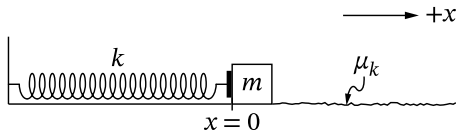


Question 3

The following information applies to parts A and B.

3. A horizontal spring of known spring constant k is attached to a wall. A block of known mass m is placed on a horizontal surface next to but not attached to the spring. The spring is at its relaxed length when the block is at position $x = 0$, as shown in Figure 1. There is friction between the block and the surface only for positions $x > 0$.

Figure 1



A student is asked to experimentally determine the coefficient of kinetic friction μ_k between the block and the surface using a graph. The student is permitted to use only measurements from a meterstick.

Part A

- i. **Indicate** quantities that could be measured by the student that would allow them to determine μ_k using a graph.
- ii. Briefly **describe** a method to reduce experimental uncertainty for the measured quantities.

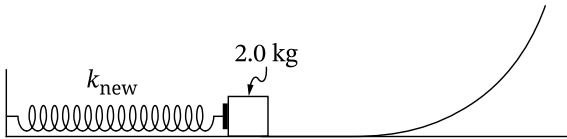
Part B

- i. **Indicate** what quantities the student could graph on the horizontal and vertical axes to create a graph that can be used to determine μ_k .
- ii. Briefly **describe** the relationship between μ_k and a feature of the graph from part B (i). Your answer may include an equation that relates μ_k and the chosen feature of the graph.

The following information applies to parts C and D.

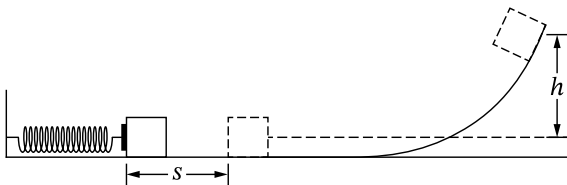
In a different experiment, the student is asked to determine the spring constant k_{new} of a new spring that is attached to a wall. A block of mass 2.0 kg is placed next to the spring on a surface where frictional forces are negligible. The surface is horizontal near the spring and slopes upward into a ramp to the right of the initial position of the block, as shown in Figure 2.

Figure 2



The student pushes the block, compressing the spring a distance s , as shown in Figure 3. The block is released from rest.

Figure 3



The student uses a meterstick to measure the maximum height h of the block on the ramp. The experiment is repeated several times with different compression distances s . The student's measurements of s and h are shown in Table 1.

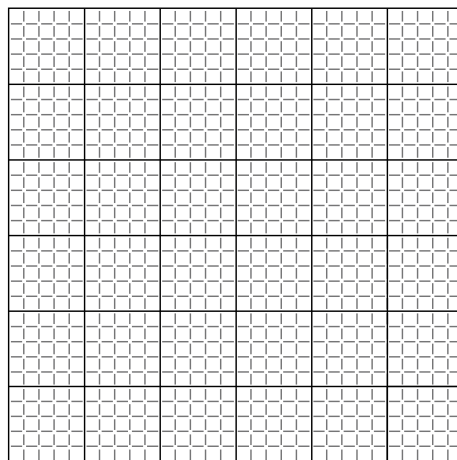
Table 1

s (m)	h (m)
0.020	0.02
0.040	0.04
0.060	0.13
0.080	0.18
0.100	0.33

Part C

- i. **Label** the axes of the grid provided with measured or calculated quantities. Include units, as appropriate. The graphed quantities should yield a linear graph that can be used to determine k_{new} .
- ii. On the grid provided, create a graph of the quantities indicated in part C (i).
 - Clearly **label** the vertical and horizontal axes with numerical scales.
 - **Plot** the corresponding data points on the grid.
 - Table 2 is provided in your booklet for scratch work and will not be scored.

Quantity (units, if appropriate)



Quantity (units, if appropriate)

- iii. **Draw** a best-fit line for the data graphed in part C (ii).

Part D

Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for k_{new} .

Question 2: Version J

2. In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

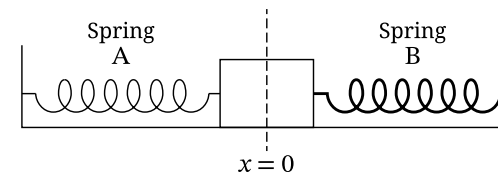


Figure 1

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

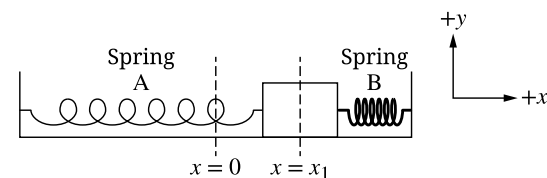


Figure 2

A. An energy bar chart can be used to represent the elastic potential energy U_A of Spring A, the elastic potential energy U_B of Spring B, and the kinetic energy K_{block} of the block. On the energy bar chart in Figure 3, **draw** shaded bars to represent the energy of the system for when the block is at $x = x_1$.

- The height of the shaded bars should be proportional to the relative values of U_A , U_B , and K_{block} .
- Any energy that is equal to zero should be represented by a distinct line on the zero-energy line.

