Question 1: Mathematical Routines (MR) — 10 points

A	**(i)**	For drawing only one arrow for the momentum of Block 2 before the collision that points leftward and is 6 units long	**Point A1**
		For drawing only identical arrows for the momentum of the two-block system before and after the collision that are equal to the sum of the momentums drawn for blocks 1 and 2 before the collision	**Point A2**

Example Response

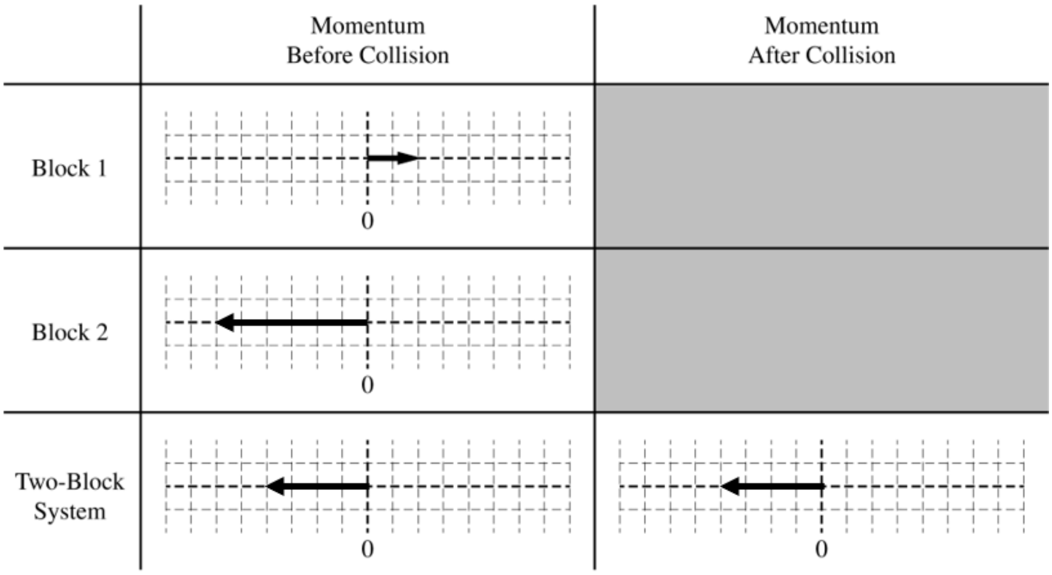


Figure 2

(ii) For a multistep derivation that includes the integral form of impulse or the differential form of Newton's second law — **Point A3**

For indicating **one** of the following: — **Point A4**

- The final speed of Block 1 or Block 2 is $\frac{4}{7}v_0$.
- The change in the momentum of Block 2 is $+\frac{18}{7}mv_0$.
- The change in the momentum of Block 1 is $-\frac{18}{7}mv_0$.
- The change in the velocity of Block 2 is $+\frac{3}{7}v_0$.
- The change in the velocity of Block 1 is $-\frac{18}{7}v_0$.

For substituting the given expression for $F(t)$ into an expression for the impulse exerted on either block or the differential form of Newton's second law	**Point A5**
For an integral with appropriate limits or a constant of integration	**Point A6**
For a correct expression for F_{max} in terms of given quantities	**Point A7**

Example Responses

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{\text{net}}(t)\, dt = \Delta\vec{p} \qquad \text{OR} \qquad \begin{gathered}\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \\ \int \vec{F}_{\text{net}}\, dt = \Delta\vec{p}\end{gathered}$$

$$\sum \vec{p}_0 = \sum \vec{p}_f$$
$$(m)(2v_0) + (6m)(-v_0) = (m + 6m)(v_f)$$
$$v_f = -\frac{4}{7}v_0$$

$$\Delta p_{\text{Block 2}} = -\Delta p_{\text{Block 1}} = 6m\left(-\frac{4}{7}v_0 - (-v_0)\right)$$

$$\Delta p_{\text{Block 2}} = \int_0^{t_c} F_{\text{max}} \sin(At)\, dt$$

$$\frac{18}{7}mv_0 = \frac{F_{\text{max}}}{A}\left[-\cos(At)\right]\Big|_0^{t_c}$$
$$\frac{18}{7}mv_0 = \frac{F_{\text{max}}}{A}\left[1 - \cos(At_c)\right]$$

$$F_{\text{max}} = \frac{18}{7}\left(\frac{Amv_0}{1 - \cos(At_c)}\right)$$

B	For a multistep derivation that includes conservation of momentum	**Point B1**
	For indicating that the magnitude of the final momentum of the two-block system is $7mv_0$	**Point B2**
	For a correct expression for v_1 in terms of v_0	**Point B3**

Example Response

$$\sum \vec{p}_0 = \sum \vec{p}_f$$

$$mv_1 + 6m(-v_0) = 7m(v_0)$$
$$mv_1 = 13mv_0$$
$$v_1 = 13v_0$$

Question 2: Free-Response Question — 15 points

(a)	For selecting "Equal to" with an attempt at a relevant justification	**1 point**
	For a correct justification	**1 point**

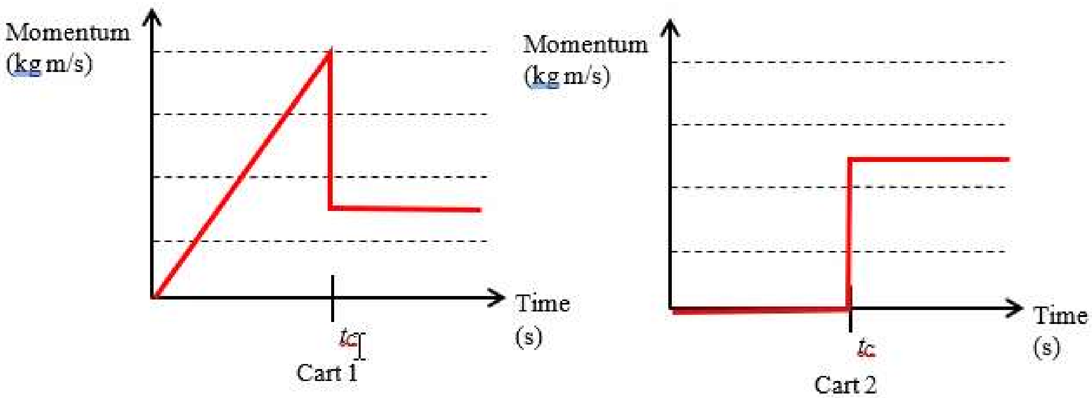
Example Response

Newton's third law says equal/opposite forces, time intervals are same during same collision, so magnitudes of impulse must be equal.

Total for part (a) 2 points

(b)	For correctly drawing the momentum for carts 1 and 2 for the time interval $0 < t < t_C$: Linear increasing momentum for Cart 1 and zero for Cart 2	**1 point**
	For drawing a horizontal line for Cart 1 when $t > t_C$ that is smaller in magnitude than the momentum of Cart 1 at time $t = t_C$	**1 point**
	For drawing a horizontal line for Cart 2 when $t > t_C$ that is greater in magnitude than the momentum of Cart 1 after time $t = t_C$	**1 point**
	For carts 1 and 2 having a change in momentum that is equal in magnitude, such that Cart 1 loses momentum and Cart 2 gains momentum or a response with changes in momentum consistent with the response in part (a)	**1 point**

Example Response



Total for part (b) 4 points

(c)	For using conservation of energy to find the speed of Cart 1 at the bottom of the incline **OR** For a correct substitution of acceleration and displacement in a kinematics equation to find the speed of Cart 1 at the bottom of the incline	**1 point**
	For using conservation of momentum to find the speed of the two-cart system after the collision	**1 point**
	For combining correct equations from above	**1 point**

Example Response

Conservation of energy:

$$m_1 gH = \frac{1}{2} m_1 v_1^2$$

$$v_1 = \sqrt{2gH}$$

OR

$$v_f^2 = v_i^2 + 2g\sin(\theta)L$$
$$H = L\sin(\theta)$$
$$v_f^2 = v_i^2 + 2g\sin(\theta)\left(\frac{H}{\sin\theta}\right)$$
$$v_f^2 = 2gH$$

Conservation of momentum:

$$m_1 v_1 = (m_1 + m_2)v_f$$

$$v_f = \frac{m_1}{m_1 + m_2} v_1$$

Combining:

