Question 2: Translation Between Representations (TBR) — 12 points

A	For indicating that K_{block} is zero	**Point A1**
	For drawing bars with positive heights for U_A and U_B	**Point A2**
	For drawing a bar for U_B with a height that is twice the height of the bar drawn for U_A **Scoring Note:** This point may be earned regardless of the signs of either bar.	**Point A3**

Example Response

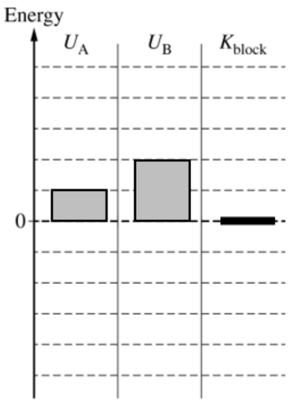


Figure 3

B	For a multistep derivation that includes energy conservation or simple harmonic motion	**Point B1**
	For relating the presence of both springs to the behavior of the system	**Point B2**
	For relating positions $x = x_1$ and $x = \frac{1}{2}x_1$ to the oscillation of the block	**Point B3**
	For a correct expression for v in terms of given quantities	**Point B4**

Example Responses

$$E_{\text{tot},0} = U_A(x_1) + U_B(x_1) + K_{\text{block}}(x_1)$$

$$E_{\text{tot},f} = U_A\left(\frac{1}{2}x_1\right) + U_B\left(\frac{1}{2}x_1\right) + K_{\text{block}}\left(\frac{1}{2}x_1\right)$$

$$E_{\text{tot},0} = \frac{1}{2}(k)(x_1)^2 + \frac{1}{2}(2k)(x_1)^2 + 0$$

$$E_{\text{tot},f} = \frac{1}{2}(k)\left(\frac{1}{2}x_1\right)^2 + \frac{1}{2}(2k)\left(\frac{1}{2}x_1\right)^2 + K_{\text{block},\frac{1}{2}x_1}$$

$$E_{\text{tot},0} = E_{\text{tot},f}$$

$$\frac{1}{2}kx_1^2 + kx_1^2 = \frac{1}{8}kx_1^2 + \frac{1}{4}kx_1^2 + K_{\text{block},\frac{1}{2}x_1}$$

$$\frac{3}{2}kx_1^2 = \frac{3}{8}kx_1^2 + K_{\text{block},\frac{1}{2}x_1}$$

$$K_{\text{block},\frac{1}{2}x_1} = \frac{9}{8}kx_1^2$$

$$\frac{1}{2}mv^2 = \frac{9}{8}kx_1^2$$

$$v = \frac{3}{2}x_1\sqrt{\frac{k}{m}}$$

OR

$$x = x_{\max}\cos(\omega t + \phi)$$

$$x(t) = x_1\cos\omega t$$

$$v(t) = -x_1\omega\sin\omega t$$

$$k_{\text{eff}} = k + 2k = 3k$$

$$\omega = \sqrt{\frac{3k}{m}}$$

$$\frac{x_1}{2} = x_1\cos\omega t_{\frac{1}{2}x_1}$$

$$\cos\omega t_{\frac{1}{2}x_1} = \frac{1}{2}$$

$$t_{\frac{1}{2}x_1} = \frac{\pi}{3\omega}$$

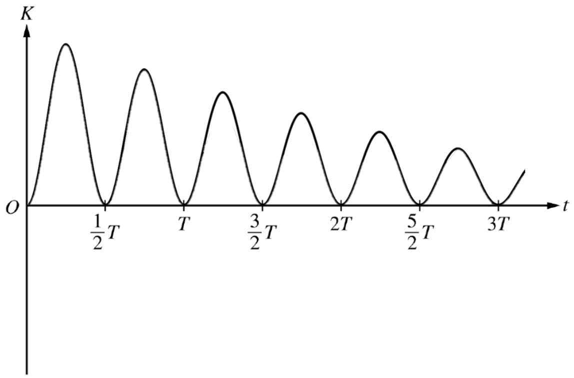
$$v_{\frac{1}{2}x_1} = -x_1\omega\sin\omega t_{\frac{1}{2}x_1}$$

$$v_{\frac{1}{2}x_1} = -x_1\sqrt{\frac{3k}{m}}\sin\frac{\pi}{3}$$

$$v = \left|v_{\frac{1}{2}x_1}\right| = \frac{3}{2}x_1\sqrt{\frac{k}{m}}$$

C	For sketching a curve that starts at zero and is always positive or zero	**Point C1**
	For sketching a periodic curve with zeros that have a period of $\frac{1}{2}T$	**Point C2**
	For sketching a periodic curve with a decreasing amplitude	**Point C3**

Example Response



Scenario 2

Figure 5

D	For indicating **one** of the following: • The period increases. • The rate at which the graph approaches zero increases. • The maximum value of K over each period is less than the graph drawn.	**Point D1**
	For a correct justification relevant to the feature indicated	**Point D2**

Example Responses

The period of the graph increases. The period of a simple harmonic oscillator increases with increasing mass.

OR

The rate at which the graph approaches zero amplitude increases. Because the mass of the block is larger, the force of friction is greater, which causes the block to lose energy at a greater rate.

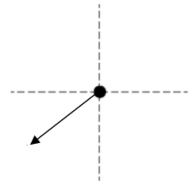
Question 2: Free-Response Question — 15 points

(a) For a single acceleration vector pointing down and to the left and attempting a justification — **1 point**

For a correct justification — **1 point**

Example response for part (a)

The block is changing direction with a centripetal component of the acceleration toward the center of the circle and a gravitational acceleration downward. Therefore, the acceleration of the block will be down and to the left.



Total for part (a) — 2 points

(b) i. For correctly using conservation of energy for the block at point B — **1 point**

$$K_A + U_{gA} = K_B + U_{gB}$$

$$\frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B$$

For correctly substituting into the above equation — **1 point**

$$0 + mgh = \frac{1}{2}mv_B^2 + mgR$$

$$v_B = \sqrt{2g(h-R)}$$

ii. For correctly substituting the expression for v_B from part (b)(i) into an expression for centripetal force — **1 point**

$$F_c = \frac{mv^2}{r} = \frac{mv_B^2}{R} = \frac{m\left(\sqrt{2g(h-R)}\right)^2}{R}$$

For correctly substituting into an equation for vector addition to derive an expression for the net force at point B — **1 point**

$$F_{net} = \sqrt{F_c^2 + (mg)^2}$$

$$F_{net} = \sqrt{\left(\frac{mv_B^2}{R}\right)^2 + (mg)^2} = \sqrt{\left(\frac{2mg}{R}(h-R)\right)^2 + (mg)^2}$$

Total for part (b) — 4 points

(c)	For correctly using conservation of energy for the speed of the block at point C	**1 point**

$$K_A + U_{gA} = K_C + U_{gC}$$
$$\frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_C^2 + mgh_C$$
$$0 + mgh = \frac{1}{2}mv_C^2 + mg(2R)$$
$$v_C = \sqrt{2g(h-2R)}$$

For correctly applying Newton's second law at point C	**1 point**

$$F_c = \frac{mv_C^2}{r}$$
$$F_N + mg = \frac{mv_C^2}{r}$$

Set the normal force equal to zero

$$0 + mg = \frac{mv_C^2}{R} \therefore v_C = \sqrt{Rg}$$

For combining the two equations above	**1 point**

$$\sqrt{Rg} = \sqrt{2g(h-2R)} \therefore R = 2(h-2R) \therefore R/2 = h - 2R$$
$$h = 2.5R$$

Total for part (c) 3 points

(d)	For correctly using conservation of energy for the block compressing the spring	**1 point**
	For correctly substituting the gravitational and elastic potential energies into an equation for conservation of energy	**1 point**

$$mgh = \frac{1}{2}kx_{MAX}^2$$

For correctly substituting for the spring constant k into the equation above	**1 point**

$$x_{MAX} = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2mgh}{mg/(2R)}} = \sqrt{4hR} = \sqrt{(4)(0.30\text{ m})(0.10\text{ m})} = 0.35\text{ m}$$

Total for part (d) 3 points

(e) i.	For a correct justification	**1 point**

Example response for part (e)(i)
Because the block does not make it through the loop at this height, it will not compress the spring.

ii.	For indicating that as the height increases, the compression of the spring increases	**1 point**
	For indicating that the height is proportional to the square of the compression of the spring	**1 point**

Example response for part (e)(ii)
From the equation in part (c), the compression of the spring is directly proportional to the square root of the height that the block is released. Thus, the graph would be an x-axis parabola, as shown.

Total for part (e) 3 points

Total for question 2 15 points

2019

AP® CollegeBoard

AP® Physics C: Mechanics

Scoring Guidelines Set 1

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

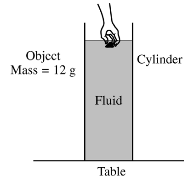
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

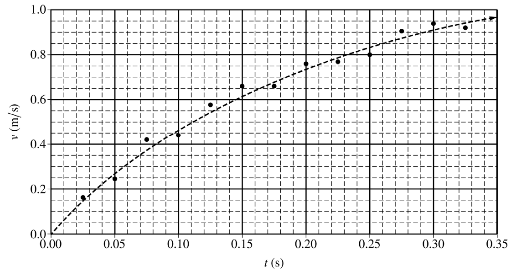
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 1

15 points



In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of $m = 12$ g is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.



(a)

i. LO CHA-1.C, SP 7.A
1 point

Does the speed of the object increase, decrease, or remain the same?

____ Increase ____ Decrease ____ Remain the same

For selecting "Increase"		1 point

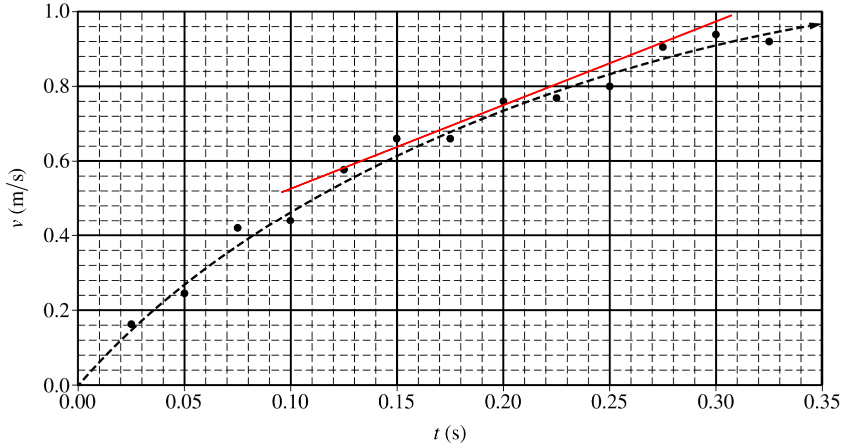
ii. LO CHA-1.C, SP 4.D
2 points

In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.