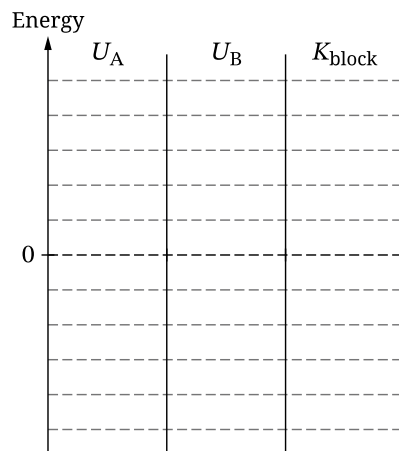
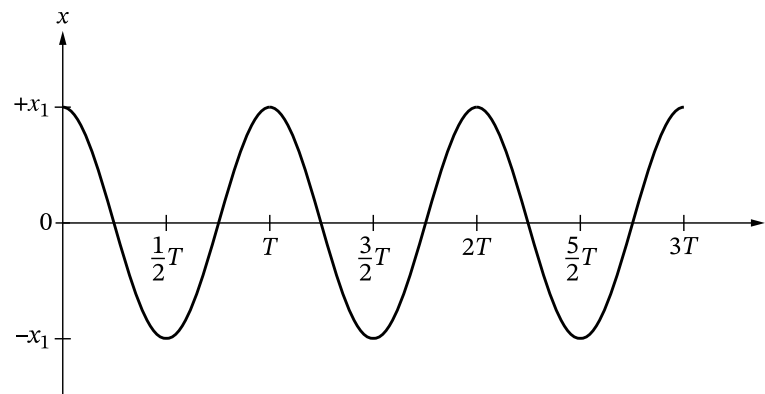


Figure 3

B. The block is released from rest at $x = x_1$ and begins to oscillate. **Derive** an expression for the speed v of the block as the block passes through $x = \frac{1}{2}x_1$. Express your answer in terms of m , k , x_1 , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

C. In Scenario 1, the block oscillates with period T . The position x of the block in Scenario 1 as a function of time t is shown in Figure 4.

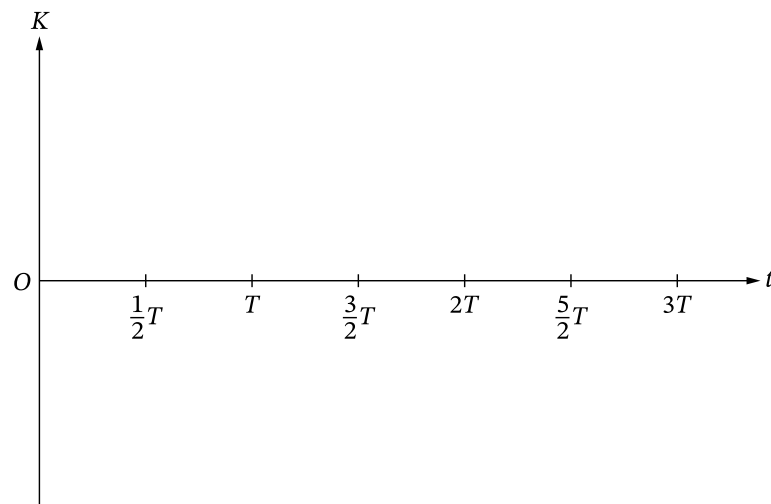


Scenario 1

Figure 4

In Scenario 2, the block-springs system is placed on a new surface. There is friction between the block and the new surface. The block is again moved to the same position $x = x_1$ and released from rest. The block completes multiple oscillations with the same period as in Scenario 1 before coming to rest.

On the axes shown in Figure 5, **sketch** a graph of the kinetic energy K of the block as a function of t for Scenario 2.



Scenario 2

Figure 5

- D. In Scenario 3, the block is replaced with a new block of larger mass. The coefficient of kinetic friction between the new block and the surface in Scenario 3 is the same as the coefficient of kinetic friction between the original block and the surface in Scenario 2.

The new block is moved to position $x = x_1$ and released from rest. The kinetic energy of the new block is plotted as a function of time.

Describe how one feature of the graph of K as a function of t in Scenario 3 would differ from the graph you drew in Figure 5 for Scenario 2.

Briefly **justify** your answer.

Question 3

3. A box is connected to one end of a rigid rod. Both the box and the rod have negligible mass. The other end of the rod is connected to a pivot. The box is open on one side, and a block is placed inside the box.

The center of mass of the block is displaced a vertical distance h , as shown in Figure 1. The block-box system is then released from rest and swings downward. There is negligible friction about the pivot. When the system is at the lowest point of its swing, the rod collides with a rigid stopper, as shown in Figure 2. The box comes to rest, and the block is launched horizontally out of the box. The block moves across a horizontal surface toward a motion sensor that measures the speed of the block. All frictional forces are negligible.

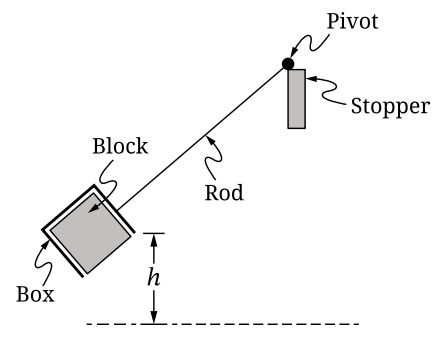


Figure 1

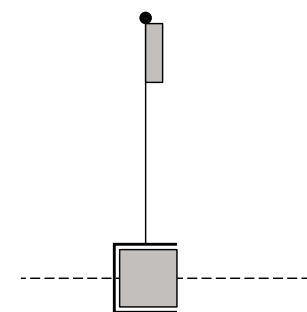


Figure 2

- A. Students are asked to experimentally determine the acceleration due to gravity g using a linear graph. To determine g , the students are permitted to use measurements from only a meterstick and the motion sensor.
- Describe** an experimental procedure using the described setup to collect data that would allow the students to determine an experimental value of g using a linear graph. Include any steps necessary to reduce experimental uncertainty.
- B. **Describe** how the data collected in part A could be graphed and how that graph would be analyzed to determine the value of g .