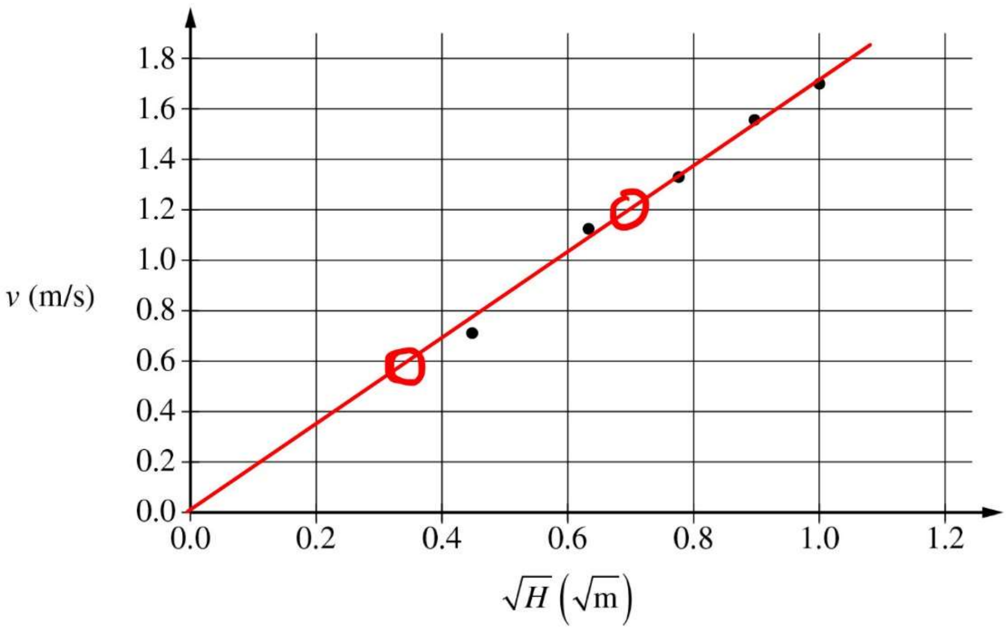
$$v_f = \frac{m_1}{m_1 + m_2}\sqrt{2gH}$$

$$v_f = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}$$

Total for part (c) 3 points

(d)(i)	For drawing an appropriate best-fit line including approximately the same number of points above and below the line	**1 point**

Example Response



(d)(ii)	For calculating the slope using two points on the best-fit line	**1 point**
	For correctly relating the slope of the best-fit line to the mass of Cart 2	**1 point**
	For a correct mass of Cart 2	**1 point**

Example Response

$$slope = \frac{(1.20 - 0.60)\text{ m/s}}{(0.70 - 0.35)\sqrt{\text{m}}} = 1.72\,\frac{\sqrt{\text{m}}}{\text{s}}$$

$$slope = \frac{m_1}{m_1 + m_2}\sqrt{2g}$$

$$m_2 = \frac{m_1\sqrt{2g}}{slope} - m_1$$

$$m_2 = \frac{(0.25\text{ kg})\sqrt{2\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)}}{1.72\,\frac{\sqrt{\text{m}}}{\text{s}}} - (0.25\text{ kg})$$

$$\therefore m_2 = 0.39\text{ kg}$$

Scoring Note: Acceptable responses for mass are 0.30 to 0.60 kg

Total for part (d) 4 points

(e)	For selecting "$m_1' < 0.250$ kg" with an attempt at a relevant justification	**1 point**
	For a correct justification	**1 point**

Example Response

A smaller m_2 indicates that the initial energy and momentum was smaller. With identical slope and height H this means that the mass m_1' must be smaller.

Total for part (e) 2 points

Total for question 2 15 points

Question 2: Free-Response Question 15 points

(a)	For selecting "$J_S > J_2$" with an attempt at a relevant justification	**1 point**
	For a correct justification	**1 point**

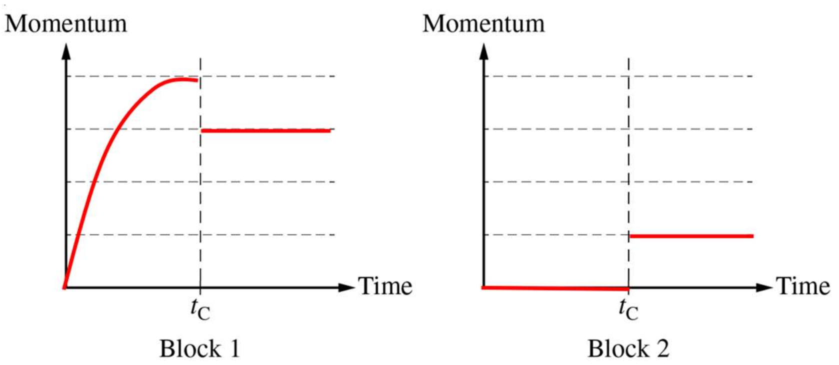
Example Response

The impulse between the two blocks is the same, so because Block 1 is moving at the end, the impulse given by the spring must be greater than the impulse given to Block 2 by Block 1.

Total for part (a) 2 points

(b)	For correctly drawing the momentum for blocks 1 and 2 for the time interval $0 < t < t_C$: Block 1 shows an increasing curve with a decreasing slope (concave down) to highest value (technically a cosine curve) and Block 2 shows zero momentum up to t_C (a horizontal line along the time axis).	**1 point**
	For drawing a horizontal line for Block 1 when $t > t_C$ that is smaller in magnitude than the momentum of Block 1 at time $t = t_C$	**1 point**
	For drawing a horizontal line for Block 2 when $t > t_C$ that is smaller in magnitude than the momentum of Block 1 after time $t = t_C$	**1 point**
	For blocks 1 and 2 having a change in momentum that is equal in magnitude such that Block 1 loses momentum and Block 2 gains momentum	**1 point**

Example Response



Total for part (b) 4 points

(c)	For using conservation of energy to find the speed of Block 1 at $x = 0$	**1 point**
	For using conservation of momentum to find the speed of the two-block system after the collision	**1 point**
	For combining correct equations from above	**1 point**

Example Response

$$\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}m_1 v_1^2$$

$$v_1 = \sqrt{\frac{k}{m_1}}\Delta x$$

$$m_1 v_1 = (m_1 + m_2)v$$

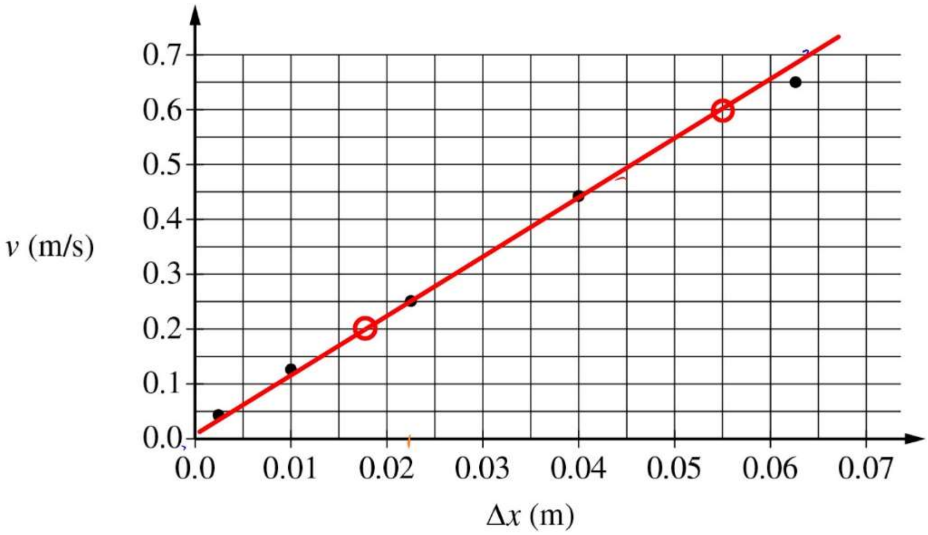
$$v = \frac{m_1 v_1}{m_1 + m_2}$$

$$\therefore\ v = \frac{\Delta x\sqrt{k m_1}}{m_1 + m_2}$$

Total for part (c) 3 points

(d)(i)	For drawing an appropriate best-fit line including approximately the same number of points above and below the line	**1 point**

Example Response



(d)(ii)	For calculating the slope using two points on the line	**1 point**
	For correctly relating the slope of the best-fit line to the mass of Block 2	**1 point**
	For a correct mass of Block 2	**1 point**

Example Response

$$slope = \frac{(0.60 - 0.02)\text{ m/s}}{(0.055 - 0.0175)\text{ m}} = 10.67\text{ s}^{-1}$$

$$slope = \frac{\sqrt{km_1}}{m_1 + m_2}$$

$$m_2 = \frac{\sqrt{km_1}}{slope} - m_1$$

$$m_2 = \frac{\sqrt{\left(150\,\frac{\text{N}}{\text{m}}\right)(0.50\text{ kg})}}{10.67\text{ s}^{-1}} - 0.50\text{ kg}$$

$$m_2 = 0.31\text{ kg}$$

Scoring Note: Acceptable responses for mass are $0.27\text{ kg} - 0.37\text{ kg}$.

