

Question 1: Mathematical Routines (MR)**10 points**

- A (i)** For drawing only one arrow for the momentum of Block 2 before the collision that points leftward and is 6 units long **Point A1**

For drawing only identical arrows for the momentum of the two-block system before and after the collision that are equal to the sum of the momentums drawn for blocks 1 and 2 before the collision **Point A2**

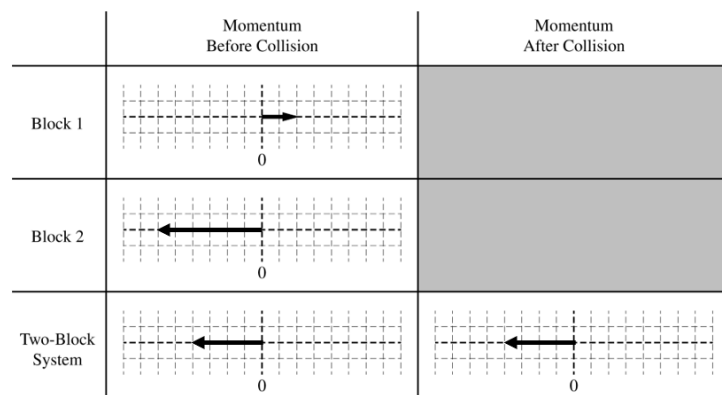
Example Response

Figure 2

- (ii)** For a multistep derivation that includes the integral form of impulse or the differential form of Newton's second law **Point A3**

For indicating **one** of the following: **Point A4**

- The final speed of Block 1 or Block 2 is $\frac{4}{7}v_0$.
- The change in the momentum of Block 2 is $+\frac{18}{7}mv_0$.
- The change in the momentum of Block 1 is $-\frac{18}{7}mv_0$.
- The change in the velocity of Block 2 is $+\frac{3}{7}v_0$.
- The change in the velocity of Block 1 is $-\frac{18}{7}v_0$.

For substituting the given expression for $F(t)$ into an expression for the impulse exerted on either block or the differential form of Newton's second law **Point A5**

For an integral with appropriate limits or a constant of integration **Point A6**

For a correct expression for $F_{\max}$ in terms of given quantities **Point A7**

Example Responses

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{\text{net}}(t) dt = \Delta \vec{p} \quad \text{OR} \quad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \\ \int \vec{F}_{\text{net}} dt = \Delta \vec{p}$$

$$\sum \vec{p}_0 = \sum \vec{p}_f \\ (m)(2v_0) + (6m)(-v_0) = (m + 6m)(v_f) \\ v_f = -\frac{4}{7}v_0$$

$$\Delta p_{\text{Block 2}} = -\Delta p_{\text{Block 1}} = 6m\left(-\frac{4}{7}v_0 - (-v_0)\right)$$

$$\Delta p_{\text{Block 2}} = \int_0^{t_c} F_{\max} \sin(At) dt$$

$$\frac{18}{7}mv_0 = \frac{F_{\max}}{A}[-\cos(At)] \Big|_0^{t_c}$$

$$\frac{18}{7}mv_0 = \frac{F_{\max}}{A}[1 - \cos(At_c)]$$

$$F_{\max} = \frac{18}{7} \left(\frac{Amv_0}{1 - \cos(At_c)} \right)$$

- B** For a multistep derivation that includes conservation of momentum **Point B1**

For indicating that the magnitude of the final momentum of the two-block system is $7mv_0$ **Point B2**

For a correct expression for v_1 in terms of v_0 **Point B3**

Example Response

$$\sum \vec{p}_0 = \sum \vec{p}_f$$

$$mv_1 + 6m(-v_0) = 7m(v_0)$$

$$mv_1 = 13mv_0$$

$$v_1 = 13v_0$$

Question 1: Free-Response Question

15 points

(a)(i) For a multi-step derivation that includes **one** of the following: **1 point**

- An application of conservation of energy that indicates that the initial mechanical energy of the system is U_g
- An application of Newton's second law that includes the frictional force, including sign