- C. The experiment is repeated, but the horizontal surface on which the block slides is replaced with a new rough surface, as shown in Figure 3. The coefficient of kinetic friction between the block and the new surface is μ .

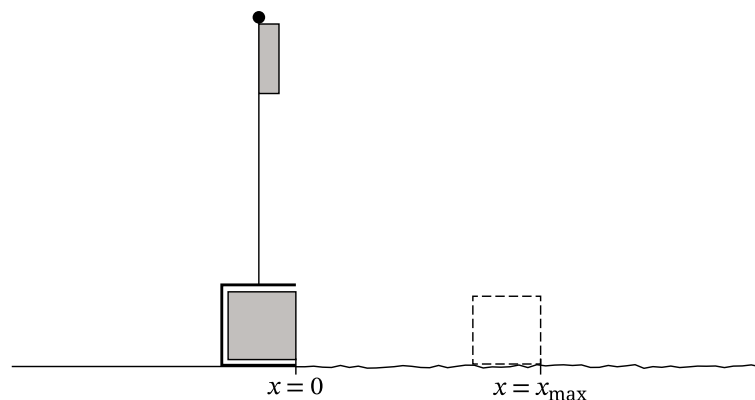


Figure 3

The block-box system is pulled aside so that the center of mass of the block is displaced various vertical distances h and then released from rest. For each vertical distance, students measure the position $x = x_{\max}$ at which the block comes to rest.

The students' measurements of h and $x_{\max}$ are shown in Table 1.

Table 1

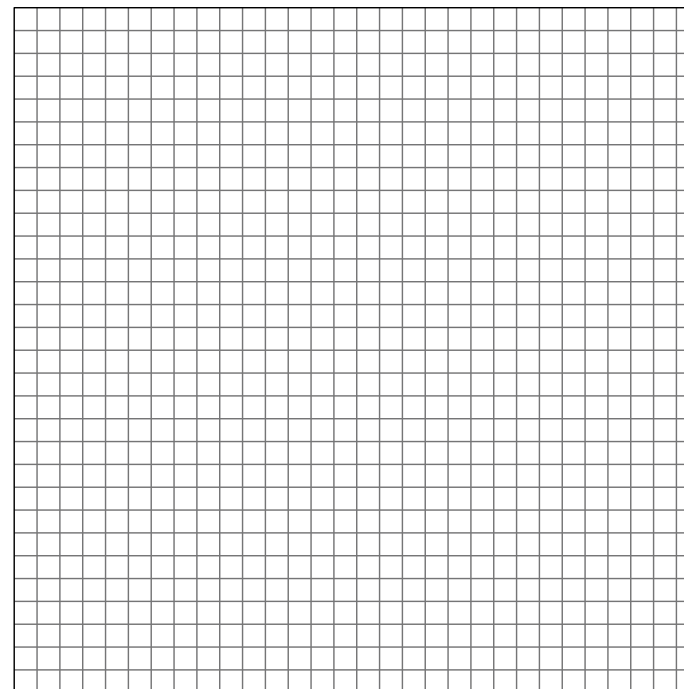
h (m)	$x_{\max}$ (m)
0.30	0.76
0.45	1.10
0.60	1.40
0.75	1.90
0.90	2.30

- i. **Indicate** two quantities, either measured quantities from Table 1 or additional calculated quantities, that could be graphed to produce a straight line that could be used to determine μ .

Vertical axis: _____ Horizontal axis: _____

- ii. On the grid provided, create a graph of the quantities indicated in part C (i).

- Use Table 2 to record the measured or calculated quantities that you will plot.
- Clearly **label** the axes, including units as appropriate.
- **Plot** the points you recorded in Table 2.

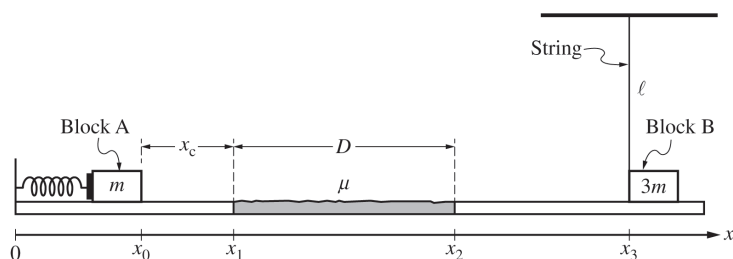


- iii. **Draw** a best-fit line to the data graphed in part C (ii).

- D. Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for μ .

Begin your response to **QUESTION 1** on this page.**PHYSICS C: MECHANICS****SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

Figure 1

1. Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D , where the coefficient of kinetic friction between Block A and the surface is μ . Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c .

GO ON TO THE NEXT PAGE.Continue your response to **QUESTION 1** on this page.

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c . Block A moves with speed v , and the spring is at its equilibrium position.
- At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction.
- At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m , k , D , μ , x_c , and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1 .

ii. **Derive** an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3 .

