Question 2: Translation Between Representations (TBR) — 12 points

A	For indicating that K_{block} is zero	**Point A1**
	For drawing bars with positive heights for U_A and U_B	**Point A2**
	For drawing a bar for U_B with a height that is twice the height of the bar drawn for U_A **Scoring Note:** This point may be earned regardless of the signs of either bar.	**Point A3**

Example Response

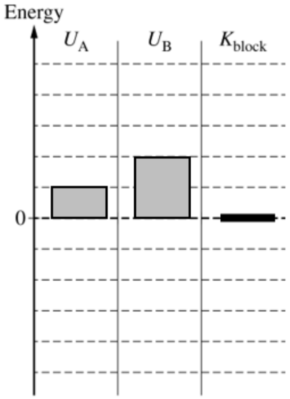


Figure 3

B	For a multistep derivation that includes energy conservation or simple harmonic motion	**Point B1**
	For relating the presence of both springs to the behavior of the system	**Point B2**
	For relating positions $x = x_1$ and $x = \frac{1}{2}x_1$ to the oscillation of the block	**Point B3**
	For a correct expression for v in terms of given quantities	**Point B4**

Example Responses

$$E_{\text{tot},0} = U_A(x_1) + U_B(x_1) + K_{\text{block}}(x_1)$$
$$E_{\text{tot},f} = U_A\left(\frac{1}{2}x_1\right) + U_B\left(\frac{1}{2}x_1\right) + K_{\text{block}}\left(\frac{1}{2}x_1\right)$$

$$E_{\text{tot},0} = \frac{1}{2}(k)(x_1)^2 + \frac{1}{2}(2k)(x_1)^2 + 0$$
$$E_{\text{tot},f} = \frac{1}{2}(k)\left(\frac{1}{2}x_1\right)^2 + \frac{1}{2}(2k)\left(\frac{1}{2}x_1\right)^2 + K_{\text{block},\frac{1}{2}x_1}$$

$$E_{\text{tot},0} = E_{\text{tot},f}$$
$$\frac{1}{2}kx_1^2 + kx_1^2 = \frac{1}{8}kx_1^2 + \frac{1}{4}kx_1^2 + K_{\text{block},\frac{1}{2}x_1}$$
$$\frac{3}{2}kx_1^2 = \frac{3}{8}kx_1^2 + K_{\text{block},\frac{1}{2}x_1}$$

$$K_{\text{block},\frac{1}{2}x_1} = \frac{9}{8}kx_1^2$$
$$\frac{1}{2}mv^2 = \frac{9}{8}kx_1^2$$

$$v = \frac{3}{2}x_1\sqrt{\frac{k}{m}}$$

OR

$$x = x_{\max}\cos(\omega t + \phi)$$

$$x(t) = x_1\cos\omega t$$
$$v(t) = -x_1\omega\sin\omega t$$

$$k_{\text{eff}} = k + 2k = 3k$$
$$\omega = \sqrt{\frac{3k}{m}}$$

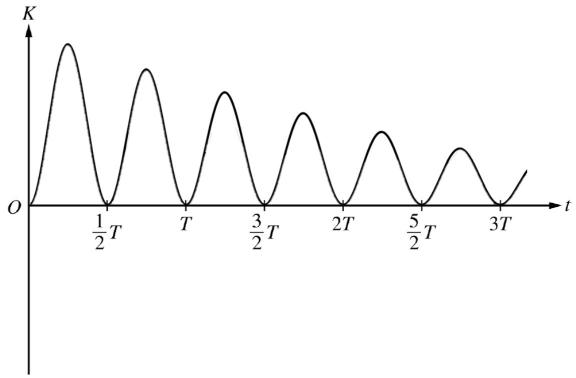
$$\frac{x_1}{2} = x_1\cos\omega t_{\frac{1}{2}x_1}$$
$$\cos\omega t_{\frac{1}{2}x_1} = \frac{1}{2}$$
$$t_{\frac{1}{2}x_1} = \frac{\pi}{3\omega}$$

$$v_{\frac{1}{2}x_1} = -x_1\omega\sin\omega t_{\frac{1}{2}x_1}$$
$$v_{\frac{1}{2}x_1} = -x_1\sqrt{\frac{3k}{m}}\sin\frac{\pi}{3}$$

$$v = \left|v_{\frac{1}{2}x_1}\right| = \frac{3}{2}x_1\sqrt{\frac{k}{m}}$$

C	For sketching a curve that starts at zero and is always positive or zero	**Point C1**
	For sketching a periodic curve with zeros that have a period of $\frac{1}{2}T$	**Point C2**
	For sketching a periodic curve with a decreasing amplitude	**Point C3**

Example Response



Scenario 2

Figure 5

D	For indicating **one** of the following: • The period increases. • The rate at which the graph approaches zero increases. • The maximum value of K over each period is less than the graph drawn.	**Point D1**
	For a correct justification relevant to the feature indicated	**Point D2**

Example Responses

The period of the graph increases. The period of a simple harmonic oscillator increases with increasing mass.

OR

The rate at which the graph approaches zero amplitude increases. Because the mass of the block is larger, the force of friction is greater, which causes the block to lose energy at a greater rate.

