Question 1: Free-Response Question — 15 points

(a)(i) For a multi-step derivation with an application of the conservation of mechanical energy that indicates that all of the energy of the system is initially U_s — **1 point**

Example Response

$$E_{\text{initial}} = E_{\text{final}}$$
$$\frac{1}{2}kx_c^{\ 2} = \frac{1}{2}mv^2$$

For a correct solution for v — **1 point**

Example Response

$$v = x_c\sqrt{\frac{k}{m}}$$

Example Solution

$$E_{\text{initial}} = E_{\text{final}}$$
$$U_s = K$$
$$\frac{1}{2}kx_c^{\ 2} = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{kx_c^{\ 2}}{m}}$$
$$v = x_c\sqrt{\frac{k}{m}}$$

(a)(ii) For a derivation to solve for the speed at x_2 that includes **one** of the following: **1 point**

- An appropriate application of the conservation of energy
- An appropriate kinematics equation

Example Responses

$K_{\text{initial}} - \Delta E_{\text{friction}} = K_{\text{final}}$ **OR** $v^2 = v_0^2 + 2a\Delta x$

For **one** of the following that is consistent with the previous point in the response for part (a)(ii): **1 point**

- A correct expression for the energy dissipated by friction
- A correct expression for the acceleration of the block in the region with nonnegligible friction

Example Responses

$\Delta E_{\text{friction}} = \mu mgD$ **OR** $a = -\mu g$

For attempting to derive an expression for $v_{\text{A,B}}$ by using the conservation of momentum **1 point**

Example Response

$m_{\text{initial}} v_{\text{initial}} = m_{\text{A,B}} v_{\text{A,B}}$

For substituting the expression for the speed at x_2 that is consistent with the first point of the response in part (a)(ii) and substituting the correct masses into an expression for conservation of momentum **1 point**

Example Response

$$v_{\text{A,B}} = \frac{m\sqrt{\dfrac{k{x_c}^2}{m} - 2\mu gD}}{4m}$$

Example Solutions

$$E_{x_1} = E_{\text{before collision}}$$
$$K_{x_1} - \Delta E_{\text{friction}} = K_{\text{before collision}}$$
$$\frac{1}{2}m\left(\sqrt{\frac{kx_c^2}{m}}\right)^2 - \mu mgD = \frac{1}{2}mv_2^2$$
$$v_2 = \sqrt{\frac{kx_c^2}{m} - 2\mu gD}$$

$$\Sigma p_{\text{before collision}} = \Sigma p_{\text{after collision}}$$
$$m_A v_2 = m_{A,B} v_{A,B}$$
$$m_A v_2 = (m + 3m)v_{A,B}$$
$$v_{A,B} = \frac{m\sqrt{\frac{kx_c^2}{m} - 2\mu gD}}{4m}$$
$$v_{A,B} = \frac{1}{4}\sqrt{\frac{kx_c^2}{m} - 2\mu gD}$$

OR

$$v_2^2 = v^2 + 2a\Delta x$$
$$v_2^2 = \left(x_c\sqrt{\frac{k}{m}}\right)^2 + 2aD$$
$$v_2 = \sqrt{\frac{kx_c^2}{m} + 2aD}$$

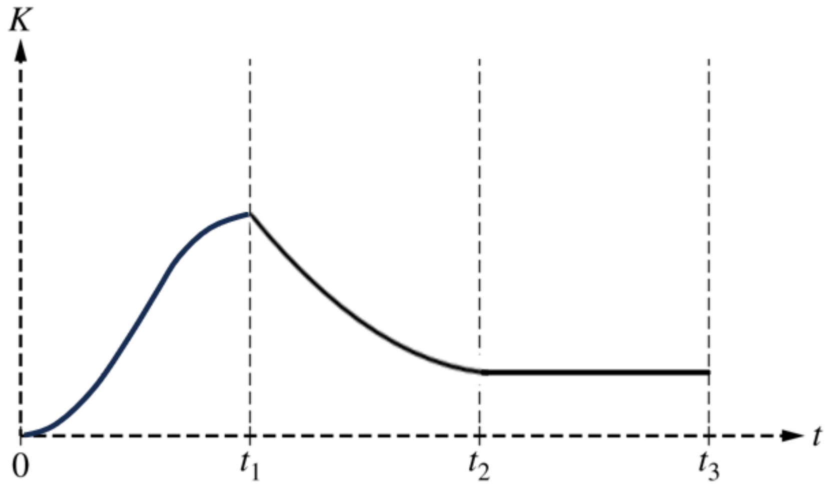
$$\Sigma F_x = -F_f = ma$$
$$-\mu mg = ma$$
$$a = -\mu g$$
$$v_2 = \sqrt{\frac{kx_c^2}{m} - 2\mu gD}$$