Total for part (d) 4 points

(e)	For selecting "$k' > 150\text{ N/m}$" with an attempt at relevant justification	**1 point**
	For a correct justification	**1 point**

Example Response

A greater m_2 *indicates the spring constant* k' *should be greater than* 150 N/m. *The slope of the graph is the same so as* m_2 *increases* k *must be smaller.*

Total for part (e) 2 points

Total for question 2 15 points

2019

AP® CollegeBoard

AP® Physics C: Mechanics

Scoring Guidelines Set 2

2019 SCORING GUIDELINES

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

2019 SCORING GUIDELINES

Question 1

15 points

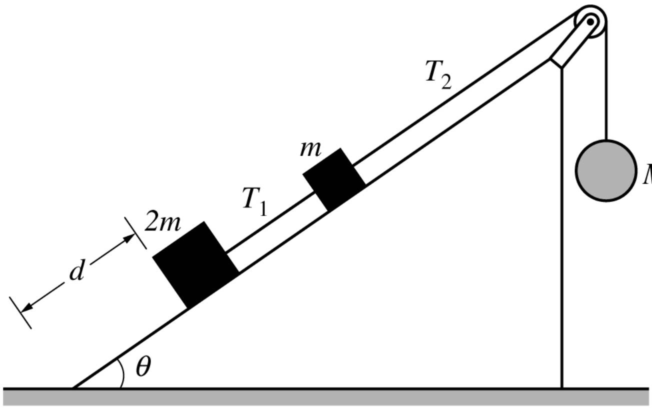
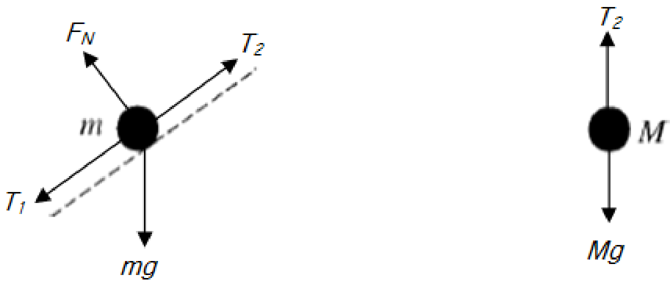


Figure 1

Blocks of mass m and $2m$ are connected by a light string and placed on a frictionless inclined plane that makes an angle θ with the horizontal, as shown in Figure 1 above. Another light string connecting the block of mass m to a hanging sphere of mass M passes over a pulley of negligible mass and negligible friction. The entire system is initially at rest and in equilibrium.

(a) LO INT-1.A, SP 3.D
3 points

On the dots below that represent the block of mass m and the sphere of mass M, draw and label the forces (not components) that act on each of the objects shown. Each force must be represented by a distinct arrow starting on and pointing away from the dot.



For correctly drawing and labeling vectors representing the normal force and the gravitational force on the block of mass m		1 point
For correctly drawing and labeling vectors representing the forces of tension on the block of mass m		1 point
For correctly drawing and labeling vectors representing the tension force and the gravitational force on the sphere of mass M		1 point
Note: A maximum of two points can be earned if there are any extraneous vectors.		

2019 SCORING GUIDELINES

Question 1 (continued)

(b)

Derive expressions for the magnitude of each of the following. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figures in part (a).

i. LO INT-1.C.e, SP 1.D, 5.E
2 points

The force T_2 exerted on the block of mass m by the string. Express your answers in terms of m, θ , and physical constants, as appropriate.

For using an attempt at a correct statement of Newton's second law for the two blocks		1 point
$F_{net} = (m + 2m)a = 0 \therefore T_2 - (m + 2m)g\sin\theta = 0$		
For a correct answer with supporting work		1 point
$T_2 = 3mg\sin\theta$		

ii. LO INT-1.D, SP 5.E
1 point

The mass M for which the system can remain in equilibrium. Express your answers in terms of m, θ , and physical constants, as appropriate.

For using a correct statement of Newton's second law for the whole system to derive an expression for M		1 point
$F_{net} = m_{tot}a \therefore Mg - 2mg\sin\theta - mg\sin\theta = 0$		
$M = 3m\sin\theta$		
Alternate Solution		*Alternate Points*
For a correct statement of Newton's second law for the sphere and an answer consistent with part (b)(i)		1 point
$T_2 - Mg = 0 \therefore Mg = T_2 \therefore M = T_2/g$		
$M = 3m\sin\theta$		

2019 SCORING GUIDELINES

Question 1 (continued)

(c)

Now suppose that mass M is large enough to descend and that the sphere reaches the floor before the blocks reach the pulley. Answer the following for the moment immediately after the sphere reaches the floor.

i. LO INT-1.D, SP 7.A
1 point

Does the tension T_1 increase, decrease to a nonzero value, decrease to zero, or stay the same?

____ Increase ____ Decrease to a nonzero value

____ Decrease to zero ____ Stay the same

For correctly stating that the magnitude of T_1 drops to zero		1 point

ii. LO INT-1.D, SP 7.A
1 point

Is the velocity of the block of mass m up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

For selecting "Up the ramp"		1 point

iii. LO INT-1.D, SP 7.A
1 point

Is the acceleration of the block of mass m up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

For selecting "Down the ramp"		1 point

(d) LO INT-1.D, SP 5.A, 5.E
3 points

Consider the initial setup in Figure 1. Now suppose the surface of the incline is rough and the coefficient of static friction between the blocks and the inclined plane is μ_s. Derive an expression for the minimum possible value of M that will keep the blocks from moving down the incline. Express your answer in terms of m, μ_s, θ, and fundamental constants, as appropriate.

For an attempt at a correct statement of Newton's second law for the system		1 point
$F_{\text{net}} = m_{\text{tot}}a \therefore 2mg\sin\theta - f_{s2} + mg\sin\theta - f_{s1} - Mg = 0$		
For attempting to substitute in for the force of friction		1 point
$3mg\sin\theta - \mu_s F_{N2} - \mu_s F_{N1} = Mg$		
$3mg\sin\theta - \mu_s(2mg\cos\theta) - \mu_s(mg\cos\theta) = Mg$		
For a correct answer with supporting work		1 point
$M = 3m(\sin\theta - \mu_s\cos\theta)$		

2019 SCORING GUIDELINES

Question 1 (continued)

(e) LO INT-1.D, CHA-1.A.b, SP 5.A, 5.E
3 points

The string connecting block *m* and the sphere of mass *M* then breaks, and the blocks begin to move from rest down the incline. The lower block starts a distance *d* from the bottom of the incline, as shown in Figure 1. The coefficient of kinetic friction between the blocks and the inclined plane is μ_k. Derive an expression for the speed of the blocks when the lower block reaches the bottom of the incline. Express your answer in terms of *m*, *d*, μ_k, θ, and fundamental constants, as appropriate.

