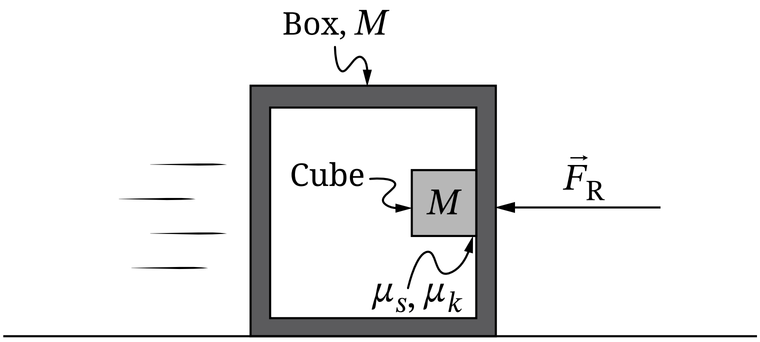
## Question 1: Version J

**1.** A box of mass $M$ slides to the right on a horizontal surface. A small cube, also of mass $M$, is inside the box. The cube is against the right wall of the box and above the bottom of the box, as shown in Figure 1. The coefficients of static and kinetic friction between the cube and the wall of the box are $\mu_s$ and $\mu_k$, respectively, where $\mu_s > \mu_k$.

**Figure 1**




The speed of the box is decreasing as a result of a resistive force that is exerted on the box. The resistive force $\vec{F}_R$ is modeled by $\vec{F}_R = -b\vec{v}$, where $b$ is a positive constant and $\vec{v}$ is the velocity of the box. Friction between the box and horizontal surface is negligible.

Consider the motion of the box and cube for the following times.

- At time $t = 0$, the box-cube system slides to the right with speed $v_0$.
- For $0 \le t < t_{crit}$, the cube remains in contact with the wall of the box at a constant height above the bottom of the box.
- At $t = t_{crit}$, the cube begins to slide downward along the wall of the box.

**Part A**

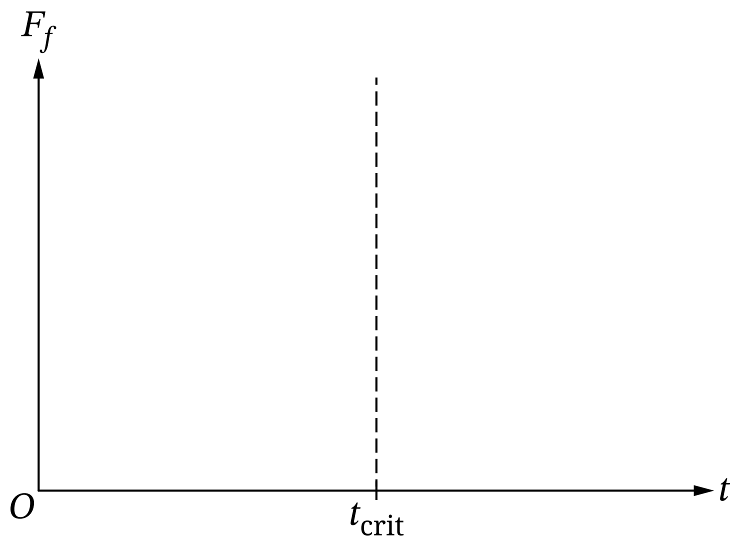
The magnitude of the normal force exerted on the cube by the wall of the box is $F_N$.

i. **Determine** an expression for the magnitude of the acceleration of the cube. Express your answer in terms of $M$, $F_N$, and physical constants, as appropriate.

ii. **Derive** an expression for $F_N$ as a function of $t$ from $t = 0$ until immediately before the cube reaches the bottom of the box. Express your answer in terms of $M$, $b$, $v_0$, $t$, and physical constants, as appropriate. Begin your derivation with a fundamental physics principle or an equation from the reference information.

iii. On the axes shown in Figure 2, **sketch** a graph of the magnitude $F_f$ of the frictional force exerted on the cube by the wall of the box as a function of $t$ from $t = 0$ until immediately before the cube reaches the bottom of the box.

**Figure 2**




**Part B**

**Derive** an expression for $t_{crit}$ in terms of $M$, $b$, $\mu_s$, $v_0$, and physical constants, as appropriate. Begin your derivation with a fundamental physics principle or an equation from the reference information.

Begin your response to **QUESTION 2** on this page.