$$\Sigma p_{\text{before collision}} = \Sigma p_{\text{after collision}}$$
$$m_A v_2 = m_{A,B} v_{A,B}$$
$$m_A v_2 = (m + 3m)v_{A,B}$$
$$v_{A,B} = \frac{m\sqrt{\frac{kx_c^2}{m} - 2\mu gD}}{4m}$$
$$v_{A,B} = \frac{1}{4}\sqrt{\frac{kx_c^2}{m} - 2\mu gD}$$

Total for part (a) 6 points

(b)(i)	For a nonlinear sketch that begins at zero and increases for the entire time interval $0 \le t \le t_1$	**1 point**
	For a sketch that decreases for the entire time interval $t_1 \le t \le t_2$ but does not go to zero	**1 point**
	For a sketch that is concave up for the time interval $t_1 \le t \le t_2$	**1 point**
	For a continuous function for the time interval $t_1 \le t \le t_3$ that has a horizontal line that is greater than zero for the time interval $t_2 \le t \le t_3$	**1 point**

Example Response



(b)(ii)	For a statement about the change in kinetic energy that is consistent with the graph drawn in the response for part (b)(i)	**1 point**
	For a correct explanation for why the kinetic energy is increasing, such as **one** of the following: • An increasing graph means positive work is being done on the block. • An external force is exerted on Block A, causing the velocity of the block to increase and the kinetic energy of the block to increase. • Mechanical energy is conserved and/or there is no work done for the block-spring system, and the potential energy decreases.	**1 point**
	For a correct explanation for why the graph is nonlinear, such as **one** of the following: • The rate at which the slope of the graph changes is related to the rate at which work is being done on the block. • The external force exerted on Block A is changing, which causes a nonuniform change in the velocity of Block A, which results in a nonuniform change in kinetic energy.	**1 point**

Example Response

From $0 < t < t_1$, the kinetic energy of Block A increases. The force exerted on the block by the compressed spring transfers the elastic potential energy in the block-spring system to the kinetic energy of the block. Because the force exerted by the spring is not applied at a constant rate, the kinetic energy of the block does not increase at a constant rate.

Total for part (b) **7 points**

(c)	For selecting $f_{2\ell} < f_{\ell}$ with an attempt at a relevant justification	**1 point**
	For correctly applying an equation that relates the length of a pendulum to the period or frequency of the pendulum	**1 point**

Example Response

The period of a pendulum is calculated by using $T = 2\pi\sqrt{\frac{l}{g}}$ *. Therefore, as the length is increased, the period will also increase. Because frequency and period are inversely related, an increase in period will result in a decrease in frequency.*

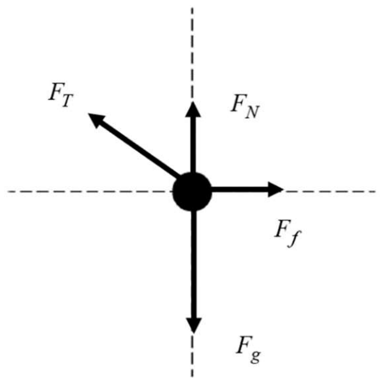
Total for part (c) 2 points

Total for question 1 15 points

Question 1: Free-Response Question — 15 points

(a)	For correctly drawing and labeling the force of gravity and the normal force on the block of mass m	**1 point**
	For correctly drawing and labeling the force of friction on the block of mass m	**1 point**
	For correctly drawing and labeling the force of tension on the block of mass m	**1 point**

Example Response



Scoring Notes:

- Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth" $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force. F_{string}, F_s, F_T, F_{Tension}, T, "string force," "tension force," or similar labels may be used for the tension force exerted by the string.
- A response with extraneous forces or vectors can earn a maximum of two points.