For **one** of the following that is consistent with the previous point: **1 point**

- An expression for the energy dissipated by friction, including the correct sign
- A substitution of acceleration in a kinematics equation

Example Responses

$$\Delta E_{\text{friction}} = -F_f D \quad \text{OR} \quad v^2 = v_0^2 + 2\left(\frac{F_{g,x} - F_f}{2m}\right)\Delta x$$

For a correct expression for v **1 point**

Example Response

$$v = \sqrt{2gD(\sin\theta - \mu\cos\theta)}$$

Example Solutions

$$\begin{aligned} E_{\text{initial}} &= E_{\text{final}} \\ U_g - \Delta E_{\text{friction}} &= K \\ m_A g D \sin\theta - \mu m_A g D \cos\theta &= \frac{1}{2} m_A v^2 \quad \text{OR} \\ g D \sin\theta - \mu g D \cos\theta &= \frac{1}{2} v^2 \\ v &= \sqrt{2gD(\sin\theta - \mu\cos\theta)} \end{aligned}$$

$$\begin{aligned} a &= \frac{F_{\text{net}}}{m} \\ 2ma &= F_{g,x} - F_f \\ 2ma &= 2mg \sin\theta - \mu(2m)g \cos\theta \\ a &= g \sin\theta - \mu g \cos\theta \\ v^2 &= v_0^2 + 2a\Delta x \\ v^2 &= 0^2 + 2(g \sin\theta - \mu g \cos\theta)D \\ v &= \sqrt{2gD(\sin\theta - \mu\cos\theta)} \end{aligned}$$

(a)(ii) For using the conservation of momentum to find $v_{A,B}$ **1 point**For equating the kinetic energy after the collision between the blocks to the maximum elastic potential energy of the compressed spring **1 point**

Example Response

$$K_{\text{after collision}} = U_{s, \text{max}}$$

For indicating v before the collision between the blocks and the spring is equal to $v_{A,B}$ **1 point**

Example Solution

$$p_{\text{before collision}} = p_{\text{after collision}}$$

$$2mv = (2m + m)v_{A,B}$$

$$v_{A,B} = \frac{2m\sqrt{2gD(\sin\theta - \mu\cos\theta)}}{(3m)}$$

$$v_{A,B} = \frac{2}{3}\sqrt{2gD(\sin\theta - \mu\cos\theta)}$$

$$E_{\text{after collision}} = E_{\text{max compression of spring}}$$

$$K_{\text{after collision}} = U_{s, \text{max}}$$

$$\frac{1}{2}(2m + m)(v_{A,B})^2 = \frac{1}{2}kx^2$$

$$(3m)\left(\frac{2}{3}\sqrt{2gD(\sin\theta - \mu\cos\theta)}\right)^2 = kx_c^2$$

$$k = \frac{(3m)}{x_c^2}\left(\frac{2}{3}\sqrt{2gD(\sin\theta - \mu\cos\theta)}\right)^2$$

$$k = \frac{8}{3}\frac{mgD(\sin\theta - \mu\cos\theta)}{x_c^2}$$

Total for part (a) **6 points**

(b)(i) For a sketch that increases linearly during the time interval $0 \leq t < t_1$ **1 point**

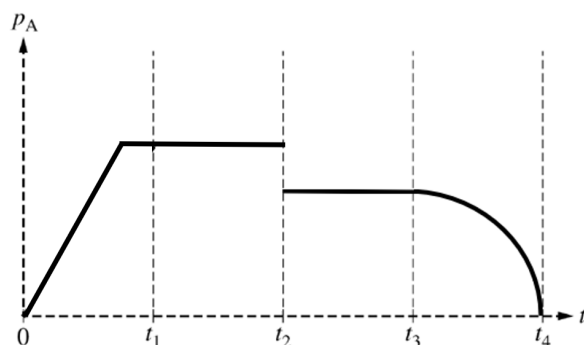
Scoring Note: A sketch that only increases linearly from $t = 0$ to $t = t_1$ earns this point.