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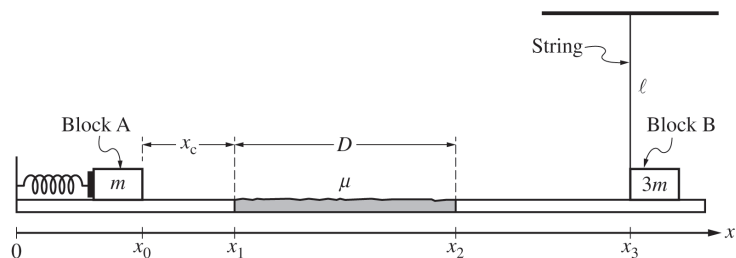
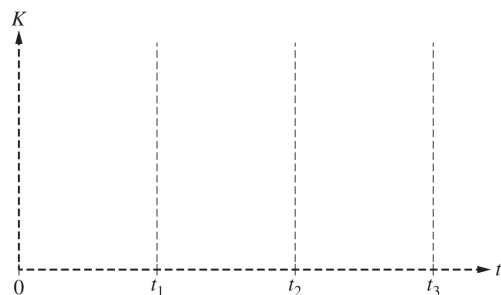
Continue your response to **QUESTION 1** on this page.Note: Figure not drawn to scale.

Figure 1

(b)

- i. On the following axes, **sketch** a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to time t_3 .



- ii. Use principles of work and energy to **justify** the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

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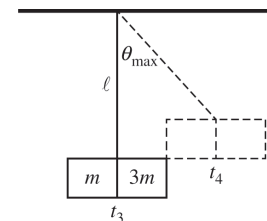
Continue your response to **QUESTION 1** on this page.Note: Figure not drawn to scale.

Figure 2

After the collision, the two-block system instantaneously comes to rest at time t_4 , which occurs when the string makes a small angle $\theta_{\max}$ with the vertical, as shown in Figure 2. For times $t > t_4$, the system oscillates with frequency f_ℓ . The support holding the string is raised, and the procedure is then repeated using a new string of length 2ℓ .

- (c) **Indicate** how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length 2ℓ will compare to the frequency of oscillation f_ℓ from the original procedure.

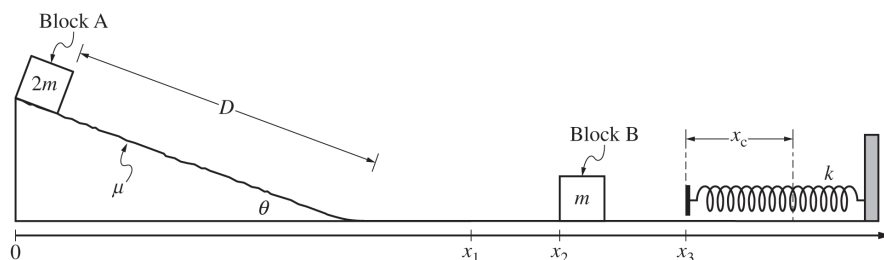
_____ $f_{2\ell} > f_\ell$ _____ $f_{2\ell} < f_\ell$ _____ $f_{2\ell} = f_\ell$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 1** on this page.**PHYSICS C: MECHANICS****SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

1. Blocks A and B of masses $2m$ and m , respectively, are arranged in a setup consisting of a ramp that makes an angle θ with a smooth horizontal table and an ideal spring of spring constant k fixed to a wall, as shown. Block A is held at rest a distance D up the ramp, and Block B is at rest on the horizontal table. The coefficient of kinetic friction between Block A and the rough ramp is μ in the region of length D , and there is negligible friction between the blocks and the smooth table.

GO ON TO THE NEXT PAGE.Continue your response to **QUESTION 1** on this page.

At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.

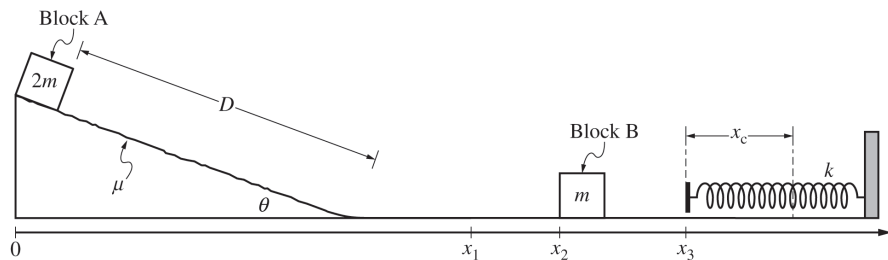
- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$.
- At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B.
- At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position.
- At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.

- (a) For parts (a)(i) and (a)(ii), express your answer in terms of m , θ , D , μ , x_c , and physical constants, as appropriate.

- i. **Derive** an expression for the speed v of Block A at time t_1 .

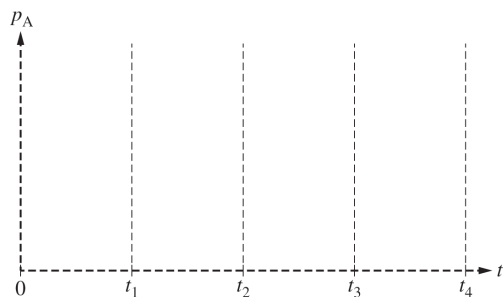
- ii. **Derive** an expression for the spring constant k of the spring.

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Continue your response to **QUESTION 1** on this page.Note: Figure not drawn to scale.

(b)

- i. On the following axes, **sketch** a graph of the magnitude of the momentum p_A of Block A as a function of time t from $t = 0$ to t_4 .



- ii. Use principles of forces to **justify** the graph drawn in part (b)(i) for the time interval $t = t_3$ to $t = t_4$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

GO ON TO THE NEXT PAGE.Continue your response to **QUESTION 1** on this page.

For times $t > t_4$, the two-block-spring system oscillates with period T_O . The procedure is then repeated using a new ramp, where there is negligible friction between Block A and the ramp.

