Question 1: Version J

1. Two blocks, 1 and 2, slide toward each other on a horizontal surface. Block 1 has mass m and slides in the $+x$-direction with constant speed $2v_0$. Block 2 has mass $6m$ and slides in the $-x$-direction with constant speed v_0, as shown in Figure 1. The blocks then collide and stick together. The collision occurs from time $t = 0$ to $t = t_c$. After the collision, where $t > t_c$, the blocks move together with the same constant speed.

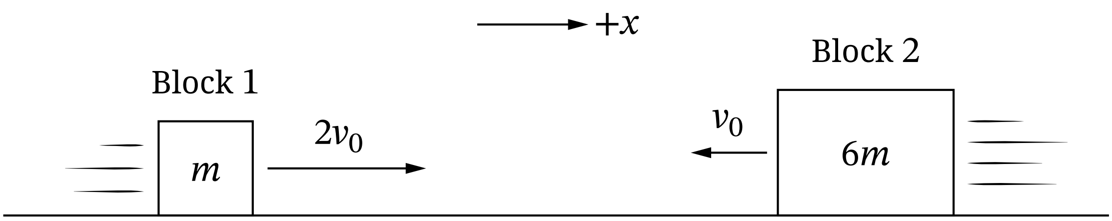


Figure 1

A. The diagrams in Figure 2 can be used to represent the momentum of blocks 1 and 2 before and after the collision. The momentum vector diagram for Block 1 before the collision is shown.

i. **Draw** arrows on the grids to represent the momentum vectors of Block 2 before the collision and the two-block system before and after the collision.

- Arrows should start at the zero-momentum line.
- The length of the arrows should be proportional to the relative magnitudes of the vectors.
- Represent an arrow of zero length by drawing a dot at zero.

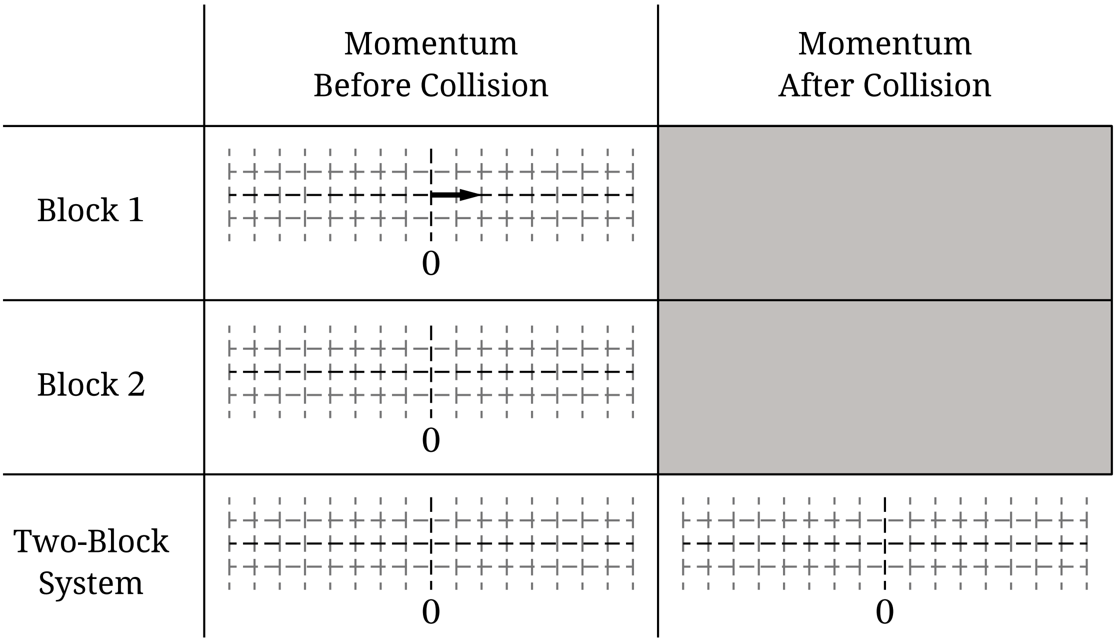


Figure 2

ii. During the time interval $0 \le t \le t_c$, a force F is exerted on Block 2 by Block 1 along the x-direction as a function of t that is modeled by $F(t) = F_{max} \sin(At)$, where A is a positive constant and F_{max} is the magnitude of the maximum force exerted on Block 2 by Block 1 during the collision.

