

Question 2: Version J

2. In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

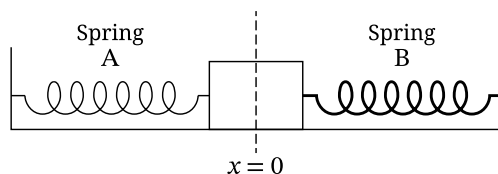


Figure 1

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

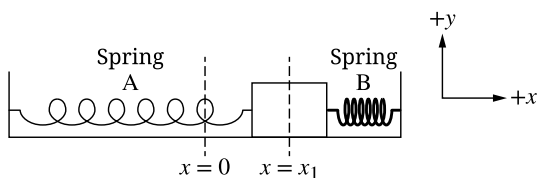


Figure 2

- A. An energy bar chart can be used to represent the elastic potential energy U_A of Spring A, the elastic potential energy U_B of Spring B, and the kinetic energy K_{block} of the block. On the energy bar chart in Figure 3, **draw** shaded bars to represent the energy of the system for when the block is at $x = x_1$.

- The height of the shaded bars should be proportional to the relative values of U_A , U_B , and K_{block} .
- Any energy that is equal to zero should be represented by a distinct line on the zero-energy line.

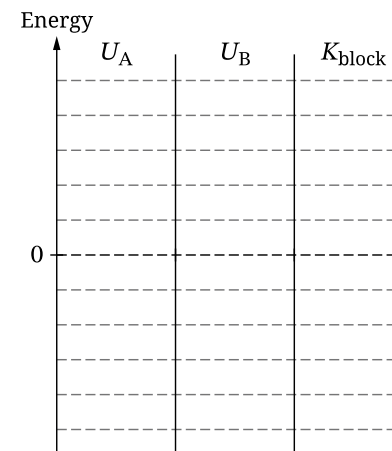
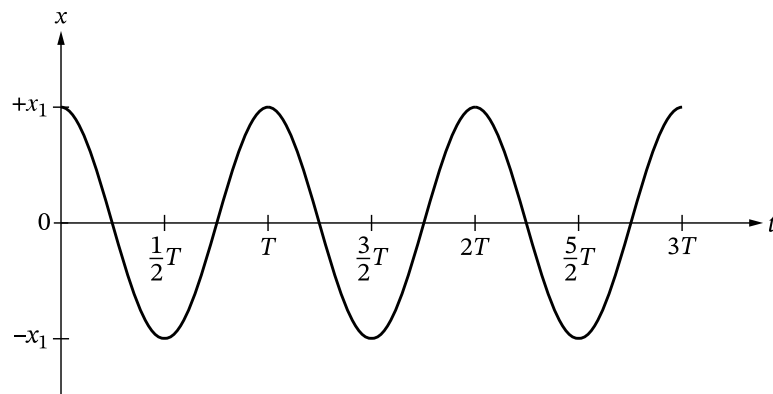


Figure 3

- B. The block is released from rest at $x = x_1$ and begins to oscillate. **Derive** an expression for the speed v of the block as the block passes through $x = \frac{1}{2}x_1$. Express your answer in terms of m , k , x_1 , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

- C. In Scenario 1, the block oscillates with period T . The position x of the block in Scenario 1 as a function of time t is shown in Figure 4.