Question 3: Experimental Design and Analysis (LAB) — 10 points

A	For describing a procedure that includes measuring h and the speed of the block as the block moves across the surface	**Point A1**
	For a procedure that indicates a reasonable method of reducing experimental uncertainty Examples of acceptable responses may include the following: • Repeating the experiment multiple times for the same value of h • Repeating the experiment for multiple values of h	**Point A2**

Example Response

Measure the height h at which the block-box system is released. Measure the speed of the block as the block slides across the horizontal surface. Repeat the measurement of the speed of the block for varying release heights.

B For describing a graph that has linear trend that can be used to find g — **Point B1**

Examples of acceptable responses include the following:

- v^2 vs. $2h$
- $\frac{1}{2}v^2$ vs. h
- v^2 vs. h
- v vs. $\sqrt{h}$

Scoring Notes:

- Reponses that include the reciprocals of the preceding examples, in addition to other equivalent graphs, also earn this point.
- This point may be earned independently of the response in part A.

For correctly relating the slope of the best-fit line to g — **Point B2**

Examples of acceptable responses include the following:

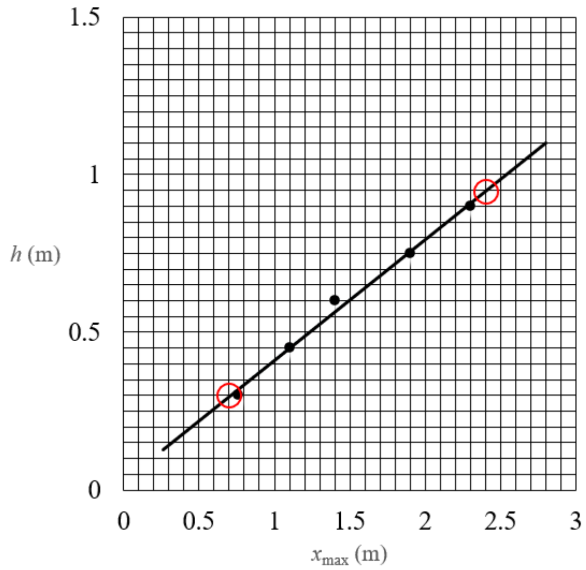
Graph	Analysis
v^2 vs. $2h$	slope $= g$
$\frac{1}{2}v^2$ vs. h	slope $= g$
v^2 vs. h	slope $= 2g$
v vs. $\sqrt{h}$	slope $= \sqrt{2g}$

Example Response

Plot v^2 on the vertical axis and $2h$ on the horizontal axis. The slope of the best-fit line is equal to g.

C	**(i)**	For indicating appropriate quantities that could be plotted to produce a linear graph that can be used to determine μ, such as h **vs.** $x_{\max}$ **Scoring Note:** Reponses that include the reciprocal of the preceding example, in addition to other equivalent graphs, also earn this point.	**Point C1**
	(ii)	For labeling the axes (including units) with a linear scale	**Point C2**
		For plotting data points consistent with **one** of the following: • The quantities indicated in part C (i) • The quantities provided in Table 2 • The axes indicated on the grid	**Point C3**
	(iii)	For drawing a line or curve that approximates the trend of the plotted data	**Point C4**

Example Response



AP Physics C: **Mechanics: 2025**

D For correctly relating the slope of the best-fit line to the value of μ **Point D1**

Examples of acceptable responses includes the following:

Graph	**Analysis**
h vs. $x_{\max}$	slope $= \mu$

For a value of μ within a range of 0.30 to 0.50 **Point D2**

Example Response

$$E_0 + W = E_f$$
$$mgh - F_f x_{\max} = 0$$
$$mgh = \mu mg x_{\max}$$
$$h = \mu x_{\max}$$

$$y = mx + b$$
$$h = \mu x_{\max}$$
$$\text{slope} = \mu$$
$$\text{slope} = \frac{0.95\text{ m} - 0.30\text{ m}}{2.4\text{ m} - 0.70\text{ m}}$$
$$\text{slope} = 0.38$$
$$\mu = \text{slope} = 0.38$$

Question 1: Free-Response Question — 15 points

(a)(i) For a multi-step derivation with an application of the conservation of mechanical energy that indicates that all of the energy of the system is initially U_s **1 point**

Example Response

$$E_{\text{initial}} = E_{\text{final}}$$
$$\frac{1}{2}kx_c^2 = \frac{1}{2}mv^2$$

For a correct solution for v **1 point**

Example Response

$$v = x_c\sqrt{\frac{k}{m}}$$

Example Solution

$$E_{\text{initial}} = E_{\text{final}}$$
$$U_s = K$$
$$\frac{1}{2}kx_c^2 = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{kx_c^2}{m}}$$
$$v = x_c\sqrt{\frac{k}{m}}$$

(a)(ii) For a derivation to solve for the speed at x_2 that includes **one** of the following: **1 point**

- An appropriate application of the conservation of energy
- An appropriate kinematics equation

Example Responses

$K_{\text{initial}} - \Delta E_{\text{friction}} = K_{\text{final}}$ **OR** $v^2 = v_0^2 + 2a\Delta x$

For **one** of the following that is consistent with the previous point in the response for part (a)(ii): **1 point**

- A correct expression for the energy dissipated by friction
- A correct expression for the acceleration of the block in the region with nonnegligible friction

Example Responses

$\Delta E_{\text{friction}} = \mu mgD$ **OR** $a = -\mu g$

For attempting to derive an expression for $v_{\text{A,B}}$ by using the conservation of momentum **1 point**