Derive an expression for F_{max}. Express your answer in terms of m, v_0, A, t_c, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

B. Consider a new scenario where Block 1 initially slides in the $+x$-direction with a new constant speed v_1 and Block 2 again initially slides in the $-x$-direction with constant speed v_0. The blocks collide and stick together. In this new scenario, the two-block system has constant speed v_0 after the collision.

Derive an expression for v_1 in terms of v_0. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

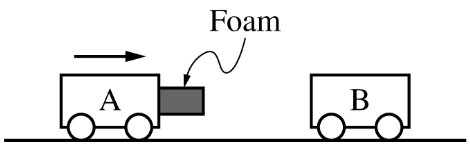
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

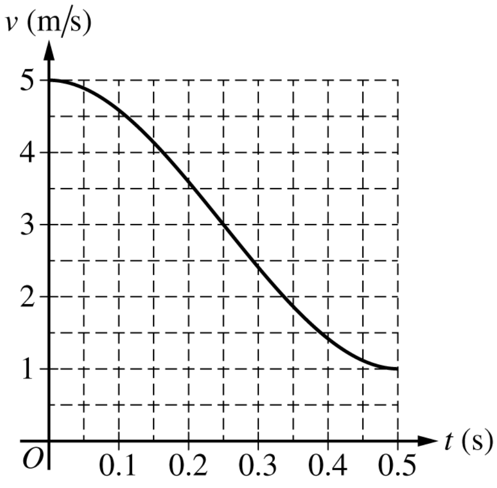
irections: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, hich are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this ooklet in the spaces provided after each part.



. Scientists have created a new type of lightweight foam and are performing experiments to investigate the properties of the foam. The mass of Cart A is 1000 kg and the mass of Cart B is 2000 kg. A piece of foam with negligible mass is attached to the front of Cart A, as shown. Cart A moves with a constant speed toward Cart B, which is initially at rest. At time $t = 0$ s, the foam connected to Cart A makes contact with Cart B. The foam remains in contact with Cart B for 0.5 s, after which the carts separate and both carts move with constant velocities.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

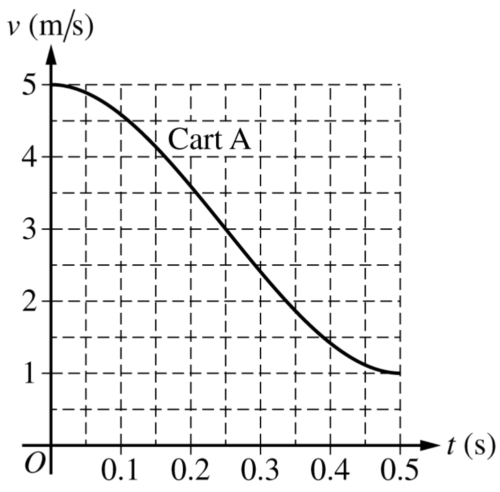


(a) The graph shows the velocity v of Cart A as a function of time t for the time interval when the foam and Cart B are in contact.

i. What feature(s) of the graph could be used to estimate the displacement of Cart A during the collision?

ii. Using the information shown in the graph, determine the speed of Cart B at $t = 0.5$ s.

iii. On the following grid, draw a smooth curve of the velocity of Cart B as a function of time.