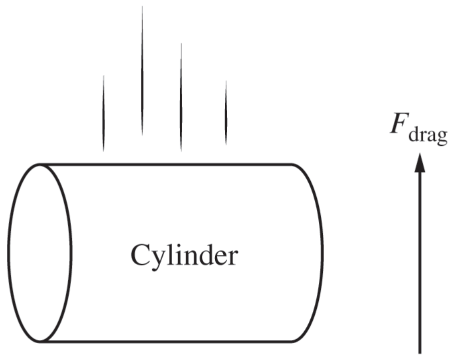



Figure 1

2. A student drops a cylinder of mass $m$ from rest. The air exerts a drag force of magnitude $F_{drag}$ on the cylinder, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv^2$, where $v$ is the speed of the cylinder and $b$ is a positive constant with appropriate units.

(a) **Derive**, but do NOT solve, a differential equation that could be used to determine the speed $v$ of the cylinder as a function of time $t$. Express your answer in terms of given quantities and physical constants, as appropriate.

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Continue your response to **QUESTION 2** on this page.

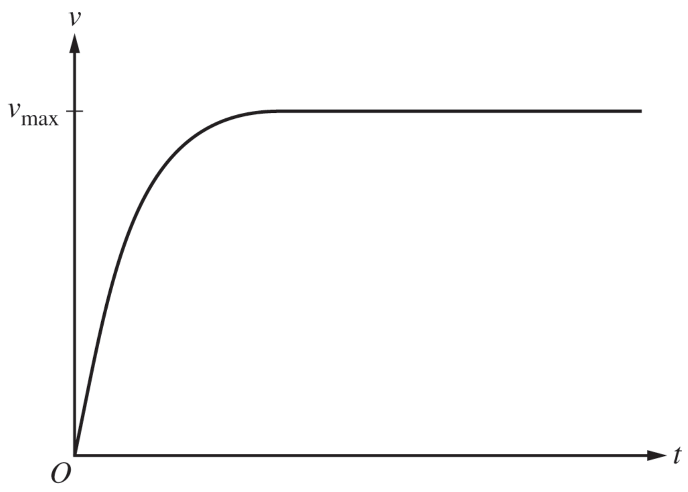


Figure 2

(b) The student correctly sketches the speed $v$ of the cylinder as a function of time $t$, as shown in Figure 2.

i. **Draw** a vertical line on the sketch in Figure 2 to indicate the earliest time at which $F_{drag}$ on the cylinder is equal to the magnitude of the weight of the cylinder. Label this time as $t_1$ on the time axis.

ii. **Justify** the location of $t_1$. Explicitly reference appropriate features of the sketch in Figure 2.

(c) Rather than dropping the cylinder from rest, the student throws the cylinder upward with a nonzero initial speed. The cylinder is in the same orientation as when the cylinder was previously dropped. The student allows the cylinder to fall toward the ground.

**Indicate** whether the magnitude of the cylinder's maximum downward speed after being thrown upward would be greater than, less than, or equal to the maximum speed $v_{max}$ in Figure 2.

______ Greater than ______ Less than ______ Equal to

Briefly **justify** your answer.

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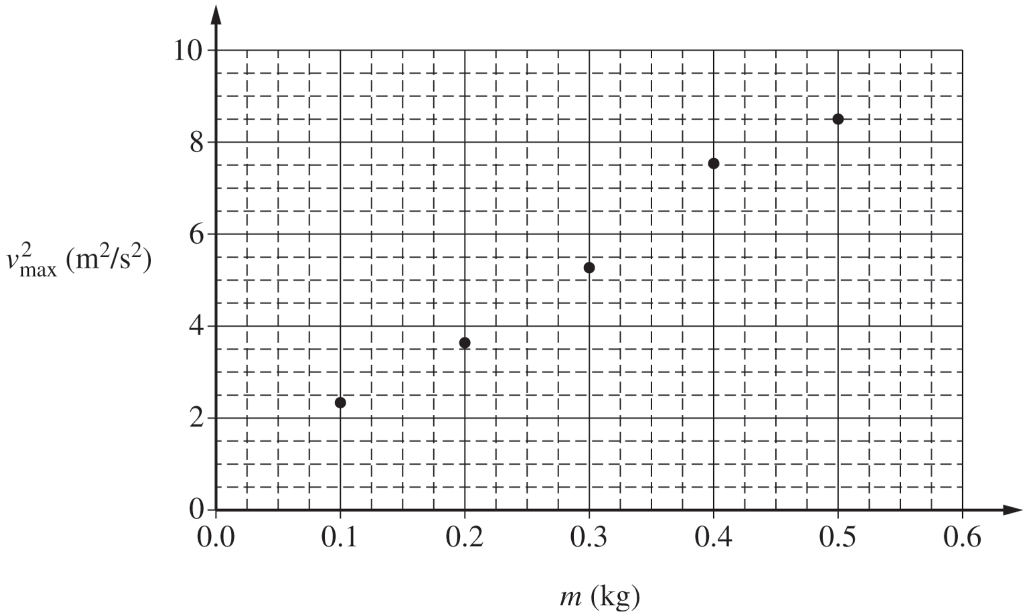
Continue your response to **QUESTION 2** on this page.

