Question 1: Mathematical Routines (MR) — 10 points

A **(i)** For drawing only one arrow for the momentum of Block 2 before the collision that points leftward and is 6 units long — **Point A1**

For drawing only identical arrows for the momentum of the two-block system before and after the collision that are equal to the sum of the momentums drawn for blocks 1 and 2 before the collision — **Point A2**

Example Response

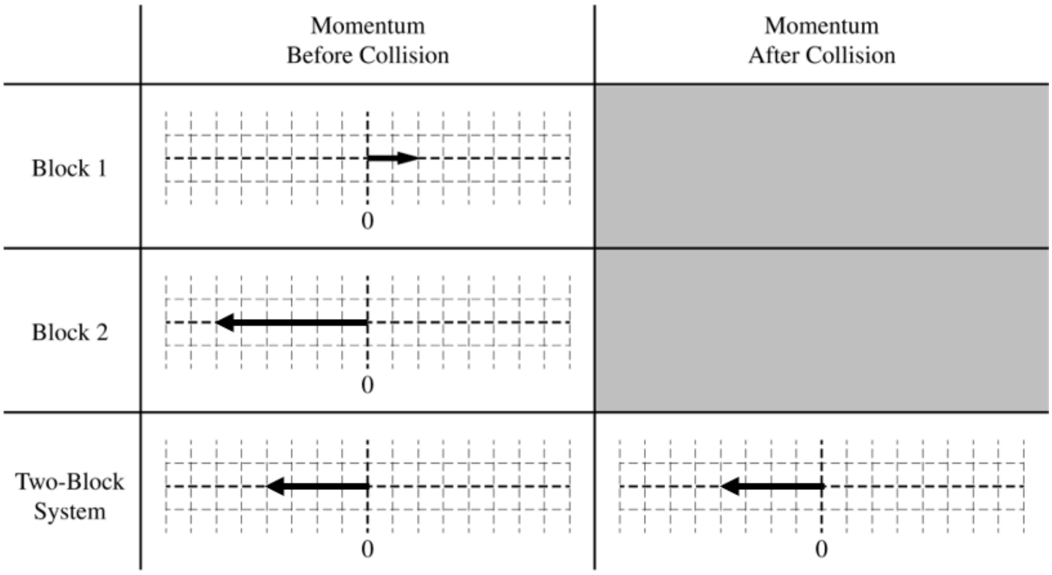


Figure 2

(ii) For a multistep derivation that includes the integral form of impulse or the differential form of Newton's second law — **Point A3**

For indicating **one** of the following: — **Point A4**

- The final speed of Block 1 or Block 2 is $\frac{4}{7}v_0$.
- The change in the momentum of Block 2 is $+\frac{18}{7}mv_0$.
- The change in the momentum of Block 1 is $-\frac{18}{7}mv_0$.
- The change in the velocity of Block 2 is $+\frac{3}{7}v_0$.
- The change in the velocity of Block 1 is $-\frac{18}{7}v_0$.

For substituting the given expression for $F(t)$ into an expression for the impulse exerted on either block or the differential form of Newton's second law — **Point A5**

For an integral with appropriate limits or a constant of integration — **Point A6**

For a correct expression for F_{max} in terms of given quantities — **Point A7**

Example Responses

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{\text{net}}(t)\, dt = \Delta\vec{p} \qquad \text{OR} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$$

$$\int \vec{F}_{\text{net}}\, dt = \Delta\vec{p}$$

$$\sum \vec{p}_0 = \sum \vec{p}_f$$
$$(m)(2v_0) + (6m)(-v_0) = (m + 6m)(v_f)$$
$$v_f = -\frac{4}{7}v_0$$

$$\Delta p_{\text{Block 2}} = -\Delta p_{\text{Block 1}} = 6m\left(-\frac{4}{7}v_0 - (-v_0)\right)$$

$$\Delta p_{\text{Block 2}} = \int_0^{t_c} F_{\text{max}} \sin(At)\, dt$$

$$\frac{18}{7}mv_0 = \frac{F_{\text{max}}}{A}\left[-\cos(At)\right]\Big|_0^{t_c}$$
$$\frac{18}{7}mv_0 = \frac{F_{\text{max}}}{A}\left[1 - \cos(At_c)\right]$$

$$F_{\text{max}} = \frac{18}{7}\left(\frac{Amv_0}{1 - \cos(At_c)}\right)$$

B	For a multistep derivation that includes conservation of momentum	**Point B1**
	For indicating that the magnitude of the final momentum of the two-block system is $7mv_0$	**Point B2**
	For a correct expression for v_1 in terms of v_0	**Point B3**