For an attempt at a correct statement of Newton's second law for the two blocks		1 point
$F_{\text{net}} = m_{tot}a \therefore 2mg\sin\theta - f_{k2} + mg\sin\theta - f_{k1} = 3ma$		
Solve for the acceleration		
$3mg\sin\theta - \mu_k(2mg\cos\theta) - \mu_k mg\cos\theta = 3ma$		
$a = g(\sin\theta - \mu_k\cos\theta)$		
For using a correct kinematics equation to solve for the final velocity		1 point
$v_2^2 = v_1^2 + 2ad$		
$v_2^2 = 0^2 + 2g(\sin\theta - \mu_k\cos\theta)d$		
For a correct answer with supporting work		1 point
$v = \sqrt{2gd(\sin\theta - \mu_k\cos\theta)}$		
Alternate Solution		*Alternate Points*
For an attempt at a correct statement of conservation of energy for the two blocks		*1 point*
$U_1 + K_1 = U_2 + K_2 + E_{\text{lost}}$		
$2mgh + mgh + 0 = 0 + \frac{1}{2}(2m)v^2 + \frac{1}{2}mv^2 + fd$		
For correct substitutions of height and friction		*1 point*
$3mgd\sin\theta = \frac{1}{2}(3m)v^2 + \mu_k(2mg\cos\theta)d + \mu_k(mg\cos\theta)d$		
$3gd\sin\theta - 3\mu_k gd\cos\theta = \frac{3}{2}v^2$		
For a correct answer with supporting work		*1 point*
$v = \sqrt{2gd(\sin\theta - \mu_k\cos\theta)}$		

Question 1 (continued)

Learning Objectives

CHA-1.A.b: Calculate unknown variables of motion such as acceleration, velocity, or positions for an object undergoing uniformly accelerated motion in one dimension.
INT-1.A: Describe an object (either in a state of equilibrium or acceleration) in different types of physical situations such as inclines, falling through air resistance, Atwood machines, or circular tracks).
INT-1.C.e: Derive a complete Newton's second law statement (in the appropriate direction) for an object in various physical dynamic situations (e.g., mass on incline, mass in elevator, strings/pulleys, or Atwood machines).
INT-1.D: Calculate a value for an unknown force acting on an object accelerating in a dynamic situation (e.g., inclines, Atwood Machines, falling with air resistance, pulley systems, mass in elevator, etc.)

Science Practices

1.D: Select relevant features of a representation to answer a question or solve a problem.
3.C: Sketch a graph that shows a functional relationship between two quantities.
3.D: Create appropriate diagrams to represent physical situations.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
7.A: Make a scientific claim.

2019 SCORING GUIDELINES

Question 2

15 points

A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0\ \text{m/s}^2$, $L = 0.25\ \text{m/s}^4$, and t is the time in seconds. At $t = 6.0$ s, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

(a) LO INT-5.E, SP 6.B, 6.C
2 points

Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0$ s.

For an expression for calculating impulse and correct substitution of $a(t)$ and m into the correct expression.		1 point
$J = \int F(t)dt$		
$J = \int ma(t)dt = (0.50)\int(9.0 - 0.25t^2)dt$		
For integrating with correct limits or including a constant of integration		1 point
$J = (0.50)\int_{t=0}^{t=6}(9.0 - 0.25t^2)dt = (0.50)\left[9.0t - \frac{1}{12}t^3\right]_{t=0}^{t=6}$		
$J = (0.50)\left[\left((9.0)(6) - \left(\frac{1}{12}\right)(6)^3\right)\right] - 0 = 18\ \text{N•s}$		
Alternate Solution (using an alternate solution from part (b))		*Alternate Points*
For correctly relating impulse to the speed of the rocket		1 point
$J = \Delta p = m(v_2 - v_1)$ OR $J = m\Delta v = mv_f$		
For correctly substituting answer from part (b) into equation above		1 point
$J = (0.50\ \text{kg})(36 - 0) \therefore J = 18 N \bullet s$		

Question 2 (continued)

(b) LO INT-5.A.a, SP 6.A, 6.C
2 points

Calculate the speed of the rocket at $t = 6.0$ s.

For correctly relating impulse to the speed of the rocket		1 point
$J = \Delta p = m(v_2 - v_1)$ OR $J = m\Delta v = mv_f$		
For correctly substituting answer from part (a) into equation above		1 point
$(18\ \text{N}\bullet\text{s}) = (0.50\ \text{kg})(v_2 - 0)\ \therefore\ v_2 = 36\ \text{m/s}$		
Alternate Solution		*Alternate Points*
Integrate expression for a(t) (This may have already been done in solving part (a).)		1 point
$v = \int a dt = \int (9.0 - 0.25t^2) dt$		
For integrating with correct limits or including a constant of integration		1 point
$v = \int_{t=0}^{t=6} (9.0 - 0.25t^2) dt = \left[9.0t - \frac{1}{12}t^3\right]_{t=0}^{t=6} = 36\ \text{m/s}$		

(c)
i. LO INT-4.C.c, SP 6.B, 6.C
2 points

Calculate the kinetic energy of the rocket at $t = 6.0$ s.

For substituting the mass of the rocket into the equation for kinetic energy		1 point
For substituting the answer from part (b) into the equation for kinetic energy		1 point
$K = \frac{1}{2}mv^2 = \left(\frac{1}{2}\right)(0.50\ \text{kg})(36\ \text{m/s})^2 = 324\ \text{J}$		

ii. LO CHA-1.B, CON-1.E, SP 6.A, 6.C
3 points

Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.

For integrating the acceleration twice to derive an expression for position		1 point
For integrating with correct limits or including a constant of integration		1 point
$v(t) = \int a(t) dt = \int (9.0 - 0.25t^2) dt = 9.0t - \frac{1}{12}t^3 + v_0 = 9.0t - \frac{1}{12}t^3$		
$\Delta y = \int v(t) dt = \int_{t=0}^{t=6} \left(9.0t - \frac{1}{12}t^3\right) dt = \left[\frac{9}{2}t^2 - \frac{1}{48}t^4\right]_{t=0}^{t=6}$		
$\Delta y = \left[\left(\frac{9}{2}\right)(6)^2 - \left(\frac{1}{48}\right)(6)^4\right] - 0 = 135\ \text{m}$		
For substituting into equation for potential energy		1 point
$\Delta U_g = mg\Delta y = (0.50\ \text{kg})(9.8\ \text{m/s}^2)(135\ \text{m}) = 660\ \text{J}$		

Question 2 (continued)

(d) LO CHA-1.B, CON-2.B, SP 6.B, 6.C
3 points

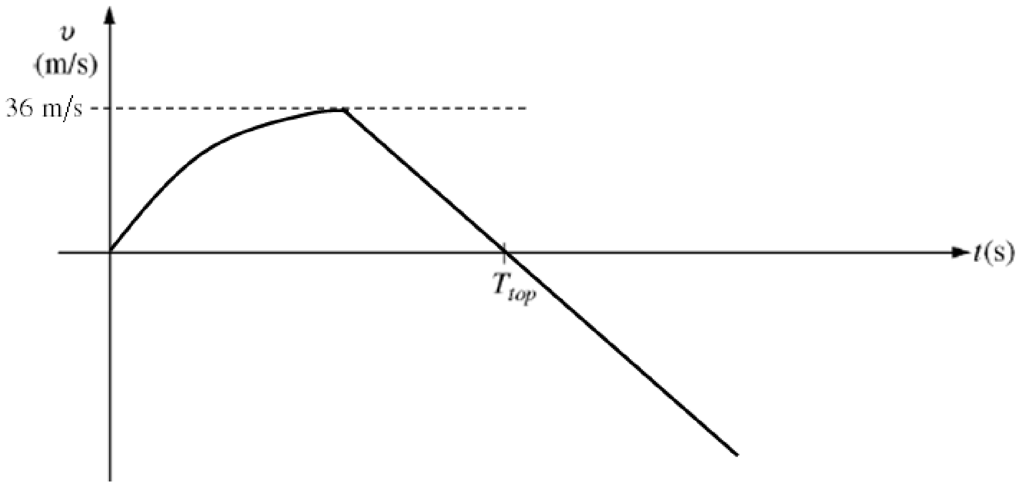
Calculate the maximum height reached by the rocket relative to its launching point.

