Question 2: Free-Response Question — 15 points

(a) For a multi-step derivation that includes Newton's second law of motion — **1 point**

For indicating that the net force exerted on the cylinder includes only the gravitational force and a drag force — **1 point**

Example Response

$F_{\text{net}} = F_g - F_{\text{drag}}$

For a correct differential equation that is in terms of the given variables — **1 point**

Scoring Note: Variables do not have to be separated for this point to be earned.

Example Response

$m\dfrac{dv}{dt} = mg - bv^2$

Example Solution

$\Sigma F = ma$

$F_g - F_{\text{drag}} = ma_y$

$mg - bv^2 = ma_y$

$m\dfrac{dv}{dt} = mg - bv^2$

Total for part (a) — 3 points

(b)(i) For a vertical line labeled t_1 at the approximate location at which the line becomes horizontal — **1 point**

Example Response

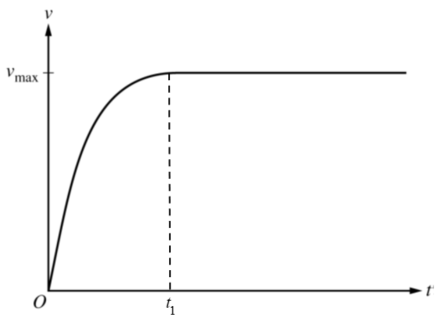


Figure 2

(b)(ii) For relating t_1 to the time at which the velocity versus time graph is constant or the slope of the line is zero — **1 point**

For indicating that a constant velocity indicates that the net force is zero — **1 point**

Example Response

Because the sketched line is horizontal after t_1, the velocity is constant. If the velocity is constant, then the acceleration is zero. Therefore, the net force is zero, which means that the gravitational and drag forces are equal in magnitude.

Total for part (b) — 3 points

(c) For selecting "Equal to" with an attempt at a relevant justification — **1 point**

For a correct justification — **1 point**

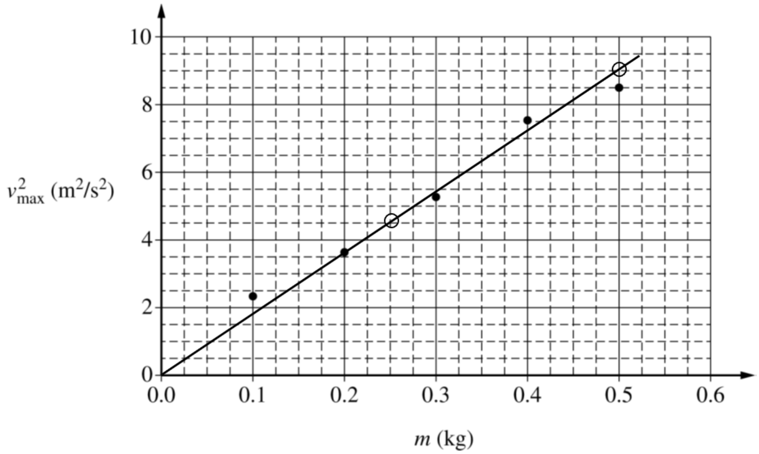
Example Response

At its peak, the cylinder will have a speed of 0 m/s. Therefore, the cylinder would reach the same v_{max} as if the student had dropped the cylinder from rest at that height. Because the cylinder reached v_{max} from the initial drop, the two max speeds are equal.

Total for part (c) — 2 points

(d)(i) For drawing an appropriate line of best fit that approximates the data — **1 point**

Example Response



(d)(ii)	For calculating a value for the slope of the line using two points on the best-fit line **Scoring Note:** Using data points that fall on the best-fit line is acceptable.	**1 point**

Example Response

$$\text{slope} = \frac{9\ \text{m}^2/\text{s}^2 - 4.5\ \text{m}^2/\text{s}^2}{0.5\,\text{kg} - 0.25\,\text{kg}}$$

For using the correct relationship between the slope of the best-fit line and the value of b — **1 point**

Example Response

$$\text{slope} = \frac{g}{b}$$

For a calculated value of b that is $0.45\ \text{kg/m} \le b \le 0.75\ \text{kg/m}$ — **1 point**

Example Response

$b = 0.544\ \text{kg/m}$

Example Solution

$$mg - bv^2 = 0$$
$$bv^2 = mg$$
$$\frac{v^2}{m} = \frac{g}{b}$$
$$\text{slope} = \frac{g}{b}$$
$$b = \frac{g}{\text{slope}}$$
$$b = \frac{9.8\ \text{m/s}^2}{\dfrac{9\ \text{m}^2/\text{s}^2 - 4.5\ \text{m}^2/\text{s}^2}{0.5\ \text{kg} - 0.25\ \text{kg}}}$$
$$b = 0.544\ \text{kg/m}$$

Total for part (d) 4 points

(e)(i)	For indicating that the length of the cylinder should be graphed	**1 point**
	For indicating that the maximum velocity of the cylinder should be graphed	**1 point**
(e)(ii)	For describing how the quantities graphed are related to the conclusions of the experiment	**1 point**

Example Response

The slope of the length vs. maximum velocity graph can be used to determine if length affects terminal velocity.