Example Response

$$\sum \vec{p}_0 = \sum \vec{p}_f$$

$$mv_1 + 6m(-v_0) = 7m(v_0)$$
$$mv_1 = 13mv_0$$
$$v_1 = 13v_0$$

Question 1: Free-Response Question — 15 points

(a)(i) For a multi-step derivation that includes **one** of the following: **1 point**

- An application of conservation of energy that indicates that the initial mechanical energy of the system is U_g
- An application of Newton's second law that includes the frictional force, including sign

For **one** of the following that is consistent with the previous point: **1 point**

- An expression for the energy dissipated by friction, including the correct sign
- A substitution of acceleration in a kinematics equation

Example Responses

$\Delta E_{\text{friction}} = -F_f D$ **OR** $v^2 = v_0^2 + 2\left(\dfrac{F_{g,x} - F_f}{2m}\right)\Delta x$

For a correct expression for v **1 point**

Example Response

$v = \sqrt{2gD(\sin\theta - \mu\cos\theta)}$

Example Solutions

$$E_{\text{initial}} = E_{\text{final}}$$
$$U_g - \Delta E_{\text{friction}} = K$$
$$m_A gD\sin\theta - \mu m_A gD\cos\theta = \frac{1}{2}m_A v^2$$
$$gD\sin\theta - \mu gD\cos\theta = \frac{1}{2}v^2$$
$$v = \sqrt{2gD(\sin\theta - \mu\cos\theta)}$$

OR

$$a = \frac{F_{\text{net}}}{m}$$
$$2ma = F_{g,x} - F_f$$
$$2ma = 2mg\sin\theta - \mu(2m)g\cos\theta$$
$$a = g\sin\theta - \mu g\cos\theta$$

$$v^2 = v_0^2 + 2a\Delta x$$
$$v^2 = 0^2 + 2(g\sin\theta - \mu g\cos\theta)D$$
$$v = \sqrt{2gD(\sin\theta - \mu\cos\theta)}$$

(a)(ii)	For using the conservation of momentum to find $v_{A,B}$	**1 point**
	For equating the kinetic energy after the collision between the blocks to the maximum elastic potential energy of the compressed spring	**1 point**

Example Response

$$K_{\text{after collision}} = U_{s,\,\text{max}}$$

For indicating v before the collision between the blocks and the spring is equal to $v_{A,B}$	**1 point**

Example Solution

$$p_{\text{before collision}} = p_{\text{after collision}}$$

$$2mv = (2m + m)v_{A,B}$$

$$v_{A,B} = \frac{2m\sqrt{2gD(\sin\theta - \mu\cos\theta)}}{(3m)}$$

$$v_{A,B} = \frac{2}{3}\sqrt{2gD(\sin\theta - \mu\cos\theta)}$$

$$E_{\text{after collision}} = E_{\text{max compression of spring}}$$

$$K_{\text{after collision}} = U_{s,\,\text{max}}$$

$$\frac{1}{2}(2m + m)\left(v_{A,B}\right)^2 = \frac{1}{2}kx^2$$

$$(3m)\left(\frac{2}{3}\sqrt{2gD(\sin\theta - \mu\cos\theta)}\right)^2 = kx_c^{\,2}$$