For a horizontal line for the time interval $t_1 \leq t \leq t_2$ that is continuous at t_1 **1 point**

For a horizontal line for the time interval $t_2 \leq t \leq t_3$ that has a smaller magnitude than the previous time interval **1 point**

For drawing a concave down curve, continuous at t_3 , in the interval $t_3 \leq t < t_4$ that reaches zero at t_4 **1 point**

Example Response



(b)(ii) For a statement about the change in momentum that is consistent with the graph drawn in the response for part (b)(i) **1 point**

For indicating that a decreasing graph means that the force exerted on Block A is in a direction opposite to the motion of Block A **1 point**

Scoring Note: A response that indicates that an increasing graph means that the force exerted on Block A is in the same direction as the motion of Block A also earns this point.

For relating the change in momentum to the magnitude of the force exerted on Block A **1 point**

Example Response

The momentum of Block A decreases between t_3 and t_4 because the spring exerts a force on the blocks in the opposite direction of the velocity of the blocks, causing the blocks to slow to a stop. The spring force increases the more the spring compresses, so the momentum of Block A decreases at an increasing rate, which is shown in the slope of the curve becoming steeper with time.

Total for part (b) 7 points

(c) For selecting $T_N = T_O$ with an attempt at a relevant justification **1 point**

For a correct justification that includes **one** of the following: **1 point**

- The period of a spring-block oscillator is only dependent on the mass on the spring and spring constant, which do not change.
- The period of a spring-block oscillator is not dependent on increasing amplitude, velocity, or compression distance.

Example Response

Repeating the experiment on a smooth ramp will only affect the compression distance of the spring. The period of oscillation of a spring-block system depends only on mass and the spring constant, therefore the period of oscillation will not change.

Total for part (c) 2 points

Total for question 1 15 points

Question 2: Free-Response Question

15 points

- (a) For selecting “Equal to” with an attempt at a relevant justification 1 point
- For a correct justification 1 point

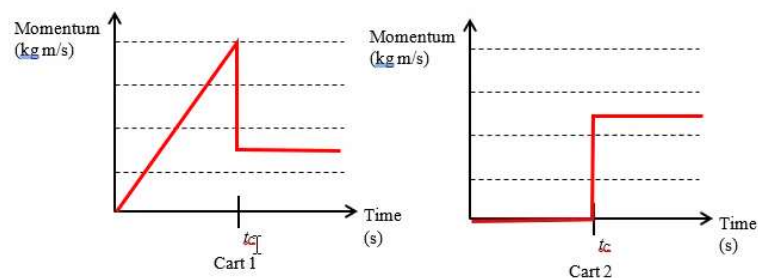
Example Response

Newton's third law says equal/opposite forces, time intervals are same during same collision, so magnitudes of impulse must be equal.

Total for part (a) 2 points

- (b) For correctly drawing the momentum for carts 1 and 2 for the time interval $0 < t < t_C$: 1 point
- Linear increasing momentum for Cart 1 and zero for Cart 2
- For drawing a horizontal line for Cart 1 when $t > t_C$ that is smaller in magnitude than the momentum of Cart 1 at time $t = t_C$ 1 point
- For drawing a horizontal line for Cart 2 when $t > t_C$ that is greater in magnitude than the momentum of Cart 1 after time $t = t_C$ 1 point
- For carts 1 and 2 having a change in momentum that is equal in magnitude, such that Cart 1 loses momentum and Cart 2 gains momentum or a response with changes in momentum consistent with the response in part (a) 1 point

Example Response



Total for part (b) 4 points

- (c) For using conservation of energy to find the speed of Cart 1 at the bottom of the incline 1 point

OR

For a correct substitution of acceleration and displacement in a kinematics equation to find the speed of Cart 1 at the bottom of the incline

For using conservation of momentum to find the speed of the two-cart system after the collision 1 point