Total for part (e) 3 points

Total for question 2 15 points

Question 2: Free-Response Question — 15 points

(a)	For a multi-step derivation that includes Newton's second law of motion	**1 point**
	For indicating that the net force exerted on the sphere includes only the gravitational force and a drag force	**1 point**

Example Response

$F_{\text{net}} = F_g - F_{\text{drag}}$

For a correct differential equation that is in terms of the given variables — **1 point**

Scoring Note: Variables do not have to be separated for this point to be earned.

Example Response

$$m\frac{dv}{dt} = mg - bv$$

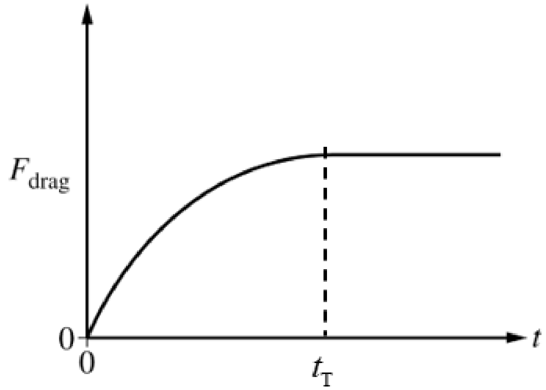
Example Solution

$$\Sigma F = ma$$
$$F_g - F_{\text{drag}} = ma$$
$$mg - bv = ma$$
$$m\frac{dv}{dt} = mg - bv$$

Total for part (a) 3 points

(b)(i)	For a vertical line labeled t_T at the approximate location at which the line becomes horizontal	**1 point**

Example Response



(b)(ii)	For a response that references the slope of the graph or the rate at which the slope changes	**1 point**
	For correctly relating a feature of the graph to the forces exerted on the sphere as the sphere reaches terminal speed	**1 point**

Example Response

For the times leading up to t_T, the slope of the graph is positive which means that the magnitude of the drag force is still increasing. After t_T, the slope of the graph is zero which means that the magnitude of the drag force is constant and equal to the downward gravitational force, which indicates that the net force is zero and that the sphere has reached a constant terminal velocity.

Total for part (b) 3 points

(c)	For selecting "Equal to" with an attempt at a relevant justification	**1 point**
	For a correct justification	**1 point**

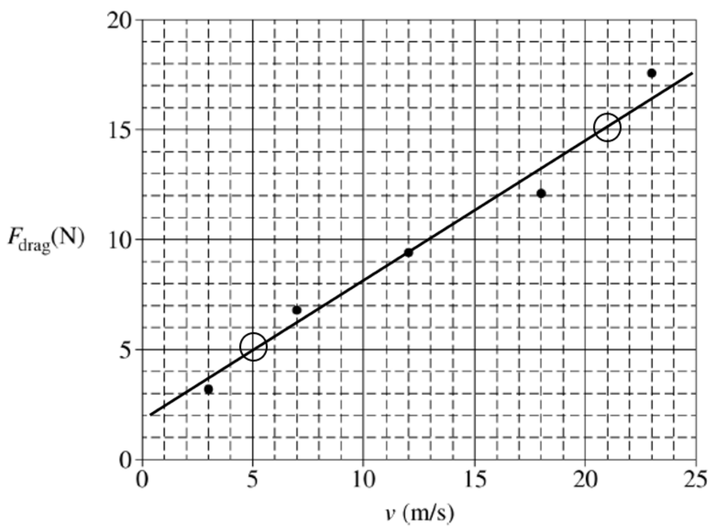
Example Response

The magnitude of the drag force at terminal speed does not change since the mass of the sphere is not changed and the drag force at terminal speed does not depend on the initial speed of the sphere.

Total for part (c) 2 points

(d)(i)	For drawing an appropriate line of best fit that approximates the data	**1 point**

Example Response



(d)(ii)	For calculating a value for the slope of the line using two points on the best-fit line	**1 point**

Scoring Note: Using data points that fall on the best-fit line earns this point.

Example Response

$$\text{slope} = \frac{15\text{ N} - 5\text{ N}}{21\text{ m/s} - 5\text{ m/s}}$$

For using the correct relationship between the slope of the best-fit line and the value of b — **1 point**

Example Response

$$\text{slope} = \frac{|F_{drag}|}{v}$$
$$\text{slope} = b$$

For a calculated value of b that is $0.6\text{ kg/s} < b < 0.8\text{ kg/s}$ — **1 point**

Example Response

$b = 0.625\text{ kg/s}$

Example Solution

$$|F_{drag}| = bv$$
$$\frac{|F_{drag}|}{v} = b$$
$$\text{slope} = b$$
$$b = \frac{15\text{ N} - 5\text{ N}}{21\text{ m/s} - 5\text{ m/s}}$$
$$b = 0.625\text{ kg/s}$$

Total for part (d) 4 points

(e)(i)	For indicating the diameter of the sphere should be graphed	**1 point**
	For indicating the terminal velocity of the sphere should be graphed	**1 point**
(e)(ii)	For describing how the quantities graphed are related to the conclusion of the experiment	**1 point**

Example Response

The slope of the diameter vs terminal velocity graph can be used to determine if sphere diameter affects terminal velocity.