Example Response

$m_{\text{initial}}v_{\text{initial}} = m_{\text{A,B}}v_{\text{A,B}}$

For substituting the expression for the speed at x_2 that is consistent with the first point of the response in part (a)(ii) and substituting the correct masses into an expression for conservation of momentum **1 point**

Example Response

$$v_{\text{A,B}} = \frac{m\sqrt{\frac{kx_c^2}{m} - 2\mu gD}}{4m}$$

Example Solutions

$E_{x_1} = E_{\text{before collision}}$

$K_{x_1} - \Delta E_{\text{friction}} = K_{\text{before collision}}$

$$\frac{1}{2}m\left(\sqrt{\frac{kx_c^2}{m}}\right)^2 - \mu mgD = \frac{1}{2}mv_2^2$$

$$v_2 = \sqrt{\frac{kx_c^2}{m} - 2\mu gD}$$

$\Sigma p_{\text{before collision}} = \Sigma p_{\text{after collision}}$

$m_{\text{A}}v_2 = m_{\text{A,B}}v_{\text{A,B}}$

$m_{\text{A}}v_2 = (m + 3m)v_{\text{A,B}}$

$$v_{\text{A,B}} = \frac{m\sqrt{\frac{kx_c^2}{m} - 2\mu gD}}{4m}$$

$$v_{\text{A,B}} = \frac{1}{4}\sqrt{\frac{kx_c^2}{m} - 2\mu gD}$$

OR

$v_2^2 = v^2 + 2a\Delta x$

$$v_2^2 = \left(x_c\sqrt{\frac{k}{m}}\right)^2 + 2aD$$

$$v_2 = \sqrt{\frac{kx_c^2}{m} + 2aD}$$

$\Sigma F_x = -F_f = ma$

$-\mu mg = ma$

$a = -\mu g$

$$v_2 = \sqrt{\frac{kx_c^2}{m} - 2\mu gD}$$

$\Sigma p_{\text{before collision}} = \Sigma p_{\text{after collision}}$

$m_{\text{A}}v_2 = m_{\text{A,B}}v_{\text{A,B}}$

$m_{\text{A}}v_2 = (m + 3m)v_{\text{A,B}}$

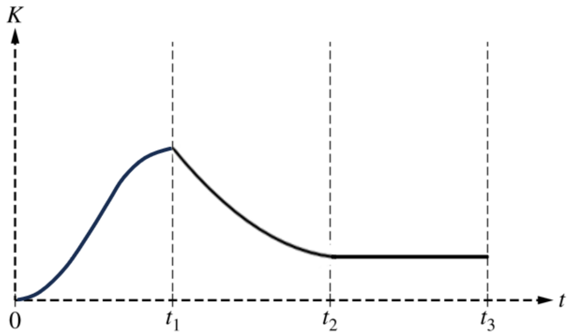
$$v_{\text{A,B}} = \frac{m\sqrt{\frac{kx_c^2}{m} - 2\mu gD}}{4m}$$

$$v_{\text{A,B}} = \frac{1}{4}\sqrt{\frac{kx_c^2}{m} - 2\mu gD}$$

Total for part (a) 6 points

(b)(i)	For a nonlinear sketch that begins at zero and increases for the entire time interval $0 \le t \le t_1$	**1 point**
	For a sketch that decreases for the entire time interval $t_1 \le t \le t_2$ but does not go to zero	**1 point**
	For a sketch that is concave up for the time interval $t_1 \le t \le t_2$	**1 point**
	For a continuous function for the time interval $t_1 \le t \le t_3$ that has a horizontal line that is greater than zero for the time interval $t_2 \le t \le t_3$	**1 point**

Example Response



(b)(ii)	For a statement about the change in kinetic energy that is consistent with the graph drawn in the response for part (b)(i)	**1 point**
	For a correct explanation for why the kinetic energy is increasing, such as **one** of the following: • An increasing graph means positive work is being done on the block. • An external force is exerted on Block A, causing the velocity of the block to increase and the kinetic energy of the block to increase. • Mechanical energy is conserved and/or there is no work done for the block-spring system, and the potential energy decreases.	**1 point**
	For a correct explanation for why the graph is nonlinear, such as **one** of the following: • The rate at which the slope of the graph changes is related to the rate at which work is being done on the block. • The external force exerted on Block A is changing, which causes a nonuniform change in the velocity of Block A, which results in a nonuniform change in kinetic energy.	**1 point**

Example Response

From $0 < t < t_1$, the kinetic energy of Block A increases. The force exerted on the block by the compressed spring transfers the elastic potential energy in the block-spring system to the kinetic energy of the block. Because the force exerted by the spring is not applied at a constant rate, the kinetic energy of the block does not increase at a constant rate.

Total for part (b) 7 points

(c)	For selecting $f_{2\ell} < f_\ell$ with an attempt at a relevant justification	**1 point**
	For correctly applying an equation that relates the length of a pendulum to the period or frequency of the pendulum	**1 point**

Example Response

The period of a pendulum is calculated by using $T = 2\pi\sqrt{\frac{l}{g}}$. Therefore, as the length is increased, the period will also increase. Because frequency and period are inversely related, an increase in period will result in a decrease in frequency.