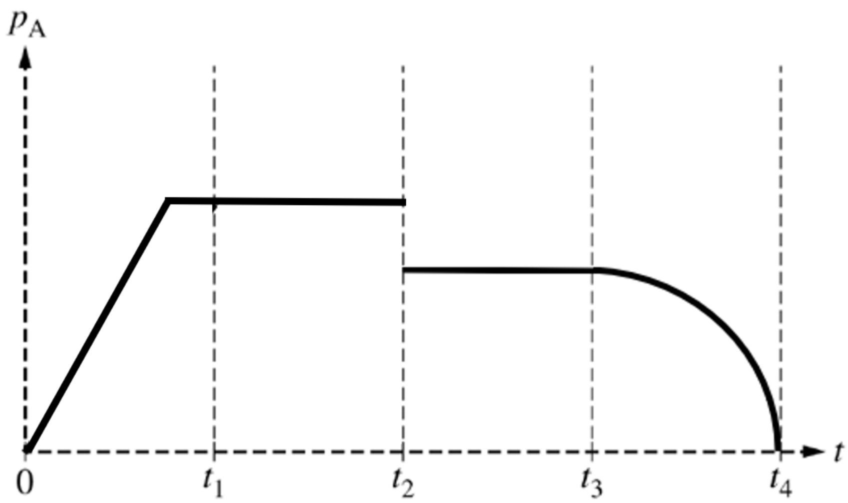
$$k = \frac{(3m)}{x_c^{\,2}}\left(\frac{2}{3}\sqrt{2gD(\sin\theta - \mu\cos\theta)}\right)^2$$

$$k = \frac{8}{3}\frac{mgD(\sin\theta - \mu\cos\theta)}{x_c^{\,2}}$$

Total for part (a) 6 points

(b)(i)	For a sketch that increases linearly during the time interval $0 \le t < t_1$ **Scoring Note:** A sketch that only increases linearly from $t = 0$ to $t = t_1$ earns this point.	**1 point**
	For a horizontal line for the time interval $t_1 \le t \le t_2$ that is continuous at t_1	**1 point**
	For a horizontal line for the time interval $t_2 \le t \le t_3$ that has a smaller magnitude than the previous time interval	**1 point**
	For drawing a concave down curve, continuous at t_3, in the interval $t_3 \le t < t_4$ that reaches zero at t_4	**1 point**

Example Response



(b)(ii)	For a statement about the change in momentum that is consistent with the graph drawn in the response for part (b)(i)	**1 point**
	For indicating that a decreasing graph means that the force exerted on Block A is in a direction opposite to the motion of Block A **Scoring Note:** A response that indicates that an increasing graph means that the force exerted on Block A is in the same direction as the motion of Block A also earns this point.	**1 point**
	For relating the change in momentum to the magnitude of the force exerted on Block A	**1 point**

Example Response

The momentum of Block A *decreases between* t_3 *and* t_4 *because the spring exerts a force on the blocks in the opposite direction of the velocity of the blocks, causing the blocks to slow to a stop. The spring force increases the more the spring compresses, so the momentum of Block* A *decreases at an increasing rate, which is shown in the slope of the curve becoming steeper with time.*

Total for part (b) 7 points

(c)	For selecting $T_N = T_O$ with an attempt at a relevant justification	**1 point**
	For a correct justification that includes **one** of the following: • The period of a spring-block oscillator is only dependent on the mass on the spring and spring constant, which do not change. • The period of a spring-block oscillator is not dependent on increasing amplitude, velocity, or compression distance.	**1 point**
	Example Response *Repeating the experiment on a smooth ramp will only affect the compression distance of the spring. The period of oscillation of a spring-block system depends only on mass and the spring constant, therefore the period of oscillation will not change.*	
	Total for part (c)	**2 points**
	Total for question 1	**15 points**

Question 2: Free-Response Question — 15 points

(a) For selecting "Equal to" with an attempt at a relevant justification — **1 point**

For a correct justification — **1 point**

Example Response

Newton's third law says equal/opposite forces, time intervals are same during same collision, so magnitudes of impulse must be equal.