For using $a = g$ in a correct kinematics equation to solve for height		1 point
$v_2^2 = v_1^2 + 2a(y_2 - y_1) \therefore 0 = v_1^2 + 2a(y_2 - y_1)$		
$y_2 - y_1 = -\dfrac{v_1^2}{2a} \therefore y_2 = -\dfrac{v_1^2}{2a} + y_1$		
For substituting the speed from part (b)		1 point
$y_2 = -\dfrac{(36\ \text{m/s})^2}{2\left(-9.8\ \text{m/s}^2\right)} + y_1$		
For substituting the height from part (c)		1 point
$y_2 = 66\ \text{m} + 135\ \text{m} = 201\ \text{m}$ (199.8 m if $g = 10\ \text{m/s}^2$)		
Alternate Solution 1		*Alternate Points*
For using energy conservation to find maximum height, consistent with the speed found in part (b).		1 point
$mg\Delta y = \dfrac{1}{2}mv^2 \therefore \Delta y = \dfrac{v^2}{2g}$		
For substituting the speed from part (b)		1 point
$\Delta y = \dfrac{(36\ \text{m/s})^2}{2\left(9.8\ \text{m/s}^2\right)} = 66\ \text{m}$		
For substituting the height from part (c)		1 point
$y_2 = 66\ \text{m} + 135\ \text{m} = 201\ \text{m}$ (199.8 m if $g = 10\ \text{m/s}^2$)		
Alternate Solution 2		*Alternate Points*
For using energy conservation to find maximum height, from kinetic and potential energies found in part (c)		1 point
$K_1 + U_1 = 0 + U_{top}$		
For substituting the kinetic energy from part (c)(i) into the equation above		1 point
For substituting the potential energy from part (c)(ii) into the equation above		1 point
$324 + 660 = (0.5)(9.8)\Delta y$		
$\Delta y = 201\ \text{m}$ (199.8 m if g = 10m/s^2)		

Question 2 (continued)

(e) LO CHA-1.C, SP 3.C
3 points

On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



For an initial concave down curve that starts at the origin.		1 point
For a transition that occurs before T_{top} into a straight line with a negative slope.		1 point
For labeling the maximum value of the velocity, consistent with part (b), and a line that crosses the x-axis at T_{top}		1 point

Question 2 (continued)

Learning Objectives

CHA-1.B: Determine functions of position, velocity, and acceleration that are consistent with each other, for the motion of an object with a nonuniform acceleration.
CHA-1.C: Describe the motion of an object in terms of the consistency that exists between position and time, velocity and time, and acceleration and time.
CON-1.E: Calculate the potential energy of a system consisting of an object in a uniform gravitational field.
CON-2.B: Describe kinetic energy, potential energy, and total energy in relation to time (or position) for a "conservative" mechanical system.
INT-4.C.c: Calculate changes in an object's kinetic energy or changes in speed that result from the application of specified forces.
INT-5.A.a: Calculate the total momentum of an object or system of objects.
INT-5.E: Calculate the change in momentum of an object given a nonlinear function, $F(t)$, for a net force acting on the object.

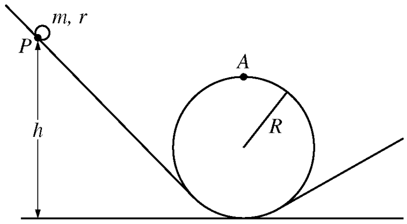
Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.
6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

2019 SCORING GUIDELINES

Question 3

15 points



Note: Figure not drawn to scale.

The rotational inertia of a rolling object may be written in terms of its mass m and radius r as $I = bmr^2$, where b is a numerical value based on the distribution of mass within the rolling object. Students wish to conduct an experiment to determine the value of b for a partially hollowed sphere. The students use a looped track of radius $R >> r$, as shown in the figure above. The sphere is released from rest a height h above the floor and rolls around the loop.

(a) LO INT-2.D, SP 5.A, 5.E
2 points

Derive an expression for the minimum speed of the sphere's center of mass that will allow the sphere to just pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

For an expression relating the gravitational force to the centripetal force		1 point
$F_C = mg + F_N = \frac{mv^2}{R}$		
$mg + 0 = \frac{mv^2}{R}$		
For a correct substitution into a correct expression.		1 point
$v = \sqrt{Rg}$		

2019 SCORING GUIDELINES

Question 3 (continued)

(b) LO INT-7.E, SP 5.A, 5.E
3 points

Suppose the sphere is released from rest at some point *P* and rolls without slipping. Derive an equation for the minimum release height *h* that will allow the sphere to pass point *A* without losing contact with the track. Express your answer in terms of *b, m, R,* and fundamental constants, as appropriate.

For using a correct expression of the conservation of energy for the object		1 point
$K_1 + U_1 = K_2 + U_2$		
For correct substitutions, including rotational kinetic energy		1 point
$0 + mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh_2$		
$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(bmr^2\right)\left(\frac{v}{r}\right)^2 + mgh_2$		
For correctly substituting for the final height and the answer from part (a) for the velocity		1 point
$gh = \frac{1}{2}\left(\sqrt{Rg}\right)^2 + \frac{1}{2}(b)\left(\sqrt{Rg}\right)^2 + g(2R)$		
$h = \frac{1}{2}R + \frac{1}{2}bR + 2R = \frac{1}{2}(b+5)R$		

2019 SCORING GUIDELINES

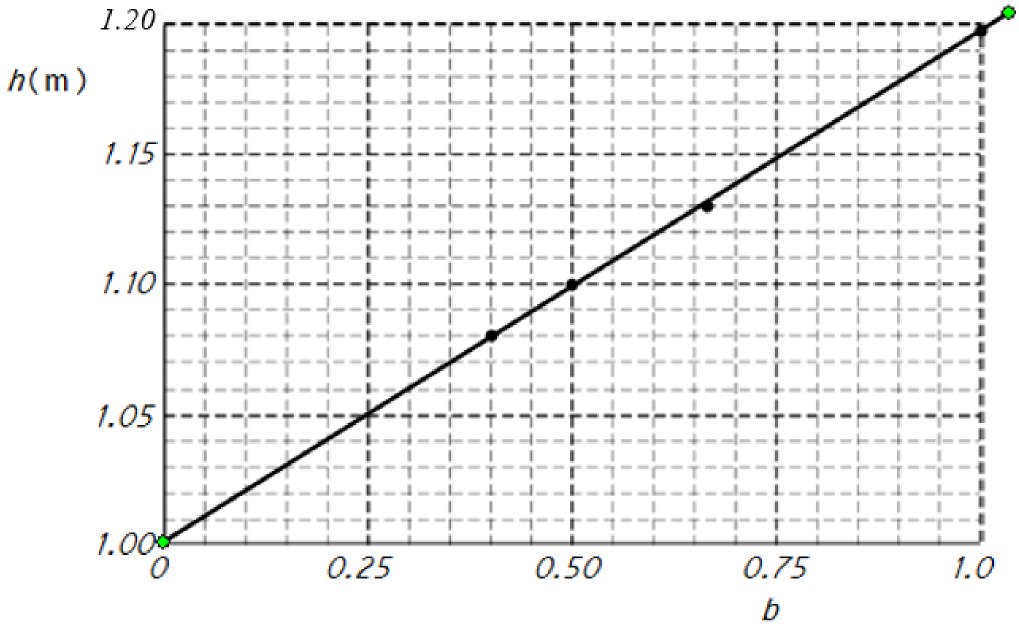
Question 3 (continued)

The students perform an experiment by determining the minimum release height h for various other objects of radius r and known values of b. They collect the following data.

Object	b	h (m)
Solid sphere	0.40	1.08
Hollow sphere	0.67	1.13
Solid cylinder	0.50	1.10
Hollow cylinder	1.0	1.20

(c) LO INT-7.E, SP 3.A, 4.C
4 points

On the grid below, plot the release height h as a function of b. Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



For correctly labeling both axes with variables, units for h, no units for b		1 point
For correctly using and labeling the scale of the axes so that the data uses at least half the grid		1 point
For correctly plotting the data		1 point
For drawing a best-fit straight line that represents the data		1 point

2019 SCORING GUIDELINES

Question 3 (continued)

(d) LO INT-7.E, SP 4.D, 6.C
2 points

The students repeat the experiment with the partially hollowed sphere and determine the minimum release height to be 1.16 m. Using the straight line from part (c), determine the value of *b* for the partially hollowed sphere.

For correctly calculating the slope from the best-fit straight line and substituting height into the line equation		1 point
slope $= \dfrac{1.20 - 1.00}{1.0 - 0.0} = 0.20$ m		
$y = mx + b \therefore h = 0.20b + 1.00$		
$1.16 = 0.20b + 1.00$		
For a correct answer for *b*		1 point
$b = 0.80$		
Note: Full credit can be earned for the correct answer if there is an indication that this was read from the graph.		