Total for part (c) 2 points

Total for question 1 15 points

Question 1: Free-Response Question 15 points

(a)(i)	For a multi-step derivation that includes **one** of the following: • An application of conservation of energy that indicates that the initial mechanical energy of the system is U_g • An application of Newton's second law that includes the frictional force, including sign	**1 point**
	For **one** of the following that is consistent with the previous point: • An expression for the energy dissipated by friction, including the correct sign • A substitution of acceleration in a kinematics equation	**1 point**

Example Responses

$\Delta E_{\text{friction}} = -F_f D$ **OR** $v^2 = v_0^2 + 2\left(\frac{F_{g,x} - F_f}{2m}\right)\Delta x$

	For a correct expression for v	**1 point**

Example Response

$v = \sqrt{2gD(\sin\theta - \mu\cos\theta)}$

Example Solutions

$$E_{\text{initial}} = E_{\text{final}}$$
$$U_g - \Delta E_{\text{friction}} = K$$
$$m_A gD\sin\theta - \mu m_A gD\cos\theta = \frac{1}{2}m_A v^2$$
$$gD\sin\theta - \mu gD\cos\theta = \frac{1}{2}v^2$$
$$v = \sqrt{2gD(\sin\theta - \mu\cos\theta)}$$

OR

$$a = \frac{F_{\text{net}}}{m}$$
$$2ma = F_{g,x} - F_f$$
$$2ma = 2mg\sin\theta - \mu(2m)g\cos\theta$$
$$a = g\sin\theta - \mu g\cos\theta$$
$$v^2 = v_0^2 + 2a\Delta x$$
$$v^2 = 0^2 + 2(g\sin\theta - \mu g\cos\theta)D$$
$$v = \sqrt{2gD(\sin\theta - \mu\cos\theta)}$$

(a)(ii) For using the conservation of momentum to find $v_{A,B}$ — **1 point**

For equating the kinetic energy after the collision between the blocks to the maximum elastic potential energy of the compressed spring — **1 point**

Example Response

$K_{\text{after collision}} = U_{s,\text{ max}}$

For indicating v before the collision between the blocks and the spring is equal to $v_{A,B}$ — **1 point**

Example Solution

$$p_{\text{before collision}} = p_{\text{after collision}}$$
$$2mv = (2m + m)v_{A,B}$$
$$v_{A,B} = \frac{2m\sqrt{2gD(\sin\theta - \mu\cos\theta)}}{(3m)}$$
$$v_{A,B} = \frac{2}{3}\sqrt{2gD(\sin\theta - \mu\cos\theta)}$$

$$E_{\text{after collision}} = E_{\text{max compression of spring}}$$
$$K_{\text{after collision}} = U_{s,\text{ max}}$$
$$\frac{1}{2}(2m + m)\left(v_{A,B}\right)^2 = \frac{1}{2}kx^2$$
$$(3m)\left(\frac{2}{3}\sqrt{2gD(\sin\theta - \mu\cos\theta)}\right)^2 = kx_c^{\,2}$$
$$k = \frac{(3m)}{x_c^{\,2}}\left(\frac{2}{3}\sqrt{2gD(\sin\theta - \mu\cos\theta)}\right)^2$$
$$k = \frac{8}{3}\frac{mgD(\sin\theta - \mu\cos\theta)}{x_c^{\,2}}$$

Total for part (a) 6 points

(b)(i) For a sketch that increases linearly during the time interval $0 \le t < t_1$ — **1 point**

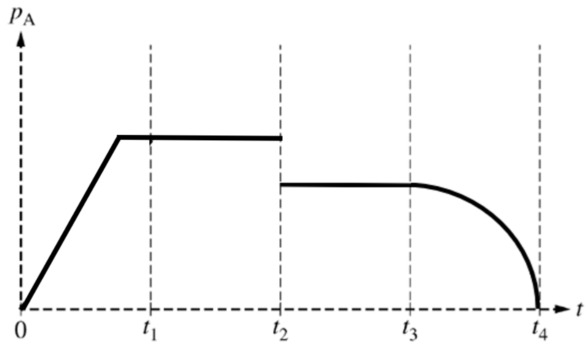
Scoring Note: A sketch that only increases linearly from $t = 0$ to $t = t_1$ earns this point.

For a horizontal line for the time interval $t_1 \le t \le t_2$ that is continuous at t_1 — **1 point**

For a horizontal line for the time interval $t_2 \le t \le t_3$ that has a smaller magnitude than the previous time interval — **1 point**

For drawing a concave down curve, continuous at t_3, in the interval $t_3 \le t < t_4$ that reaches zero at t_4 — **1 point**

Example Response



(b)(ii) For a statement about the change in momentum that is consistent with the graph drawn in the response for part (b)(i) — **1 point**

For indicating that a decreasing graph means that the force exerted on Block A is in a direction opposite to the motion of Block A — **1 point**

Scoring Note: A response that indicates that an increasing graph means that the force exerted on Block A is in the same direction as the motion of Block A also earns this point.

For relating the change in momentum to the magnitude of the force exerted on Block A — **1 point**

Example Response

The momentum of Block A *decreases between* t_3 *and* t_4 *because the spring exerts a force on the blocks in the opposite direction of the velocity of the blocks, causing the blocks to slow to a stop. The spring force increases the more the spring compresses, so the momentum of Block* A *decreases at an increasing rate, which is shown in the slope of the curve becoming steeper with time.*

Total for part (b) 7 points

(c) For selecting $T_N = T_O$ with an attempt at a relevant justification — **1 point**

For a correct justification that includes **one** of the following: — **1 point**

- The period of a spring-block oscillator is only dependent on the mass on the spring and spring constant, which do not change.
- The period of a spring-block oscillator is not dependent on increasing amplitude, velocity, or compression distance.