Total for part (e) 3 points

Total for question 2 15 points

2019

AP® CollegeBoard

AP® Physics C: Mechanics

Scoring Guidelines Set 1

2019 SCORING GUIDELINES

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

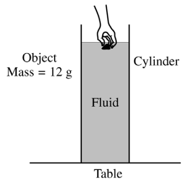
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

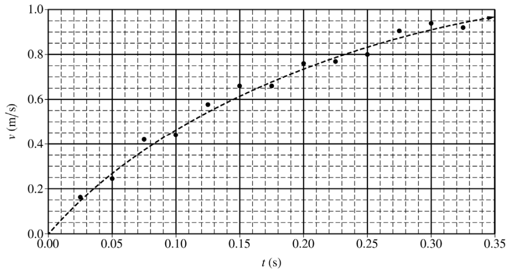
2019 SCORING GUIDELINES

Question 1

15 points



In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of $m = 12$ g is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.



(a)

i. LO CHA-1.C, SP 7.A
1 point

Does the speed of the object increase, decrease, or remain the same?

____ Increase ____ Decrease ____ Remain the same

For selecting "Increase"		1 point

ii. LO CHA-1.C, SP 4.D
2 points

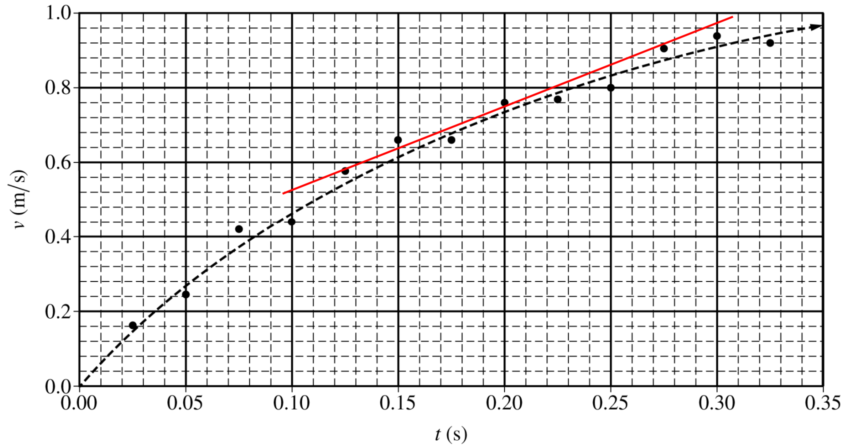
In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.

For a description that includes the direction of the acceleration as being "downwards"		1 point
For a description that includes the decrease in the magnitude of the acceleration		1 point
Example: Because the object is moving downwards and speeding up, the acceleration must be downwards. Because the slope of the graph of speed as a function of time is decreasing, the magnitude of the acceleration must be decreasing.		

2019 SCORING GUIDELINES

Question 1 (continued)

(a) continued



iii. LO CHA-1.C, SP 4.D, 6.C
2 points

Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20$ s.

For calculating the slope of a trend line at $t = 0.20$ s		1 point
$\text{slope} = a = \dfrac{\Delta v}{\Delta t} = \dfrac{(0.8 - 0.6)\ \text{m/s}}{(0.226 - 0.136)\ \text{s}}$		
For a correct answer using points from a tangent line		1 point
$a = 2.22\ \text{m/s}^2$		

The students use the equation $v = A\left(1 - e^{-Bt}\right)$ to model the speed of the falling object and find the best-fit coefficients to be $A = 1.18\ \text{m/s}$ and $B = 5\ \text{s}^{-1}$.

Question 1 (continued)

(b) Use the above equation to:

i. LO CHA-1.B, SP 6.B, 6.C
3 points

Derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time t.

For indicating that the vertical displacement is the integration of the velocity		1 point
$\Delta y = \int v dt = \int A\left(1 - e^{-Bt}\right) dt$		
For the equation for speed using appropriate limits or constant of integration		1 point
$\Delta y = \int_{t'=0}^{t'=t} A\left(1 - e^{-Bt'}\right) dt' = A\left[t' - \frac{1}{-B}e^{-Bt'}\right]_{t'=0}^{t'=t} A\left[\left(t + \frac{1}{B}e^{-Bt}\right) - \left(0 + \frac{1}{B}e^{0}\right)\right]$		
For a correct answer		1 point
$\Delta y = A\left(t + \frac{1}{B}\left(e^{-Bt} - 1\right)\right) = (1.18)\left(t + \frac{1}{5}\left(e^{-5t} - 1\right)\right)$		
<u>Note:</u> Credit given for using A and B or plugging in the given values		

ii. LO INT-1.C.d, SP 6.B, 6.C
3 points

Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time t.

For attempting the derivative of the equation for speed		1 point
$a = \frac{dv}{dt} = \frac{d}{dt}\left[A\left(1 - e^{-Bt}\right)\right]$		
For a correct equation for the acceleration		1 point
$a = ABe^{-Bt}$		
For multiplying the equation above by the mass of the object		1 point
$F = ma = m\left(ABe^{-Bt}\right) = mABe^{-Bt}$		
$F = (.012)(1.18)(5)e^{-5t} = 0.071e^{-5t}$		
<u>Note:</u> Credit given for using A, B, and m or plugging in the given values		

Question 1 (continued)

(c) The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed.

i. LO INT-1.I, SP 7.A, 7.C
2 points

Determine the constant speed of the object.
Justify your answer.

For stating the constant speed is $v = A$		1 point
For indicating that the constant speed can be determined by setting the time equal to infinity		1 point
Example: After a long time, the falling object will reach a terminal constant speed in the fluid. This can be determined by setting the time t in the equation for speed equal to infinity. By doing this, the constant speed is determined to be $v = A$.		
<u>Note:</u> Credit given for solving mathematically		

ii. LO INT-1.H.b, SP 7.A, 7.C
2 points

Determine the force exerted by the fluid on the object at this time. Justify your answer.

