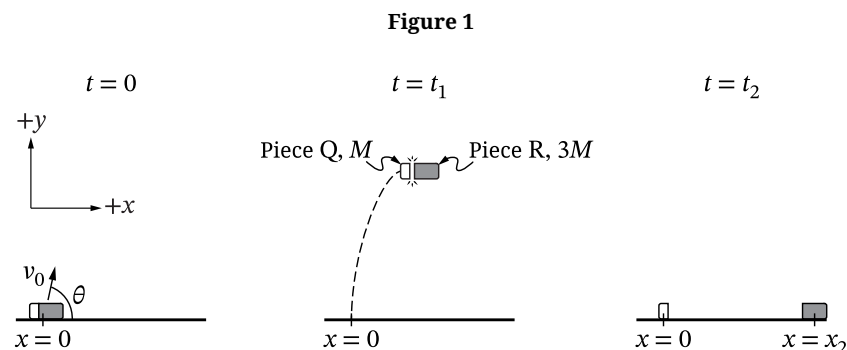


Question 2: Version J

2. A projectile of total mass $4M$ is launched from the ground at position $x = 0$ and time $t = 0$. The projectile is launched with an initial speed v_0 at an angle θ above the horizontal. When the projectile is at the highest point in its trajectory, it breaks into Pieces Q and R of masses M and $3M$, respectively.

The motion of the projectile is described for the following times.

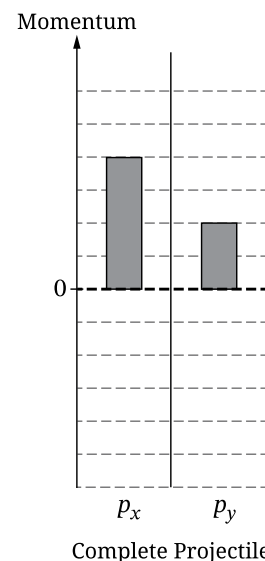
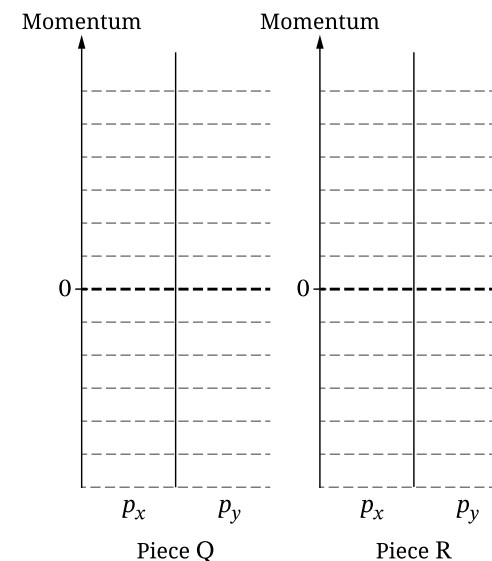
- At $t = t_1$, immediately after the projectile breaks apart, the two pieces are moving away from each other horizontally.
- At $t = t_2$, Piece Q reaches the ground at $x = 0$ and Piece R reaches the ground at $x = x_2$, as shown in Figure 1.



Note: Figure not drawn to scale.

Part A

The horizontal and vertical components of a momentum vector are represented by p_x and p_y , respectively. The shaded bars in Figure 2 represent p_x and p_y of the projectile immediately after $t = 0$.

Figure 2 $t = 0$ **Figure 3** $t = t_1$ 

On Figure 3, **draw** shaded bars to represent p_x and p_y of Pieces Q and R at $t = t_1$.

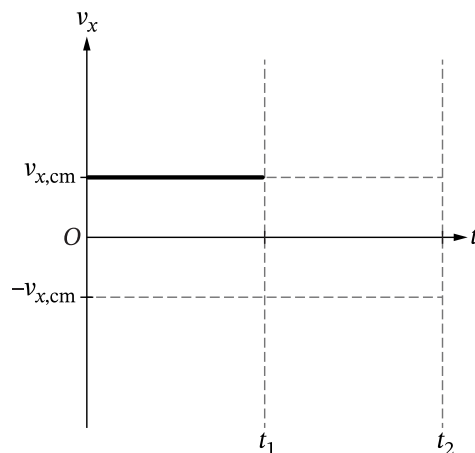
- Start the shaded bars at the dashed line that represents zero momentum.
- Represent the magnitudes of the components of the momentum by using the heights of the shaded bars, consistent with the scale used in Figure 2.
- Represent any momentum component that is equal to zero by drawing a distinct line on the zero-momentum line.