(d) The student conducts an experiment to better understand the relationship between maximum speed $v_{max}$ and mass. The student collects data to determine the maximum speed for cylinders dropped from rest, each with the same physical size and shape but a different mass $m$. The student then graphs $v_{max}^2$ as a function of mass.




i. **Draw** the best-fit line for the data.

ii. Use the best-fit line to **calculate** an experimental value for $b$.

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Continue your response to **QUESTION 2** on this page.

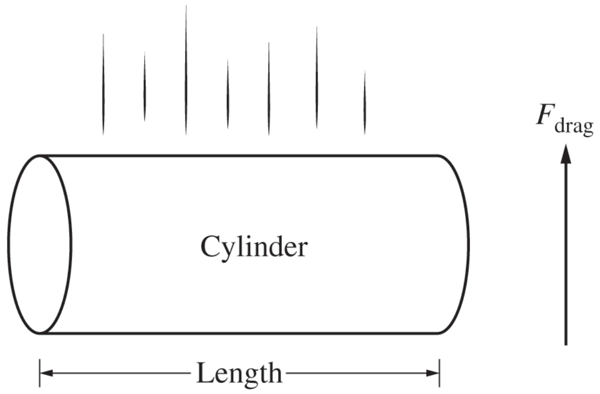


Figure 3

A student claims that the magnitude of the maximum speed of a cylinder dropped from rest depends on the length of the cylinder. The student designs an experiment to collect data that can be used to provide evidence to support the claim. The student drops cylinders with the orientation shown in Figure 3.

(e) The student has access to but does not have to use all of the following equipment.

- Cylinder Set 1: cylinders of the same known length with different known masses
- Cylinder Set 2: cylinders of the same known mass with different known lengths
- A motion detector that can measure velocity as a function of time

i. **Indicate** two quantities that when graphed could be used to determine whether the length of the cylinder affects the maximum speed.

Vertical axis: ______________ Horizontal axis: ______________

ii. Briefly **describe** how the quantities graphed could be used to determine the relationship between cylinder length and maximum speed.

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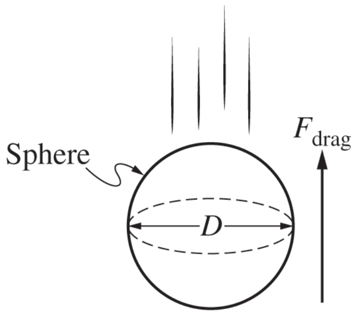


Figure 1

2. A student drops a sphere of mass $m$ from rest. The air exerts a drag force of magnitude $F_{drag}$ on the sphere, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv$, where $v$ is the speed of the sphere and $b$ is a positive constant with appropriate units.

   (a) **Derive**, but do NOT solve, a differential equation that could be used to determine the speed $v$ of the sphere as a function of time $t$. Express your answer in terms of given quantities and physical constants, as appropriate.

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Continue your response to **QUESTION 2** on this page.

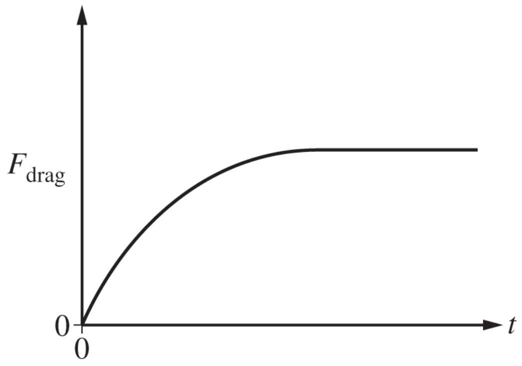


Figure 2

(b) The student sketches the drag force $F_{drag}$ exerted on the sphere as a function of time $t$, as shown in Figure 2.

i. **Draw** a vertical line on the sketch in Figure 2 to indicate the earliest time at which $F_{drag}$ is equal to the magnitude of the weight of the sphere, which occurs when the sphere reaches terminal speed. Label this time as $t_T$ on the time axis.

ii. **Justify** the location of $t_T$. Explicitly reference appropriate features of the sketch in Figure 2.