- (c) **Indicate** how the new period of oscillation T_N in the procedure that uses the new ramp compares with the period of oscillation T_O from the original procedure.

_____ $T_N > T_O$ _____ $T_N < T_O$ _____ $T_N = T_O$

Briefly **justify** your answer.**GO ON TO THE NEXT PAGE.**

PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

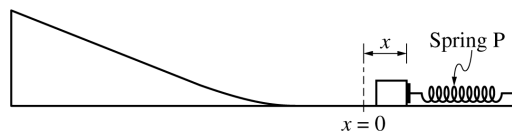


Figure 1

A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x , as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\max,P}$. Frictional forces between the block and all surfaces are negligible.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

A student compares $h_{\max,P}$ for Spring P with the maximum height $h_{\max,Q}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x that the spring is compressed, as shown in Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0 \text{ N/m}$, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0 \text{ N/m}^{1/2}$.

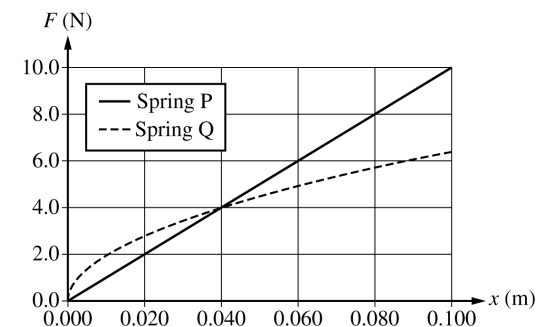
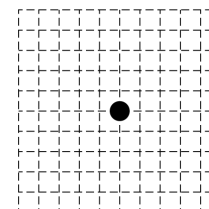
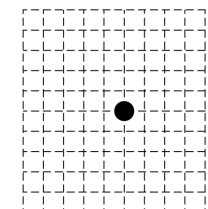


Figure 2

- (a) For the experiment, the block is pushed against one of the springs, compressing the spring a distance $x = 0.010 \text{ m}$. The block is then released from rest. In Trial 1, Spring P is used, and in Trial 2, Spring Q is used. On the following dots, representing the block in Trials 1 and 2, draw and label the forces (not components) that are exerted on the block at the instant the block is released. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. The lengths of the horizontal vectors should represent the relative magnitude of the horizontal forces and the lengths of the vertical vectors should represent the relative magnitude of the vertical forces.

Trial 1
(Spring P)Trial 2
(Spring Q)

GO ON TO THE NEXT PAGE.

(b)

i. What feature(s) of the graph in Figure 2 could be used to estimate the work done on the block by each spring as each spring is compressed?

ii. There is one compression distance x_0 for which the maximum height $h_{\max}$ reached by the block is the same regardless of which spring, Spring P or Spring Q, is used. Predict whether the value of x_0 is greater than, less than, or equal to 0.040 m. Use the graph in Figure 2 to justify your answer.

iii. On the axes provided, for both Spring P and Spring Q, sketch a graph of the maximum height $h_{\max}$ reached by the block as a function of the distance each spring is compressed for values of x ranging from 0 to 0.100 m. Clearly label the curve for Spring P and Spring Q.

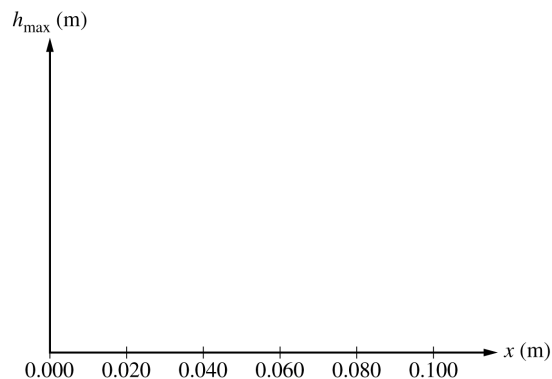
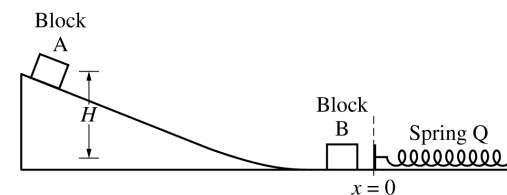
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Figure 3

(c) Spring Q is attached to the wall and is at equilibrium, as shown in Figure 3. Block A has a mass of 0.120 kg and Block B has a mass of 0.070 kg. Block A is held at rest at the top of the ramp and Block B is at rest on the horizontal surface. The student releases Block A, and it moves down the ramp and collides with Block B. The change in vertical height of Block A is $H = 0.75$ m. After the collision, the blocks stick together and move to the right, compressing the spring. Frictional forces between the blocks and all surfaces are negligible.

i. Calculate the velocity of the two-block system immediately after the collision between Blocks A and B.

ii. Calculate the maximum compression of the spring.

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Continue your response to Question 1 on this page.

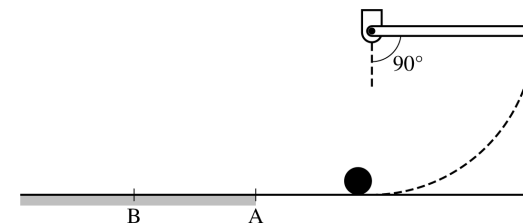
- (d) Spring Q where $F_Q(x) = Cx^{1/2}$ is replaced by a different nonlinear spring, Spring R, and the procedure described in part (c) is repeated. For Spring R, $F_R(x) = Dx^{1/2}$. The maximum compression of Spring R is greater than the maximum compression of Spring Q. Which of the following correctly compares the constants C and D ?