For a description that includes the direction of the acceleration as being "downwards"		1 point
For a description that includes the decrease in the magnitude of the acceleration		1 point
Example: Because the object is moving downwards and speeding up, the acceleration must be downwards. Because the slope of the graph of speed as a function of time is decreasing, the magnitude of the acceleration must be decreasing.		

Question 1 (continued)

(a) continued



iii. LO CHA-1.C, SP 4.D, 6.C
2 points

Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20$ s.

For calculating the slope of a trend line at $t = 0.20$ s		1 point
slope $= a = \dfrac{\Delta v}{\Delta t} = \dfrac{(0.8 - 0.6)\ \text{m/s}}{(0.226 - 0.136)\ \text{s}}$		
For a correct answer using points from a tangent line		1 point
$a = 2.22\ \text{m/s}^2$		

The students use the equation $v = A\left(1 - e^{-Bt}\right)$ to model the speed of the falling object and find the best-fit coefficients to be $A = 1.18\ \text{m/s}$ and $B = 5\ \text{s}^{-1}$.

Question 1 (continued)

(b) Use the above equation to:

i. LO CHA-1.B, SP 6.B, 6.C
3 points

Derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time t.

For indicating that the vertical displacement is the integration of the velocity		1 point
$\Delta y = \int v dt = \int A\left(1 - e^{-Bt}\right)dt$		
For the equation for speed using appropriate limits or constant of integration		1 point
$\Delta y = \int_{t'=0}^{t'=t} A\left(1 - e^{-Bt'}\right)dt' = A\left[t' - \frac{1}{-B}e^{-Bt'}\right]_{t'=0}^{t'=t} A\left[\left(t + \frac{1}{B}e^{-Bt}\right) - \left(0 + \frac{1}{B}e^{0}\right)\right]$		
For a correct answer		1 point
$\Delta y = A\left(t + \frac{1}{B}\left(e^{-Bt} - 1\right)\right) = (1.18)\left(t + \frac{1}{5}\left(e^{-5t} - 1\right)\right)$		
<u>Note:</u> Credit given for using A and B or plugging in the given values		

ii. LO INT-1.C.d, SP 6.B, 6.C
3 points

Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time t.

For attempting the derivative of the equation for speed		1 point
$a = \frac{dv}{dt} = \frac{d}{dt}\left[A\left(1 - e^{-Bt}\right)\right]$		
For a correct equation for the acceleration		1 point
$a = ABe^{-Bt}$		
For multiplying the equation above by the mass of the object		1 point
$F = ma = m\left(ABe^{-Bt}\right) = mABe^{-Bt}$		
$F = (.012)(1.18)(5)e^{-5t} = 0.071e^{-5t}$		
<u>Note:</u> Credit given for using A, B, and m or plugging in the given values		

Question 1 (continued)

(c)
The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed.

i. LO INT-1.I, SP 7.A, 7.C
2 points

Determine the constant speed of the object.
Justify your answer.

For stating the constant speed is $v = A$		1 point
For indicating that the constant speed can be determined by setting the time equal to infinity		1 point
Example: After a long time, the falling object will reach a terminal constant speed in the fluid. This can be determined by setting the time t in the equation for speed equal to infinity. By doing this, the constant speed is determined to be $v = A$.		
<u>Note:</u> Credit given for solving mathematically		

ii. LO INT-1.H.b, SP 7.A, 7.C
2 points

Determine the force exerted by the fluid on the object at this time. Justify your answer.

For indicating that the net force is equal to zero when the object moves with constant speed		1 point
For indicating the resistive force is equal to the weight of the object at this time		1 point
Example: When the falling object reaches a constant speed in the fluid, the net force must be zero. Because the only vertical forces acting on the object are Earth's gravitational pull and the resistive force of the fluid, these two forces must be equal. So, the resistive force must be equal to the weight of the object or 0.12 N.		
<u>Note:</u> Credit given for solving mathematically		

Question 1 (continued)

Learning Objectives

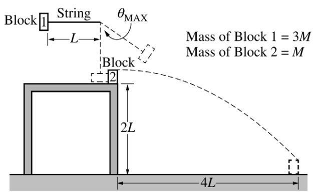
CHA-1.B: Determine functions of position, velocity, and acceleration that are consistent with each other, for the motion of an object with a nonuniform acceleration.
CHA-1.C: Describe the motion of an object in terms of the consistency that exists between position and time, velocity and time, and acceleration and time.
INT-1.C.d: Derive an expression for the net force on an object in translational motion.
INT-1.H.b: Describe the acceleration, velocity, or position in relation to time for an object subject to a resistive force (with different initial conditions, i.e., falling from rest or projected vertically).
INT-1.I: Calculate the terminal velocity of an object moving vertically under the influence of a resistive force of a given relationship.

Science Practices

4.D: Select relevant features of a graph to describe a physical situation or solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

Question 2

15 points



Note: Figure not drawn to scale.

A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

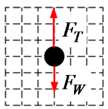
(a) LO CON-2.C.a, SP 5.E
1 point

Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

For correctly calculating the speed of block 1		1 point
$U_{g1} = K_2 \therefore mgh = \frac{1}{2}mv^2 \therefore gL = \frac{1}{2}v^2$		
$v = \sqrt{2gL}$		
Note: Credit is earned even if no work is shown.		

(b) LO INT-2.B.b, SP 3.D
2 points

On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



For correctly drawing and labeling the weight of the block and the tension in the string		1 point
For drawing the tension longer than the weight of the block		1 point
Note: A maximum of one point can be earned if there are any extraneous vectors.		

Question 2 (continued)

(c) LO INT-2.B.b, SP 5.A, 5.E
2 points

Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

For using an appropriate equation to calculate the tension in the string		1 point
$F_T = mg + ma_C = mg + \frac{mv^2}{r} = 3Mg + \frac{(3M)\left(\sqrt{2gL}\right)^2}{L}$		
For a correct answer		1 point
$F_T = 9Mg$		

For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

(d) LO CHA-2.C, SP 6.A, 6.C
2 points

Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

For using a correct kinematic equation to calculate the time		1 point
$\Delta y = v_1 t + \frac{1}{2}at^2 = \frac{1}{2}at^2 \therefore t = \sqrt{\frac{2\Delta y}{a}} = \sqrt{\frac{2(2L)}{g}} = \sqrt{\frac{(4)(0.75\ \text{m})}{\left(9.8\ \text{m/s}^2\right)}}$		
For a correct answer		1 point
$t = 0.55$ s		

(e) LO CHA-2.C, SP 6.A, 6.C
2 points

Calculate the speed of block 2 as it leaves the table.

For using a correct kinematic equation to calculate the speed		1 point
$\Delta x = vt \therefore v = \frac{\Delta x}{t} = \frac{4L}{t}$		
For substituting the time from part (d) into equation above		1 point
$v = \frac{(4)(0.75\ \text{m})}{(0.55\ \text{s})} = 5.45\ \text{m/s}$		

Question 2 (continued)

(f) LO CON-4.E.a, SP 6.B, 6.C
3 points

Calculate the speed of block 1 just after it collides with block 2.

For indicating the use of the correct conservation of momentum to calculate the speed		1 point
$p_i = p_f \therefore m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$		
For correctly substituting the mass into equation above		1 point
$(3M)v_{1i} + 0 = (3M)v_{1f} + (M)v_{2f}$		
For substituting the speeds from parts (a) and (e) into equation above		1 point
$v_{1f} = \dfrac{(3M)v_{1i} - (M)v_{2f}}{(3M)} = \dfrac{(3M)\sqrt{2gL} - (M)v_{2f}}{(3M)}$		
$v_{1f} = \dfrac{(3)\sqrt{(2)(9.8 \text{ m/s}^2)(0.75 \text{ m})} - (1)(5.45 \text{ m/s})}{(3)} = 2.02 \text{ m/s}$		
($v_{1f} = 2.06$ m/s when using $g = 10 \text{ m/s}^2$)		

(g) LO CON-2.C.a, SP 6.B, 6.C
3 points

Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.

For using conservation of energy to calculate the angle		1 point
$K_1 = U_{g2}$		
$\frac{1}{2}mv^2 = mgh$		
For correctly substituting $h = L(1 - \cos\theta)$ into the equation above		1 point
For correctly substituting the speed from part (f) into the equation above		1 point
$\frac{1}{2}mv^2 = mgL(1 - \cos\theta) \therefore \dfrac{v^2}{2gL} = 1 - \cos\theta$		
$\theta = \cos^{-1}\left(1 - \dfrac{v^2}{2gL}\right) = \cos^{-1}\left(1 - \dfrac{(2.02 \text{ m/s})^2}{(2)(9.8 \text{ m/s}^2)(0.75 \text{ m})}\right) = 43.5°$		
($\theta = 44.1°$ when using $g = 10 \text{ m/s}^2$)		

Question 2 (continued)

Learning Objectives

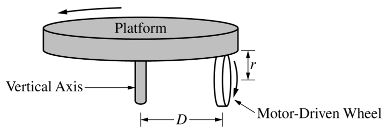
CHA-2.C: Calculate kinematic quantities of an object in projectile motion, such as displacement, velocity, speed, acceleration, and time, given initial conditions of various launch angles, including a horizontal launch at some point in its trajectory.
INT-2.B.b: Describe forces that are exerted on objects undergoing horizontal circular motion, vertical circular motion, or horizontal circular motion on a banked curve.
CON-2.C.a: Calculate unknown quantities (e.g., speed or positions of an object) that are in a conservative system of connected objects, such as the masses in an Atwood machine, masses connected with pulley/string combinations, or the masses in a modified Atwood machine.
CON-4.E.a: Calculate the changes in speeds, changes in velocities, changes in kinetic energy, or changes in momenta of objects in all types of collisions (elastic or inelastic) in one dimension, given the initial conditions of the objects.