For indicating that the net force is equal to zero when the object moves with constant speed		1 point
For indicating the resistive force is equal to the weight of the object at this time		1 point
Example: When the falling object reaches a constant speed in the fluid, the net force must be zero. Because the only vertical forces acting on the object are Earth's gravitational pull and the resistive force of the fluid, these two forces must be equal. So, the resistive force must be equal to the weight of the object or 0.12 N.		
<u>Note:</u> Credit given for solving mathematically		

Question 1 (continued)

Learning Objectives

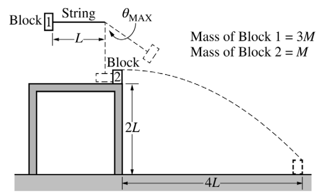
CHA-1.B: Determine functions of position, velocity, and acceleration that are consistent with each other, for the motion of an object with a nonuniform acceleration.
CHA-1.C: Describe the motion of an object in terms of the consistency that exists between position and time, velocity and time, and acceleration and time.
INT-1.C.d: Derive an expression for the net force on an object in translational motion.
INT-1.H.b: Describe the acceleration, velocity, or position in relation to time for an object subject to a resistive force (with different initial conditions, i.e., falling from rest or projected vertically).
INT-1.I: Calculate the terminal velocity of an object moving vertically under the influence of a resistive force of a given relationship.

Science Practices

4.D: Select relevant features of a graph to describe a physical situation or solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

Question 2

15 points



Note: Figure not drawn to scale.

A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

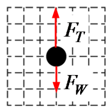
(a) LO CON-2.C.a, SP 5.E
1 point

Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

For correctly calculating the speed of block 1		1 point
$U_{g1} = K_2 \therefore mgh = \frac{1}{2}mv^2 \therefore gL = \frac{1}{2}v^2$		
$v = \sqrt{2gL}$		
Note: Credit is earned even if no work is shown.		

(b) LO INT-2.B.b, SP 3.D
2 points

On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



For correctly drawing and labeling the weight of the block and the tension in the string		1 point
For drawing the tension longer than the weight of the block		1 point
Note: A maximum of one point can be earned if there are any extraneous vectors.		

Question 2 (continued)

(c) LO INT-2.B.b, SP 5.A, 5.E
2 points

Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

For using an appropriate equation to calculate the tension in the string		1 point
$F_T = mg + ma_C = mg + \frac{mv^2}{r} = 3Mg + \frac{(3M)\left(\sqrt{2gL}\right)^2}{L}$		
For a correct answer		1 point
$F_T = 9Mg$		

For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

(d) LO CHA-2.C, SP 6.A, 6.C
2 points

Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

For using a correct kinematic equation to calculate the time		1 point
$\Delta y = v_1 t + \frac{1}{2}at^2 = \frac{1}{2}at^2 \therefore t = \sqrt{\frac{2\Delta y}{a}} = \sqrt{\frac{2(2L)}{g}} = \sqrt{\frac{(4)(0.75\text{ m})}{\left(9.8\text{ m/s}^2\right)}}$		
For a correct answer		1 point
$t = 0.55$ s		

(e) LO CHA-2.C, SP 6.A, 6.C
2 points

Calculate the speed of block 2 as it leaves the table.

For using a correct kinematic equation to calculate the speed		1 point
$\Delta x = vt \therefore v = \frac{\Delta x}{t} = \frac{4L}{t}$		
For substituting the time from part (d) into equation above		1 point
$v = \frac{(4)(0.75\text{ m})}{(0.55\text{ s})} = 5.45\text{ m/s}$		

Question 2 (continued)

(f) LO CON-4.E.a, SP 6.B, 6.C
3 points

Calculate the speed of block 1 just after it collides with block 2.

For indicating the use of the correct conservation of momentum to calculate the speed		1 point
$p_i = p_f \therefore m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$		
For correctly substituting the mass into equation above		1 point
$(3M)v_{1i} + 0 = (3M)v_{1f} + (M)v_{2f}$		
For substituting the speeds from parts (a) and (e) into equation above		1 point
$v_{1f} = \frac{(3M)v_{1i} - (M)v_{2f}}{(3M)} = \frac{(3M)\sqrt{2gL} - (M)v_{2f}}{(3M)}$		
$v_{1f} = \frac{(3)\sqrt{(2)\left(9.8\text{ m/s}^2\right)(0.75\text{ m})} - (1)(5.45\text{ m/s})}{(3)} = 2.02\text{ m/s}$		
($v_{1f} = 2.06$ m/s when using $g = 10\text{ m/s}^2$)		

(g) LO CON-2.C.a, SP 6.B, 6.C
3 points

Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.