☐ $C < D$ ☐ $C > D$ ☐ $C = D$

Briefly justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to Question 1 on this page.



Note: Figure not drawn to scale.

Figure 1

3. A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.
- (a) Derive an expression for the angular speed of the rod just before striking the sphere in terms of the length ℓ and physical constants as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

- (b) Derive an expression for the linear speed v_0 of the sphere immediately after colliding with the rod in terms of the length ℓ and physical constants as appropriate.

After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ , beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

- (c) In the following diagram, which represents the sphere while the sphere is traveling between Points A and B, draw and label the forces (not components) that act on the sphere. Each force must be represented by a distinct arrow starting on, and pointing away from, the point of application on the sphere.



- (d) Derive an expression for each of the following as the sphere is rotating and sliding between points A and B in terms of v_0 , μ , R , t , and physical constants as appropriate.

i. The linear velocity v of the center of mass of the sphere as a function of time t

ii. The angular velocity ω of the sphere as a function of time t

GO ON TO THE NEXT PAGE.Continue your response to **QUESTION 3** on this page.

(e)

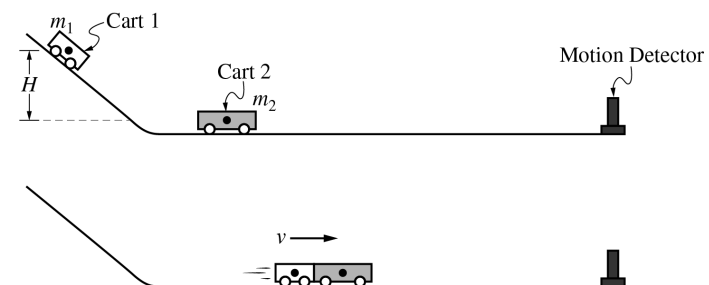
- i. Derive an expression for the time it takes the sphere to travel from Point A to Point B in terms of v_0 , μ , and physical constants as appropriate.

ii. Derive an expression for the linear velocity of the sphere upon reaching Point B in terms of v_0 .

GO ON TO THE NEXT PAGE.

STOP**END OF EXAM**

Begin your response to question 2 on this page.



Cart 1 of mass m_1 is held at rest above the bottom of an incline. Cart 2 has mass m_2 , where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and smoothly transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance of H , as shown. At time t_C , Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v . Frictional and rotational effects are negligible.

(a) During the collision, is the impulse on Cart 1 from Cart 2 greater than, less than, or equal to the magnitude of the impulse on Cart 2 from Cart 1 ?

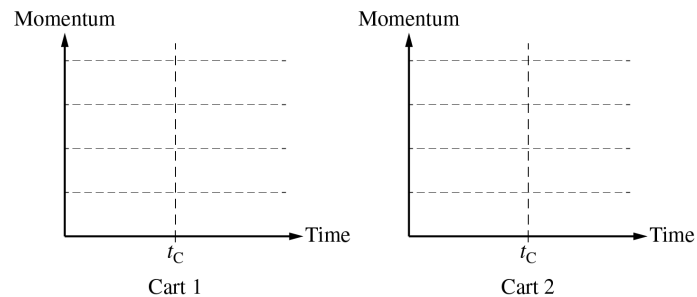
___ Greater than ___ Less than ___ Equal to

Justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

- (b) On the following axes, draw graphs of the magnitude of the momentum of each cart as a function of time t , before and after t_C . The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.



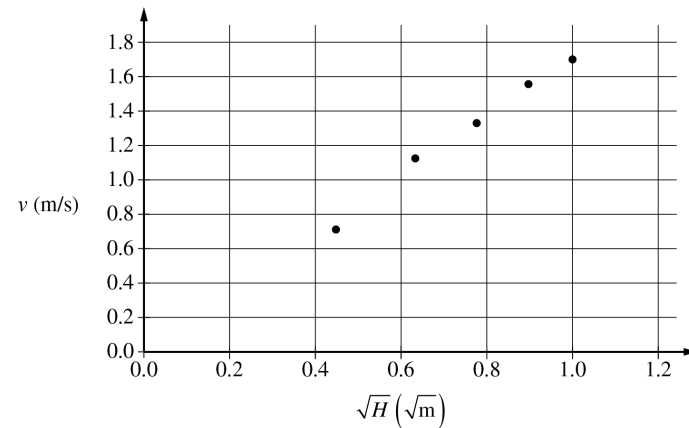
- (c) Show that the velocity v of the two-cart system after the collision is given by the equation

$$v = \sqrt{2g} \left(\frac{m_1}{m_1 + m_2} \right) \sqrt{H}.$$

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

- (d) A group of students use the setup to perform an experiment. They measure the mass of Cart 1 to be $m_1 = 0.250$ kg. The mass of Cart 2 is unknown. The students perform several trials and in each trial, Cart 1 is released from a different height H and the final velocity of the two-cart system is measured. The students graph v as a function of $\sqrt{H}$, as shown below.



- Draw a line that represents the best fit to the data points shown.
- Use the best-fit line to calculate the mass of Cart 2.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

- (e) After the experiment, the students use a balance to measure the mass of Cart 2 and find it to be less than what was determined in part (d). To explain this discrepancy, one of the students proposes that the mass of Cart 1 was incorrectly measured at the beginning of the experiment. The students measure the mass of Cart 1 again and record a new value, m_1' .

Should the students expect that m_1' will be greater than 0.250 kg, less than 0.250 kg, or equal to 0.250 kg ?