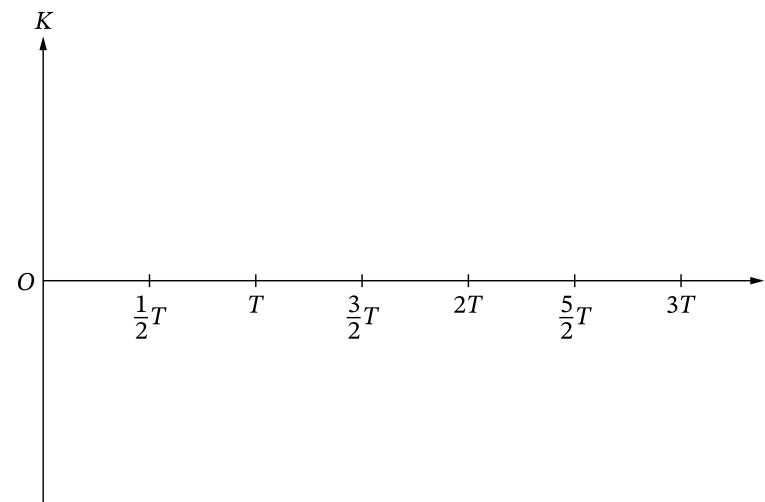


Scenario 1

Figure 4

In Scenario 2, the block-springs system is placed on a new surface. There is friction between the block and the new surface. The block is again moved to the same position $x = x_1$ and released from rest. The block completes multiple oscillations with the same period as in Scenario 1 before coming to rest.

On the axes shown in Figure 5, **sketch** a graph of the kinetic energy K of the block as a function of t for Scenario 2.



Scenario 2

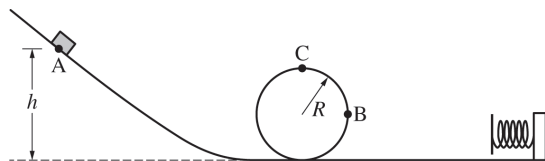
Figure 5

- D. In Scenario 3, the block is replaced with a new block of larger mass. The coefficient of kinetic friction between the new block and the surface in Scenario 3 is the same as the coefficient of kinetic friction between the original block and the surface in Scenario 2.

The new block is moved to position $x = x_1$ and released from rest. The kinetic energy of the new block is plotted as a function of time.

Describe how one feature of the graph of K as a function of t in Scenario 3 would differ from the graph you drew in Figure 5 for Scenario 2.

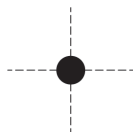
Briefly **justify** your answer.



Note: Figure not drawn to scale.

2. A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \frac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m , h , R , and physical constants, as appropriate.

- (a) On the dot below, which represents the block, draw an arrow that represents the direction of the acceleration of the block at point B in the figure above. The arrow must start on and point away from the dot.



Justify your answer.

- (b)
- Derive an expression for the speed v of the block at point B.
 - Derive an expression for the magnitude of the net force F on the block at point B.

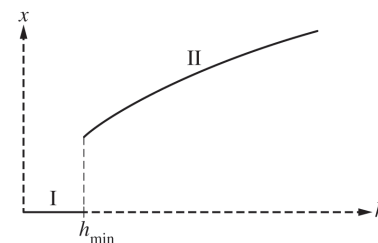
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- (c) In terms of R , derive an expression for the minimum height $h_{\min}$ necessary for the block to maintain contact with the track through point C.

- (d) It is determined that $h = 0.30$ m and $R = 0.10$ m. If the block is released from a height greater than that found in part (c), what would be the maximum compression x_{MAX} of the spring?

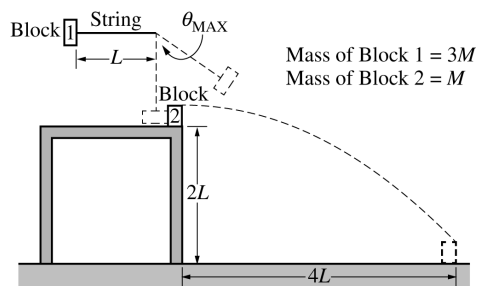
- (e) A graph of the maximum compression of the spring as a function of height is shown below. The height $h_{\min}$ is the height calculated in part (c).



- Explain why section I appears as a horizontal line segment on the horizontal axis.
- Explain the reason for the shape of section II on the graph.

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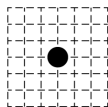
Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.



Note: Figure not drawn to scale.

2. A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M , which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M , L , and physical constants, as appropriate.