(c) Suppose the student throws the same sphere downward with a nonzero initial speed. The magnitude of the new drag force at terminal speed after being thrown downward is $F_{new}$.

**Indicate** whether $F_{new}$ would be greater than, less than, or equal to the magnitude of $F_{drag}$ at terminal speed represented in Figure 2.

_____ Greater than _____ Less than _____ Equal to

Briefly **justify** your answer.

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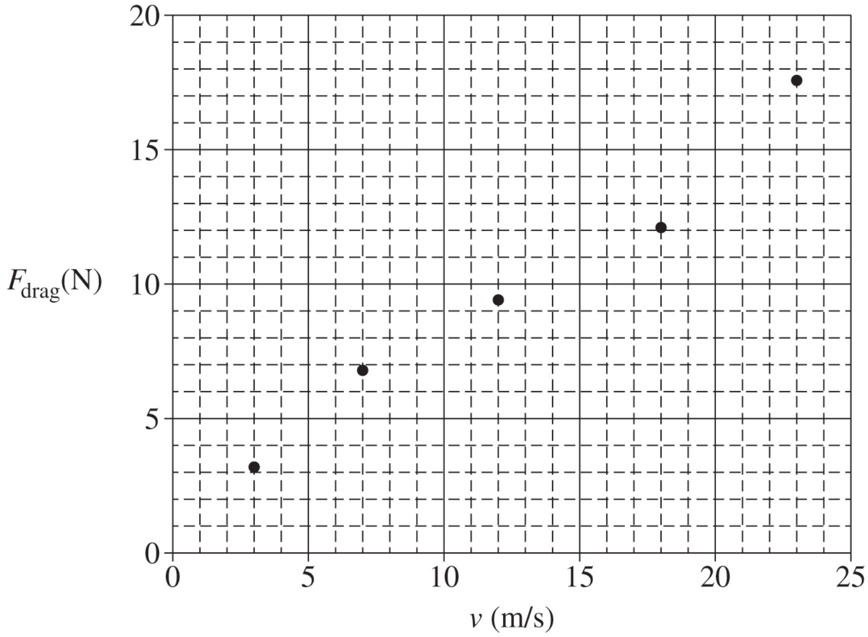
Continue your response to **QUESTION 2** on this page.

(d) The student conducts an experiment to better understand the relationship between $F_{drag}$ and $v$. The student makes measurements to calculate and graph the magnitude of $F_{drag}$ as a function of $v$ for the falling sphere.




i. **Draw** the best-fit line for the data.

ii. Use the best-fit line to **calculate** an experimental value for $b$.

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Continue your response to **QUESTION 2** on this page.

A student claims that the terminal speed $v_T$ of the sphere depends on the diameter $D$ of the sphere. The student designs an experiment to collect data that can be used to provide evidence to support the claim.

(e) The student has access to but does not have to use all of the following equipment.

- Sphere Set 1: spheres of the same known mass with different known diameters
- Sphere Set 2: spheres of the same known diameter with different known masses
- A motion detector that can measure velocity as a function of time

i. **Indicate** two quantities that when graphed could be used to determine whether the diameter of the sphere affects the terminal speed.

Vertical axis: ________________ Horizontal axis: ________________

ii. Briefly **describe** how the quantities graphed could be used to determine the relationship between sphere diameter and terminal speed.

