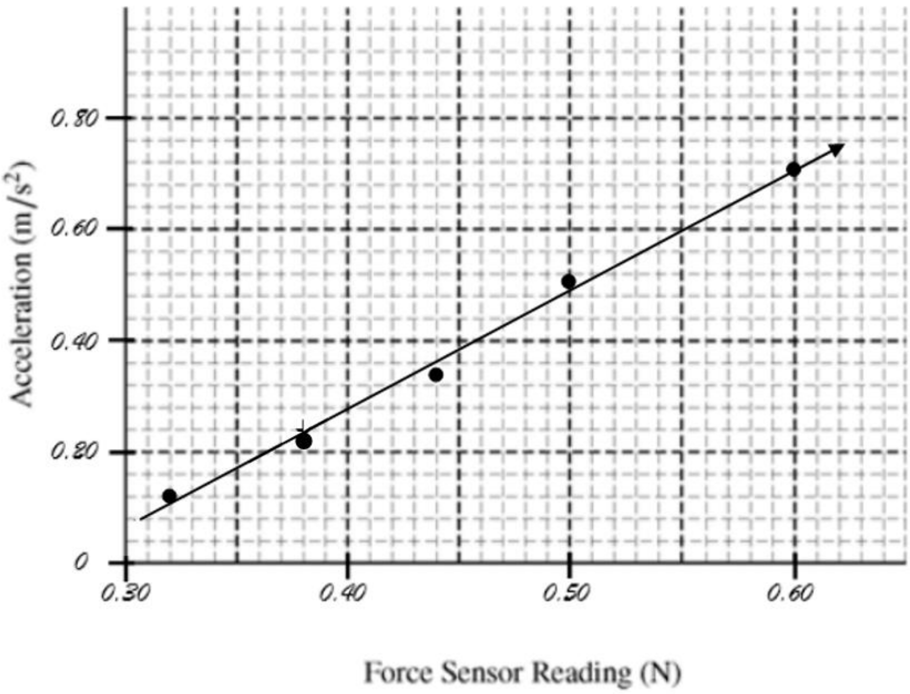
3 points



For correctly drawing and labeling the force of tension	1 point
For correctly drawing and labeling the force of friction	1 point
For correctly drawing and labeling both forces in the vertical direction	1 point

Note: A maximum of two points may be earned if there are any extraneous vectors.

3 points



For a correct scale that uses more than half the grid	1 point
For correctly plotting the given data	1 point
For drawing a straight line consistent with the given data	1 point

Note: Full credit can be earned if the axes are switched.

SCORING GUIDELINES

Question 1 (continued)

Distribution of points

2 points

For correctly calculating slope using the best-fit straight line and not data points — 1 point

$$\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(0.70 - 0.16)}{(0.60 - 0.35)}\,\text{kg}^{-1} = 2.16\ \text{kg}^{-1}$$

Note: Linear regression gives slope $= 2.12\ \text{kg}^{-1}$.

For correctly calculating the mass of the cart using the slope — 1 point

$$m = \frac{1}{\text{slope}} = \frac{1}{\left(2.16\ \text{kg}^{-1}\right)}$$

Correct answer:

$m = 0.463\ \text{kg}$ (Note: linear regression gives $m = 0.472\ \text{kg}$)

1 point

For an answer with correct units consistent with the x-intercept of the graph from (b) i. — 1 point

$f = 0.272\ \text{N}$

(

1 point

plying Newton's second law and substituting the values from part (b)

$$\sum F = ma = F_a - f$$

$$a = \frac{F_a - f}{m} = \frac{0.45\ \text{N} - 0.272\ \text{N}}{0.463\ \text{kg}}$$

For an answer with correct units consistent with part (b), either from the graph or calculated using the mass and frictional force. — 1 point

$a = 0.376\ \text{m/s}^2$

SCORING GUIDELINES

Question 1 (continued)

Distribution of points

3 points

For using a correct equation to solve for the speed of the cart when the string breaks, with an acceleration consistent with part (c) i. — 1 point