For using conservation of energy to calculate the angle		1 point
$K_1 = U_{g2}$		
$\frac{1}{2}mv^2 = mgh$		
For correctly substituting $h = L(1 - \cos\theta)$ into the equation above		1 point
For correctly substituting the speed from part (f) into the equation above		1 point
$\frac{1}{2}mv^2 = mgL(1 - \cos\theta) \therefore \frac{v^2}{2gL} = 1 - \cos\theta$		
$\theta = \cos^{-1}\left(1 - \frac{v^2}{2gL}\right) = \cos^{-1}\left(1 - \frac{(2.02\text{ m/s})^2}{(2)\left(9.8\text{ m/s}^2\right)(0.75\text{ m})}\right) = 43.5°$		
($\theta = 44.1°$ when using $g = 10\text{ m/s}^2$)		

Question 2 (continued)

Learning Objectives

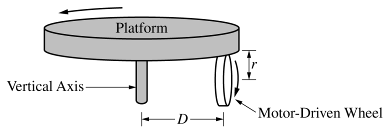
CHA-2.C: Calculate kinematic quantities of an object in projectile motion, such as displacement, velocity, speed, acceleration, and time, given initial conditions of various launch angles, including a horizontal launch at some point in its trajectory.
INT-2.B.b: Describe forces that are exerted on objects undergoing horizontal circular motion, vertical circular motion, or horizontal circular motion on a banked curve.
CON-2.C.a: Calculate unknown quantities (e.g., speed or positions of an object) that are in a conservative system of connected objects, such as the masses in an Atwood machine, masses connected with pulley/string combinations, or the masses in a modified Atwood machine.
CON-4.E.a: Calculate the changes in speeds, changes in velocities, changes in kinetic energy, or changes in momenta of objects in all types of collisions (elastic or inelastic) in one dimension, given the initial conditions of the objects.

Science Practices

3.D: Create appropriate diagrams to represent physical situations.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

Question 3

15 points



A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the wheel until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

(a) LO INT-7.A.b, CHA-4.A.b, SP 5.A, 5.E
2 points

Derive an expression for the angular speed ω_P of the platform. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.

For correctly substituting into the rotational form of Newton's second law			1 point
$\tau = I\alpha \therefore FD = I_P\alpha$			
$\alpha = \dfrac{FD}{I_P}$			
For correctly substituting into a rotational kinematic equation to calculate the angular speed			1 point
$\omega_2 = \omega_1 + \alpha\Delta t = 0 + \left(\dfrac{FD}{I_P}\right)\Delta t$			
$\omega_P = \dfrac{FD\Delta t}{I_P}$			

(b) LO INT-7.D.a, SP 5.A, 5.E
2 points

Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed ω_P. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.

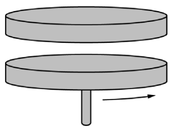
For using the equation for rotational kinetic energy		1 point
$K = \dfrac{1}{2}I\omega_P^2 = \left(\dfrac{1}{2}\right)(I)\left(\dfrac{FD\Delta t}{I}\right)^2$		
For an answer consistent with part (a)		1 point
$K = \dfrac{(FD\Delta t)^2}{2I}$		

Question 3 (continued)

(c) LO INT-7.C, SP 5.A, 5.E
2 points

Derive an expression for the angular speed of the wheel ω_W when the platform has reached angular speed ω_P. Express your answer in terms of D, r, ω_P, and physical constants, as appropriate.

For indicating that the linear speed of the platform is equal to the linear speed of the wheel		1 point
$v_P = v_W$ OR $(r\omega)_P = (r\omega)_W$		
For correctly relating the linear speeds to the angular speeds in the above equation		1 point
$D\omega_P = r\omega_W$		
$\omega_W = \dfrac{D\omega_P}{r}$		



When the platform is spinning at angular speed ω_P, the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia I_P as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed ω_f.

(d) LO CON-5.D.c, SP 5.A, 5.E
2 points

Derive an expression for ω_f. Express your answer in terms of ω_P, I_P, and physical constants, as appropriate.

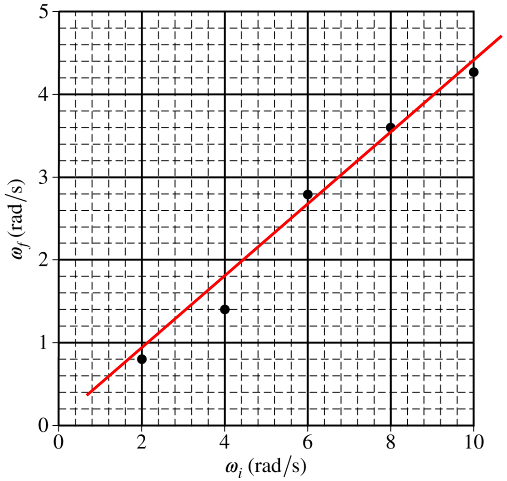
For using an expression for the conservation of angular momentum		1 point
$L_1 = L_2 \therefore I_1\omega_1 = I_2\omega_2$		
For correctly substituting into the above equation		1 point
$I\omega_P = (2I)\omega_F$		
$\omega_F = \dfrac{1}{2}\omega_P$		

Question 3 (continued)

A student now uses the rotating platform $\left(I_P = 3.1\ \text{kg} \bullet \text{m}^2\right)$ to determine the rotational inertia I_U of an unknown object about a vertical axis that passes through the object's center of mass. The platform is rotating at an initial angular speed ω_i when the unknown object is dropped with its center of mass directly above the center of the platform. The platform and object are observed to be rotating together at angular speed ω_f. Trials are repeated for different values of ω_i. A graph of ω_f as a function of ω_i is shown on the axes below.

(e) i. LO CON-5.D.c, SP 4.C
1 points

On the graph on the previous page, draw a best-fit line for the data.



For an appropriate best-fit line for the graph above		1 point

Question 3 (continued)

(e) continued
ii. LO CON-5.D.c, SP 4.D, 6.C
2 points

Using the straight line, calculate the rotational inertia of the unknown object I_U about a vertical axis passing through its center of mass.