Science Practices

3.D: Create appropriate diagrams to represent physical situations.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

Question 3

15 points



A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the wheel until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

(a) LO INT-7.A.b, CHA-4.A.b, SP 5.A, 5.E
2 points

Derive an expression for the angular speed ω_P of the platform. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.

For correctly substituting into the rotational form of Newton's second law			1 point
$\tau = I\alpha \therefore FD = I_P\alpha$			
$\alpha = \dfrac{FD}{I_P}$			
For correctly substituting into a rotational kinematic equation to calculate the angular speed			1 point
$\omega_2 = \omega_1 + \alpha\Delta t = 0 + \left(\dfrac{FD}{I_P}\right)\Delta t$			
$\omega_P = \dfrac{FD\Delta t}{I_P}$			

(b) LO INT-7.D.a, SP 5.A, 5.E
2 points

Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed ω_P. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.

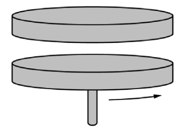
For using the equation for rotational kinetic energy		1 point
$K = \dfrac{1}{2}I\omega_P^2 = \left(\dfrac{1}{2}\right)(I)\left(\dfrac{FD\Delta t}{I}\right)^2$		
For an answer consistent with part (a)		1 point
$K = \dfrac{(FD\Delta t)^2}{2I}$		

Question 3 (continued)

(c) LO INT-7.C, SP 5.A, 5.E
2 points

Derive an expression for the angular speed of the wheel ω_W when the platform has reached angular speed ω_P. Express your answer in terms of D, r, ω_P, and physical constants, as appropriate.

For indicating that the linear speed of the platform is equal to the linear speed of the wheel		1 point
$v_P = v_W$ OR $(r\omega)_P = (r\omega)_W$		
For correctly relating the linear speeds to the angular speeds in the above equation		1 point
$D\omega_P = r\omega_W$		
$\omega_W = \dfrac{D\omega_P}{r}$		



When the platform is spinning at angular speed ω_P, the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia I_P as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed ω_f.

(d) LO CON-5.D.c, SP 5.A, 5.E
2 points

Derive an expression for ω_f. Express your answer in terms of ω_P, I_P, and physical constants, as appropriate.

For using an expression for the conservation of angular momentum		1 point
$L_1 = L_2 \therefore I_1\omega_1 = I_2\omega_2$		
For correctly substituting into the above equation		1 point
$I\omega_P = (2I)\omega_F$		
$\omega_F = \dfrac{1}{2}\omega_P$		

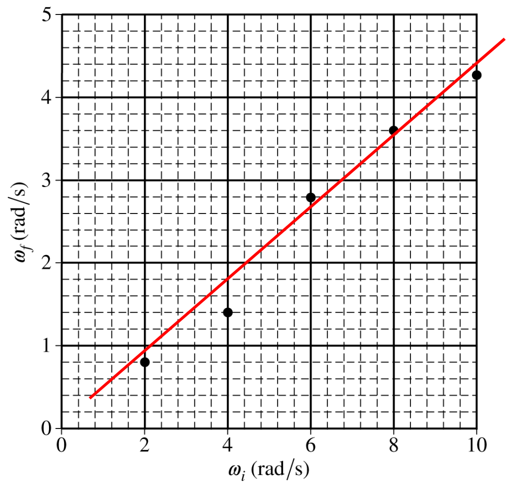
Question 3 (continued)

A student now uses the rotating platform $\left(I_P = 3.1 \text{ kg} \bullet \text{m}^2\right)$ to determine the rotational inertia I_U of an unknown object about a vertical axis that passes through the object's center of mass. The platform is rotating at an initial angular speed ω_i when the unknown object is dropped with its center of mass directly above the center of the platform. The platform and object are observed to be rotating together at angular speed ω_f. Trials are repeated for different values of ω_i. A graph of ω_f as a function of ω_i is shown on the axes below.

(e)

i. LO CON-5.D.c, SP 4.C
1 points

On the graph on the previous page, draw a best-fit line for the data.



For an appropriate best-fit line for the graph above		1 point

Question 3 (continued)

(e) continued

ii. LO CON-5.D.c, SP 4.D, 6.C
2 points

Using the straight line, calculate the rotational inertia of the unknown object I_U about a vertical axis passing through its center of mass.

For using conservation of angular momentum to derive an expression that includes I_U		1 point
$L_i = L_f \therefore I_P\omega_i = (I_P + I_U)\omega_f$		
$I_U = I_P\left(\dfrac{\omega_i}{\omega_f}\right) - I_P$		
For substituting points from the best-fit line into the expression above		1 point
$I_U = \left(3.1 \text{ kg}{\bullet}\text{m}^2\right)\left(\dfrac{(4.0 - 1.0) \text{ rad/s}}{(9.3 - 2.4) \text{ rad/s}}\right) - \left(3.1 \text{ kg}{\bullet}\text{m}^2\right) = 4.1 \text{ kg}{\bullet}\text{m}^2$		
Note: The point (0, 0) can be used implicitly if the best-fit line goes through the origin.		

(f) LO CON-5.D.c, SP 7.A, 7.C
2 points

The kinetic energy of the spinning platform before the object is dropped on it is K_i. The total kinetic energy of the platform-object system when it reaches angular speed ω_f is K_f. Which of the following expressions is true?

____ $K_f < K_i$ ____ $K_f = K_i$ ____ $K_f > K_i$

Justify your answer.

For selecting $K_f < K_i$ with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because the two disks will be rotating with the same final angular speed, this is an inelastic collision, and kinetic energy will be lost during the collision.		

2019 SCORING GUIDELINES

Question 3 (continued)

(g) LO INT-6.E, SP 7.A, 7.C
2 points

One of the students observes that the center of mass of the object is not actually aligned with the axis of the platform. Is the experimental value of I_U obtained in part (e) greater than, less than, or equal to the actual value of the rotational inertia of the unknown object about a vertical axis that passes through its center of mass?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

For selecting "Greater than" with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because the center of mass of the object is off the axis of the platform, the parallel axis theorem would be used to calculate the total rotational inertia of the platform-object system. Using $I = I_{CM} + Mh^2$, the experimental value will be increased by Mh^2.		

Learning Objectives

INT-6.E: Derive the moments of inertia of an extended rigid body for different rotational axes (parallel to an axis that goes through the object's center of mass) if the moment of inertia is known about an axis through the object's center of mass.
CHA-4.A.b: Calculate unknown quantities such as angular positions, displacement, angular speeds, or angular acceleration of a rigid body in uniformly accelerated motion, given initial conditions.
INT-7.A.b: Calculate unknown quantities such as net torque, angular acceleration, or moment of inertia for a rigid body undergoing rotational acceleration.
INT-7.C: Derive expressions for physical systems such as Atwood Machines, pulleys with rotational inertia, or strings connecting discs or strings connecting multiple pulleys that relate linear or translational motion characteristics to the angular motion characteristics of rigid bodies in the system that are: **(a)** rolling (or rotating on a fixed axis) without slipping. **(b)** rotating and sliding simultaneously.
INT-7.D.a: Calculate the rotational kinetic energy of a rotating rigid body.
CON-5.D.c: Calculate the changes of angular momentum of each disc in a rotating system of two rotating discs that collide with each other inelastically about a common rotational axis.

Science Practices

4.C: Linearize data and/or determine a best-fit line or curve.
4.D: Select relevant features of a graph to describe a physical situation or solve problems.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

2019

AP CollegeBoard

AP® Physics C: Mechanics

Scoring Guidelines Set 2

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 1

15 points

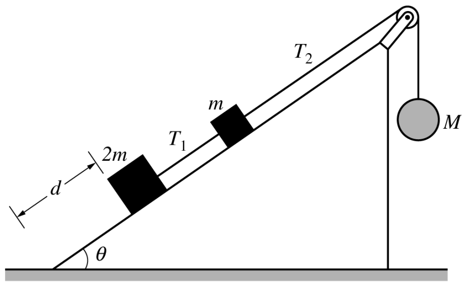
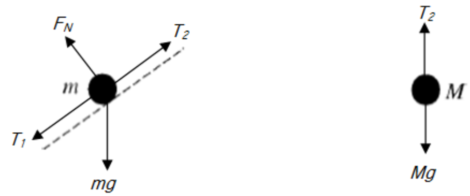


Figure 1

Blocks of mass m and $2m$ are connected by a light string and placed on a frictionless inclined plane that makes an angle θ with the horizontal, as shown in Figure 1 above. Another light string connecting the block of mass m to a hanging sphere of mass M passes over a pulley of negligible mass and negligible friction. The entire system is initially at rest and in equilibrium.

(a) LO INT-1.A, SP 3.D
3 points

On the dots below that represent the block of mass m and the sphere of mass M, draw and label the forces (not components) that act on each of the objects shown. Each force must be represented by a distinct arrow starting on and pointing away from the dot.



For correctly drawing and labeling vectors representing the normal force and the gravitational force on the block of mass m		1 point
For correctly drawing and labeling vectors representing the forces of tension on the block of mass m		1 point
For correctly drawing and labeling vectors representing the tension force and the gravitational force on the sphere of mass M		1 point
Note: A maximum of two points can be earned if there are any extraneous vectors.		

Question 1 (continued)

(b)

Derive expressions for the magnitude of each of the following. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figures in part (a).

i. LO INT-1.C.e, SP 1.D, 5.E
2 points

The force T_2 exerted on the block of mass m by the string. Express your answers in terms of m, θ, and physical constants, as appropriate.

For using an attempt at a correct statement of Newton's second law for the two blocks		1 point
$F_{net} = (m+2m)a = 0 \therefore T_2 - (m+2m)g\sin\theta = 0$		
For a correct answer with supporting work		1 point
$T_2 = 3mg\sin\theta$		

ii. LO INT-1.D, SP 5.E
1 point

The mass M for which the system can remain in equilibrium. Express your answers in terms of m, θ, and physical constants, as appropriate.