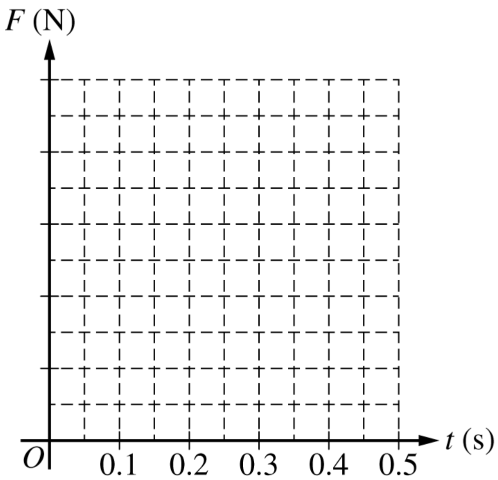
GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(b) For $0 \le t \le 0.50$ s, the velocity v of Cart A can be described by the function $v(t) = 64t^3 - 48t^2 + 5$.

i. Calculate the magnitude of the maximum net force acting on Cart A during this interval.

ii. On the following grid, draw a smooth curve of the magnitude of the force acting on Cart A as a function of time. Clearly indicate the value of the maximum force on the vertical axis.



GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

The foam is removed from the front of Cart A and the experiment is repeated. The carts collide, with both Cart A and Cart B having the same initial and final velocities as in the original collision. The time intervals during which the carts are in contact are different in the collision with the foam and the collision without the foam. In the collision without the foam, Cart A is in contact with Cart B for a shorter duration than in the original collision, when the foam was present.

For the original collision when the foam is present, the magnitude of the average net force exerted on Cart B is F_1. For the collision without the foam, the magnitude of the average net force exerted on Cart B is F_2.

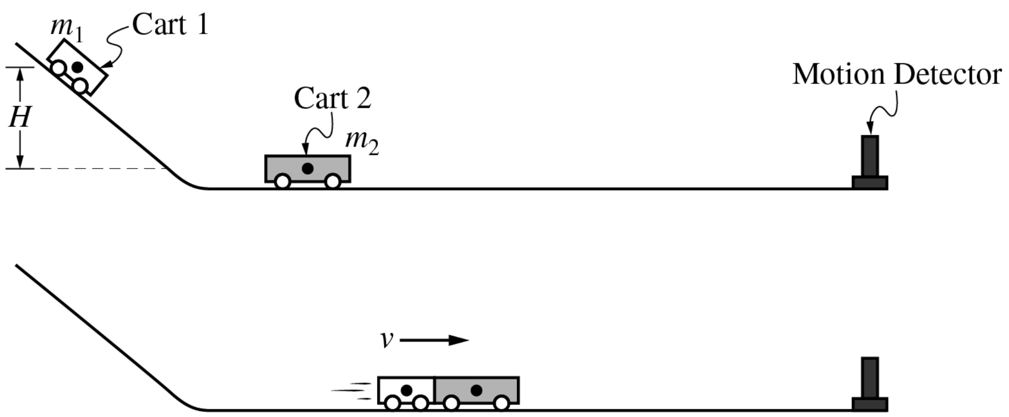
(c) What is the relationship between the magnitude of the average net force F_1 exerted on Cart B for the collision with the foam and the magnitude of the average net force F_2 exerted on Cart B for the collision without the foam?

_____ $F_1 > F_2$ _____ $F_1 < F_2$ _____ $F_1 = F_2$

Justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 2** on this page.



. Cart 1 of mass m_1 is held at rest above the bottom of an incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and smoothly transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance of H, as shown. At time t_C, Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(a) During the collision, is the impulse on Cart 1 from Cart 2 greater than, less than, or equal to the magnitude of the impulse on Cart 2 from Cart 1 ?

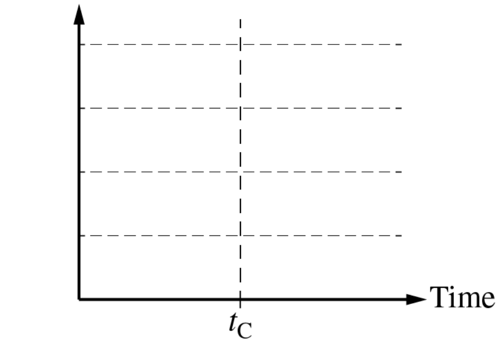
___ Greater than ___ Less than ___ Equal to

Justify your answer.