Example Response

Repeating the experiment on a smooth ramp will only affect the compression distance of the spring. The period of oscillation of a spring-block system depends only on mass and the spring constant, therefore the period of oscillation will not change.

Total for part (c) 2 points

Total for question 1 15 points

Question 2: Free-Response Question — 15 points

(a) For selecting "Equal to" with an attempt at a relevant justification — **1 point**

For a correct justification — **1 point**

Example Response

Newton's third law says equal/opposite forces, time intervals are same during same collision, so magnitudes of impulse must be equal.

Total for part (a) 2 points

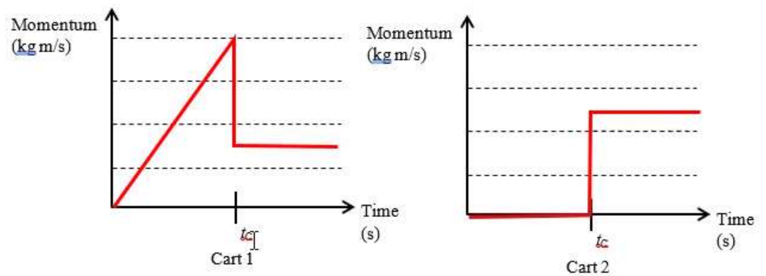
(b) For correctly drawing the momentum for carts 1 and 2 for the time interval $0 < t < t_C$: Linear increasing momentum for Cart 1 and zero for Cart 2 — **1 point**

For drawing a horizontal line for Cart 1 when $t > t_C$ that is smaller in magnitude than the momentum of Cart 1 at time $t = t_C$ — **1 point**

For drawing a horizontal line for Cart 2 when $t > t_C$ that is greater in magnitude than the momentum of Cart 1 after time $t = t_C$ — **1 point**

For carts 1 and 2 having a change in momentum that is equal in magnitude, such that Cart 1 loses momentum and Cart 2 gains momentum or a response with changes in momentum consistent with the response in part (a) — **1 point**

Example Response



Total for part (b) 4 points

(c) For using conservation of energy to find the speed of Cart 1 at the bottom of the incline — **1 point**

OR

For a correct substitution of acceleration and displacement in a kinematics equation to find the speed of Cart 1 at the bottom of the incline

For using conservation of momentum to find the speed of the two-cart system after the collision — **1 point**

For combining correct equations from above — **1 point**

Example Response

Conservation of energy:

$$m_1 gH = \frac{1}{2}m_1 v_1^2$$

$$v_1 = \sqrt{2gH}$$

OR

$$v_f^2 = v_i^2 + 2g\sin(\theta)L$$

$$H = L\sin(\theta)$$

$$v_f^2 = v_i^2 + 2g\sin(\theta)\left(\frac{H}{\sin\theta}\right)$$

$$v_f^2 = 2gH$$

Conservation of momentum:

$$m_1 v_1 = (m_1 + m_2)v_f$$

$$v_f = \frac{m_1}{m_1 + m_2}v_1$$

Combining:

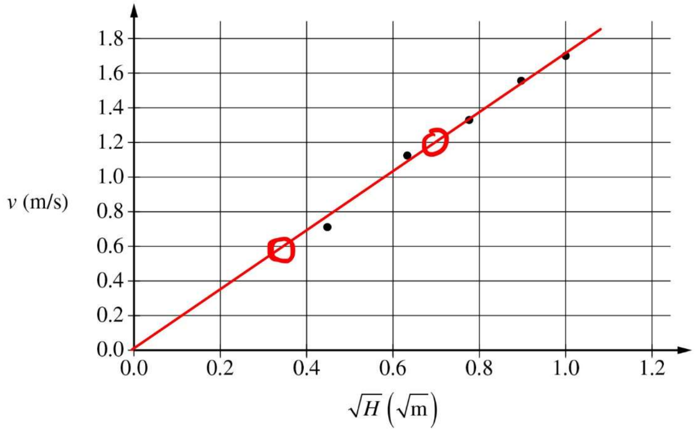
$$v_f = \frac{m_1}{m_1 + m_2}\sqrt{2gH}$$

$$v_f = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}$$

Total for part (c) 3 points

(d)(i)	For drawing an appropriate best-fit line including approximately the same number of points above and below the line	**1 point**

Example Response



(d)(ii)	For calculating the slope using two points on the best-fit line	**1 point**
	For correctly relating the slope of the best-fit line to the mass of Cart 2	**1 point**
	For a correct mass of Cart 2	**1 point**

Example Response

$$slope = \frac{(1.20 - 0.60)\text{ m/s}}{(0.70 - 0.35)\sqrt{\text{m}}} = 1.72\frac{\sqrt{\text{m}}}{\text{s}}$$

$$slope = \frac{m_1}{m_1 + m_2}\sqrt{2g}$$

$$m_2 = \frac{m_1\sqrt{2g}}{slope} - m_1$$

$$m_2 = \frac{(0.25\text{ kg})\sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)}}{1.72\frac{\sqrt{\text{m}}}{\text{s}}} - (0.25\text{ kg})$$

$$\therefore m_2 = 0.39\text{ kg}$$

Scoring Note: Acceptable responses for mass are 0.30 to 0.60 kg

Total for part (d) 4 points

(e)	For selecting "$m_1' < 0.250$ kg" with an attempt at a relevant justification	**1 point**
	For a correct justification	**1 point**

Example Response

A smaller m_2 indicates that the initial energy and momentum was smaller. With identical slope and height H this means that the mass m_1' must be smaller.