Part B

Derive an expression for x_2 in terms of v_0 , θ , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

The horizontal component of a velocity vector is represented by v_x . Figure 4 shows the horizontal component $v_{x,\text{cm}}$ of the velocity of the center of mass of the projectile as a function of t during the time interval $0 < t < t_1$.

Figure 4

On Figure 4, **sketch** a line or curve to represent v_x as a function of t for the time interval $t_1 < t < t_2$ for each of the following.

- Piece Q
- Piece R
- The center of mass of the two-piece system

Clearly **label** all lines or curves.

Part D

Consider a case in which the projectile is launched at the same angle and initial speed as initially described. When the projectile breaks into Pieces Q and R, Piece Q falls straight down. In this case, Piece R reaches the ground at $x = x_{\text{new}}$.

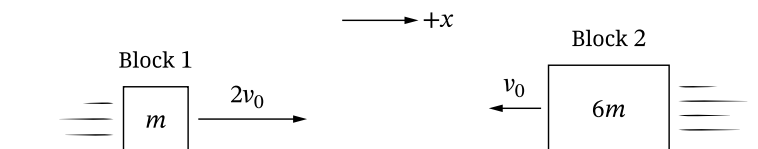
Indicate whether x_{new} is greater than, less than, or equal to x_2 by writing one of the following.

- $x_{\text{new}} > x_2$
- $x_{\text{new}} < x_2$
- $x_{\text{new}} = x_2$

Briefly **justify** your answer either by referencing a feature of the representations you drew in part A or C or by using conceptual reasoning beyond algebraic solutions.

Question 1: Version J

1. Two blocks, 1 and 2, slide toward each other on a horizontal surface. Block 1 has mass m and slides in the $+x$ -direction with constant speed $2v_0$. Block 2 has mass $6m$ and slides in the $-x$ -direction with constant speed v_0 , as shown in Figure 1. The blocks then collide and stick together. The collision occurs from time $t = 0$ to $t = t_c$. After the collision, where $t > t_c$, the blocks move together with the same constant speed.

**Figure 1**

- A. The diagrams in Figure 2 can be used to represent the momentum of blocks 1 and 2 before and after the collision. The momentum vector diagram for Block 1 before the collision is shown.

- i. **Draw** arrows on the grids to represent the momentum vectors of Block 2 before the collision and the two-block system before and after the collision.
- Arrows should start at the zero-momentum line.
 - The length of the arrows should be proportional to the relative magnitudes of the vectors.
 - Represent an arrow of zero length by drawing a dot at zero.

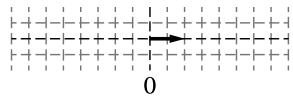
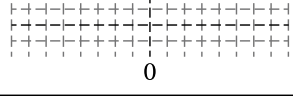
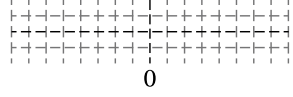
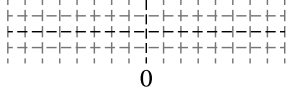
	Momentum Before Collision	Momentum After Collision
Block 1		
Block 2		
Two-Block System		

Figure 2

- ii. During the time interval $0 \leq t \leq t_c$, a force F is exerted on Block 2 by Block 1 along the x -direction as a function of t that is modeled by $F(t) = F_{\max} \sin(At)$, where A is a positive constant and $F_{\max}$ is the magnitude of the maximum force exerted on Block 2 by Block 1 during the collision.

Derive an expression for $F_{\max}$. Express your answer in terms of m , v_0 , A , t_c , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

- B. Consider a new scenario where Block 1 initially slides in the $+x$ -direction with a new constant speed v_1 and Block 2 again initially slides in the $-x$ -direction with constant speed v_0 . The blocks collide and stick together. In this new scenario, the two-block system has constant speed v_0 after the collision.