For using a correct statement of Newton's second law for the whole system to derive an expression for M		1 point
$F_{net} = m_{tot}a \therefore Mg - 2mg\sin\theta - mg\sin\theta = 0$		
$M = 3m\sin\theta$		
Alternate Solution		*Alternate Points*
For a correct statement of Newton's second law for the sphere and an answer consistent with part (b)(i)		1 point
$T_2 - Mg = 0 \therefore Mg = T_2 \therefore M = T_2/g$		
$M = 3m\sin\theta$		

Question 1 (continued)

(c)

Now suppose that mass M is large enough to descend and that the sphere reaches the floor before the blocks reach the pulley. Answer the following for the moment immediately after the sphere reaches the floor.

i. LO INT-1.D, SP 7.A
1 point

Does the tension T_1 increase, decrease to a nonzero value, decrease to zero, or stay the same?

____ Increase ____ Decrease to a nonzero value

____ Decrease to zero ____ Stay the same

For correctly stating that the magnitude of T_1 drops to zero		1 point

ii. LO INT-1.D, SP 7.A
1 point

Is the velocity of the block of mass m up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

For selecting "Up the ramp"		1 point

iii. LO INT-1.D, SP 7.A
1 point

Is the acceleration of the block of mass m up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

For selecting "Down the ramp"		1 point

(d) LO INT-1.D, SP 5.A, 5.E
3 points

Consider the initial setup in Figure 1. Now suppose the surface of the incline is rough and the coefficient of static friction between the blocks and the inclined plane is μ_s. Derive an expression for the minimum possible value of M that will keep the blocks from moving down the incline. Express your answer in terms of m, μ_s, θ, and fundamental constants, as appropriate.

For an attempt at a correct statement of Newton's second law for the system		1 point
$F_{net} = m_{tot}a \therefore 2mg\sin\theta - f_{s2} + mg\sin\theta - f_{s1} - Mg = 0$		
For attempting to substitute in for the force of friction		1 point
$3mg\sin\theta - \mu_s F_{N2} - \mu_s F_{N1} = Mg$		
$3mg\sin\theta - \mu_s(2mg\cos\theta) - \mu_s(mg\cos\theta) = Mg$		
For a correct answer with supporting work		1 point
$M = 3m(\sin\theta - \mu_s\cos\theta)$		

Question 1 (continued)

(e) LO INT-1.D, CHA-1.A.b, SP 5.A, 5.E
3 points

The string connecting block m and the sphere of mass M then breaks, and the blocks begin to move from rest down the incline. The lower block starts a distance d from the bottom of the incline, as shown in Figure 1. The coefficient of kinetic friction between the blocks and the inclined plane is μ_k. Derive an expression for the speed of the blocks when the lower block reaches the bottom of the incline. Express your answer in terms of m, d, μ_k, θ, and fundamental constants, as appropriate.

For an attempt at a correct statement of Newton's second law for the two blocks		1 point
$F_{net} = m_{tot}a \therefore 2mg\sin\theta - f_{k2} + mg\sin\theta - f_{k1} = 3ma$		
Solve for the acceleration		
$3mg\sin\theta - \mu_k(2mg\cos\theta) - \mu_k mg\cos\theta = 3ma$		
$a = g(\sin\theta - \mu_k\cos\theta)$		
For using a correct kinematics equation to solve for the final velocity		1 point
$v_2^2 = v_1^2 + 2ad$		
$v_2^2 = 0^2 + 2g(\sin\theta - \mu_k\cos\theta)d$		
For a correct answer with supporting work		1 point
$v = \sqrt{2gd(\sin\theta - \mu_k\cos\theta)}$		
Alternate Solution		*Alternate Points*
For an attempt at a correct statement of conservation of energy for the two blocks		*1 point*
$U_1 + K_1 = U_2 + K_2 + E_{lost}$		
$2mgh + mgh + 0 = 0 + \frac{1}{2}(2m)v^2 + \frac{1}{2}mv^2 + fd$		
For correct substitutions of height and friction		*1 point*
$3mgd\sin\theta = \frac{1}{2}(3m)v^2 + \mu_k(2mg\cos\theta)d + \mu_k(mg\cos\theta)d$		
$3gd\sin\theta - 3\mu_k gd\cos\theta = \frac{3}{2}v^2$		
For a correct answer with supporting work		*1 point*
$v = \sqrt{2gd(\sin\theta - \mu_k\cos\theta)}$		

Question 1 (continued)

Learning Objectives

CHA-1.A.b: Calculate unknown variables of motion such as acceleration, velocity, or positions for an object undergoing uniformly accelerated motion in one dimension.
INT-1.A: Describe an object (either in a state of equilibrium or acceleration) in different types of physical situations such as inclines, falling through air resistance, Atwood machines, or circular tracks).
INT-1.C.e: Derive a complete Newton's second law statement (in the appropriate direction) for an object in various physical dynamic situations (e.g., mass on incline, mass in elevator, strings/pulleys, or Atwood machines).
INT-1.D: Calculate a value for an unknown force acting on an object accelerating in a dynamic situation (e.g., inclines, Atwood Machines, falling with air resistance, pulley systems, mass in elevator, etc.)

Science Practices

1.D: Select relevant features of a representation to answer a question or solve a problem.
3.C: Sketch a graph that shows a functional relationship between two quantities.
3.D: Create appropriate diagrams to represent physical situations.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
7.A: Make a scientific claim.

Question 2

15 points

A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0\ \text{m/s}^2$, $L = 0.25\ \text{m/s}^4$, and t is the time in seconds. At $t = 6.0$ s, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

(a) LO INT-5.E, SP 6.B, 6.C
2 points

Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0$ s.

For an expression for calculating impulse and correct substitution of $a(t)$ and m into the correct expression.		1 point
$J = \int F(t)dt$		
$J = \int ma(t)dt = (0.50)\int\left(9.0 - 0.25t^2\right)dt$		
For integrating with correct limits or including a constant of integration		1 point
$J = (0.50)\int_{t=0}^{t=6}\left(9.0 - 0.25t^2\right)dt = (0.50)\left[9.0t - \frac{1}{12}t^3\right]_{t=0}^{t=6}$		
$J = (0.50)\left[\left((9.0)(6) - \left(\frac{1}{12}\right)(6)^3\right)\right] - 0 = 18\ \text{N•s}$		
Alternate Solution (using an alternate solution from part (b))		*Alternate Points*
For correctly relating impulse to the speed of the rocket		1 point
$J = \Delta p = m(v_2 - v_1)$ OR $J = m\Delta v = mv_f$		
For correctly substituting answer from part (b) into equation above		1 point
$J = (0.50\ \text{kg})(36 - 0)\ \therefore\ J = 18N•s$		

Question 2 (continued)

(b) LO INT-5.A.a, SP 6.A, 6.C
2 points

Calculate the speed of the rocket at $t = 6.0$ s.

For correctly relating impulse to the speed of the rocket		1 point
$J = \Delta p = m(v_2 - v_1)$ OR $J = m\Delta v = mv_f$		
For correctly substituting answer from part (a) into equation above		1 point
$(18\ \text{N•s}) = (0.50\ \text{kg})(v_2 - 0)\ \therefore\ v_2 = 36\ \text{m/s}$		
Alternate Solution		*Alternate Points*
Integrate expression for a(t) (This may have already been done in solving part (a).)		1 point
$v = \int a dt = \int\left(9.0 - 0.25t^2\right)dt$		
For integrating with correct limits or including a constant of integration		1 point
$v = \int_{t=0}^{t=6}\left(9.0 - 0.25t^2\right)dt = \left[9.0t - \frac{1}{12}t^3\right]_{t=0}^{t=6} = 36\ \text{m/s}$		

(c)

i. LO INT-4.C.c, SP 6.B, 6.C
2 points

Calculate the kinetic energy of the rocket at $t = 6.0$ s.

For substituting the mass of the rocket into the equation for kinetic energy		1 point
For substituting the answer from part (b) into the equation for kinetic energy		1 point
$K = \frac{1}{2}mv^2 = \left(\frac{1}{2}\right)(0.50\ \text{kg})(36\ \text{m/s})^2 = 324\ \text{J}$		

ii. LO CHA-1.B, CON-1.E, SP 6.A, 6.C
3 points

Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.

For integrating the acceleration twice to derive an expression for position		1 point
For integrating with correct limits or including a constant of integration		1 point
$v(t) = \int a(t)dt = \int\left(9.0 - 0.25t^2\right)dt = 9.0t - \frac{1}{12}t^3 + v_0 = 9.0t - \frac{1}{12}t^3$		
$\Delta y = \int v(t)dt = \int_{t=0}^{t=6}\left(9.0t - \frac{1}{12}t^3\right)dt = \left[\frac{9}{2}t^2 - \frac{1}{48}t^4\right]_{t=0}^{t=6}$		
$\Delta y = \left[\left(\frac{9}{2}\right)(6)^2 - \left(\frac{1}{48}\right)(6)^4\right] - 0 = 135\ \text{m}$		
For substituting into equation for potential energy		1 point
$\Delta U_g = mg\Delta y = (0.50\ \text{kg})\left(9.8\ \text{m/s}^2\right)(135\ \text{m}) = 660\ \text{J}$		

Question 2 (continued)

(d) LO CHA-1.B, CON-2.B, SP 6.B, 6.C
3 points

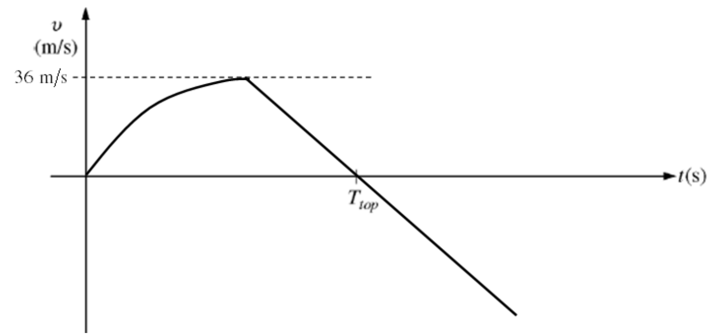
Calculate the maximum height reached by the rocket relative to its launching point.