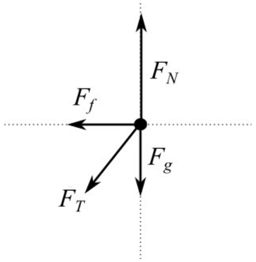
Total for part (e) 2 points

Total for question 2 15 points

Question 1: Free-Response Question — 15 points

(a)	For correctly drawing and labeling the force of gravity and the normal force on the sled	**1 point**
	For correctly drawing and labeling the force of friction on the on the sled	**1 point**
	For correctly drawing and labeling the force of tension on the on the sled	**1 point**

Example Response



Scoring Notes:

- Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force. F_{string}, F_s, F_T, F_{Tension}, T, "string force," "tension force," or similar labels may be used for the tension force exerted by the string.
- A response with extraneous forces or vectors can earn a maximum of two points.

Total for part (a) 3 points

(b)	For any correct trigonometric expression for θ in terms of the given quantities	**1 point**

$$\sin\theta = \frac{x}{\sqrt{y^2 + x^2}} \qquad \therefore \theta = \sin^{-1}\left(\frac{x}{\sqrt{y^2 + x^2}}\right)$$

OR

$$\cos\theta = \frac{y}{\sqrt{y^2 + x^2}} \qquad \therefore \theta = \cos^{-1}\frac{y}{\sqrt{y^2 + x^2}}$$

OR

$$\tan\theta = \frac{x}{y} \qquad \therefore \theta = \tan^{-1}\frac{x}{y}$$

Total for part (b) 1 point

(c)(i) For beginning the derivation with Newton's second law to write an equation that is consistent with part (a). **1 point**

$$\Sigma F = ma$$
$$F_N - F_{T,y} - F_g = 0$$
$$F_N = F_{T,y} + F_g$$

Scoring Note: Derivation must start with a statement of Newton's second law to earn this point.

For correctly substituting the vertical component of the tension in terms of the given variables consistent with part (b) **1 point**

$$F_N = mg + F_T \cos\theta$$
$$F_N = mg + F_T\left(\frac{y}{\sqrt{y^2 + x^2}}\right)$$

(c)(ii) For summing the forces in the horizontal direction, consistent with parts (a) and (b) **1 point**

$$F_{net} = -F_{T,x} - F_f$$

For correctly substituting the horizontal component of the force of tension in terms of the given variables **1 point**

$$F_{net} = -F_T \sin\theta - F_f$$
$$F_{net} = -F_T\left(\frac{x}{\sqrt{y^2 + x^2}}\right) - F_f$$

For correctly substituting the expression for the normal force from part (c)(i) into the expression for the force of friction **1 point**

$$F_{net} = -F_T\left(\frac{x}{\sqrt{y^2 + x^2}}\right) - \mu_k F_N$$
$$F_{net} = -F_T\left(\frac{x}{\sqrt{y^2 + x^2}}\right) - \mu_k\left(mg + F_T\left(\frac{y}{\sqrt{y^2 + x^2}}\right)\right)$$

Total for part (c) 5 points

(d) For using the integral definition of work to derive an expression for the work done by the force of tension **1 point**

$$W = \int F \cdot dx$$

For any indication that the work done on the sled by the string is due only to the horizontal component of the tension in the string **1 point**

$$W = \int F_{T,x} dx$$

For using the horizontal component of the force of tension consistent with part (b) or (c) **1 point**

$$W = \int_{x=0}^{x=L} -F_T\left(\frac{x}{\sqrt{y^2 + x^2}}\right)dx$$

For correct limits of integration, x= 0 to x= L, and indicating that the work done is negative **1 point**

let $u = y^2 + x^2$, then $du = 2x\ dx$

$$W = -F_T \int_{x=0}^{x=L} \frac{1}{2}\left(\frac{1}{\sqrt{u}}\right)du$$
$$W = -F_T\left(\sqrt{y^2 + x^2}\right)\Big|_{x=0}^{x=L}$$
$$W = -F_T\left(\sqrt{y^2 + L^2} - \sqrt{y^2}\right)$$

Total for part (d) 4 points

(e) For selecting "$E_1 > E_2$" with an attempt at a relevant justification **1 point**

For a justification that correctly relates the force of friction to the normal force, and the normal force to the position or the angle **1 point**

Example Response

The vertical component of the string force is the largest when the string is more vertical ($0 < x < L$). A larger vertical component of the string tension leads to a larger normal force and, hence, a larger friction force.

Total for part (e) 2 points

Total for question 1 15 points

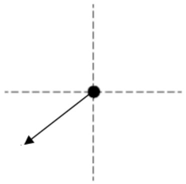
Question 2: Free-Response Question — 15 points

(a) For a single acceleration vector pointing down and to the left and attempting a justification — **1 point**

For a correct justification — **1 point**

Example response for part (a)

The block is changing direction with a centripetal component of the acceleration toward the center of the circle and a gravitational acceleration downward. Therefore, the acceleration of the block will be down and to the left.