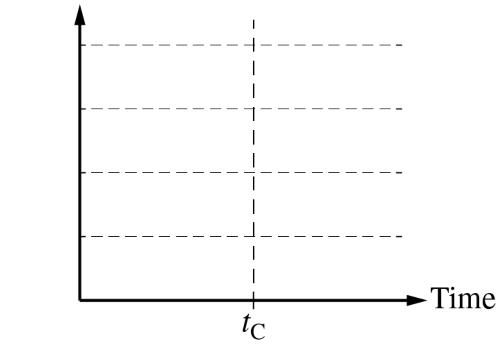
GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b) On the following axes, draw graphs of the magnitude of the momentum of each cart as a function of time t, before and after t_C. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.



Cart 1



Cart 2

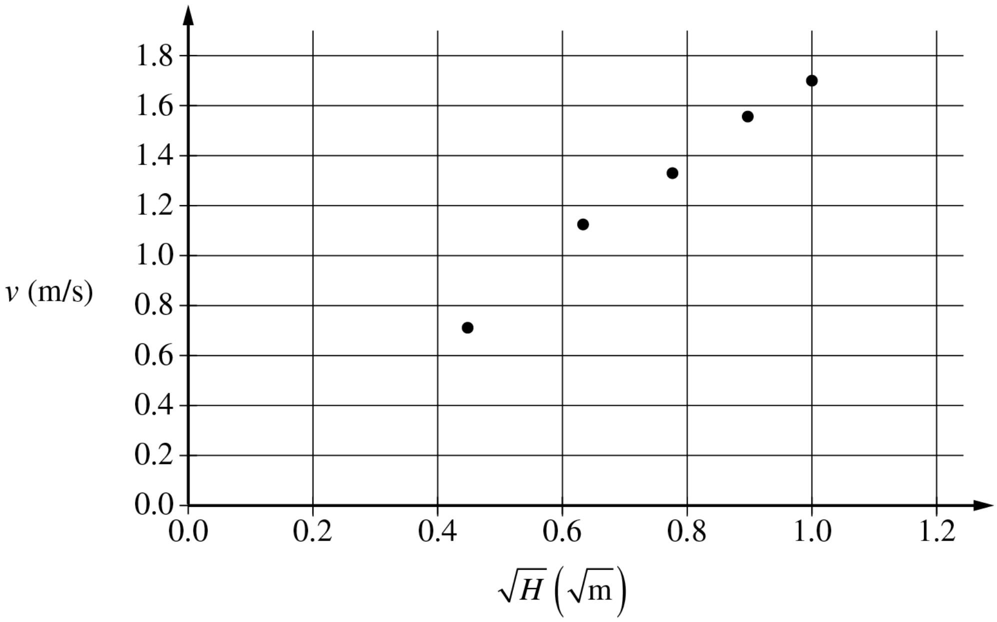
(c) Show that the velocity v of the two-cart system after the collision is given by the equation

$$v = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}.$$

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) A group of students use the setup to perform an experiment. They measure the mass of Cart 1 to be $m_1 = 0.250\,\text{kg}$. The mass of Cart 2 is unknown. The students perform several trials and in each trial, Cart 1 is released from a different height H and the final velocity of the two-cart system is measured. The students graph v as a function of $\sqrt{H}$, as shown below.



i. Draw a line that represents the best fit to the data points shown.

ii. Use the best-fit line to calculate the mass of Cart 2.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(e) After the experiment, the students use a balance to measure the mass of Cart 2 and find it to be less than what was determined in part (d). To explain this discrepancy, one of the students proposes that the mass of Cart 1 was incorrectly measured at the beginning of the experiment. The students measure the mass of Cart 1 again and record a new value, m_1'.