Total for part (a) — 3 points

(b)	For a trigonometric expression relating the angle to the horizontal distance of the block from the left corner of the table, x.	**1 point**
	For any correct trigonometric expression for θ in terms of the given quantities	**1 point**

Example Responses

First Point	Second Point
$\sin\theta = \dfrac{H}{\sqrt{H^2 + x^2}}$	$\theta = \sin^{-1}\left(\dfrac{H}{\sqrt{H^2 + x^2}}\right)$
$\cos\theta = \dfrac{x}{\sqrt{H^2 + x^2}}$	$\theta = \cos^{-1}\left(\dfrac{x}{\sqrt{H^2 + x^2}}\right)$
$\tan\theta = \dfrac{H}{x}$	$\theta = \tan^{-1}\left(\dfrac{H}{x}\right)$

Total for part (b) — 2 points

(c)(i) For using Newton's second law to sum the forces in the vertical direction and write an equation that is consistent with part (a) — **1 point**

$$\Sigma F_y = ma$$
$$F_{net,y} = F_N + F_{T,y} - F_g = 0$$
$$F_N = F_g - F_{T,y}$$

For correctly substituting the vertical component of the tension in terms of the given variables consistent with part (b) — **1 point**

$$F_N = mg - F_T \sin\theta$$
$$F_N = mg - F_T\left(\frac{H}{\sqrt{H^2 + x^2}}\right)$$

(c)(ii) For using Newton's second law to sum the forces in the horizontal direction and write an equation that is consistent with part (a) — **1 point**

$$F_{net,x} = F_{T,x} - F_f$$

For correctly substituting the horizontal component of the force of tension in terms of the given variables consistent with part (b) — **1 point**

$$F_{net,x} = F_T \cos\theta - F_f$$
$$F_{net,x} = F_T\left(\frac{(x)}{\sqrt{H^2 + x^2}}\right) - F_f$$

For correctly substituting the expression for the normal force from part (c)(i) into the expression for the force of friction — **1 point**

$$F_{net,x} = F_T\left(\frac{(x)}{\sqrt{H^2 + x^2}}\right) - \mu_k F_N$$
$$F_{net,x} = F_T\left(\frac{(x)}{\sqrt{H^2 + x^2}}\right) - \mu_k\left(mg - F_T\left(\frac{H}{\sqrt{H^2 + x^2}}\right)\right)$$

Total for part (c) — 5 points

(d) For any indication that the work done on the block by the string is due only to the horizontal component of the tension in the string — **1 point**

$$W = \int F_{T,x}dx$$

For using the horizontal component of the force of tension consistent with part (c) — **1 point**

For indicating that work is the integral of the force with respect to x, including limits or a constant of integration — **1 point**

$$W = \int_{x=L}^{x=0} -F_T\left(\frac{x}{\sqrt{H^2 + x^2}}\right)dx$$

Total for part (d) — 3 points

(e)	For selecting "More work …" with an attempt at a relevant justification	**1 point**
	For a correct justification relating the smaller angle to a larger component of the force of tension, thus resulting in greater work.	**1 point**

Example Response

F_T stays the same for both halves, the displacement is the same in both halves, but from $x = L$ to $x = L/2$ the angle is smaller, resulting in a larger component of the tension force that aligns with the displacement.

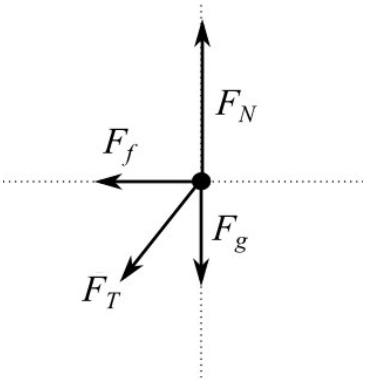
Total for part (e) 2 points

Total for question 1 15 points

Question 1: Free-Response Question — 15 points

(a)	For correctly drawing and labeling the force of gravity and the normal force on the sled	**1 point**
	For correctly drawing and labeling the force of friction on the on the sled	**1 point**
	For correctly drawing and labeling the force of tension on the on the sled	**1 point**

Example Response



Scoring Notes:

- Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force. F_{string}, F_s, F_T, F_{Tension}, T, "string force," "tension force," or similar labels may be used for the tension force exerted by the string.
- A response with extraneous forces or vectors can earn a maximum of two points.