Total for part (a) — 2 points

(b) i. For correctly using conservation of energy for the block at point B — **1 point**

$$K_A + U_{gA} = K_B + U_{gB}$$

$$\frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B$$

For correctly substituting into the above equation — **1 point**

$$0 + mgh = \frac{1}{2}mv_B^2 + mgR$$

$$v_B = \sqrt{2g(h-R)}$$

ii. For correctly substituting the expression for v_B from part (b)(i) into an expression for centripetal force — **1 point**

$$F_c = \frac{mv^2}{r} = \frac{mv_B^2}{R} = \frac{m\left(\sqrt{2g(h-R)}\right)^2}{R}$$

For correctly substituting into an equation for vector addition to derive an expression for the net force at point B — **1 point**

$$F_{net} = \sqrt{F_c^2 + (mg)^2}$$

$$F_{net} = \sqrt{\left(\frac{mv_B^2}{R}\right)^2 + (mg)^2} = \sqrt{\left(\frac{2mg}{R}(h-R)\right)^2 + (mg)^2}$$

Total for part (b) — 4 points

(c) For correctly using conservation of energy for the speed of the block at point C — **1 point**

$$K_A + U_{gA} = K_C + U_{gC}$$

$$\frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_C^2 + mgh_C$$

$$0 + mgh = \frac{1}{2}mv_C^2 + mg(2R)$$

$$v_C = \sqrt{2g(h-2R)}$$

For correctly applying Newton's second law at point C — **1 point**

$$F_c = \frac{mv_C^2}{r}$$

$$F_N + mg = \frac{mv_C^2}{r}$$

Set the normal force equal to zero

$$0 + mg = \frac{mv_C^2}{R} \therefore v_C = \sqrt{Rg}$$

For combining the two equations above — **1 point**

$$\sqrt{Rg} = \sqrt{2g(h-2R)} \therefore R = 2(h-2R) \therefore R/2 = h - 2R$$

$$h = 2.5R$$

Total for part (c) — 3 points

(d) For correctly using conservation of energy for the block compressing the spring — **1 point**

For correctly substituting the gravitational and elastic potential energies into an equation for conservation of energy — **1 point**

$$mgh = \frac{1}{2}kx_{MAX}^2$$

For correctly substituting for the spring constant k into the equation above — **1 point**

$$x_{MAX} = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2mgh}{mg/(2R)}} = \sqrt{4hR} = \sqrt{(4)(0.30\text{ m})(0.10\text{ m})} = 0.35\text{ m}$$

Total for part (d) — 3 points

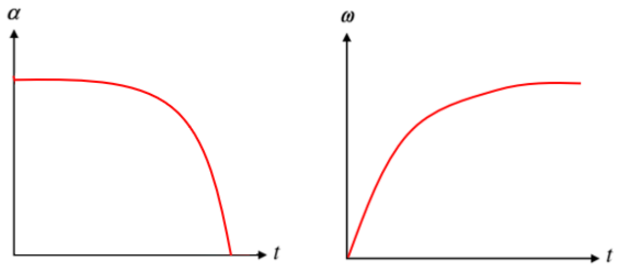
(e) i.	For a correct justification	**1 point**
	Example response for part (e)(i) *Because the block does not make it through the loop at this height, it will not compress the spring.*	
ii.	For indicating that as the height increases, the compression of the spring increases	**1 point**
	For indicating that the height is proportional to the square of the compression of the spring	**1 point**
	Example response for part (e)(ii) *From the equation in part (c), the compression of the spring is directly proportional to the square root of the height that the block is released. Thus, the graph would be an x-axis parabola, as shown.*	
	Total for part (e)	**3 points**
	Total for question 2	**15 points**

Question 2: Free-Response Question — 15 points

(a)	For integrating using the correct limits or constant of integration $I = \int_{r=0}^{r=2L} \lambda r^2 dr = \lambda \left[\frac{r^3}{3} \right]_{r=0}^{r=2L} = \frac{\lambda}{3}\left((2L)^3 - 0\right)$	**1 point**
	For correctly relating λ to M and L $\lambda = \frac{m}{\ell} = \frac{M}{2L} \therefore I = \left(\frac{1}{3}\right)\left(\frac{M}{2L}\right)\left(8L^3\right) = \frac{4}{3}ML^2$	**1 point**
	Total for part (a)	**2 points**
(b) i.	For correctly substituting into an equation for the center of mass of an object in the horizontal direction $X_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\left[\left(\frac{M}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{M}{2}\right)(L)\right]}{\left(\frac{M}{2} + \frac{M}{2}\right)} = \frac{\left(\frac{ML}{4} + \frac{ML}{2}\right)}{M} = \frac{3}{4}L$	**1 point**
ii.	For correctly substituting into an equation for the center of mass of an object in the vertical direction $Y_{CM} = \frac{\sum m_i y_i}{\sum m_i} = \frac{\left[\left(\frac{M}{2}\right)(L) + \left(\frac{M}{2}\right)\left(\frac{L}{2}\right)\right]}{\left(\frac{M}{2} + \frac{M}{2}\right)} = \frac{\left(\frac{ML}{2} + \frac{ML}{4}\right)}{M} = \frac{3}{4}L$	**1 point**
	Total for part (b)	**2 points**
(c)	For selecting "Less than" and attempting a relevant justification	**1 point**
	For a correct justification	**1 point**
	Example response for part (c) *Because object B has more of its mass closer to the pivot than object A, the rotational inertia of object B must be less than that of object A.*	
	Total for part (c)	**2 points**