For using conservation of angular momentum to derive an expression that includes I_U		1 point
$L_i = L_f \therefore I_P\omega_i = (I_P + I_U)\omega_f$		
$I_U = I_P\left(\frac{\omega_i}{\omega_f}\right) - I_P$		
For substituting points from the best-fit line into the expression above		1 point
$I_U = \left(3.1\ \text{kg}\bullet\text{m}^2\right)\left(\frac{(4.0-1.0)\ \text{rad/s}}{(9.3-2.4)\ \text{rad/s}}\right) - \left(3.1\ \text{kg}\bullet\text{m}^2\right) = 4.1\ \text{kg}\bullet\text{m}^2$		
Note: The point (0, 0) can be used implicitly if the best-fit line goes through the origin.		

(f) LO CON-5.D.c, SP 7.A, 7.C
2 points

The kinetic energy of the spinning platform before the object is dropped on it is K_i. The total kinetic energy of the platform-object system when it reaches angular speed ω_f is K_f. Which of the following expressions is true?

____ $K_f < K_i$ ____ $K_f = K_i$ ____ $K_f > K_i$

Justify your answer.

For selecting $K_f < K_i$ with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because the two disks will be rotating with the same final angular speed, this is an inelastic collision, and kinetic energy will be lost during the collision.		

Question 3 (continued)

(g) LO INT-6.E, SP 7.A, 7.C
2 points

One of the students observes that the center of mass of the object is not actually aligned with the axis of the platform. Is the experimental value of I_U obtained in part (e) greater than, less than, or equal to the actual value of the rotational inertia of the unknown object about a vertical axis that passes through its center of mass?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

For selecting "Greater than" with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because the center of mass of the object is off the axis of the platform, the parallel axis theorem would be used to calculate the total rotational inertia of the platform-object system. Using $I = I_{CM} + Mh^2$, the experimental value will be increased by Mh^2.		

Learning Objectives

INT-6.E: Derive the moments of inertia of an extended rigid body for different rotational axes (parallel to an axis that goes through the object's center of mass) if the moment of inertia is known about an axis through the object's center of mass.
CHA-4.A.b: Calculate unknown quantities such as angular positions, displacement, angular speeds, or angular acceleration of a rigid body in uniformly accelerated motion, given initial conditions.
INT-7.A.b: Calculate unknown quantities such as net torque, angular acceleration, or moment of inertia for a rigid body undergoing rotational acceleration.
INT-7.C: Derive expressions for physical systems such as Atwood Machines, pulleys with rotational inertia, or strings connecting discs or strings connecting multiple pulleys that relate linear or translational motion characteristics to the angular motion characteristics of rigid bodies in the system that are: **(a)** rolling (or rotating on a fixed axis) without slipping. **(b)** rotating and sliding simultaneously.
INT-7.D.a: Calculate the rotational kinetic energy of a rotating rigid body.
CON-5.D.c: Calculate the changes of angular momentum of each disc in a rotating system of two rotating discs that collide with each other inelastically about a common rotational axis.

Science Practices

4.C: Linearize data and/or determine a best-fit line or curve.
4.D: Select relevant features of a graph to describe a physical situation or solve problems.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

2017

AP Physics C: Mechanics
Scoring Guidelines

AP® PHYSICS
2017 SCORING GUIDELINES

General Notes About 2017 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth one point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections—Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based and AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

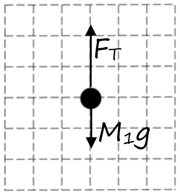
SCORING GUIDELINES

Question 1

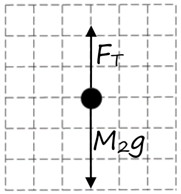
15 points total **Distribution of points**

(a) 3 points

Block of Mass M_1



Block of Mass M_2



For correctly drawing and labeling the vectors for the weight of the block and the tension for block M_1 with the tension larger than the weight	1 point
For correctly drawing and labeling the vectors for the weight of the block and the tension for block M_2 with the weight larger than the tension	1 point
For correctly drawing tension on the two blocks as equal in magnitude	1 point

Note: If any extraneous vectors are drawn, only a maximum of two points may be earned.

(b) 3 points

For correctly applying Newton's second law to block 1 — 1 point

$F_T - M_1 g = M_1 a$

For correctly applying Newton's second law to block 2 — 1 point

$M_2 g - F_T = M_2 a$

For combining the two equations above in such a way that will lead to the correct equation — 1 point

$M_2 g - M_1 g = (M_1 + M_2)a$

$$a = \frac{(M_2 - M_1)}{(M_1 + M_2)} g$$

(c) 1 point

For indicating variables that will create a straight line whose slope can be used to determine g — 1 point

Example: Vertical axis: a

Horizontal axis: $\frac{(M_2 - M_1)}{(M_1 + M_2)}$

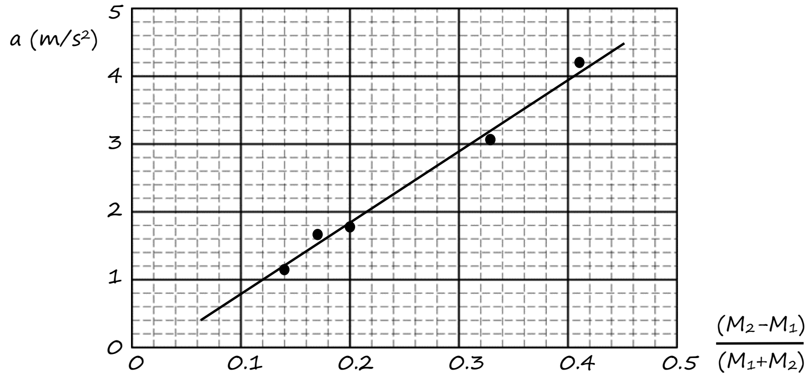
Note: Full credit is earned if axes are reversed, or if the student uses another acceptable combination.