- (a) Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.
 (b) On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



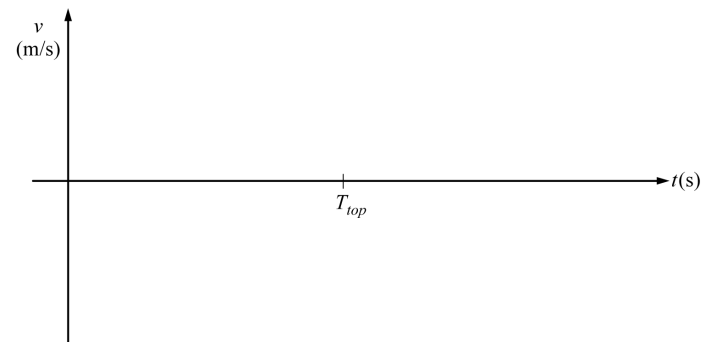
- (c) Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

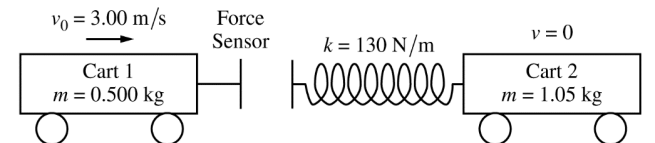
For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

- (d) Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.
 (e) Calculate the speed of block 2 as it leaves the table.
 (f) Calculate the speed of block 1 just after it collides with block 2.
 (g) Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.

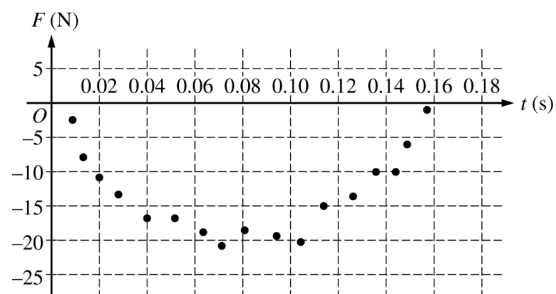
2. A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0$ m/s², $L = 0.25$ m/s⁴, and t is the time in seconds. At $t = 6.0$ s, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

- (a) Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0$ s.
 (b) Calculate the speed of the rocket at $t = 6.0$ s.
 (c)
 i. Calculate the kinetic energy of the rocket at $t = 6.0$ s.
 ii. Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.
 (d) Calculate the maximum height reached by the rocket relative to its launching point.
 (e) On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.





2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00 \text{ m/s}$ toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg , the mass of cart 2 is 1.05 kg , and the spring has negligible mass. The spring has a spring constant of $k = 130 \text{ N/m}$. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = (3200 \text{ N/s}^2)t^2 - (500 \text{ N/s})t$.

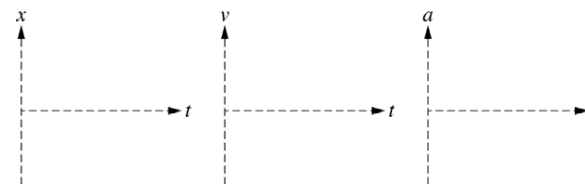


- (a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.
- (b)
- Calculate the speed of cart 1 after the collision.
 - In which direction does cart 1 move after the collision?
 ____ Left ____ Right
 ____ The direction is undefined, because the speed of cart 1 is zero after the collision.
- (c)
- Calculate the speed of cart 2 after the collision.
 - Show that the collision between the two carts is elastic.
- (d)
- Calculate the speed of the center of mass of the two-cart-spring system.
 - Calculate the maximum elastic potential energy stored in the spring.



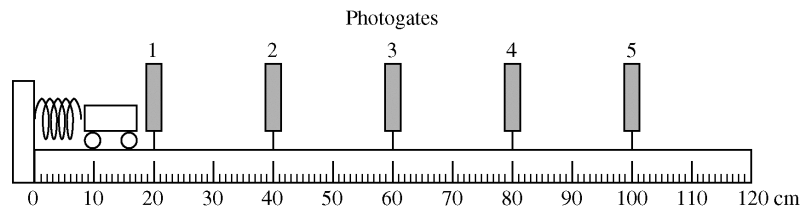
Note: Figure not drawn to scale.