(e) LO INT-7.E, SP 6.A, 6.C
2 points

Calculate *R*, the radius of the loop.

For substituting into the expression from part (b)		1 point
$h = \dfrac{1}{2}(b+5)R \therefore R = \dfrac{2(1.16\text{ m})}{(0.80+5)}$		
For an answer consistent with previous parts		1 point
$R = 0.40$ m		
Alternate Solution		*Alternate Points*
For using an expression for the y-intercept (set b = 0)		1 point
$h = \dfrac{1}{2}(b+5)R = \dfrac{1}{2}(0+5)R = \dfrac{5}{2}R$		
For determining the value of the intercept (1.0 m) from the graph		1 point
$1.0\text{ m} = \dfrac{5}{2}R \therefore R = \dfrac{2}{5}\text{ m} = 0.40\text{ m}$		

2019 SCORING GUIDELINES

Question 3 (continued)

(f) LO INT-7.E, SP 7.A, 7.C
2 points

In part (b), the radius r of the rolling sphere was assumed to be much smaller than the radius R of the loop. If the radius r of the rolling sphere was not negligible, would the value of the minimum release height h be greater, less, or the same?

____ Greater ____ Less ____ The same

Justify your answer.

For selecting "Less" and any justification		1 point
For a correct justification		1 point
Examples:		
If the radius of the object is not negligible, then the center of mass of the object travels in a radius less than R. If the radius of the circle decreases, the height needed to pass through the loop decreases according to the equation from part (b).		
If the radius of the object is not negligible, then the center of mass of the object travels in a radius less than R. If the radius of the circle decreases, the speed needed to pass through the loop decreases, and thus the height needed also decreases.		

Learning Objectives

INT-2.D: Derive expressions relating centripetal force to the minimum speed or maximum speed of an object moving in a vertical circular path.
INT-7.E: Derive expressions using energy conservation principles for physical systems such as rolling bodies on inclines, Atwood Machines, pendulums, physical pendulums, and systems with massive pulleys that relate linear or angular motion characteristics to initial conditions (such as height or position) or properties of rolling body (such as moment of inertia or mass).

Science Practices

3.A: Select and plot appropriate data.
4.C: Linearize data and/or determine a best-fit line or curve.
4.D: Select relevant features of a graph to describe a physical situation or solve problems.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

2018

AP® CollegeBoard

AP Physics C: Mechanics

Scoring Guidelines

2018 SCORING GUIDELINES

General Notes About 2018 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

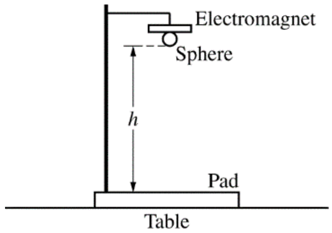
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

2018 SCORING GUIDELINES

Question 1

15 points total **Distribution of points**



A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

(a) 2 points

While taking the first data point, the student notices that the electromagnet actually releases the sphere after the timer begins. Would the value of g calculated from this one measurement be greater than, less than, or equal to the actual value of g at the student's location?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

For selecting "Less than" with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because the measured time to fall the same distance will be larger, the acceleration must be less.		
Example: Because the time is larger and $a \propto 1/t^2$ *, then g must decrease.*		

AP® PHYSICS C: MECHANICS
2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

The electromagnet is replaced so that the timer begins when the sphere is released. The student varies the distance h. The student measures and records the time Δt of the fall for each particular height, resulting in the following data table.

h (m)	0.10	0.20	0.60	0.80	1.00
Δt (s)	0.105	0.213	0.342	0.401	0.451

(b) 1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for g.

Vertical axis: ____________

Horizontal axis: ____________

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given in the table. Label each row you use and include units.

For correctly indicating two variables that will yield a straight line that could be used to determine a value for g		1 point
Example: Vertical Axis: h *Horizontal Axis:* $(\Delta t)^2$		
Note: Student earns full credit if axes are reversed or if they use another acceptable combination.		

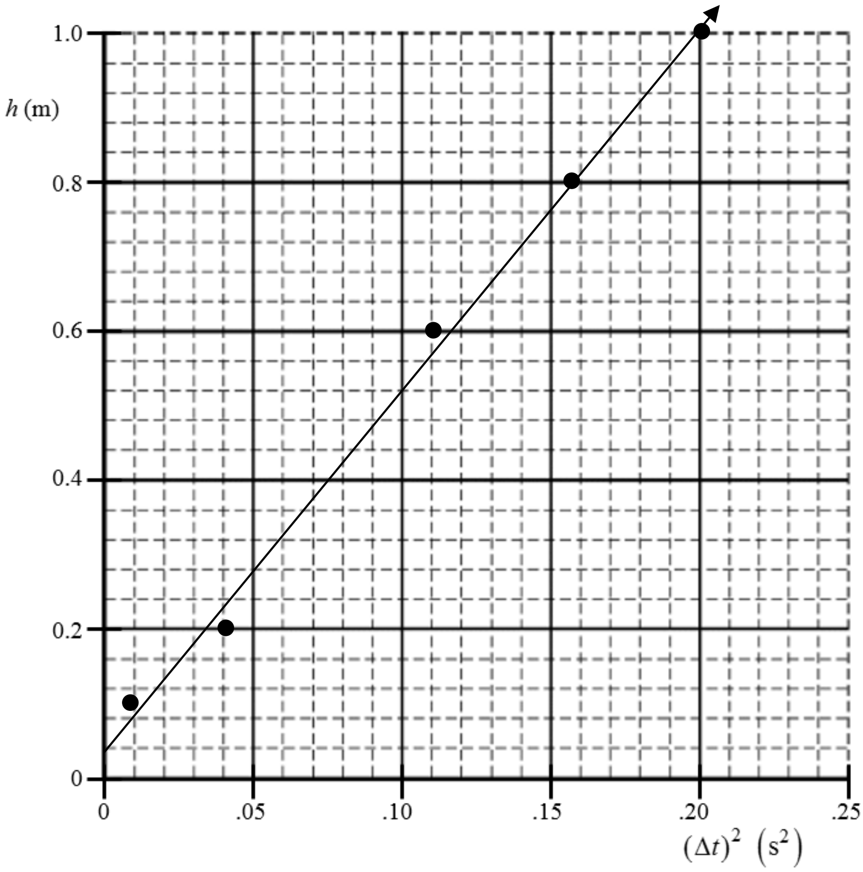
2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(c) 4 points

Plot the data points for the quantities indicated in part (b) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



For a correct scale that uses more than half the grid		1 point
For correctly labeling the axis with variables and units consistent with part (b)		1 point
For correctly plotting data consistent with part (b)		1 point
For drawing a straight line consistent with the plotted data		1 point

2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(d) 2 points

Using the straight line, calculate an experimental value for g.

For using points on the line rather than the data to calculate the slope		1 point
$\text{slope} = \dfrac{\Delta h}{\Delta(\Delta t)^2} = \dfrac{(0.80 - 0.20)\text{ m}}{(0.160 - 0.039)\text{ s}^2} = 4.96\text{ m/s}^2$ (Linear regression = 4.83 m/s^2)		
For correctly relating the slope to the acceleration due to gravity		1 point
$\text{slope} = \dfrac{1}{2}g \therefore g = 2 \times \text{slope} = 2 \times \left(4.96\text{ m/s}^2\right)$		
$g = 9.92\text{ m/s}^2$ (Linear regression = 9.66 m/s^2)		

Another student fits the data in the table to a quadratic equation. The student's equation for the distance fallen y as a function of time t is $y = At^2 + Bt + C$, where $A = 5.75\text{ m/s}^2$, $B = -0.524\text{ m/s}$, and $C = +0.080\text{ m}$. Vertically down is the positive direction.

(e) Using the student's equation above, do the following.

i. 1 point

Derive an expression for the velocity of the sphere as a function of time.

For correctly taking the time derivative of the given equation		1 point
$y = y_0 + v_1 t + \dfrac{1}{2}at^2 = At^2 + Bt + C$		
$v(t) = \dfrac{dy}{dt} = 2At + B$		
Note: Credit is earned for substituting numbers: $v(t) = \left(11.5\text{ m/s}^2\right)t - 0.524\text{ m/s}$.		

ii. 2 points

Calculate the new experimental value for g.