Derive an expression for v_1 in terms of v_0 . Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Begin your response to **QUESTION 1** on this page.

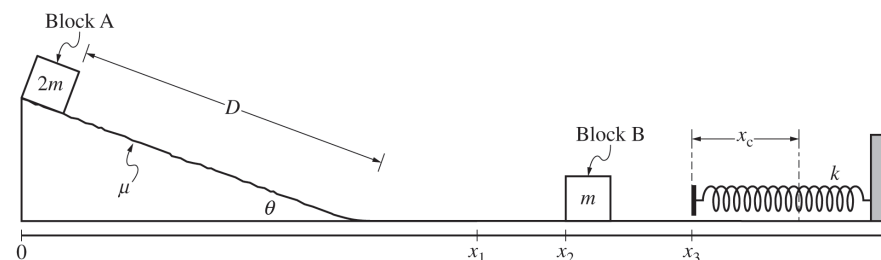
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

1. Blocks A and B of masses $2m$ and m , respectively, are arranged in a setup consisting of a ramp that makes an angle θ with a smooth horizontal table and an ideal spring of spring constant k fixed to a wall, as shown. Block A is held at rest a distance D up the ramp, and Block B is at rest on the horizontal table. The coefficient of kinetic friction between Block A and the rough ramp is μ in the region of length D , and there is negligible friction between the blocks and the smooth table.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.

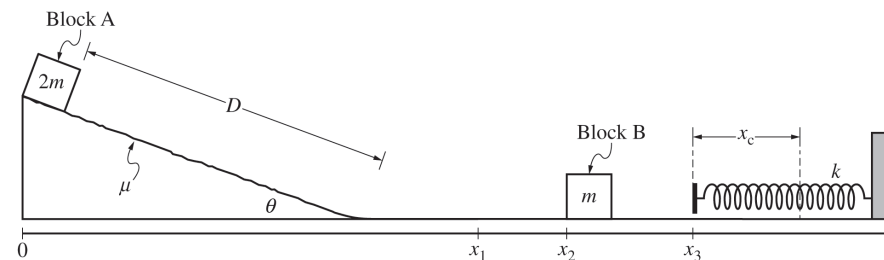
- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$.
- At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B.
- At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position.
- At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m , θ , D , μ , x_c , and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1 .

ii. **Derive** an expression for the spring constant k of the spring.

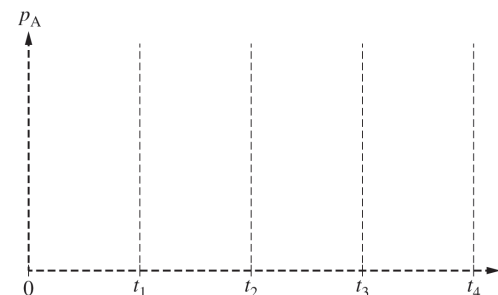
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Continue your response to **QUESTION 1** on this page.

Note: Figure not drawn to scale.

(b)

i. On the following axes, **sketch** a graph of the magnitude of the momentum p_A of Block A as a function of time t from $t = 0$ to t_4 .



ii. Use principles of forces to **justify** the graph drawn in part (b)(i) for the time interval $t = t_3$ to $t = t_4$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

For times $t > t_4$, the two-block-spring system oscillates with period T_O . The procedure is then repeated using a new ramp, where there is negligible friction between Block A and the ramp.

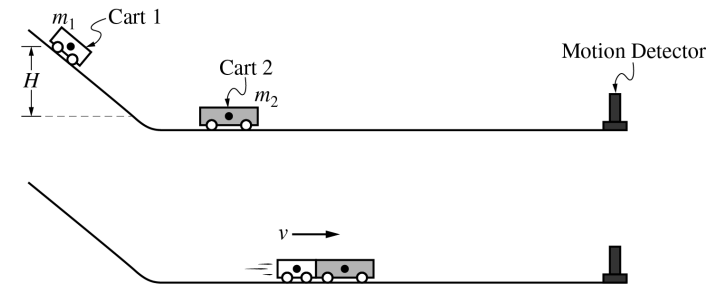
- (c) **Indicate** how the new period of oscillation T_N in the procedure that uses the new ramp compares with the period of oscillation T_O from the original procedure.