(d)	For an acceleration graph that is concave down and begins horizontally	**1 point**
	For an angular speed graph that is concave down and ends horizontally	**1 point**
	For consistency between the angular acceleration and angular speed graphs	**1 point**

Example responses for part (d)



Total for part (d) **3 points**

(e)	For selecting "Decreasing" and attempting a relevant justification	**1 point**
	For a justification that indicates the lever arm for the torque is decreasing	**1 point**

Example response for part (e)

Because the horizontal position of the center of mass for the object is moving closer to the pivot, the lever arm for the force of gravity is decreasing so the angular acceleration decreases.

Total for part (e) **2 points**

(f) For using conservation of energy **1 point**

$U_{g1} = K_2$

For correctly relating the change in rotational kinetic energy to the change in gravitational potential energy **1 point**

$mgh = \frac{1}{2}I\omega^2$

For correctly substituting for h into the equation above **1 point**

$mg\left(\frac{L}{2}\right) = \frac{1}{2}I\omega^2$

For an expression for ω that uses only the allowed symbols and is algebraically consistent with the previous steps **1 point**

$\omega = \sqrt{\frac{2mgh}{I}} = \sqrt{\frac{2Mg(L/2)}{I_B}}$

$\omega = \sqrt{\frac{MgL}{I_B}}$

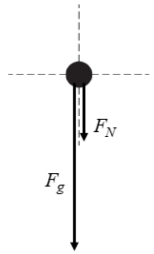
Total for part (f) **4 points**

Total for question 2 **15 points**

Question 3: Free-Response Question — 15 points

(a)	For correctly drawing and labeling the weight of the block	**1 point**
	For correctly drawing and labeling the force exerted by the track on the block	**1 point**
	For a correct justification consistent with the diagram	**1 point**

Example response for part (a)



The force F_g represents the weight of the block and always points downward. The force F_N represents the force the track exerts on the block to keep it moving in a circular path and points perpendicular to the surface of the track.

Scoring Note: Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$, $F_{\text{Block,E}}$. The labels G or g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force.

Scoring Note: If extraneous forces are present, a maximum of 2 points can be earned.

Total for part (a) **3 points**

(b) i. For using conservation of energy **1 point**

$U_1 + K_1 = U_2 + K_2 \therefore U_1 + 0 = U_2 + K_2$

For correctly relating the elastic potential energy at maximum spring compression to the gravitational potential energy at point B **1 point**

$U_{s1} + 0 = U_{g2} + K_2 \therefore K_2 = U_{s1} - U_{g2}$

For a correct substitution into the equation above **1 point**

$\frac{1}{2}mv_B^2 = \frac{1}{2}k(\Delta x)^2 - mgh_2$

$v_B^2 = \frac{k}{m}(\Delta x)^2 - 2g(3R) \therefore v_B = \sqrt{\frac{k}{m}(\Delta x)^2 - 6gR}$

	ii.	For correctly relating the centripetal force to speed from part (b)(i) $F_C = \frac{mv^2}{r} = \frac{mv_B^2}{R}$	**1 point**
		For an answer consistent with part (b)(i) $F_C = \frac{m}{R}\left(\sqrt{\frac{k}{m}(\Delta x)^2 - 6gR}\right)^2 = \frac{k(\Delta x)^2}{R} - 6mg$	**1 point**
		Total for part (b)	**5 points**
(c)		For correctly relating the net force to the diagram from part (a) and setting the normal force equal to zero	**1 point**
		For correctly substituting into the equation above $\frac{k(\Delta x)^2}{R} - 6mg = mg \therefore \frac{k(\Delta x)^2}{R} = 7mg \therefore \Delta x = \sqrt{\frac{7mgR}{k}}$	**1 point**
		Total for part (c)	**2 points**
(d)		For correctly relating the height of fall to the time of fall $y = y_0 + v_{oy}t + \frac{1}{2}a_y t^2 \therefore H = 0 + 0 + \frac{1}{2}gt^2 \therefore t = \sqrt{\frac{(2)(4R)}{g}} = \sqrt{\frac{8R}{g}}$	**1 point**
		For correctly substituting into the equation for constant velocity consistent with part (b)(i) $D = v_x t = \left(\sqrt{\frac{k}{m}(\Delta x)^2 - 6gR}\right)\left(\sqrt{\frac{8R}{g}}\right)$	**1 point**
		Total for part (d)	**2 points**
(e)	**i.**	For a correct justification	**1 point**
	ii.	For indicating that as the maximum compression of the spring increases, the distance D increases	**1 point**
		For indicating that the minimum value is due to the minimum speed needed to get through the track.	**1 point**
		Example responses for part (e) *The block needs a minimum speed to make it through point B on the track; thus, the horizontal line segment represents compressions of the spring for which the block does not make it to point B.* OR *From the equation in part (d), the compression of the spring is directly proportional to the horizontal distance traveled by the block; thus, the graph would be a straight line. The minimum value is the distance traveled when the compression of the spring generates the minimum speed needed to reach point B on the track.*	
		Total for part (e)	**3 points**
		Total for question 3	**15 points**