For correctly relating the given equation to a correct kinematic equation		1 point
$\Delta y = y - y_0 = v_1 t + \dfrac{1}{2}at^2$		
$y = y_0 + v_1 t + \dfrac{1}{2}at^2 = At^2 + Bt + C$		
For correctly relating the equation to the value of g		1 point
$A = \dfrac{1}{2}a = \dfrac{1}{2}g$		
$g = 2A = (2)\left(5.75\text{ m/s}^2\right) = 11.5\text{ m/s}^2$		

AP® PHYSICS C: MECHANICS
2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

iii. 1 point

Using 9.81 m/s^2 as the accepted value for g at this location, calculate the percent error for the value found in part (e)(ii).

For correctly calculating the percent error		1 point
$\%\text{error} = \dfrac{\lvert\text{acc} - \text{exp}\rvert}{\text{acc}} \times 100\% = \dfrac{\left\lvert\left(11.5 \text{ m/s}^2\right) - \left(9.81 \text{ m/s}^2\right)\right\rvert}{\left(9.81 \text{ m/s}^2\right)} \times 100\%$		
$\%\text{error} = 17.2\%$		
Note: Credit is earned if percent error is expressed as positive or negative.		

iv. 2 points

Assuming the sphere is at a height of 1.40 m at $t = 0$, calculate the velocity of the sphere just before it strikes the pad.

For relating the coefficients of the equation to the kinematic variables		1 point
$y = At^2 + Bt + C \;\therefore\; v_1 = B = -0.524 \text{ m/s}$		
$a = 2A = 2 \times \left(5.75 \text{ m/s}^2\right) = 11.5 \text{ m/s}^2$		
$y_0 = C = 0.080 \text{ m}$		
For correctly using an appropriate kinematics equation to determine the velocity of the sphere		1 point
$v_2^2 = v_1^2 + 2a\Delta y$		
$v = \sqrt{v_1^2 + 2a\Delta y} = \sqrt{(-0.524 \text{ m/s})^2 + (2)\left(11.5 \text{ m/s}^2\right)(1.40 \text{ m} - 0.080 \text{ m})}$		
$v = 5.54 \text{ m/s}$		
Alternate solution:		*Alternate Points*
For correctly determining the time of fall for the sphere		*1 point*
$y = At^2 + Bt + C$		
$1.40 = \left(5.75 \text{ m/s}^2\right)t^2 + (-0.524 \text{ m/s})t + (0.080 \text{ m})$		
$5.75t^2 - 0.524t - 1.32 = 0$		
$t = 0.53 \text{ s or } -0.44 \text{ s} \;\therefore\; t = 0.53 \text{ s}$		
For correctly using the equation from part (e)(i)		*1 point*
$v(t) = \left(11.5 \text{ m/s}^2\right)(0.53 \text{ s}) - 0.524 \text{ m/s}$		
$v = 5.54 \text{ m/s}$		

AP® PHYSICS C: MECHANICS
2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(e)

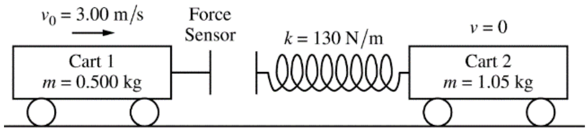
iv. (continued)

Alternate Solution — Conservation of Energy		
For relating the coefficients of the equation to the initial height and speed		1 point
$y = At^2 + Bt + C \;\therefore v_1 = B = -0.524 \text{ m/s}$		
$y_0 = C = 0.080 \text{ m}$		
For correctly using conservation of energy to determine the velocity of the sphere		1 point
$U_i + K_i = U_f + K_f$		
$mgh + \frac{1}{2}mv_1^2 = \frac{1}{2}mv^2$		
$v = \sqrt{2(11.5)(1.40 - 0.080) + (-0.524)^2} = 5.54 \text{ m/s}$		

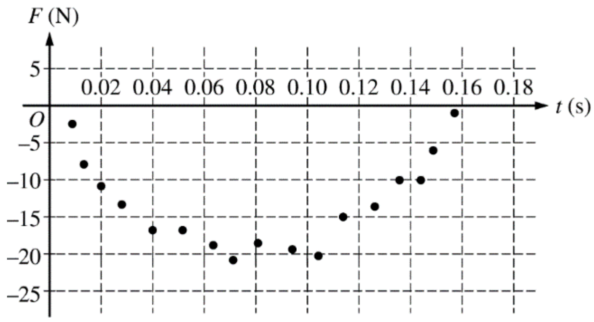
2018 SCORING GUIDELINES

Question 2

15 points total | **Distribution of points**



Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00\ \text{m/s}$ toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130\ \text{N/m}$. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200\ \text{N/s}^2\right)t^2 - \left(500\ \text{N/s}\right)t$.



(a) 3 points

Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

For using the given force equation in the integral to determine the impulse delivered to the cart		1 point
$J = \int F \bullet dt = \int \left(3200t^2 - 500t\right) dt$		
For integrating the force using the correct limits or constant of integration		1 point
$J = \int_{t=0}^{t=0.16\ \text{s}} \left(3200t^2 - 500t\right)dt = \left[\frac{3200}{3}t^3 - 250t^2\right]_{t=0}^{t=0.16}$		
$J = \left(\left(\frac{3200}{3}\right)(0.16)^3 - (250)(0.16)^2\right) - 0$		
For a correct answer		1 point
$J = -2.03\ \text{N}\bullet\text{s}$		

Question 2 (continued)

Distribution of points

(b)

i. 1 point

Calculate the speed of cart 1 after the collision.

For correct substitution into the impulse-momentum equation of the answer from part (a) to determine the speed of cart 1		1 point
$J = m_1\left(v_{1f} - v_{1i}\right) \therefore v_{1f} = \dfrac{J}{m_1} + v_{1i} = \dfrac{(-2.03\ \text{N•s})}{(0.500\ \text{kg})} + (3.00\ \text{m/s})$		
$v_{1f} = -1.06\ \text{m/s}$		

ii. 1 point

In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

For correctly selecting "Left"		1 point

(c)

i. 2 points

Calculate the speed of cart 2 after the collision.

For using a correct equation to determine the speed of cart 2		1 point
$-J = -m_1\left(v_{1f} - v_{1i}\right) = m_2 v_{2f} \therefore v_{2f} = \dfrac{-J}{m_2}$		
For correct substitution of the answer from part (a)		1 point
$v_{2f} = \dfrac{(2.03\ \text{N•s})}{(1.05\ \text{kg})}$		
$v_{2f} = 1.93\ \text{m/s}$		

AP® PHYSICS C: MECHANICS

2018 SCORING GUIDELINES

Question 2 (continued)

Distribution of points

(c)

i. (continued)

Alternate solution		*Alternate Points*
For using the conservation of momentum to calculate the speed of cart 2 after the collision		*1 point*
$m_1v_{1i} = m_1v_{1f} + m_2v_{2f} \therefore v_{2f} = \dfrac{m_1(v_{1i} - v_{1f})}{m_2}$		
For correct substitution of the answer from part (b)(i)		*1 point*
$v_{2f} = \dfrac{(0.500 \text{ kg})((3.00 \text{ m/s}) - (-1.06 \text{ m/s}))}{(1.05 \text{ kg})} = 1.93 \text{ m/s}$		

ii. 2 points

Show that the collision between the two carts is elastic.

For indicating that the initial and final kinetic energies must be equal		1 point
$K_{1i} = K_{1f} + K_{2f}$		
For correct substitutions of answers from parts (b)(i) and (c)(i) into the calculations of the initial and final kinetic energies		1 point
$\left(\frac{1}{2}\right)(0.500 \text{ kg})(3.00 \text{ m/s})^2 = \left(\frac{1}{2}\right)(0.500 \text{ kg})(-1.06 \text{ m/s})^2 + \left(\frac{1}{2}\right)(1.05 \text{ kg})(1.93 \text{ m/s})^2$		
$2.25 \text{ J} \approx 2.24 \text{ J}$		

(d)

i. 2 points

Calculate the speed of the center of mass of the two-cart-spring system.