_____ $T_N > T_O$ _____ $T_N < T_O$ _____ $T_N = T_O$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 2** on this page.



Cart 1 of mass m_1 is held at rest above the bottom of an incline. Cart 2 has mass m_2 , where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and smoothly transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance of H , as shown. At time t_C , Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v . Frictional and rotational effects are negligible.

- (a) During the collision, is the impulse on Cart 1 from Cart 2 greater than, less than, or equal to the magnitude of the impulse on Cart 2 from Cart 1?

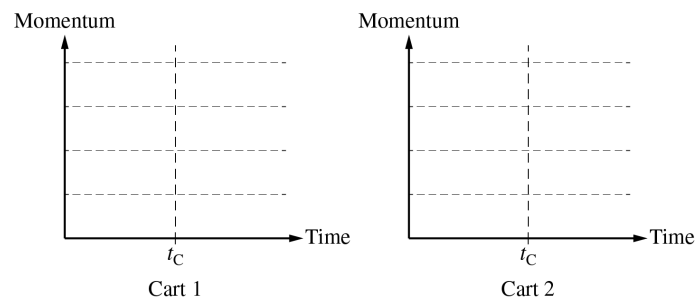
___ Greater than ___ Less than ___ Equal to

Justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

- (b) On the following axes, draw graphs of the magnitude of the momentum of each cart as a function of time t , before and after t_C . The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.



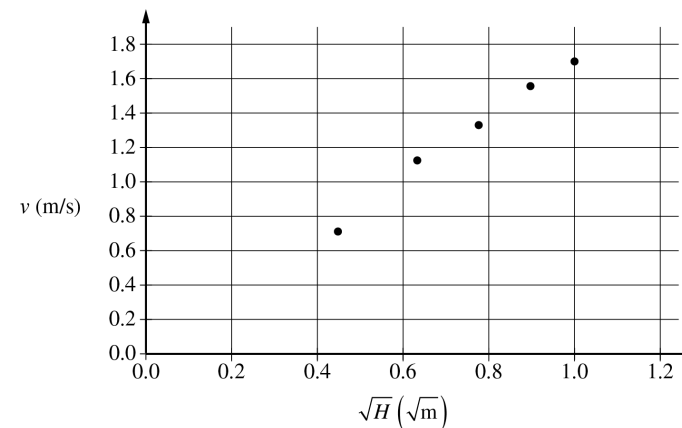
- (c) Show that the velocity v of the two-cart system after the collision is given by the equation

$$v = \sqrt{2g} \left(\frac{m_1}{m_1 + m_2} \right) \sqrt{H}.$$

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

- (d) A group of students use the setup to perform an experiment. They measure the mass of Cart 1 to be $m_1 = 0.250$ kg. The mass of Cart 2 is unknown. The students perform several trials and in each trial, Cart 1 is released from a different height H and the final velocity of the two-cart system is measured. The students graph v as a function of $\sqrt{H}$, as shown below.



- Draw a line that represents the best fit to the data points shown.
- Use the best-fit line to calculate the mass of Cart 2.

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Continue your response to **QUESTION 2** on this page.

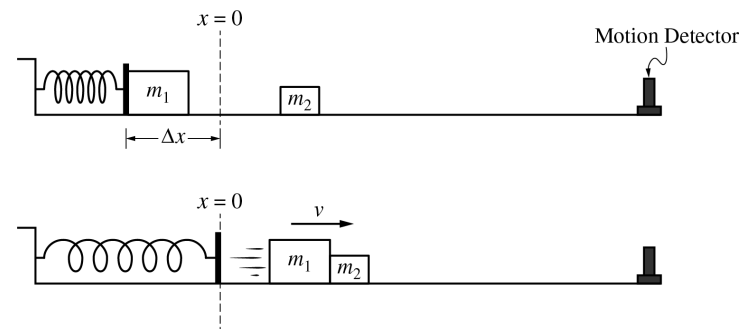
- (e) After the experiment, the students use a balance to measure the mass of Cart 2 and find it to be less than what was determined in part (d). To explain this discrepancy, one of the students proposes that the mass of Cart 1 was incorrectly measured at the beginning of the experiment. The students measure the mass of Cart 1 again and record a new value, m_1' .