2017 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(d) 3 points



For using a correct scale that uses more than half the grid and for correctly labeling the axes, including units as appropriate	1 point
For correctly plotting the data	1 point
For drawing a straight best-fit line consistent with the plotted data	1 point

(e) 2 points

For correctly calculating the slope using the best-fit line and not the data points, unless the points fall on the best-fit line — 1 point

(Note: Credit may be given for the linear regression only if the student states linear regression is used.)

$$\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(3.0 - 1.0)\ \text{m/s}^2}{(0.31 - 0.12)} = 10.5\ \text{m/s}^2$$

For correctly relating the slope to g — 1 point

$g = \text{slope} = 10.5\ \text{m/s}^2$ (using linear regression yields $10.1\ \text{m/s}^2$)

2017 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(f) 2 points

Correct answer: "Lower"

For including a correct statement about the acceleration of the blocks — 1 point
For including a correct statement about the forces on the blocks — 1 point

Example: Because the block M_1 is on the table, the net force on the system increases, and therefore the acceleration increases. Because the acceleration of M_2 increases, the net force on M_2 must increase. Therefore, there must be a greater difference in the magnitude of the two forces on M_2. Because the weight of block M_2 stays the same, the retarding force — the tension in the string — must decrease.

(g) 1 point

For a correct justification — 1 point

Example: Block M_1 will experience friction with the table. The acceleration of the system will decrease and this will decrease the slope of the line; therefore, the value of g is determined by the experiment.

SCORING GUIDELINES

Question

15 points total

Distribution of points

(a)

i. 2 points

For correctly using conservation of energy for the block moving down the incline — 1 point

$U_g = K$

$mgh = \frac{1}{2}mv^2$

For a correct answer — 1 point

$v = \sqrt{2gh}$

ii. 1 point

Correct answer: "Greater than"

For a correct justification — 1 point

Example: The speed is proportional to the square root of the change in height. So if the height is reduced by a factor of 2, the speed is reduced by a factor of $\sqrt{2} \approx 1.41$. Therefore, the speed halfway down the ramp is more than half the speed at the bottom of the ramp.

Note: If the incorrect selection is made, the justification cannot earn credit.

(b) 1 point

For correctly using conservation of energy, consistent with part (a), for the block compressing the spring — 1 point

$U_g = U_S$

$mgh = \frac{1}{2}kx_{\max}^2$

$x_{\max} = \sqrt{\frac{2mgh}{k}}$

(c) 2 points

For indicating a simple harmonic motion approach — 1 point
For a correct answer — 1 point

$t = \frac{1}{4}T = \left(\frac{1}{4}\right)\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{\pi}{2}\sqrt{\frac{m}{k}}$

2017 SCORING GUIDELINES

Question continued)

Distribution of points

(d)

i. 3 points

For correctly applying Newton's second law for the horizontally sliding block	1 point
For correctly indicating that the direction of F_{net} is opposite the direction of motion	1 point

$F_{net} = ma$

$-\beta v^2 = ma$

For expressing the equation as a differential equation	1 point

$-\beta v^2 = m\dfrac{dv}{dt}$

ii. 3 points

For correctly separating variables	1 point

$-\dfrac{\beta}{m}dt = \dfrac{1}{v^2}dv$

For correctly integrating the equation above	1 point

$\int -\dfrac{\beta}{m}dt = \int \dfrac{1}{v^2}dv$

$-\dfrac{\beta}{m}[t] = \left[-\dfrac{1}{v}\right]$

For using the correct limits or constant of integration	1 point

$-\dfrac{\beta}{m}[t]_0^t = \left[-\dfrac{1}{v}\right]_{v_0}^{v(t)}$

$-\dfrac{\beta t}{m} = -\dfrac{1}{v(t)} - \left(-\dfrac{1}{v_0}\right)$

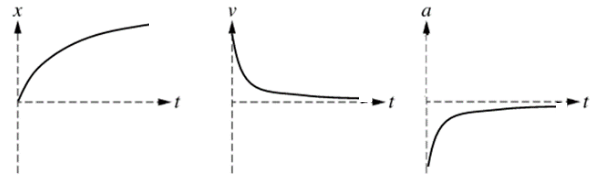
$\dfrac{1}{v(t)} = \dfrac{1}{v_0} + \dfrac{\beta t}{m}$

2017 SCORING GUIDELINES

Question continued)

Distribution of points

(e) 3 points



For a displacement curve that is concave down and approaching a horizontal asymptote	1 point
For a velocity curve that is concave up and has the horizontal axis as an asymptote	1 point
For an acceleration curve that is concave down and has the horizontal axis as an asymptote	1 point

Note: If an incorrect nonlinear velocity graph is generated, 1 point is earned if the position and acceleration graphs are consistent with the velocity graph.