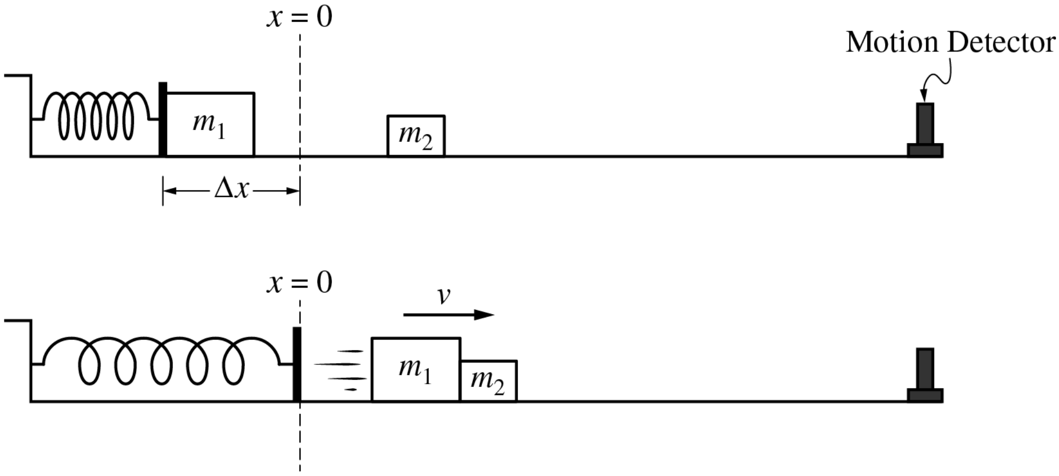
Should the students expect that m_1' will be greater than 0.250 kg, less than 0.250 kg, or equal to 0.250 kg ?

_____ $m_1' > 0.250$ kg   _____ $m_1' < 0.250$ kg   _____ $m_1' = 0.250$ kg

Justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 2** on this page.



. Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v, as shown. Frictional effects are negligible.

(a) The impulse on Block 1 from the spring during the time interval $0 < t < t_C$ is J_S. The impulse on Block 1 from Block 2 during the collision is J_2. Which of the following expressions correctly compares the magnitudes of J_S and J_2 ?

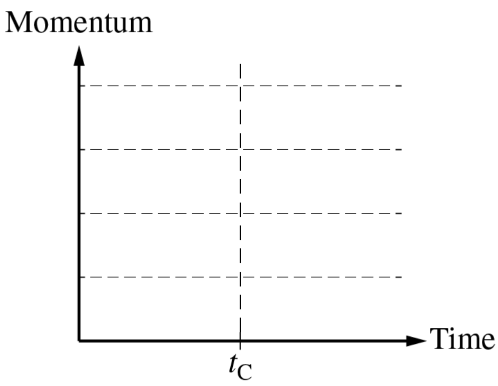
___ $J_S > J_2$ ___ $J_S < J_2$ ___ $J_S = J_2$

Justify your answer.

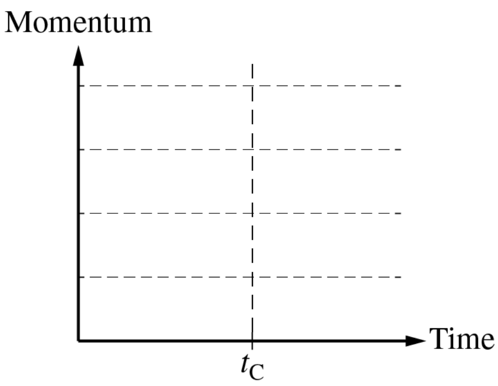
GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b) On the following axes, draw graphs of the magnitude of the momentum of each block as a function of time, before and after t_C. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.