Should the students expect that m_1' will be greater than 0.250 kg, less than 0.250 kg, or equal to 0.250 kg ?

☐ $m_1' > 0.250 \text{ kg}$ ☐ $m_1' < 0.250 \text{ kg}$ ☐ $m_1' = 0.250 \text{ kg}$

Justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 2** on this page.

Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx . The spring constant of the spring is k . Block 2 has mass m_2 , where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C , the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v , as shown. Frictional effects are negligible.

- (a) The impulse on Block 1 from the spring during the time interval $0 < t < t_C$ is J_S . The impulse on Block 1 from Block 2 during the collision is J_2 . Which of the following expressions correctly compares the magnitudes of J_S and J_2 ?

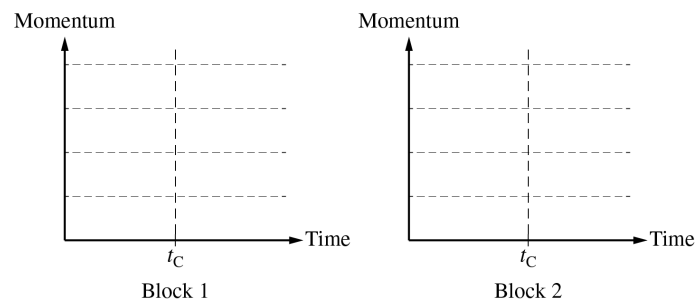
☐ $J_S > J_2$ ☐ $J_S < J_2$ ☐ $J_S = J_2$

Justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

- (b) On the following axes, draw graphs of the magnitude of the momentum of each block as a function of time, before and after t_C . The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.

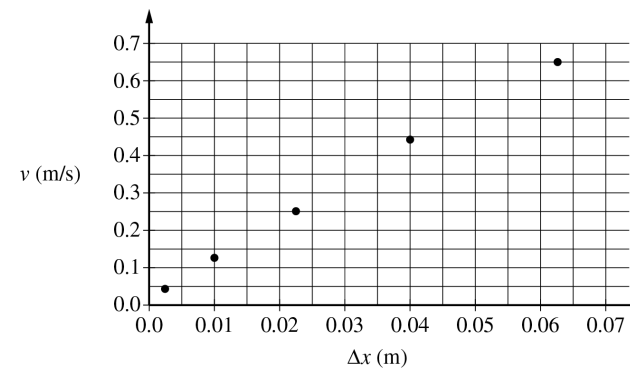


- (c) Show that the velocity v of the two-block system after the collision is given by the equation $v = \frac{\sqrt{km_1}}{m_1 + m_2} \Delta x$.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

- (d) A group of students use the setup to perform an experiment. They measure the mass of Block 1 to be $m_1 = 0.500$ kg, and the spring constant k of the spring to be 150 N/m. The mass of Block 2 is unknown. They perform several trials and in each trial the spring is compressed a different distance Δx and the final velocity v of the two-block system is measured. They graph v as a function of Δx , as shown below.



- Draw a line that represents the best fit to the data points shown.
- Use the best-fit line to calculate the mass of Block 2.

GO ON TO THE NEXT PAGE.

Name: _____

Continue your response to **QUESTION 2** on this page.

- (e) After the experiment, the students use a balance to measure the mass of Block 2 and find it to be greater than what was determined in part (d). To explain this discrepancy, one of the students proposes that the spring constant was incorrectly measured at the beginning of the experiment. The students measure the spring constant again and record a new value, k' .

Should the students expect that k' be greater than 150 N/m , less than 150 N/m , or equal to 150 N/m ?

_____ $k' > 150 \text{ N/m}$ _____ $k' < 150 \text{ N/m}$ _____ $k' = 150 \text{ N/m}$

Justify your answer.

GO ON TO THE NEXT PAGE.