Total for part (a) — 2 points

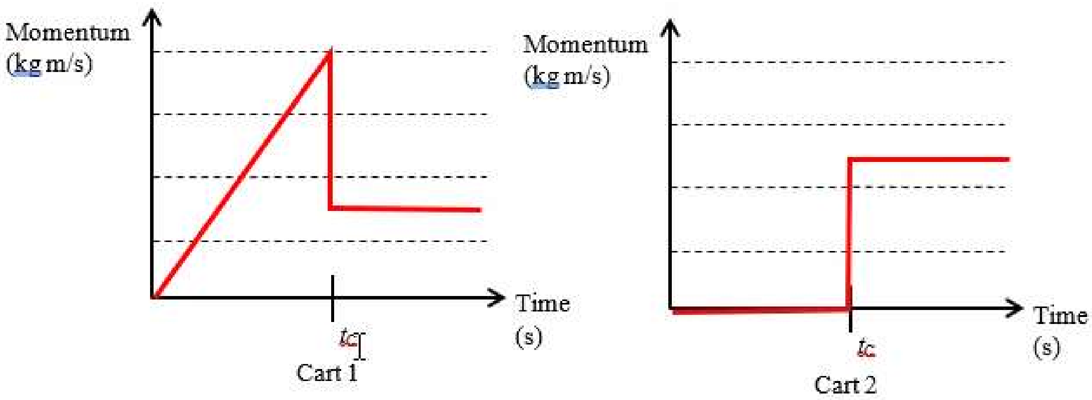
(b) For correctly drawing the momentum for carts 1 and 2 for the time interval $0 < t < t_C$: Linear increasing momentum for Cart 1 and zero for Cart 2 — **1 point**

For drawing a horizontal line for Cart 1 when $t > t_C$ that is smaller in magnitude than the momentum of Cart 1 at time $t = t_C$ — **1 point**

For drawing a horizontal line for Cart 2 when $t > t_C$ that is greater in magnitude than the momentum of Cart 1 after time $t = t_C$ — **1 point**

For carts 1 and 2 having a change in momentum that is equal in magnitude, such that Cart 1 loses momentum and Cart 2 gains momentum or a response with changes in momentum consistent with the response in part (a) — **1 point**

Example Response



Total for part (b) — 4 points

(c) For using conservation of energy to find the speed of Cart 1 at the bottom of the incline — **1 point**

OR

For a correct substitution of acceleration and displacement in a kinematics equation to find the speed of Cart 1 at the bottom of the incline

For using conservation of momentum to find the speed of the two-cart system after the collision — **1 point**

For combining correct equations from above — **1 point**

Example Response

Conservation of energy:

$$m_1 gH = \frac{1}{2} m_1 v_1^2$$

$$v_1 = \sqrt{2gH}$$

OR

$$v_f^2 = v_i^2 + 2g\sin(\theta)L$$

$$H = L\sin(\theta)$$

$$v_f^2 = v_i^2 + 2g\sin(\theta)\left(\frac{H}{\sin\theta}\right)$$

$$v_f^2 = 2gH$$

Conservation of momentum:

$$m_1 v_1 = (m_1 + m_2)v_f$$

$$v_f = \frac{m_1}{m_1 + m_2} v_1$$

Combining:

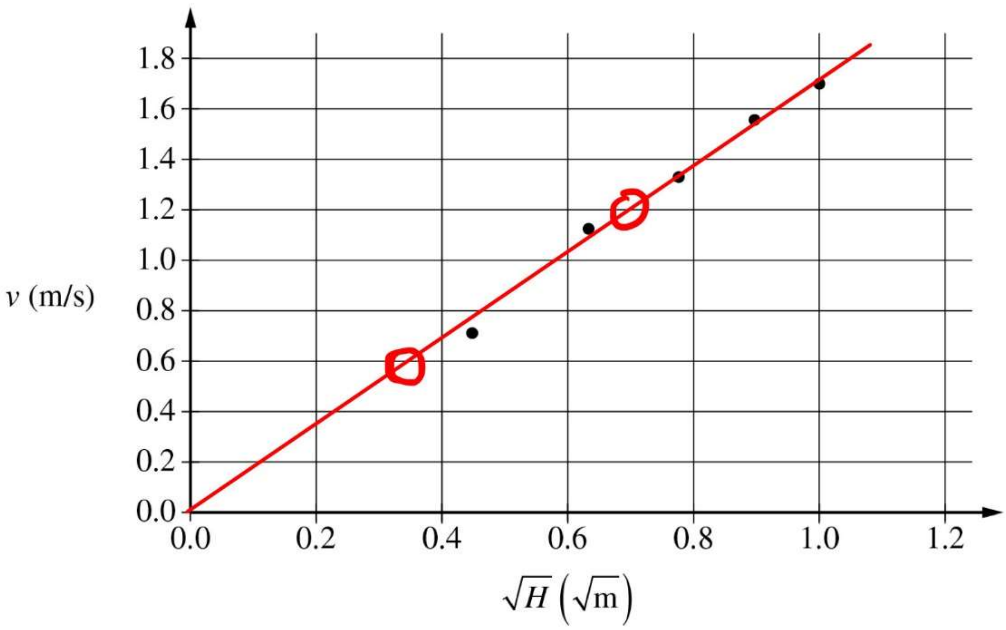
$$v_f = \frac{m_1}{m_1 + m_2}\sqrt{2gH}$$