Total for part (a) 3 points

(b)	For any correct trigonometric expression for θ in terms of the given quantities	**1 point**

$$\sin\theta = \frac{x}{\sqrt{y^2 + x^2}} \qquad \therefore \theta = \sin^{-1}\left(\frac{x}{\sqrt{y^2 + x^2}}\right)$$

OR

$$\cos\theta = \frac{y}{\sqrt{y^2 + x^2}} \qquad \therefore \theta = \cos^{-1}\frac{y}{\sqrt{y^2 + x^2}}$$

OR

$$\tan\theta = \frac{x}{y} \qquad \therefore \theta = \tan^{-1}\frac{x}{y}$$

Total for part (b) 1 point

(c)(i)	For beginning the derivation with Newton's second law to write an equation that is consistent with part (a). $\Sigma F = ma$ $F_N - F_{T,y} - F_g = 0$ $F_N = F_{T,y} + F_g$ **Scoring Note:** Derivation must start with a statement of Newton's second law to earn this point.	**1 point**
	For correctly substituting the vertical component of the tension in terms of the given variables consistent with part (b) $F_N = mg + F_T \cos\theta$ $F_N = mg + F_T\left(\frac{y}{\sqrt{y^2 + x^2}}\right)$	**1 point**
(c)(ii)	For summing the forces in the horizontal direction, consistent with parts (a) and (b) $F_{net} = -F_{T,x} - F_f$	**1 point**
	For correctly substituting the horizontal component of the force of tension in terms of the given variables $F_{net} = -F_T \sin\theta - F_f$ $F_{net} = -F_T\left(\frac{x}{\sqrt{y^2 + x^2}}\right) - F_f$	**1 point**
	For correctly substituting the expression for the normal force from part (c)(i) into the expression for the force of friction $F_{net} = -F_T\left(\frac{x}{\sqrt{y^2 + x^2}}\right) - \mu_k F_N$ $F_{net} = -F_T\left(\frac{x}{\sqrt{y^2 + x^2}}\right) - \mu_k\left(mg + F_T\left(\frac{y}{\sqrt{y^2 + x^2}}\right)\right)$	**1 point**
	Total for part (c)	**5 points**

(d)	For using the integral definition of work to derive an expression for the work done by the force of tension $W = \int F \cdot dx$	**1 point**
	For any indication that the work done on the sled by the string is due only to the horizontal component of the tension in the string $W = \int F_{T,x} dx$	**1 point**
	For using the horizontal component of the force of tension consistent with part (b) or (c) $W = \int_{x=0}^{x=L} -F_T \left(\frac{x}{\sqrt{y^2 + x^2}} \right) dx$	**1 point**
	For correct limits of integration, x= 0 to x= L, and indicating that the work done is negative let $u = y^2 + x^2$, then $du = 2x\, dx$ $W = -F_T \int_{x=0}^{x=L} \frac{1}{2} \left(\frac{1}{\sqrt{u}} \right) du$ $W = -F_T \left(\sqrt{y^2 + x^2} \right) \Big\|_{x=0}^{x=L}$ $W = -F_T \left(\sqrt{y^2 + L^2} - \sqrt{y^2} \right)$	**1 point**
	Total for part (d)	**4 points**
(e)	For selecting "$E_1 > E_2$" with an attempt at a relevant justification	**1 point**
	For a justification that correctly relates the force of friction to the normal force, and the normal force to the position or the angle	**1 point**
	Example Response *The vertical component of the string force is the largest when the string is more vertical* *($0 < x < L$). A larger vertical component of the string tension leads to a larger normal force and, hence, a larger friction force.*	
	Total for part (e)	**2 points**
	Total for question 1	**15 points**