Block 1



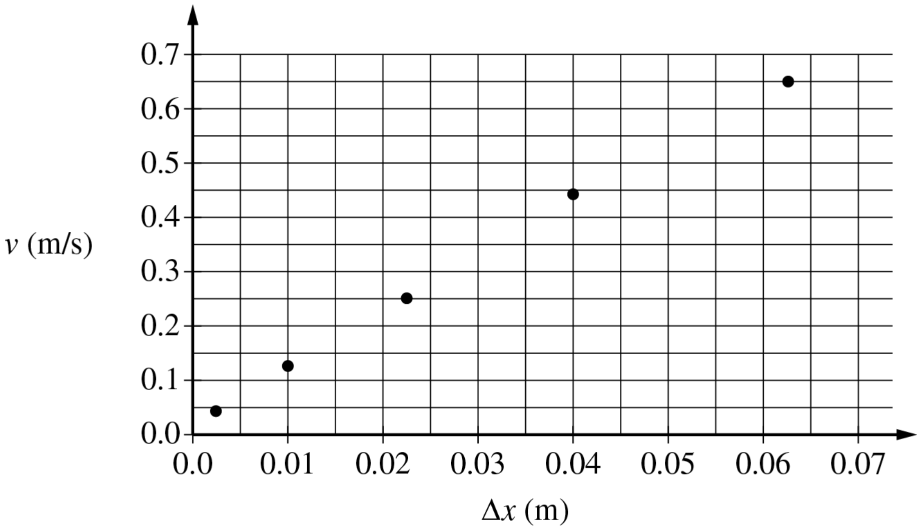
Block 2

(c) Show that the velocity v of the two-block system after the collision is given by the equation $v = \dfrac{\sqrt{km_1}}{m_1 + m_2}\Delta x$.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) A group of students use the setup to perform an experiment. They measure the mass of Block 1 to be $m_1 = 0.500\ \text{kg}$, and the spring constant k of the spring to be $150\ \text{N/m}$. The mass of Block 2 is unknown. They perform several trials and in each trial the spring is compressed a different distance Δx and the final velocity v of the two-block system is measured. They graph v as a function of Δx, as shown below.



i. Draw a line that represents the best fit to the data points shown.

ii. Use the best-fit line to calculate the mass of Block 2.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(e) After the experiment, the students use a balance to measure the mass of Block 2 and find it to be greater than what was determined in part (d). To explain this discrepancy, one of the students proposes that the spring constant was incorrectly measured at the beginning of the experiment. The students measure the spring constant again and record a new value, k'.

Should the students expect that k' be greater than 150 N / m, less than 150 N / m, or equal to 150 N / m ?

_____ $k' > 150$ N / m   _____ $k' < 150$ N / m   _____ $k' = 150$ N / m

Justify your answer.

GO ON TO THE NEXT PAGE.

2. A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0\ \text{m/s}^2$, $L = 0.25\ \text{m/s}^4$, and t is the time in seconds. At $t = 6.0$ s, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

(a) Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0$ s.

(b) Calculate the speed of the rocket at $t = 6.0$ s.

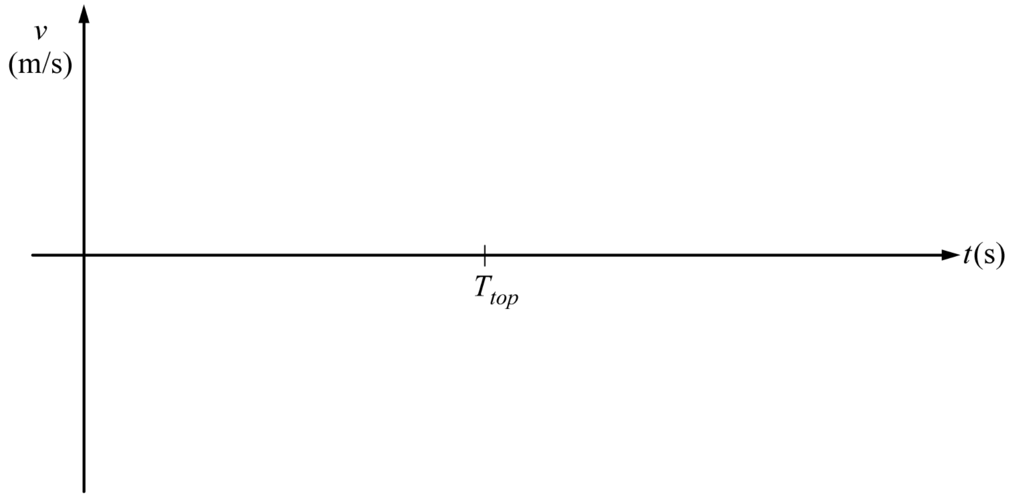
(c)