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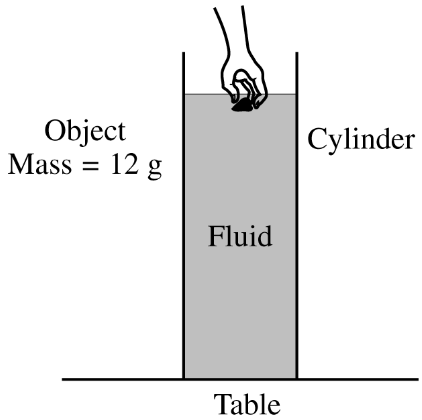
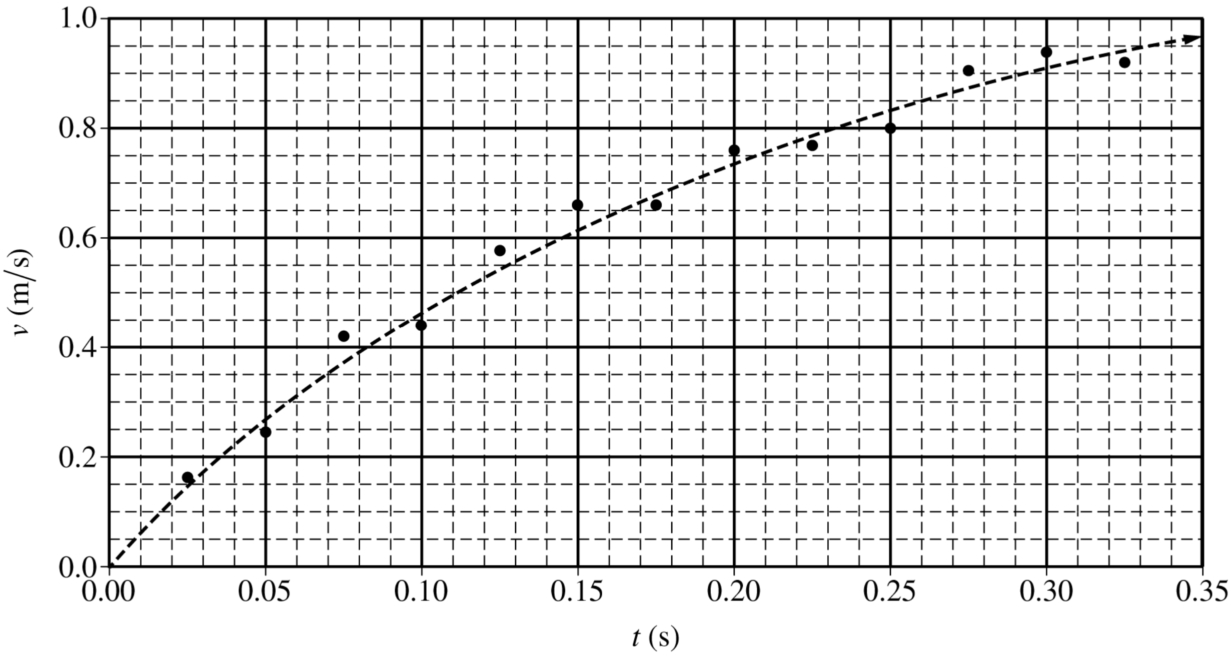
# PHYSICS C: MECHANICS
## SECTION II
## Time—45 minutes
## 3 Questions

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.




1. In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of $m$ = 12 g is released from rest at the top of the column of fluid, as shown above. The data for the speed $v$ of the falling object as a function of time $t$ are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.



(a)

i. Does the speed of the object increase, decrease, or remain the same?

____ Increase ____ Decrease ____ Remain the same

ii. In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.

iii. Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20$ s.

The students use the equation $v = A\left(1 - e^{-Bt}\right)$ to model the speed of the falling object and find the best fit coefficients to be $A = 1.18\ \text{m/s}$ and $B = 5\ \text{s}^{-1}$.

(b) Use the above equation to:

i. Derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time $t$.

ii. Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time $t$.

The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed.

(c)

i. Determine the constant speed of the object.

Justify your answer.

ii. Determine the force exerted by the fluid on the object at this time.

Justify your answer.


Note: Figure not drawn to scale.

2. A block of mass $m$ starts at rest at the top of an inclined plane of height $h$, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant $k$. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of $m$, $h$, $k$, and physical constants, as appropriate.

(a)

i. Derive an expression for the speed of the block just before it collides with the spring.

ii. Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

____ Greater than ______ Less than ____ Equal to

Justify your answer.

(b) Derive an expression for the maximum compression of the spring.

(c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed $v_0$. Now suppose the block experiences a resistive force as it slides on the horizontal surface. The magnitude of the resistive force $F$ is given as a function of speed $v$ by $F = \beta v^2$, where $\beta$ is a positive constant with units of $\mathrm{kg/m}$.

(d)

i. Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time $t$ before it reaches the spring. Express your answer in terms of $m$, $h$, $k$, $\beta$, $v$, and physical constants, as appropriate.

ii. Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time $t$ can be written in the form $\dfrac{1}{v(t)} = \dfrac{1}{v_0} + \dfrac{\beta t}{m}$, where $v_0$ is the speed at $t = 0$.

(e) Sketch graphs of position $x$ as a function of time $t$, velocity $v$ as a function of time $t$, and acceleration $a$ as a function of time $t$ for the block as it is moving on the horizontal surface before it reaches the spring.