☐ $m_1' > 0.250 \text{ kg}$ ☐ $m_1' < 0.250 \text{ kg}$ ☐ $m_1' = 0.250 \text{ kg}$

Justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 1** on this page.**PHYSICS C: MECHANICS****SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

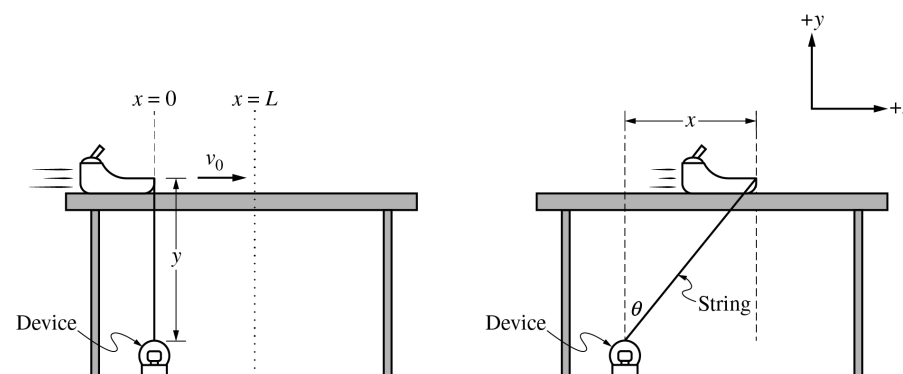


Figure 1

Figure 2

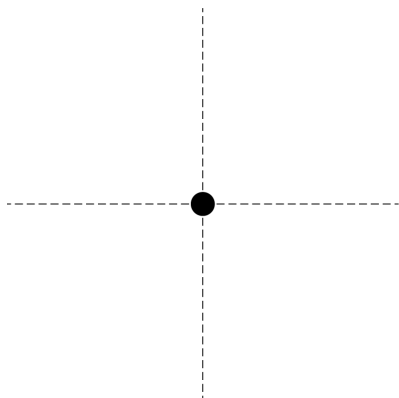
Note: Figures not drawn to scale.

A small sled slides across a rough horizontal table with an initial velocity v_0 . The coefficient of kinetic friction between the sled and the table is μ_k . A string connects the sled to a device on the ground. The device maintains constant tension F_T in the string by unwinding the string as the sled slides to the right. The total mass of the sled is m . The string is attached to the device at $x = 0$ and at a height of y , as shown in Figure 1. The horizontal position of the sled is represented by x , as shown in Figure 2. Express all algebraic answers in terms of m , μ_k , F_T , x , y , and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

- (a) On the dot below that represents the sled, draw and label the forces (not components) that are exerted on the sled a short time after $t = 0$ but before the sled has come to rest. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



- (b) Determine an expression for the angle θ that the string makes with the vertical when the sled has traveled a horizontal distance x .

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(c)

- i. Derive an expression for the normal force F_N exerted on the sled by the table as a function of the position x .

- ii. Derive an expression for the magnitude of the net horizontal force F_{net} exerted on the sled as a function of the position x .

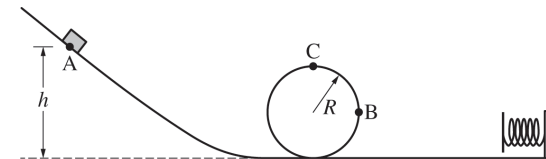
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Continue your response to **QUESTION 1** on this page.(d) Derive an expression for the work W done by the string on the sled as the sled moves from $x = 0$ to $x = L$.

(e) The sled comes to rest after traveling a horizontal distance $x = 2L$. As the system slides from $x = 0$ to $x = L$, the energy dissipated by friction is E_1 . As the sled slides from $x = L$ to $x = 2L$, the energy dissipated by friction is E_2 . Is E_1 greater than, less than, or equal to E_2 ?

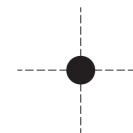
___ $E_1 > E_2$ ___ $E_1 < E_2$ ___ $E_1 = E_2$

Justify your answer.

GO ON TO THE NEXT PAGE.Note: Figure not drawn to scale.

2. A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \frac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m , h , R , and physical constants, as appropriate.

(a) On the dot below, which represents the block, draw an arrow that represents the direction of the acceleration of the block at point B in the figure above. The arrow must start on and point away from the dot.



Justify your answer.

(b)

i. Derive an expression for the speed v of the block at point B.

ii. Derive an expression for the magnitude of the net force F on the block at point B.

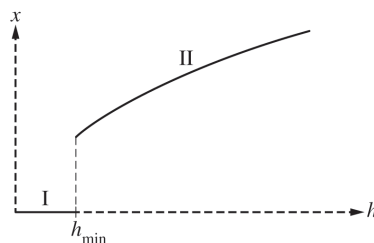
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(c) In terms of R , derive an expression for the minimum height $h_{\min}$ necessary for the block to maintain contact with the track through point C.

(d) It is determined that $h = 0.30 \text{ m}$ and $R = 0.10 \text{ m}$. If the block is released from a height greater than that found in part (c), what would be the maximum compression x_{MAX} of the spring?

(e) A graph of the maximum compression of the spring as a function of height is shown below. The height $h_{\min}$ is the height calculated in part (c).

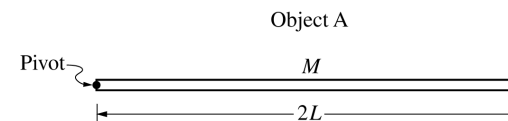


i. Explain why section I appears as a horizontal line segment on the horizontal axis.

ii. Explain the reason for the shape of section II on the graph.