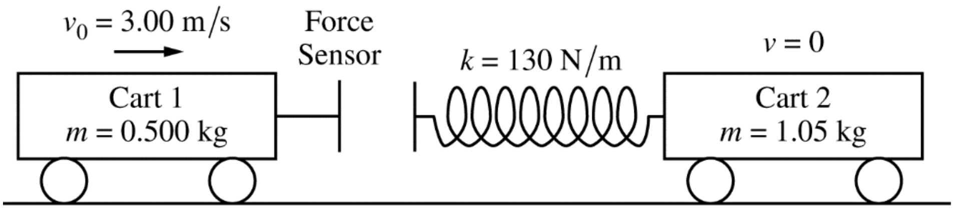
i. Calculate the kinetic energy of the rocket at $t = 6.0$ s.

ii. Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.

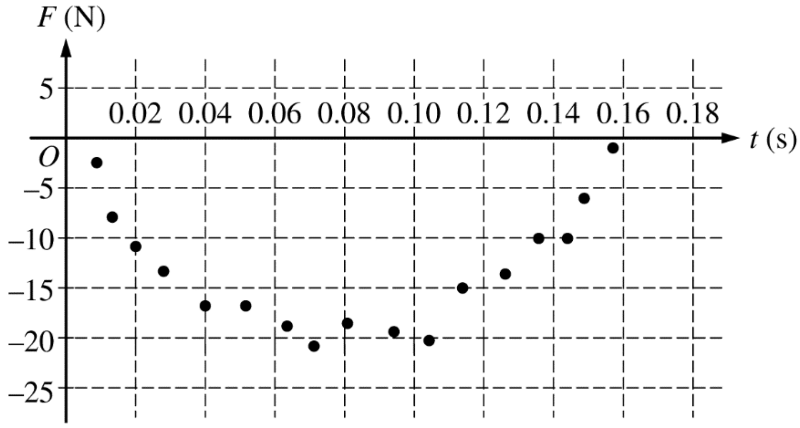
(d) Calculate the maximum height reached by the rocket relative to its launching point.

(e) On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.





2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00\ \mathrm{m/s}$ toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130\ \mathrm{N/m}$. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200\ \mathrm{N/s^2}\right)t^2 - (500\ \mathrm{N/s})t$.



(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)

i. Calculate the speed of cart 1 after the collision.

ii. In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)

i. Calculate the speed of cart 2 after the collision.

ii. Show that the collision between the two carts is elastic.

(d)

i. Calculate the speed of the center of mass of the two-cart–spring system.

ii. Calculate the maximum elastic potential energy stored in the spring.

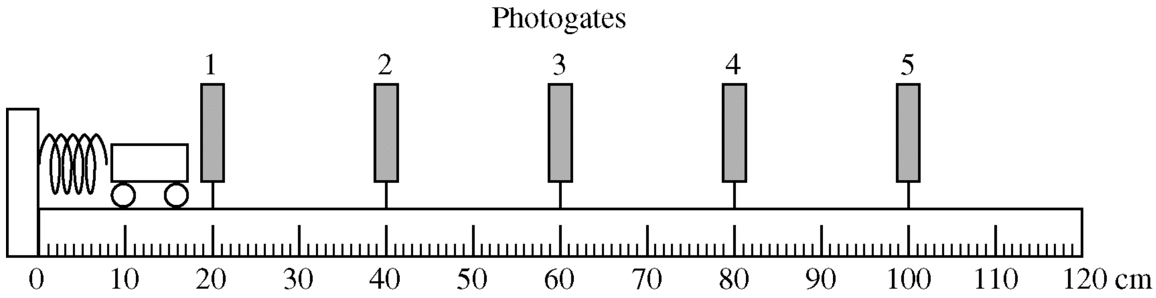
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Mech. 1.