$v_2 = v_1 + at$

$v_2 = 0 + (0.376\ \text{m/s}^2)(2.0\ \text{s})$

$v_2 = 0.752\ \text{m/s}$

For recognizing that the acceleration of the cart after the string breaks is due to the frictional force determined in part (b) — 1 point

Use a correct equation to solve for the time for the cart to stop after the string breaks

$v_2 = v_1 + at$

$0 = v_1 - \dfrac{f}{m}t$

For using the final velocity before the string breaks as the initial velocity for the cart stopping in the correct equation for time — 1 point

$t = \dfrac{mv_1}{f}$

$t = \dfrac{(0.463\ \text{kg})(0.752\ \text{m/s})}{(0.272\ \text{N})}$

Correct answer

$t = 1.28\ \text{s}$

Alternate solution — *Alternate points*

For setting the magnitude of the impulse before the string breaks equal to the magnitude of the impulse after the string breaks — *1 point*

$F_1t_1 = F_2t_2$

For correctly using the proper force (e.g., the tension force minus the friction force) for F_1 — *1 point*

For correctly using the proper force (e.g., the friction force) for F_2 — *1 point*

Correct answer

$t = 1.28\ \text{s}$

1 point

For selecting "Equal to" — 1 point

1 point

For selecting "Greater than" — 1 point

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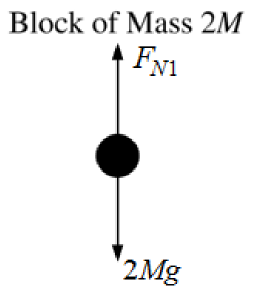
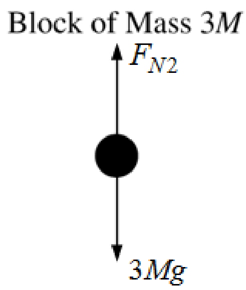
Question

15 points total **Distribution of points**

2 points





For correctly drawing and labeling vectors on the block of mass $2M$ — 1 point

For correctly drawing and labeling vectors on the block of mass $3M$ using symbols for the vectors that are physically correct and different from those on the block of mass $2M$ — 1 point

Note: A maximum of one point can be earned if there are any extraneous vectors.

2 points

Using a proper expression for conservation of momentum

$p_1 = p_2$

For correctly substituting into the above equation — 1 point

$3Mv_0 = (3M + 2M)v_f$

For a correct answer — 1 point

$v_f = \frac{3}{5}v_0$

(1 point

Using a proper expression for kinetic energy of the two-block system

$K = \frac{1}{2}mv^2$

$K = \frac{1}{2}(5M)\left(\frac{3}{5}v_0\right)^2$

For an answer consistent with part (b) — 1 point

$K = \frac{9}{10}Mv_0^2$

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Question continued)

Distribution of points

4 points

For a correct expression of the conservation of energy — 1 point

$\Delta K_{\text{system}} + \Delta U_{\text{system}} = 0$

$K_0 = U_{\text{final}}$

For attempting to integrate the spring force equation — 1 point

$$\frac{9}{10}Mv_0^2 = -\int_{x_1}^{x_2} -Bx^3 dx$$

$$\frac{9}{10}Mv_0^2 = \int_{x_1}^{x_2} Bx^3 dx$$

For using the correct limits of integration or an appropriate constant of integration — 1 point

$$\frac{9}{10}Mv_0^2 = \int_0^D Bx^3 dx$$

$$\frac{9}{10}Mv_0^2 = \left[\frac{Bx^4}{4}\right]_0^D$$

For an answer consistent with the speed from (b) or the kinetic energy from part (c) — 1 point

$$D = \sqrt[4]{\frac{18Mv_0^2}{5B}}$$

2 points

For selecting the correct answer “Right” with a reasonable attempt at a justification — 1 point

If the incorrect selection is made, no points are earned for the justification.

For an indication that at maximum compression the block of mass $2M$ has an acceleration to the right due to the forces acting on the block of mass $2M$ or an acceleration to the right due to the external spring force acting on the system of blocks — 1 point

Example: At maximum compression the two-block system is instantaneously at rest. The only horizontal external force acting on the system is due to the spring. This force is directed to the right. The system and therefore the block of mass $2M$ is accelerated to the right, which implies that the net force acting on the block of mass $2M$ is also to the right.