For combining correct equations from above 1 point

Example Response

Conservation of energy:

$$m_1 g H = \frac{1}{2} m_1 v_1^2$$

$$v_1 = \sqrt{2gH}$$

OR

$$v_f^2 = v_i^2 + 2g \sin(\theta)L$$

$$H = L \sin(\theta)$$

$$v_f^2 = v_i^2 + 2g \sin(\theta) \left(\frac{H}{\sin \theta} \right)$$

$$v_f^2 = 2gH$$

Conservation of momentum:

$$m_1 v_1 = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1}{m_1 + m_2} v_1$$

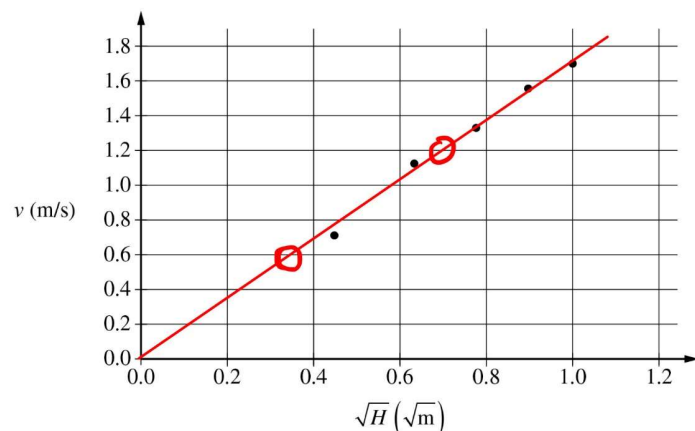
Combining:

$$v_f = \frac{m_1}{m_1 + m_2} \sqrt{2gH}$$

$$v_f = \sqrt{2g} \left(\frac{m_1}{m_1 + m_2} \right) \sqrt{H}$$

Total for part (c) 3 points

- (d)(i) For drawing an appropriate best-fit line including approximately the same number of points above and below the line **1 point**

Example Response

- (d)(ii) For calculating the slope using two points on the best-fit line **1 point**

For correctly relating the slope of the best-fit line to the mass of Cart 2 **1 point**

For a correct mass of Cart 2 **1 point**

Example Response

$$\text{slope} = \frac{(1.20 - 0.60) \text{ m/s}}{(0.70 - 0.35) \sqrt{\text{m}}} = 1.72 \frac{\sqrt{\text{m}}}{\text{s}}$$

$$\text{slope} = \frac{m_1}{m_1 + m_2} \sqrt{2g}$$

$$m_2 = \frac{m_1 \sqrt{2g}}{\text{slope}} - m_1$$

$$m_2 = \frac{(0.25 \text{ kg}) \sqrt{2 \left(9.8 \frac{\text{m}}{\text{s}^2} \right)}}{1.72 \frac{\sqrt{\text{m}}}{\text{s}}} - (0.25 \text{ kg})$$

$$\therefore m_2 = 0.39 \text{ kg}$$

Scoring Note: Acceptable responses for mass are 0.30 to 0.60 kg

Total for part (d) 4 points

- (e) For selecting “ $m_1' < 0.250 \text{ kg}$ ” with an attempt at a relevant justification **1 point**

For a correct justification **1 point**

Example Response

A smaller m_2 indicates that the initial energy and momentum was smaller. With identical slope and height H this means that the mass m_1' must be smaller.

Total for part (e) 2 points

Total for question 2 15 points

Question 2: Free-Response Question**15 points**

- (a) For selecting “ $J_S > J_2$ ” with an attempt at a relevant justification **1 point**

For a correct justification **1 point**

Example Response

The impulse between the two blocks is the same, so because Block 1 is moving at the end, the impulse given by the spring must be greater than the impulse given to Block 2 by Block 1.

Total for part (a) 2 points

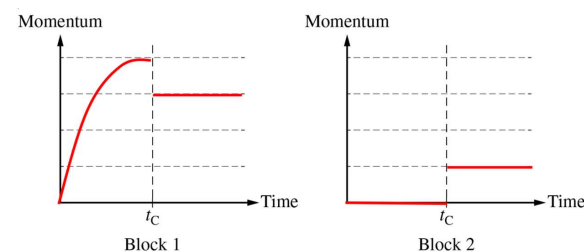
- (b) For correctly drawing the momentum for blocks 1 and 2 for the time interval $0 < t < t_C$: **1 point**

Block 1 shows an increasing curve with a decreasing slope (concave down) to highest value (technically a cosine curve) and Block 2 shows zero momentum up to t_C (a horizontal line along the time axis).