Note: Full credit is earned if all three graphs are flipped vertically.

2017 SCORING GUIDELINES

Question 3

15 points total | **Distribution of points**

(a) 2 points

For correctly applying conservation of energy to the cylinder rolling down the incline — 1 point

$U_{g\_top} = K_{table}$

$K_{table} = mgh = (0.50\ \text{kg})\left(9.8\ \text{m/s}^2\right)(1.0\ \text{m})(\sin 30)$

For a correct answer with units — 1 point

$K_{table} = 2.45\ \text{J}$ (or 2.5 J using $g = 10\ \text{m/s}^2$)

(b) 3 points

For correctly setting the kinetic energy of the cylinder equal to the sum of both the linear and rotational kinetic energy — 1 point

$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

For correctly substituting into the above equation for the linear velocity and moment of inertia of the cylinder — 1 point

$K = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{2}M(R\omega)^2 + \frac{1}{4}M(R\omega)^2 = \frac{3}{4}M(R\omega)^2$

$\omega = \sqrt{\frac{4K}{3MR^2}} = \sqrt{\frac{(4)(2.45\ \text{J})}{(3)(0.50\ \text{kg})(0.10\ \text{m})^2}}$

For correct substitution into the equation above — 1 point

$\omega = 25.6\ \text{rad/s}$ (or $26.0\ \text{rad/s}$ using $g = 10\ \text{m/s}^2$)

(c) 2 points

For using a correct expression for the ratio of the rotational kinetic energy to the total kinetic energy of the cylinder — 1 point

$$\frac{K_{rot}}{K_{tot}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh} = \frac{\frac{1}{4}M(R\omega)^2}{\frac{3}{4}M(R\omega)^2 + Mgh_{\text{table}}} = \frac{(R\omega)^2}{3(R\omega)^2 + 4gh_{\text{table}}}$$

For substituting into the above equation — 1 point

$$\frac{K_{rot}}{K_{tot}} = \frac{[(0.10\ \text{m})(25.6\ \text{rad/s})]^2}{(3)(0.10\ \text{m})^2(25.6\ \text{rad/s})^2 + (4)\left(9.81\ \text{m/s}^2\right)(0.75\ \text{m})}$$

$\frac{K_{rot}}{K_{tot}} = 0.133$ (or 0.135 using $g = 10\ \text{m/s}^2$)

2017 SCORING GUIDELINE

Question 3 (continued)

Distribution of points

(c) (continued)

Alternate Solution — *Alternate Points*

For using a correct expression for the ratio of the rotational kinetic energy to the total potential energy of the cylinder — *1 point*

$$\frac{K_{rot}}{K_{tot}} = \frac{\frac{1}{2}I\omega^2}{U_{total}} = \frac{\frac{1}{4}M(R\omega)^2}{Mg(h+y)}$$

For substituting into the above equation — *1 point*

$$\frac{K_{rot}}{U_{total}} = \frac{\frac{1}{4}(R\omega)^2}{g(h+y)} = \frac{\frac{1}{4}[(0.10\ \text{m})(25.6\ \text{rad/s})]^2}{\left(9.81\ \text{m/s}^2\right)\left((1.0\ \text{m})\sin\left(30^\circ\right) + (0.75\ \text{m})\right)}$$

$\frac{K_{rot}}{U_{total}} = 0.13$ *(or* 0.135 *using* $g = 10\ \text{m/s}^2$ *)*

(d) 2 points

For correctly using motion in the vertical direction to calculate the time for the cylinder to reach the floor — 1 point

$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

$y - y_0 = 0 - \frac{1}{2}gt^2$

Determine the time for the cylinder to reach the floor

$y - y_0 = -\frac{1}{2}gt^2$

$0 - y = -\frac{1}{2}gt^2$

$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{(2)(0.75\ \text{m})}{\left(9.81\ \text{m/s}^2\right)}} = 0.39\ \text{s}$

For correctly using the equation for constant speed in the horizontal direction — 1 point

$x - x_0 = v_{0x}t$

$x - x_0 = R\omega t$

$x - x_0 = R\omega\sqrt{\frac{2y}{g}}$

$D = R\omega t = (0.10\ \text{m})(25.6\ \text{rad/s})(0.39\ \text{s})$

$D = 1.0\ \text{m}$

2017 SCORING GUIDELINE

Question 3 (continued)

	Distribution of points
(e)	
i. 2 points	
For selecting "Equal to" and attempting a relevant justification	1 point
For a correct justification	1 point
Example: Because the sphere falls the same height as the cylinder and because they have the same mass, the sphere-Earth system has the same initial potential energy and, therefore, the same total kinetic energy when it reaches the floor.	
ii. 2 points	
For selecting "Less than" and attempting a relevant justification	1 point
For a correct justification	1 point
Example: Because the rotational inertia of the sphere is less than the rotational inertia of the cylinder, the sphere will rotate faster and, because $v = r\omega$, will move with a greater linear speed. Because the mass is the same and the linear speed is greater, the sphere will have a greater linear kinetic energy. Because the total kinetic energies of the sphere and cylinder are the same, the sphere must have less rotational kinetic energy.	
iii. 2 points	
For selecting "Greater than" and attempting a relevant justification	1 point
For a correct justification	1 point
Example: Because the sphere has a greater linear speed as it leaves the table, it will travel a greater horizontal distance before it reaches the floor.	