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Question continued)

Distribution of points

2 points

ɘ magnitude of the net force is greater on the block of mass $3M$.
If the incorrect selection is made, no points are earned for the justification.
For an indication that both blocks will have the same acceleration — 1 point
For a correct justification for why the net force is greater on the block of mass $3M$ — 1 point
Example:
Because the blocks stick together, both blocks must have the same acceleration. Because the block of mass $3M$ has more mass, the net force on it must be greater than the net force on the block of mass $2M$.

2 points

For selecting the correct answer "No," with a reasonable attempt at a justification — 1 point
If the incorrect selection is made, no points are earned for the justification.
If the correct answer is selected without any justification, the point is not earned for the selection.
For an indication that the spring does not apply a linear force and that simple harmonic motion is the resulting motion of a linear restoring force — 1 point

$$\left(a = -\frac{k}{m}\Delta x\right)$$

Example:
Because the blocks are sticking together and are attached to the spring, the spring will apply a restoring force to the blocks. However, because the restoring force exerted by a nonlinear spring is not proportional to the blocks' displacement from equilibrium, the blocks do not exhibit simple harmonic motion.

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Question

15 points total | **Distribution of points**

3 points

For an indication that the spring force equals the mass times the centripetal acceleration — 1 point

$F_S = F_C = ma_C$

$kx = mr\omega^2$

correct substitution for x in the equation — 1 point

For a correct substitution for r in the equation that can be used to solve for k — 1 point

$k\left(\frac{d}{2}\right) = m(2d)\omega^2$

rect answer:

$k = 4m\omega^2$

1 point

Substitute values for the block into the equation for rotational inertia

$I = \sum mr^2 = m(2d)^2$

For a correct answer or an answer consistent with the radius used in part (a) — 1 point

$I = 4md^2$

2 points

For an indication that the rotational inertia of the system must include the platform, the rod, and the object — 1 point

$I = I_P + I_R + I_O$

$I = \frac{m_P R^2}{2} + \frac{m_R d^2}{3} + mr^2$

For correctly substituting into the equation the rotational inertia of the platform, the rod, and the object consistent with (b) i. — 1 point

$I = \frac{5m(2d)^2}{2} + \frac{3md^2}{3} + 4md^2$

$I = (10 + 1 + 4)md^2$

rect answer:

$I = 15md^2$

1 point

Using a correct expression of angular momentum

$L = I\omega$

For an answer consistent with the answer from part (b) ii. — 1 point

$L = 15md^2\omega$

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Question continued)

Distribution of points

3 points

For indicating that the spring force equals the mass times the centripetal acceleration — 1 point

$F_S = F_C = ma_C$

$kx = mr\omega^2$

For a correct substitution for k, or a substitution consistent with part (a), in the equation — 1 point

For a correct substitution for r in the equation that can be used to solve for x — 1 point

$\left(4m\omega^2\right)x = m(d + x)\omega^2$

$4x = d + x$

$3x = d$

Correct answer:

$x = d/3$

2 points

electing "Decreasing" — 1 point

the wrong selection is made, no points are earned for the justification.

For a correct justification — 1 point

Example: The rotational inertia I of the system decreases while the angular velocity w stays the same. Because the angular momentum is Iw, it decreases.

2 points

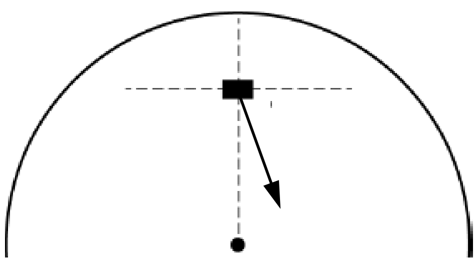
For selecting "Negative" — 1 point

If the wrong selection is made, no points are earned for the justification.

For a correct justification — 1 point

Example: Because the angular velocity is constant and the rotational inertia has decreased, the rotational kinetic energy has decreased. Therefore, work must be negative.

1 point



n arrow starting on the box and pointing down and to the right — 1 point