For using the equation for the conservation of momentum to calculate the speed for the center of mass of the system		1 point
$m_1v_{1i} = (m_1 + m_2)v_{cm} \therefore v_{cm} = \dfrac{m_1v_{1i}}{(m_1 + m_2)}$		
For correct substitution into the equation above		1 point
$v_{cm} = \dfrac{(0.500 \text{ kg})(3.00 \text{ m/s})}{(0.500 \text{ kg} + 1.05 \text{ kg})} = 0.97 \text{ m/s}$		

AP® PHYSICS C: MECHANICS
2018 SCORING GUIDELINES

Question 2 (continued)

Distribution of points

(d) (continued)

ii. 3 points

Calculate the maximum elastic potential energy stored in the spring.

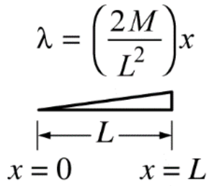
For using conservation of energy to calculate the maximum elastic potential energy stored in the spring		1 point
$K_i = K_f + U_{Sf}$		
For using the speed of the center of mass of the system for kinetic energy		1 point
$\frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}(m_1 + m_2)v_{cm}{}^2 + U_{Sf}$		
$U_{Sf} = \frac{1}{2}m_1 v_{1i}^2 - \frac{1}{2}(m_1 + m_2)v_{cm}{}^2$		
For correct substitution into above equation		1 point
$U_S = \left(\frac{1}{2}\right)(0.500\ \text{kg})(3.00\ \text{m/s})^2 - \frac{1}{2}(0.500\ \text{kg} + 1.05\ \text{kg})(0.97\ \text{m/s})^2$		
$U_S = 1.52\ \text{J}$		
Alternate Solution:		*Alternate Points*
For correctly determining the magnitude of the maximum force exerted between the carts		1 point
Set $\frac{dF}{dt} = 0 = 6400t - 500 \therefore 6400t = 500 \therefore t = 0.078\ \text{s}$		
$F_{MAX} = 3200(0.078)^2 - 500(0.078) = -19.5$		
$\lvert F_{MAX} \rvert = 19.5\ \text{N}$		
Note: Can estimate from the graph, and accept the range of 20 N to 21 N.		
For calculating the maximum compression of the spring		1 point
$F_{MAX} = -kx_{MAX}$		
$x_{MAX} = -\frac{F_{MAX}}{k} = -\frac{(-19.5\ \text{N})}{(130\ \text{N/m})} = 0.15\ \text{m}$		
Note: If estimating from the graph, accept the range from 0.15 m to 0.16 m.		
For substituting into an equation for the elastic potential energy at maximum spring compression		1 point
$U_S = \frac{1}{2}kx_{MAX}^2 = \left(\frac{1}{2}\right)(130\ \text{N/m})(0.15\ \text{m})^2$		
$U_S = 1.46\ \text{J}$		
Note: If estimating from the graph, accept range from 1.46 J to 1.70 J.		

Units point: 1 point for correct units on all calculated answers

2018 SCORING GUIDELINES

Question 3

15 points total **Distribution of points**



A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(a) 3 points

Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

For relating x to r properly in an integral to calculate the moment of inertia		1 point
$I = \int r^2 dm = \int x^2 dm$		
For correctly using the linear mass density to substitute into the equation above		1 point
$m = \int \lambda dx = \int \left(2M/L^2\right) x dx \therefore dm = \left(2M/L^2\right) x dx$		
$I = \int \left(2M/L^2\right) x^3 dx$		
For integrating using the correct limits or constant of integration		1 point
$I = \int_{x=0}^{x=L} \left(2M/L^2\right) x^3 dx = \left[\frac{\left(2M/L^2\right)x^4}{4}\right]_{x=0}^{x=L} = \frac{2M}{L^2}\left(\frac{L^4}{4} - 0\right) = ML^2/2$		

AP PHYSICS C: MECHANICS
2018 SCORING GUIDELINES

Question 3 (continued)

Distribution of points

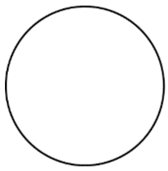


Figure 1

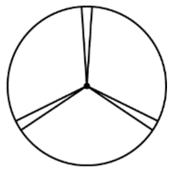


Figure 2

The thin hoop shown above in Figure 1 has a mass M, radius L, and a rotational inertia around its center of ML^2. Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

(b) 2 points

Derive an expression for the rotational inertia I_{tot} of the hoop-rods system about the center of the hoop. Express your answer in terms of M, L, and physical constants, as appropriate.

For setting the total rotational inertia for the hoop-rod system equal to the sum of the rotational inertias of the hoop and the three rods		1 point
$I = 3I_{rod} + I_{hoop}$		
$I = 3\left(ML^2/2\right) + ML^2$		
For a correct answer with work		1 point
$I = \frac{5}{2}ML^2$		

The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt.

(c) 3 points

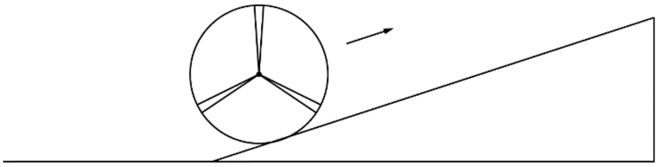
Derive an expression for the final angular speed ω of the hoop-rods system. Express your answer in terms of M, L, F, Δt, and physical constants, as appropriate.

For using an appropriate equation to determine the final angular speed of the hoop		1 point
$\tau\Delta t = I(\omega_2 - \omega_1)$		
$Fr\Delta t = I\omega$		
$\omega = \frac{Fr\Delta t}{I}$		
For relating the lever arm to the length of the rod		1 point
$\omega = \frac{FL\Delta t}{I}$		
For correct substitution from part (b)		1 point
$\omega = \frac{FL\Delta t}{\left(\frac{5}{2}\right)ML^2} = \frac{2F\Delta t}{5ML}$		

AP® PHYSICS C: MECHANICS
2018 SCORING GUIDELINES

Question 3 (continued)

Distribution of points

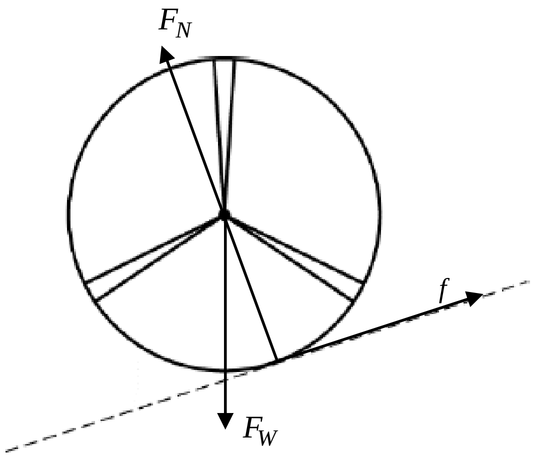


The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed ω found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

(d)

i. 3 points

On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.



For drawing the weight of the hoop-rod system in the correct direction		1 point
For drawing the normal force in the correct direction		1 point
For drawing the frictional force in the correct direction		1 point
Note: A maximum of two points can be earned if there are any extraneous vectors or any vector has an incorrect point of exertion.		

AP® PHYSICS C: MECHANICS
2018 SCORING GUIDELINES

Question 3 (continued)

Distribution of points

ii. 1 point

Justify your choice for the direction of each of the forces drawn in part (d)(i).

For a correct justification of the direction of the forces in part (d)(i)		1 point
Example: The normal force is always perpendicular to the surface, so it will be directed up and to the left. The gravitational force is always vertically downward. The friction will be opposite of the direction of rotation; therefore, it is directed up the incline.		

(e) 3 points

Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of M, L, I_{tot}, ω, and physical constants, as appropriate.

For including both linear and rotational kinetic energy in an equation for the conservation of energy to determine the final height of the hoop		1 point
$K_1 = U_{g2}$		
$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgH$		
$H = \dfrac{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2}{mg}$		
For correct substitution of $v = L\omega$		1 point
$H = \dfrac{\frac{1}{2}m(L\omega)^2 + \frac{1}{2}I_{tot}\omega^2}{mg}$		
$H = \dfrac{\frac{1}{2}\left(m_{tot}L^2 + I_{tot}\right)\omega^2}{m_{tot}g}$		
For correct substitution of inertias into energy equation		1 point
$H = \dfrac{\frac{1}{2}\left((3M + M)L^2 + I_{tot}\right)\omega^2}{(3M + M)g} = \dfrac{\left(4ML^2 + I_{tot}\right)\omega^2}{8Mg}$		