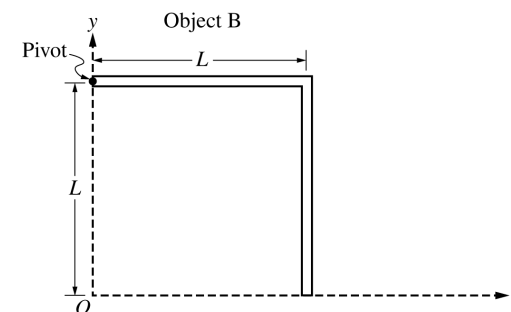
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2. Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(a) Using integral calculus, derive an expression to show that the rotational inertia I_A of object A about the pivot is given by $\frac{4}{3}ML^2$.



Object B of total mass M is formed by attaching two thin, uniform, identical rods of length L at a right angle to each other. Object B is held in place, as shown above. Express your answers in part (b) in terms of L .

(b) Determine the following for the given coordinate system shown in the figure.

i. The x -coordinate of the center of mass of object B

ii. The y -coordinate of the center of mass of object B

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Object B has a rotational inertia of I_B about its pivot.

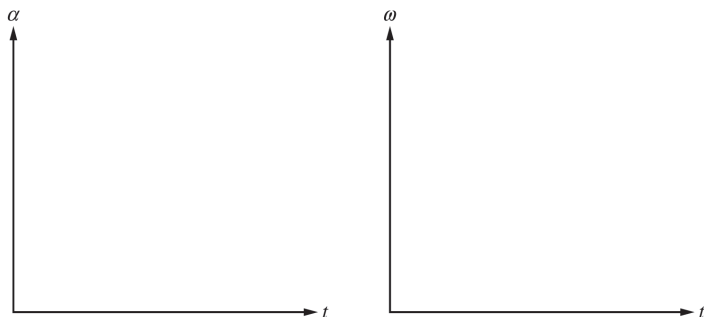
(c) Is the value of I_B greater than, less than, or equal to I_A ?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

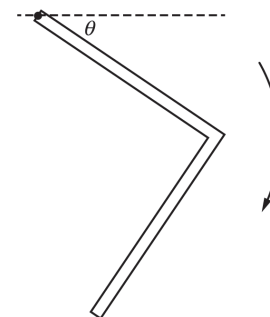
Object B is released from rest and begins to rotate about its pivot.

(d) On the axes below, sketch graphs of the magnitude of the angular acceleration α and the angular speed ω of object B as functions of time t from the time it is released to the time its center of mass reaches its lowest point.



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(e) While object B rotates from the horizontal position down through the angle θ shown above, is the magnitude of its angular acceleration increasing, decreasing, or not changing?

_____ Increasing _____ Decreasing _____ Not changing

Justify your answer.



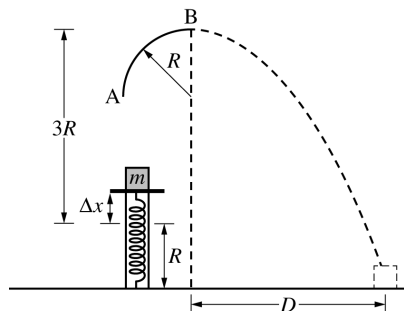
Object B rotates through the position shown above.

(f) Derive an expression for the angular speed of object B when it is in the position shown above. Express your answer in terms of M , L , I_B , and physical constants, as appropriate.

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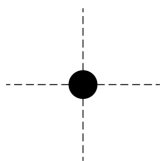
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Begin your response to QUESTION 3 on this page.

Note: Figure not drawn to scale.

3. A block of mass m is placed on top of an ideal spring of spring constant k . The block is pushed against the spring, compressing the spring a distance Δx . The block is released from rest, leaves the spring at the position shown in the figure, travels upward, and enters a track with a constant radius of curvature R that has negligible friction. The block enters the track at point A, maintains contact with the track, and exits horizontally at point B, a distance $3R$ above the point the block was released. The block then falls to the ground and lands a horizontal distance D from the end of the track. Express all algebraic answers in terms of m , k , Δx , R , and physical constants, as appropriate. The size of the block is much smaller than the radius of curvature of the track.

- (a) On the dot below, which represents the block, draw and label the forces (not components) that act on the block while still in contact with the track at point B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



Justify your choice of vectors.

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(b)

- i. Derive an expression for the speed v of the block at point B.

- ii. Derive an expression for the magnitude of the net force F on the block at point B.

- (c) Derive an expression for the minimum value of $\Delta x_{\min}$ required in order for the block to maintain contact with the track through point B.

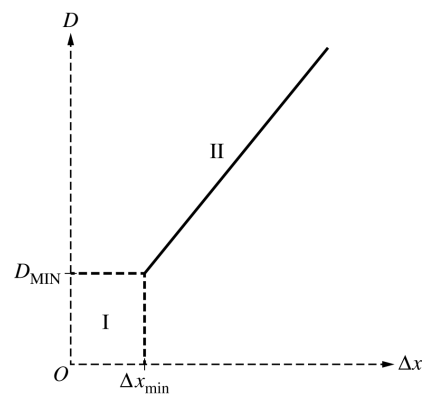
The procedure is repeated several times with the distance $\Delta x > \Delta x_{\min}$.

- (d) Calculate the distance D that the block travels.

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(e) The graph below shows the best-fit line drawn by the students through their data of D as a function of Δx .



i. Explain why there are no data for section I of the graph.

ii. Explain the reason for the shape and minimum value of section II on the graph.

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STOP

END OF EXAM