2. A block of mass m starts at rest at the top of an inclined plane of height h , as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k . There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m , h , k , and physical constants, as appropriate.
- (a)
- Derive an expression for the speed of the block just before it collides with the spring.
 - Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?
 ____ Greater than ____ Less than ____ Equal to
 Justify your answer.
- (b) Derive an expression for the maximum compression of the spring.
- (c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.
- The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0 . Now suppose the block experiences a resistive force as it slides on the horizontal surface. The magnitude of the resistive force F is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of kg/m .
- (d)
- Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m , h , k , β , v , and physical constants, as appropriate.
 - Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time t can be written in the form $\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$, where v_0 is the speed at $t = 0$.
- (e) Sketch graphs of position x as a function of time t , velocity v as a function of time t , and acceleration a as a function of time t for the block as it is moving on the horizontal surface before it reaches the spring.



PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Mech. 1.

In an experiment, a student wishes to use a spring to accelerate a cart along a horizontal, level track. The spring is attached to the left end of the track, as shown in the figure above, and produces a nonlinear restoring force of magnitude $F_s = As^2 + Bs$, where s is the distance the spring is compressed, in meters. A measuring tape, marked in centimeters, is attached to the side of the track. The student places five photogates on the track at the locations shown.

- (a) Derive an expression for the potential energy U as a function of the compression s . Express your answer in terms of A , B , s , and fundamental constants, as appropriate.

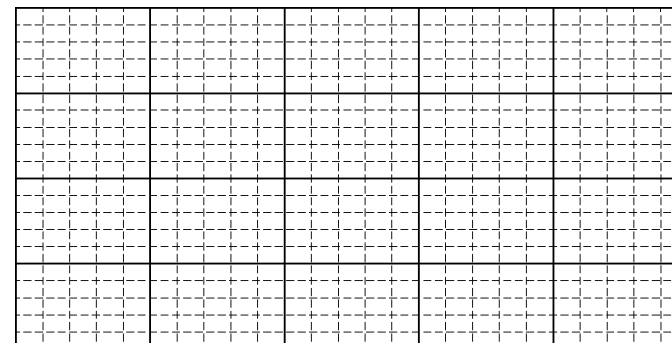
In a preliminary experiment, the student pushes the cart of mass 0.30 kg into the spring, compressing the spring 0.040 m. For this spring, $A = 200 \text{ N/m}^2$ and $B = 150 \text{ N/m}$. The cart is released from rest. Assume friction and air resistance are negligible only during the short time interval when the spring is accelerating the cart.

- (b) Calculate the following:
- The speed of the cart immediately after it loses contact with the spring
 - The impulse given to the cart by the spring

In a second experiment, the student collects data using the photogates. Each photogate measures the speed of the cart as it passes through the gate. The student calculates a spring compression that should give the cart a speed of 0.320 m/s after the cart loses contact with the spring. The student runs the experiment by pushing the cart into the spring, compressing the spring the calculated distance, and releasing the cart. The speeds are measured with a precision of $\pm 0.002 \text{ m/s}$. The positions are measured with a precision of $\pm 0.005 \text{ m}$.

Photogate	1	2	3	4	5
Cart speed (m/s)	0.412	0.407	0.399	0.374	0.338
Photogate position (m)	0.20	0.40	0.60	0.80	1.00

- (c) On the axes below, plot the data points for the speed v of the cart as a function of position x . Clearly scale and label all axes, as appropriate.



- (d)
- Compare the speed of the cart measured by photogate 1 to the predicted value of the speed of the cart just after it loses contact with the spring. List a physical source of error that could account for the difference.
 - From the measured speed values of the cart as it rolls down the track, give a physical explanation for any trend you observe.