$$v_f = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}$$

Total for part (c) 3 points

(d)(i)	For drawing an appropriate best-fit line including approximately the same number of points above and below the line	**1 point**

Example Response



(d)(ii)	For calculating the slope using two points on the best-fit line	**1 point**
	For correctly relating the slope of the best-fit line to the mass of Cart 2	**1 point**
	For a correct mass of Cart 2	**1 point**

Example Response

$$slope = \frac{(1.20 - 0.60)\text{ m/s}}{(0.70 - 0.35)\sqrt{\text{m}}} = 1.72\,\frac{\sqrt{\text{m}}}{\text{s}}$$

$$slope = \frac{m_1}{m_1 + m_2}\sqrt{2g}$$

$$m_2 = \frac{m_1\sqrt{2g}}{slope} - m_1$$

$$m_2 = \frac{(0.25\text{ kg})\sqrt{2\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)}}{1.72\,\frac{\sqrt{\text{m}}}{\text{s}}} - (0.25\text{ kg})$$

$$\therefore m_2 = 0.39\text{ kg}$$

Scoring Note: Acceptable responses for mass are 0.30 to 0.60 kg

Total for part (d) 4 points

(e)	For selecting "$m_1' < 0.250$ kg" with an attempt at a relevant justification	**1 point**
	For a correct justification	**1 point**

Example Response

A smaller m_2 indicates that the initial energy and momentum was smaller. With identical slope and height H this means that the mass m_1' must be smaller.

Total for part (e) 2 points

Total for question 2 15 points

Question 2: Free-Response Question 15 points

(a)	For selecting "$J_S > J_2$" with an attempt at a relevant justification	**1 point**
	For a correct justification	**1 point**

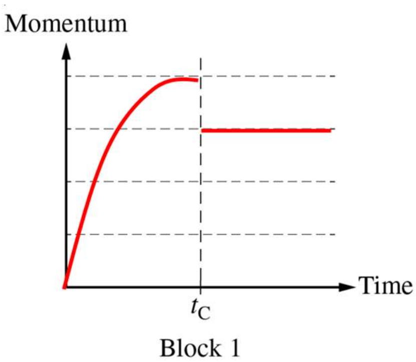
Example Response

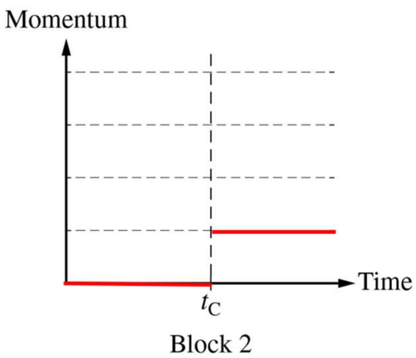
The impulse between the two blocks is the same, so because Block 1 is moving at the end, the impulse given by the spring must be greater than the impulse given to Block 2 by Block 1.