For using $a = g$ in a correct kinematics equation to solve for height		1 point
$v_2^2 = v_1^2 + 2a(y_2 - y_1) \therefore 0 = v_1^2 + 2a(y_2 - y_1)$		
$y_2 - y_1 = -\frac{v_1^2}{2a} \therefore y_2 = -\frac{v_1^2}{2a} + y_1$		
For substituting the speed from part (b)		1 point
$y_2 = -\frac{(36\ \text{m/s})^2}{2\left(-9.8\ \text{m/s}^2\right)} + y_1$		
For substituting the height from part (c)		1 point
$y_2 = 66\ \text{m} + 135\ \text{m} = 201\ \text{m}\ (199.8\ \text{m if } g = 10\ \text{m/s}^2)$		
Alternate Solution 1		*Alternate Points*
For using energy conservation to find maximum height, consistent with the speed found in part (b).		1 point
$mg\Delta y = \frac{1}{2}mv^2 \therefore \Delta y = \frac{v^2}{2g}$		
For substituting the speed from part (b)		1 point
$\Delta y = \frac{(36\ \text{m/s})^2}{2\left(9.8\ \text{m/s}^2\right)} = 66\ \text{m}$		
For substituting the height from part (c)		1 point
$y_2 = 66\ \text{m} + 135\ \text{m} = 201\ \text{m}\ (199.8\ \text{m if } g = 10\ \text{m/s}^2)$		
Alternate Solution 2		*Alternate Points*
For using energy conservation to find maximum height, from kinetic and potential energies found in part (c)		1 point
$K_1 + U_1 = 0 + U_{top}$		
For substituting the kinetic energy from part (c)(i) into the equation above		1 point
For substituting the potential energy from part (c)(ii) into the equation above		1 point
$324 + 660 = (0.5)(9.8)\Delta y$		
$\Delta y = 201\ \text{m}\ (199.8\ \text{m if g} = 10\text{m/s}^2)$		

Question 2 (continued)

(e) LO CHA-1.C, SP 3.C
3 points

On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



For an initial concave down curve that starts at the origin.		1 point
For a transition that occurs before T_{top} into a straight line with a negative slope.		1 point
For labeling the maximum value of the velocity, consistent with part (b), and a line that crosses the x-axis at T_{top}		1 point

Question 2 (continued)

Learning Objectives

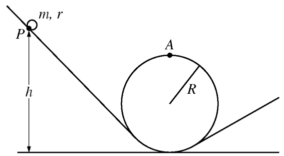
CHA-1.B: Determine functions of position, velocity, and acceleration that are consistent with each other, for the motion of an object with a nonuniform acceleration.
CHA-1.C: Describe the motion of an object in terms of the consistency that exists between position and time, velocity and time, and acceleration and time.
CON-1.E: Calculate the potential energy of a system consisting of an object in a uniform gravitational field.
CON-2.B: Describe kinetic energy, potential energy, and total energy in relation to time (or position) for a "conservative" mechanical system.
INT-4.C.c: Calculate changes in an object's kinetic energy or changes in speed that result from the application of specified forces.
INT-5.A.a: Calculate the total momentum of an object or system of objects.
INT-5.E: Calculate the change in momentum of an object given a nonlinear function, $F(t)$, for a net force acting on the object.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.
6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

Question 3

15 points



Note: Figure not drawn to scale.

The rotational inertia of a rolling object may be written in terms of its mass m and radius r as $I = bmr^2$, where b is a numerical value based on the distribution of mass within the rolling object. Students wish to conduct an experiment to determine the value of b for a partially hollowed sphere. The students use a looped track of radius $R >> r$, as shown in the figure above. The sphere is released from rest a height h above the floor and rolls around the loop.

(a) LO INT-2.D, SP 5.A, 5.E
2 points

Derive an expression for the minimum speed of the sphere's center of mass that will allow the sphere to just pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

For an expression relating the gravitational force to the centripetal force		1 point
$F_C = mg + F_N = \frac{mv^2}{R}$		
$mg + 0 = \frac{mv^2}{R}$		
For a correct substitution into a correct expression.		1 point
$v = \sqrt{Rg}$		

Question 3 (continued)

(b) LO INT-7.E, SP 5.A, 5.E
3 points

Suppose the sphere is released from rest at some point P and rolls without slipping. Derive an equation for the minimum release height h that will allow the sphere to pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

For using a correct expression of the conservation of energy for the object		1 point
$K_1 + U_1 = K_2 + U_2$		
For correct substitutions, including rotational kinetic energy		1 point
$0 + mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh_2$		
$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(bmr^2\right)\left(\frac{v}{r}\right)^2 + mgh_2$		
For correctly substituting for the final height and the answer from part (a) for the velocity		1 point
$gh = \frac{1}{2}\left(\sqrt{Rg}\right)^2 + \frac{1}{2}(b)\left(\sqrt{Rg}\right)^2 + g(2R)$		
$h = \frac{1}{2}R + \frac{1}{2}bR + 2R = \frac{1}{2}(b+5)R$		

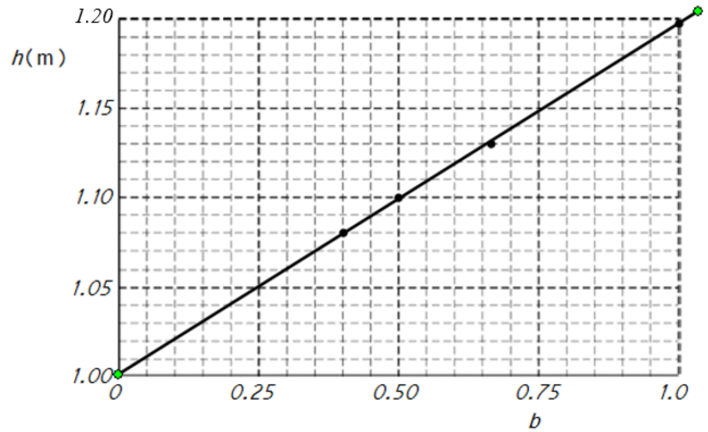
Question 3 (continued)

The students perform an experiment by determining the minimum release height h for various other objects of radius r and known values of b. They collect the following data.

Object	b	h (m)
Solid sphere	0.40	1.08
Hollow sphere	0.67	1.13
Solid cylinder	0.50	1.10
Hollow cylinder	1.0	1.20

(c) LO INT-7.E, SP 3.A, 4.C
4 points

On the grid below, plot the release height h as a function of b. Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



For correctly labeling both axes with variables, units for h, no units for b		1 point
For correctly using and labeling the scale of the axes so that the data uses at least half the grid		1 point
For correctly plotting the data		1 point
For drawing a best-fit straight line that represents the data		1 point

Question 3 (continued)

(d) LO INT-7.E, SP 4.D, 6.C
2 points

The students repeat the experiment with the partially hollowed sphere and determine the minimum release height to be 1.16 m. Using the straight line from part (c), determine the value of b for the partially hollowed sphere.

For correctly calculating the slope from the best-fit straight line and substituting height into the line equation		1 point
slope $= \dfrac{1.20 - 1.00}{1.0 - 0.0} = 0.20$ m		
$y = mx + b \therefore h = 0.20b + 1.00$		
$1.16 = 0.20b + 1.00$		
For a correct answer for b		1 point
$b = 0.80$		
Note: Full credit can be earned for the correct answer if there is an indication that this was read from the graph.		

(e) LO INT-7.E, SP 6.A, 6.C
2 points

Calculate R, the radius of the loop.

For substituting into the expression from part (b)		1 point
$h = \dfrac{1}{2}(b+5)R \therefore R = \dfrac{2(1.16\text{ m})}{(0.80+5)}$		
For an answer consistent with previous parts		1 point
$R = 0.40$ m		
Alternate Solution		*Alternate Points*
For using an expression for the y-intercept (set b = 0)		1 point
$h = \dfrac{1}{2}(b+5)R = \dfrac{1}{2}(0+5)R = \dfrac{5}{2}R$		
For determining the value of the intercept (1.0 m) from the graph		1 point
$1.0\text{ m} = \dfrac{5}{2}R \therefore R = \dfrac{2}{5}\text{ m} = 0.40\text{ m}$		

Question 3 (continued)

(f) LO INT-7.E, SP 7.A, 7.C
2 points

In part (b), the radius r of the rolling sphere was assumed to be much smaller than the radius R of the loop. If the radius r of the rolling sphere was not negligible, would the value of the minimum release height h be greater, less, or the same?

____ Greater ____ Less ____ The same

Justify your answer.

For selecting "Less" and any justification		1 point
For a correct justification		1 point
Examples:		
If the radius of the object is not negligible, then the center of mass of the object travels in a radius less than R. If the radius of the circle decreases, the height needed to pass through the loop decreases according to the equation from part (b).		
If the radius of the object is not negligible, then the center of mass of the object travels in a radius less than R. If the radius of the circle decreases, the speed needed to pass through the loop decreases, and thus the height needed also decreases.		

Learning Objectives

INT-2.D: Derive expressions relating centripetal force to the minimum speed or maximum speed of an object moving in a vertical circular path.
INT-7.E: Derive expressions using energy conservation principles for physical systems such as rolling bodies on inclines, Atwood Machines, pendulums, physical pendulums, and systems with massive pulleys that relate linear or angular motion characteristics to initial conditions (such as height or position) or properties of rolling body (such as moment of inertia or mass).

Science Practices

3.A: Select and plot appropriate data.
4.C: Linearize data and/or determine a best-fit line or curve.
4.D: Select relevant features of a graph to describe a physical situation or solve problems.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

2018

AP® CollegeBoard

AP Physics C: Mechanics

Scoring Guidelines

2018 SCORING GUIDELINES

General Notes About 2018 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

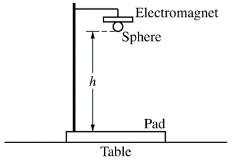
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

2018 SCORING GUIDELINES

Question 1

15 points total **Distribution of points**



A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

(a) 2 points

While taking the first data point, the student notices that the electromagnet actually releases the sphere after the timer begins. Would the value of g calculated from this one measurement be greater than, less than, or equal to the actual value of g at the student's location?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

For selecting "Less than" with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because the measured time to fall the same distance will be larger, the acceleration must be less.		
Example: Because the time is larger and $a \propto 1/t^2$ *, then g must decrease.*		

2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

The electromagnet is replaced so that the timer begins when the sphere is released. The student varies the distance h. The student measures and records the time Δt of the fall for each particular height, resulting in the following data table.

h (m)	0.10	0.20	0.60	0.80	1.00
Δt (s)	0.105	0.213	0.342	0.401	0.451

(b) 1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for g.

Vertical axis: ____________

Horizontal axis: ____________

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given in the table. Label each row you use and include units.