For drawing a horizontal line for Block 1 when $t > t_C$ that is smaller in magnitude than the momentum of Block 1 at time $t = t_C$ **1 point**

For drawing a horizontal line for Block 2 when $t > t_C$ that is smaller in magnitude than the momentum of Block 1 after time $t = t_C$ **1 point**

For blocks 1 and 2 having a change in momentum that is equal in magnitude such that Block 1 loses momentum and Block 2 gains momentum **1 point**

Example Response

Total for part (b) 4 points

- | | | |
|-----|--|---------|
| (c) | For using conservation of energy to find the speed of Block 1 at $x = 0$ | 1 point |
| | For using conservation of momentum to find the speed of the two-block system after the collision | 1 point |
| | For combining correct equations from above | 1 point |

Example Response

$$\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}m_1v_1^2$$

$$v_1 = \sqrt{\frac{k}{m_1}}\Delta x$$

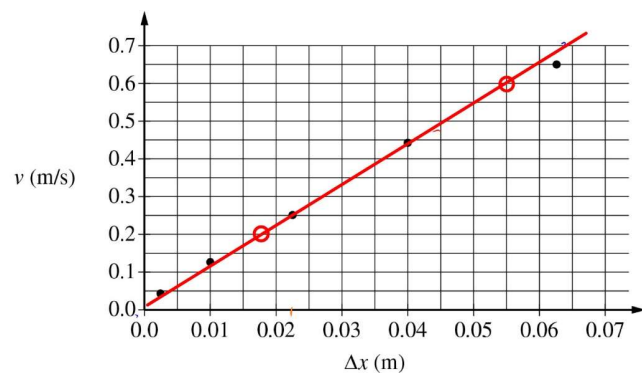
$$m_1v_1 = (m_1 + m_2)v$$

$$v = \frac{m_1v_1}{m_1 + m_2}$$

$$\therefore v = \frac{\Delta x \sqrt{km_1}}{m_1 + m_2}$$

Total for part (c) 3 points

- | | | |
|--------|---|---------|
| (d)(i) | For drawing an appropriate best-fit line including approximately the same number of points above and below the line | 1 point |
|--------|---|---------|

Example Response

- | | | |
|---------|--|---------|
| (d)(ii) | For calculating the slope using two points on the line | 1 point |
| | For correctly relating the slope of the best-fit line to the mass of Block 2 | 1 point |
| | For a correct mass of Block 2 | 1 point |

Example Response

$$\text{slope} = \frac{(0.60 - 0.02) \text{ m/s}}{(0.055 - 0.0175) \text{ m}} = 10.67 \text{ s}^{-1}$$

$$\text{slope} = \frac{\sqrt{km_1}}{m_1 + m_2}$$

$$m_2 = \frac{\sqrt{km_1}}{\text{slope}} - m_1$$

$$m_2 = \frac{\sqrt{\left(150 \frac{\text{N}}{\text{m}}\right)(0.50 \text{ kg})}}{10.67 \text{ s}^{-1}} - 0.50 \text{ kg}$$

$$m_2 = 0.31 \text{ kg}$$

Scoring Note: Acceptable responses for mass are 0.27 kg – 0.37 kg.

Total for part (d) 4 points

- | | | |
|-----|--|---------|
| (e) | For selecting “ $k' > 150 \text{ N/m}$ ” with an attempt at relevant justification | 1 point |
| | For a correct justification | 1 point |

Example Response

A greater m_2 indicates the spring constant k' should be greater than 150 N/m . The slope of the graph is the same so as m_2 increases k must be smaller.

Total for part (e) 2 points

Total for question 2 15 points