Total for part (a) 2 points

(b)	For correctly drawing the momentum for blocks 1 and 2 for the time interval $0 < t < t_C$: Block 1 shows an increasing curve with a decreasing slope (concave down) to highest value (technically a cosine curve) and Block 2 shows zero momentum up to t_C (a horizontal line along the time axis).	**1 point**
	For drawing a horizontal line for Block 1 when $t > t_C$ that is smaller in magnitude than the momentum of Block 1 at time $t = t_C$	**1 point**
	For drawing a horizontal line for Block 2 when $t > t_C$ that is smaller in magnitude than the momentum of Block 1 after time $t = t_C$	**1 point**
	For blocks 1 and 2 having a change in momentum that is equal in magnitude such that Block 1 loses momentum and Block 2 gains momentum	**1 point**

Example Response





Total for part (b) 4 points

(c)	For using conservation of energy to find the speed of Block 1 at $x = 0$	**1 point**
	For using conservation of momentum to find the speed of the two-block system after the collision	**1 point**
	For combining correct equations from above	**1 point**

Example Response

$$\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}m_1 v_1^2$$

$$v_1 = \sqrt{\frac{k}{m_1}}\Delta x$$

$$m_1 v_1 = (m_1 + m_2)v$$

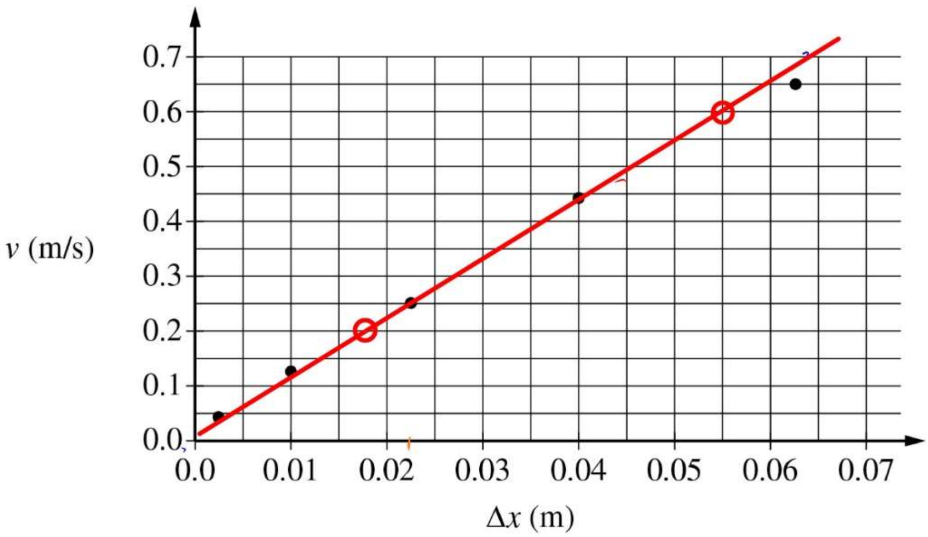
$$v = \frac{m_1 v_1}{m_1 + m_2}$$

$$\therefore v = \frac{\Delta x\sqrt{k m_1}}{m_1 + m_2}$$

Total for part (c) 3 points

(d)(i)	For drawing an appropriate best-fit line including approximately the same number of points above and below the line	**1 point**

Example Response



(d)(ii)	For calculating the slope using two points on the line	**1 point**
	For correctly relating the slope of the best-fit line to the mass of Block 2	**1 point**
	For a correct mass of Block 2	**1 point**

Example Response

$$slope = \frac{(0.60 - 0.02)\ \text{m/s}}{(0.055 - 0.0175)\ \text{m}} = 10.67\ \text{s}^{-1}$$

$$slope = \frac{\sqrt{km_1}}{m_1 + m_2}$$

$$m_2 = \frac{\sqrt{km_1}}{slope} - m_1$$

$$m_2 = \frac{\sqrt{\left(150\ \frac{\text{N}}{\text{m}}\right)(0.50\ \text{kg})}}{10.67\ \text{s}^{-1}} - 0.50\ \text{kg}$$

$$m_2 = 0.31\ \text{kg}$$

Scoring Note: Acceptable responses for mass are $0.27\ \text{kg} - 0.37\ \text{kg}$.

Total for part (d) 4 points

(e)	For selecting "$k' > 150$ N/m" with an attempt at relevant justification	**1 point**
	For a correct justification	**1 point**

Example Response

A greater m_2 indicates the spring constant k' should be greater than 150 N/m. *The slope of the graph is the same so as m_2 increases k must be smaller.*

Total for part (e) 2 points

Total for question 2 15 points