For correctly indicating two variables that will yield a straight line that could be used to determine a value for g		1 point
Example: Vertical Axis: h *Horizontal Axis:* $(\Delta t)^2$		
Note: Student earns full credit if axes are reversed or if they use another acceptable combination.		

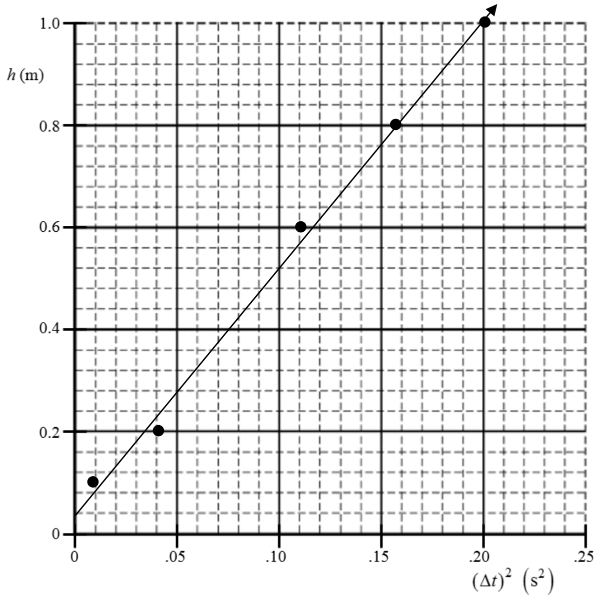
2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(c) 4 points

Plot the data points for the quantities indicated in part (b) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



For a correct scale that uses more than half the grid		1 point
For correctly labeling the axis with variables and units consistent with part (b)		1 point
For correctly plotting data consistent with part (b)		1 point
For drawing a straight line consistent with the plotted data		1 point

2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(d) 2 points

Using the straight line, calculate an experimental value for g.

For using points on the line rather than the data to calculate the slope		1 point
$\text{slope} = \dfrac{\Delta h}{\Delta(\Delta t)^2} = \dfrac{(0.80 - 0.20)\ \text{m}}{(0.160 - 0.039)\ \text{s}^2} = 4.96\ \text{m/s}^2$ (Linear regression $= 4.83\ \text{m/s}^2$)		
For correctly relating the slope to the acceleration due to gravity		1 point
$\text{slope} = \dfrac{1}{2}g \therefore g = 2 \times \text{slope} = 2 \times \left(4.96\ \text{m/s}^2\right)$		
$g = 9.92\ \text{m/s}^2$ (Linear regression $= 9.66\ \text{m/s}^2$)		

Another student fits the data in the table to a quadratic equation. The student's equation for the distance fallen y as a function of time t is $y = At^2 + Bt + C$, where $A = 5.75\ \text{m/s}^2$, $B = -0.524\ \text{m/s}$, and $C = +0.080\ \text{m}$. Vertically down is the positive direction.

(e) Using the student's equation above, do the following.

i. 1 point

Derive an expression for the velocity of the sphere as a function of time.

For correctly taking the time derivative of the given equation		1 point
$y = y_0 + v_1 t + \dfrac{1}{2}at^2 = At^2 + Bt + C$		
$v(t) = \dfrac{dy}{dt} = 2At + B$		
Note: Credit is earned for substituting numbers: $v(t) = \left(11.5\ \text{m/s}^2\right)t - 0.524\ \text{m/s}$.		

ii. 2 points

Calculate the new experimental value for g.

For correctly relating the given equation to a correct kinematic equation		1 point
$\Delta y = y - y_0 = v_1 t + \dfrac{1}{2}at^2$		
$y = y_0 + v_1 t + \dfrac{1}{2}at^2 = At^2 + Bt + C$		
For correctly relating the equation to the value of g		1 point
$A = \dfrac{1}{2}a = \dfrac{1}{2}g$		
$g = 2A = (2)\left(5.75\ \text{m/s}^2\right) = 11.5\ \text{m/s}^2$		

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2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

iii. 1 point

Using $9.81\ \text{m/s}^2$ as the accepted value for g at this location, calculate the percent error for the value found in part (e)(ii).

For correctly calculating the percent error		1 point
$\%\text{error} = \dfrac{\lvert \text{acc} - \text{exp} \rvert}{\text{acc}} \times 100\% = \dfrac{\lvert (11.5\ \text{m/s}^2) - (9.81\ \text{m/s}^2) \rvert}{(9.81\ \text{m/s}^2)} \times 100\%$		
$\%\text{error} = 17.2\%$		
Note: Credit is earned if percent error is expressed as positive or negative.		

iv. 2 points

Assuming the sphere is at a height of 1.40 m at $t = 0$, calculate the velocity of the sphere just before it strikes the pad.

For relating the coefficients of the equation to the kinematic variables		1 point
$y = At^2 + Bt + C\ \therefore v_1 = B = -0.524\ \text{m/s}$		
$a = 2A = 2 \times (5.75\ \text{m/s}^2) = 11.5\ \text{m/s}^2$		
$y_0 = C = 0.080\ \text{m}$		
For correctly using an appropriate kinematics equation to determine the velocity of the sphere		1 point
$v_2^2 = v_1^2 + 2a\Delta y$		
$v = \sqrt{v_1^2 + 2a\Delta y} = \sqrt{(-0.524\ \text{m/s})^2 + (2)(11.5\ \text{m/s}^2)(1.40\ \text{m} - 0.080\ \text{m})}$		
$v = 5.54\ \text{m/s}$		
Alternate solution:		*Alternate Points*
For correctly determining the time of fall for the sphere		*1 point*
$y = At^2 + Bt + C$		
$1.40 = (5.75\ \text{m/s}^2)t^2 + (-0.524\ \text{m/s})t + (0.080\ \text{m})$		
$5.75t^2 - 0.524t - 1.32 = 0$		
$t = 0.53\ \text{s or} -0.44\ \text{s}\ \therefore t = 0.53\ \text{s}$		
For correctly using the equation from part (e)(i)		*1 point*
$v(t) = (11.5\ \text{m/s}^2)(0.53\ \text{s}) - 0.524\ \text{m/s}$		
$v = 5.54\ \text{m/s}$		

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Question 1 (continued)

Distribution of points

(e)

iv. (continued)

Alternate Solution — Conservation of Energy		
For relating the coefficients of the equation to the initial height and speed		1 point
$y = At^2 + Bt + C\ \therefore v_1 = B = -0.524\ \text{m/s}$		
$y_0 = C = 0.080\ \text{m}$		
For correctly using conservation of energy to determine the velocity of the sphere		1 point
$U_i + K_i = U_f + K_f$		
$mgh + \dfrac{1}{2}mv_1^2 = \dfrac{1}{2}mv^2$		
$v = \sqrt{2(11.5)(1.40 - 0.080) + (-0.524)^2} = 5.54\ \text{m/s}$		

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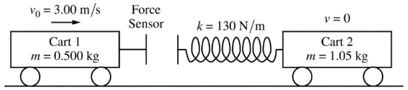
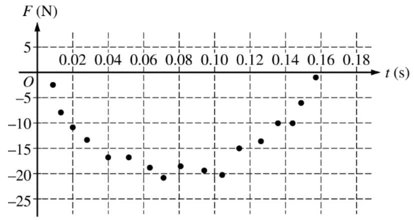
Question 2

15 points total **Distribution of points**



Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00$ m/s toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130$ N/m. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = (3200\ \text{N/s}^2)t^2 - (500\ \text{N/s})t$.



(a) 3 points

Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

For using the given force equation in the integral to determine the impulse delivered to the cart		1 point
$J = \int F \bullet dt = \int (3200t^2 - 500t)\, dt$		
For integrating the force using the correct limits or constant of integration		1 point
$J = \int_{t=0}^{t=0.16\text{ s}} (3200t^2 - 500t)\, dt = \left[\frac{3200}{3}t^3 - 250t^2\right]_{t=0}^{t=0.16}$		
$J = \left(\left(\frac{3200}{3}\right)(0.16)^3 - (250)(0.16)^2\right) - 0$		
For a correct answer		1 point
$J = -2.03\ \text{N}\bullet\text{s}$		

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Question 2 (continued)

Distribution of points

(b)
i. 1 point

Calculate the speed of cart 1 after the collision.

For correct substitution into the impulse-momentum equation of the answer from part (a) to determine the speed of cart 1		1 point
$J = m_1(v_{1f} - v_{1i}) \therefore v_{1f} = \frac{J}{m_1} + v_{1i} = \frac{(-2.03\ \text{N}\bullet\text{s})}{(0.500\ \text{kg})} + (3.00\ \text{m/s})$		
$v_{1f} = -1.06$ m/s		

ii. 1 point

In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

For correctly selecting “Left”		1 point

(c)
i. 2 points

Calculate the speed of cart 2 after the collision.

For using a correct equation to determine the speed of cart 2		1 point
$-J = -m_1(v_{1f} - v_{1i}) = m_2 v_{2f} \therefore v_{2f} = \frac{-J}{m_2}$		
For correct substitution of the answer from part (a)		1 point
$v_{2f} = \frac{(2.03\ \text{N}\bullet\text{s})}{(1.05\ \text{kg})}$		
$v_{2f} = 1.93$ m/s		

2018 SCORING GUIDELINES

Question 2 (continued)

Distribution of points

(c)

i. (continued)

Alternate solution		*Alternate Points*
For using the conservation of momentum to calculate the speed of cart 2 after the collision		*1 point*
$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \therefore v_{2f} = \dfrac{m_1 (v_{1i} - v_{1f})}{m_2}$		
For correct substitution of the answer from part (b)(i)		*1 point*
$v_{2f} = \dfrac{(0.500\text{ kg})((3.00\text{ m/s}) - (-1.06\text{ m/s}))}{(1.05\text{ kg})} = 1.93\text{ m/s}$		

ii. 2 points

Show that the collision between the two carts is elastic.

For indicating that the initial and final kinetic energies must be equal		1 point
$K_{1i} = K_{1f} + K_{2f}$		
For correct substitutions of answers from parts (b)(i) and (c)(i) into the calculations of the initial and final kinetic energies		1 point
$\left(\frac{1}{2}\right)(0.500\text{ kg})(3.00\text{ m/s})^2 = \left(\frac{1}{2}\right)(0.500\text{ kg})(-1.06\text{ m/s})^2 + \left(\frac{1}{2}\right)(1.05\text{ kg})(1.93\text{ m/s})^2$		
$2.25\text{ J} \approx 2.24\text{ J}$		

(d)

i. 2 points

Calculate the speed of the center of mass of the two-cart-spring system.