In an experiment, a student wishes to use a spring to accelerate a cart along a horizontal, level track. The spring is attached to the left end of the track, as shown in the figure above, and produces a nonlinear restoring force of magnitude $F_s = As^2 + Bs$, where s is the distance the spring is compressed, in meters. A measuring tape, marked in centimeters, is attached to the side of the track. The student places five photogates on the track at the locations shown.

(a) Derive an expression for the potential energy U as a function of the compression s. Express your answer in terms of A, B, s, and fundamental constants, as appropriate.

In a preliminary experiment, the student pushes the cart of mass 0.30 kg into the spring, compressing the spring 0.040 m. For this spring, $A = 200\ \text{N/m}^2$ and $B = 150\ \text{N/m}$. The cart is released from rest. Assume friction and air resistance are negligible only during the short time interval when the spring is accelerating the cart.

(b) Calculate the following:

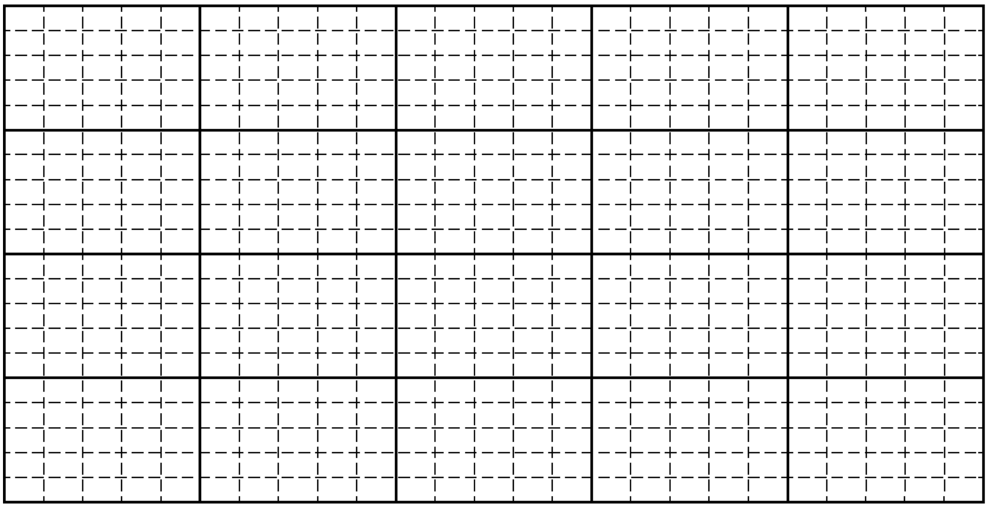
i. The speed of the cart immediately after it loses contact with the spring

ii. The impulse given to the cart by the spring

In a second experiment, the student collects data using the photogates. Each photogate measures the speed of the cart as it passes through the gate. The student calculates a spring compression that should give the cart a speed of $0.320\ \text{m/s}$ after the cart loses contact with the spring. The student runs the experiment by pushing the cart into the spring, compressing the spring the calculated distance, and releasing the cart. The speeds are measured with a precision of $\pm 0.002\ \text{m/s}$. The positions are measured with a precision of ± 0.005 m.

Photogate	1	2	3	4	5
Cart speed (m/s)	0.412	0.407	0.399	0.374	0.338
Photogate position (m)	0.20	0.40	0.60	0.80	1.00

(c) On the axes below, plot the data points for the speed v of the cart as a function of position x. Clearly scale and label all axes, as appropriate.



(d)

i. Compare the speed of the cart measured by photogate 1 to the predicted value of the speed of the cart just after it loses contact with the spring. List a physical source of error that could account for the difference.

ii. From the measured speed values of the cart as it rolls down the track, give a physical explanation for any trend you observe.