For using the equation for the conservation of momentum to calculate the speed for the center of mass of the system		1 point
$m_1 v_{1i} = (m_1 + m_2) v_{cm} \therefore v_{cm} = \dfrac{m_1 v_{1i}}{(m_1 + m_2)}$		
For correct substitution into the equation above		1 point
$v_{cm} = \dfrac{(0.500\text{ kg})(3.00\text{ m/s})}{(0.500\text{ kg} + 1.05\text{ kg})} = 0.97\text{ m/s}$		

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Question 2 (continued)

Distribution of points

(d) (continued)

ii. 3 points

Calculate the maximum elastic potential energy stored in the spring.

For using conservation of energy to calculate the maximum elastic potential energy stored in the spring		1 point
$K_i = K_f + U_{Sf}$		
For using the speed of the center of mass of the system for kinetic energy		1 point
$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2}(m_1 + m_2) v_{cm}{}^2 + U_{Sf}$		
$U_{Sf} = \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2}(m_1 + m_2) v_{cm}{}^2$		
For correct substitution into above equation		1 point
$U_S = \left(\frac{1}{2}\right)(0.500\text{ kg})(3.00\text{ m/s})^2 - \frac{1}{2}(0.500\text{ kg} + 1.05\text{ kg})(0.97\text{ m/s})^2$		
$U_S = 1.52\text{ J}$		
Alternate Solution:		*Alternate Points*
For correctly determining the magnitude of the maximum force exerted between the carts		1 point
Set $\dfrac{dF}{dt} = 0 = 6400t - 500 \therefore 6400t = 500 \therefore t = 0.078\text{ s}$		
$F_{MAX} = 3200(0.078)^2 - 500(0.078) = -19.5$		
$\lvert F_{MAX} \rvert = 19.5\text{ N}$		
Note: Can estimate from the graph, and accept the range of 20 N to 21 N.		
For calculating the maximum compression of the spring		1 point
$F_{MAX} = -kx_{MAX}$		
$x_{MAX} = -\dfrac{F_{MAX}}{k} = -\dfrac{(-19.5\text{ N})}{(130\text{ N/m})} = 0.15\text{ m}$		
Note: If estimating from the graph, accept the range from 0.15 m to 0.16 m.		
For substituting into an equation for the elastic potential energy at maximum spring compression		1 point
$U_S = \frac{1}{2} k x_{MAX}^2 = \left(\frac{1}{2}\right)(130\text{ N/m})(0.15\text{ m})^2$		
$U_S = 1.46\text{ J}$		
Note: If estimating from the graph, accept range from 1.46 J to 1.70 J.		

Units point: 1 point for correct units on all calculated answers

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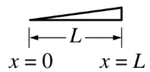
Question 3

15 points total **Distribution of points**

$$\lambda = \left(\frac{2M}{L^2}\right)x$$



A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \dfrac{2M}{L^2}x$, where x is the distance from one end of the rod.

(a) 3 points

Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

For relating x to r properly in an integral to calculate the moment of inertia		1 point
$I = \int r^2 dm = \int x^2 dm$		
For correctly using the linear mass density to substitute into the equation above		1 point
$m = \int \lambda dx = \int (2M/L^2) x dx \therefore dm = (2M/L^2) x dx$		
$I = \int (2M/L^2) x^3 dx$		
For integrating using the correct limits or constant of integration		1 point
$I = \int_{x=0}^{x=L} (2M/L^2) x^3 dx = \left[\dfrac{(2M/L^2)x^4}{4}\right]_{x=0}^{x=L} = \dfrac{2M}{L^2}\left(\dfrac{L^4}{4} - 0\right) = ML^2/2$		

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Question 3 (continued)

Distribution of points

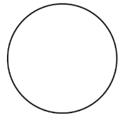


Figure 1

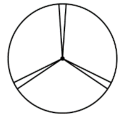


Figure 2

The thin hoop shown above in Figure 1 has a mass M, radius L, and a rotational inertia around its center of ML^2. Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

(b) 2 points

Derive an expression for the rotational inertia I_{tot} of the hoop-rods system about the center of the hoop. Express your answer in terms of M, L, and physical constants, as appropriate.

For setting the total rotational inertia for the hoop-rod system equal to the sum of the rotational inertias of the hoop and the three rods		1 point
$I = 3I_{rod} + I_{hoop}$		
$I = 3(ML^2/2) + ML^2$		
For a correct answer with work		1 point
$I = \dfrac{5}{2}ML^2$		

The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt.

(c) 3 points

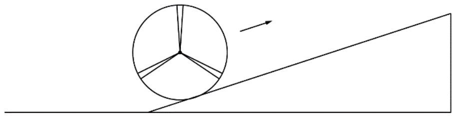
Derive an expression for the final angular speed ω of the hoop-rods system. Express your answer in terms of M, L, F, Δt, and physical constants, as appropriate.

For using an appropriate equation to determine the final angular speed of the hoop		1 point
$\tau\Delta t = I(\omega_2 - \omega_1)$		
$Fr\Delta t = I\omega$		
$\omega = \dfrac{Fr\Delta t}{I}$		
For relating the lever arm to the length of the rod		1 point
$\omega = \dfrac{FL\Delta t}{I}$		
For correct substitution from part (b)		1 point
$\omega = \dfrac{FL\Delta t}{\left(\frac{5}{2}\right)ML^2} = \dfrac{2F\Delta t}{5ML}$		

2018 SCORING GUIDELINES

Question 3 (continued)

Distribution of points

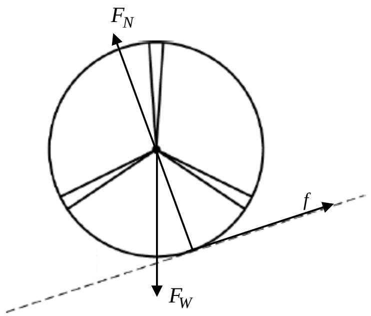


The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed ω found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

(d)

i. 3 points

On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.



For drawing the weight of the hoop-rod system in the correct direction		1 point
For drawing the normal force in the correct direction		1 point
For drawing the frictional force in the correct direction		1 point
Note: A maximum of two points can be earned if there are any extraneous vectors or any vector has an incorrect point of exertion.		

2018 SCORING GUIDELINES

Question 3 (continued)

Distribution of points

ii. 1 point

Justify your choice for the direction of each of the forces drawn in part (d)(i).

For a correct justification of the direction of the forces in part (d)(i)		1 point
Example: The normal force is always perpendicular to the surface, so it will be directed up and to the left. The gravitational force is always vertically downward. The friction will be opposite of the direction of rotation; therefore, it is directed up the incline.		

(e) 3 points

Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of M, L, I_{tot}, ω, and physical constants, as appropriate.

For including both linear and rotational kinetic energy in an equation for the conservation of energy to determine the final height of the hoop		1 point
$K_1 = U_{g2}$		
$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgH$		
$H = \dfrac{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2}{mg}$		
For correct substitution of $v = L\omega$		1 point
$H = \dfrac{\frac{1}{2}m(L\omega)^2 + \frac{1}{2}I_{tot}\omega^2}{mg}$		
$H = \dfrac{\frac{1}{2}\left(m_{tot}L^2 + I_{tot}\right)\omega^2}{m_{tot}g}$		
For correct substitution of inertias into energy equation		1 point
$H = \dfrac{\frac{1}{2}\left((3M + M)L^2 + I_{tot}\right)\omega^2}{(3M + M)g} = \dfrac{\left(4ML^2 + I_{tot}\right)\omega^2}{8Mg}$		

2017

AP CollegeBoard

AP Physics C: Mechanics
Scoring Guidelines

AP® PHYSICS
2017 SCORING GUIDELINES

General Notes About 2017 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth one point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections—Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based and AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8\ \text{m/s}^2$, but use of $10\ \text{m/s}^2$ is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

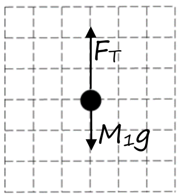
SCORING GUIDELINES

Question 1

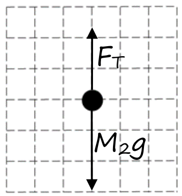
15 points total | **Distribution of points**

(a) 3 points

Block of Mass M_1



Block of Mass M_2



For correctly drawing and labeling the vectors for the weight of the block and the tension for block M_1 with the tension larger than the weight — 1 point

For correctly drawing and labeling the vectors for the weight of the block and the tension for block M_2 with the weight larger than the tension — 1 point

For correctly drawing tension on the two blocks as equal in magnitude — 1 point

Note: If any extraneous vectors are drawn, only a maximum of two points may be earned.

(b) 3 points

For correctly applying Newton's second law to block 1 — 1 point

$F_T - M_1g = M_1a$

For correctly applying Newton's second law to block 2 — 1 point

$M_2g - F_T = M_2a$

For combining the two equations above in such a way that will lead to the correct equation — 1 point

$M_2g - M_1g = (M_1 + M_2)a$

$$a = \frac{(M_2 - M_1)}{(M_1 + M_2)}g$$

(c) 1 point

For indicating variables that will create a straight line whose slope can be used to determine g — 1 point

Example: Vertical axis: a

Horizontal axis: $\dfrac{(M_2 - M_1)}{(M_1 + M_2)}$

Note: Full credit is earned if axes are reversed, or if the student uses another acceptable combination.

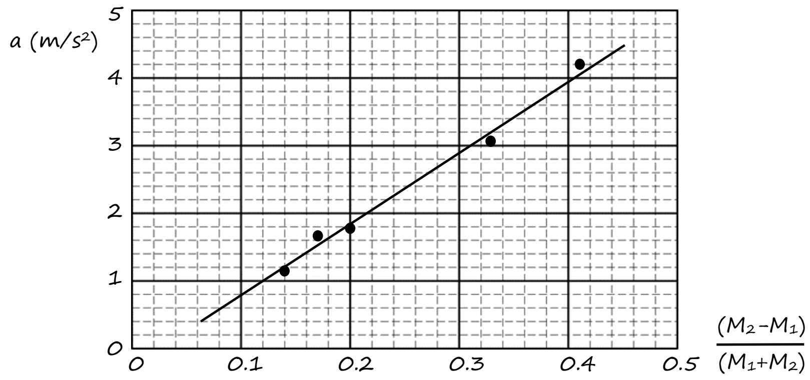
AP Physics C: **Mechanics: 2017**

2017 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(d) 3 points



For using a correct scale that uses more than half the grid and for correctly labeling the axes, including units as appropriate — 1 point

For correctly plotting the data — 1 point

For drawing a straight best-fit line consistent with the plotted data — 1 point

(e) 2 points

For correctly calculating the slope using the best-fit line and not the data points, unless the points fall on the best-fit line — 1 point

(Note: Credit may be given for the linear regression only if the student states linear regression is used.)

$$\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(3.0 - 1.0)\ \text{m/s}^2}{(0.31 - 0.12)} = 10.5\ \text{m/s}^2$$

For correctly relating the slope to g — 1 point

$g = \text{slope} = 10.5\ \text{m/s}^2$ (using linear regression yields $10.1\ \text{m/s}^2$)

2017 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(f) 2 points

Correct answer: "Lower"

For including a correct statement about the acceleration of the blocks — 1 point
For including a correct statement about the forces on the blocks — 1 point

Example: Because the block M_1 is on the table, the net force on the system increases, and therefore the acceleration increases. Because the acceleration of M_2 increases, the net force on M_2 must increase. Therefore, there must be a greater difference in the magnitude of the two forces on M_2. Because the weight of block M_2 stays the same, the retarding force — the tension in the string — must decrease.

(g) 1 point

For a correct justification — 1 point

Example: Block M_1 will experience friction with the table. The acceleration of the system will decrease and this will decrease the slope of the line; therefore, the value of g is determined by the experiment.

SCORING GUIDELINES

Question

15 points total

Distribution of points

(a)

i. 2 points

For correctly using conservation of energy for the block moving down the incline — 1 point

$U_g = K$

$mgh = \frac{1}{2}mv^2$

For a correct answer — 1 point

$v = \sqrt{2gh}$

ii. 1 point

Correct answer: "Greater than"

For a correct justification — 1 point

Example: The speed is proportional to the square root of the change in height. So if the height is reduced by a factor of 2, the speed is reduced by a factor of $\sqrt{2} \approx 1.41$. Therefore, the speed halfway down the ramp is more than half the speed at the bottom of the ramp.

Note: If the incorrect selection is made, the justification cannot earn credit.

(b) 1 point

For correctly using conservation of energy, consistent with part (a), for the block compressing the spring — 1 point

$U_g = U_S$

$mgh = \frac{1}{2}kx_{\text{max}}^2$

$x_{\text{max}} = \sqrt{\frac{2mgh}{k}}$

(c) 2 points

For indicating a simple harmonic motion approach — 1 point
For a correct answer — 1 point

$t = \frac{1}{4}T = \left(\frac{1}{4}\right)\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{\pi}{2}\sqrt{\frac{m}{k}}$

2017 SCORING GUIDELINES

Question continued)

Distribution of points

(d)

i. 3 points

For correctly applying Newton's second law for the horizontally sliding block	1 point
For correctly indicating that the direction of F_{net} is opposite the direction of motion	1 point

$F_{net} = ma$

$-\beta v^2 = ma$

For expressing the equation as a differential equation	1 point

$-\beta v^2 = m\dfrac{dv}{dt}$

ii. 3 points

For correctly separating variables	1 point

$-\dfrac{\beta}{m}dt = \dfrac{1}{v^2}dv$

For correctly integrating the equation above	1 point

$\int -\dfrac{\beta}{m}dt = \int \dfrac{1}{v^2}dv$

$-\dfrac{\beta}{m}[t] = \left[-\dfrac{1}{v}\right]$

For using the correct limits or constant of integration	1 point

$-\dfrac{\beta}{m}[t]_0^t = \left[-\dfrac{1}{v}\right]_{v_0}^{v(t)}$

$-\dfrac{\beta t}{m} = -\dfrac{1}{v(t)} - \left(-\dfrac{1}{v_0}\right)$

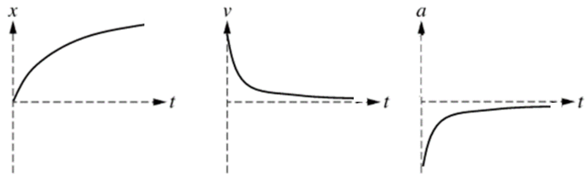
$\dfrac{1}{v(t)} = \dfrac{1}{v_0} + \dfrac{\beta t}{m}$

2017 SCORING GUIDELINES

Question continued)

Distribution of points

(e) 3 points



For a displacement curve that is concave down and approaching a horizontal asymptote	1 point
For a velocity curve that is concave up and has the horizontal axis as an asymptote	1 point
For an acceleration curve that is concave down and has the horizontal axis as an asymptote	1 point

Note: If an incorrect nonlinear velocity graph is generated, 1 point is earned if the position and acceleration graphs are consistent with the velocity graph.

Note: Full credit is earned if all three graphs are flipped vertically.

2017 SCORING GUIDELINES

Question 3

15 points total | **Distribution of points**

(a) 2 points

For correctly applying conservation of energy to the cylinder rolling down the incline — 1 point

$U_{g\_top} = K_{table}$

$K_{table} = mgh = (0.50 \text{ kg})(9.8 \text{ m/s}^2)(1.0 \text{ m})(\sin 30)$

For a correct answer with units — 1 point

$K_{table} = 2.45 \text{ J}$ (or 2.5 J using $g = 10 \text{ m/s}^2$)

(b) 3 points

For correctly setting the kinetic energy of the cylinder equal to the sum of both the linear and rotational kinetic energy — 1 point

$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

For correctly substituting into the above equation for the linear velocity and moment of inertia of the cylinder — 1 point

$K = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{2}M(R\omega)^2 + \frac{1}{4}M(R\omega)^2 = \frac{3}{4}M(R\omega)^2$

$\omega = \sqrt{\frac{4K}{3MR^2}} = \sqrt{\frac{(4)(2.45 \text{ J})}{(3)(0.50 \text{ kg})(0.10 \text{ m})^2}}$

For correct substitution into the equation above — 1 point

$\omega = 25.6 \text{ rad/s}$ (or 26.0 rad/s using $g = 10 \text{ m/s}^2$)

(c) 2 points

For using a correct expression for the ratio of the rotational kinetic energy to the total kinetic energy of the cylinder — 1 point

$$\frac{K_{rot}}{K_{tot}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh} = \frac{\frac{1}{4}M(R\omega)^2}{\frac{3}{4}M(R\omega)^2 + Mgh_{\text{table}}} = \frac{(R\omega)^2}{3(R\omega)^2 + 4gh_{\text{table}}}$$

For substituting into the above equation — 1 point

$$\frac{K_{rot}}{K_{tot}} = \frac{[(0.10 \text{ m})(25.6 \text{ rad/s})]^2}{(3)(0.10 \text{ m})^2(25.6 \text{ rad/s})^2 + (4)(9.81 \text{ m/s}^2)(0.75 \text{ m})}$$

$\frac{K_{rot}}{K_{tot}} = 0.133$ (or 0.135 using $g = 10 \text{ m/s}^2$)

2017 SCORING GUIDELINE

Question 3 (continued)

Distribution of points

(c) (continued)

Alternate Solution — *Alternate Points*

For using a correct expression for the ratio of the rotational kinetic energy to the total potential energy of the cylinder — *1 point*

$$\frac{K_{rot}}{K_{tot}} = \frac{\frac{1}{2}I\omega^2}{U_{total}} = \frac{\frac{1}{4}M(R\omega)^2}{Mg(h+y)}$$

For substituting into the above equation — *1 point*

$$\frac{K_{rot}}{U_{total}} = \frac{\frac{1}{4}(R\omega)^2}{g(h+y)} = \frac{\frac{1}{4}[(0.10 \text{ m})(25.6 \text{ rad/s})]^2}{(9.81 \text{ m/s}^2)\left((1.0 \text{ m})\sin(30^\circ) + (0.75 \text{ m})\right)}$$

$\frac{K_{rot}}{U_{total}} = 0.13$ *(or* 0.135 *using* $g = 10 \text{ m/s}^2$ *)*

(d) 2 points

For correctly using motion in the vertical direction to calculate the time for the cylinder to reach the floor — 1 point

$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

$y - y_0 = 0 - \frac{1}{2}gt^2$

Determine the time for the cylinder to reach the floor

$y - y_0 = -\frac{1}{2}gt^2$

$0 - y = -\frac{1}{2}gt^2$

$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{(2)(0.75 \text{ m})}{(9.81 \text{ m/s}^2)}} = 0.39 \text{ s}$

For correctly using the equation for constant speed in the horizontal direction — 1 point

$x - x_0 = v_{0x}t$

$x - x_0 = R\omega t$

$x - x_0 = R\omega\sqrt{\frac{2y}{g}}$

$D = R\omega t = (0.10 \text{ m})(25.6 \text{ rad/s})(0.39 \text{ s})$

$D = 1.0 \text{ m}$

2017 SCORING GUIDELINE

Question 3 (continued)

	Distribution of points
(e)	
i. 2 points	
For selecting "Equal to" and attempting a relevant justification	1 point
For a correct justification	1 point
Example: Because the sphere falls the same height as the cylinder and because they have the same mass, the sphere-Earth system has the same initial potential energy and, therefore, the same total kinetic energy when it reaches the floor.	
ii. 2 points	
For selecting "Less than" and attempting a relevant justification	1 point
For a correct justification	1 point
Example: Because the rotational inertia of the sphere is less than the rotational inertia of the cylinder, the sphere will rotate faster and, because $v = r\omega$, will move with a greater linear speed. Because the mass is the same and the linear speed is greater, the sphere will have a greater linear kinetic energy. Because the total kinetic energies of the sphere and cylinder are the same, the sphere must have less rotational kinetic energy.	
iii. 2 points	
For selecting "Greater than" and attempting a relevant justification	1 point
For a correct justification	1 point
Example: Because the sphere has a greater linear speed as it leaves the table, it will travel a greater horizontal distance before it reaches the floor.	