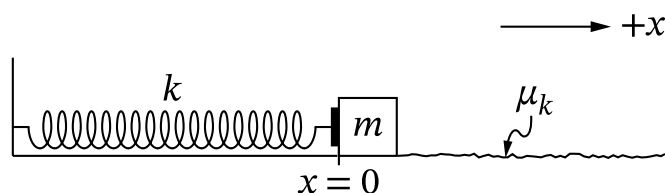


Question 3

The following information applies to parts A and B.

3. A horizontal spring of known spring constant k is attached to a wall. A block of known mass m is placed on a horizontal surface next to but not attached to the spring. The spring is at its relaxed length when the block is at position $x = 0$, as shown in Figure 1. There is friction between the block and the surface only for positions $x > 0$.

Figure 1



A student is asked to experimentally determine the coefficient of kinetic friction μ_k between the block and the surface using a graph. The student is permitted to use only measurements from a meterstick.

Part A

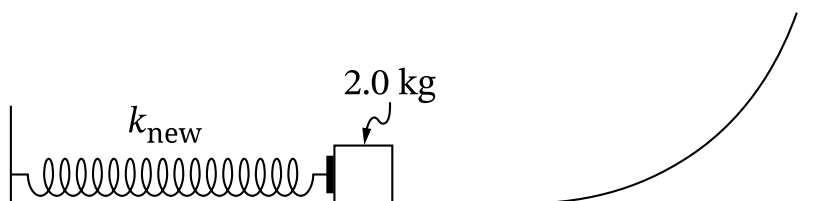
- Indicate** quantities that could be measured by the student that would allow them to determine μ_k using a graph.
- Briefly **describe** a method to reduce experimental uncertainty for the measured quantities.

Part B

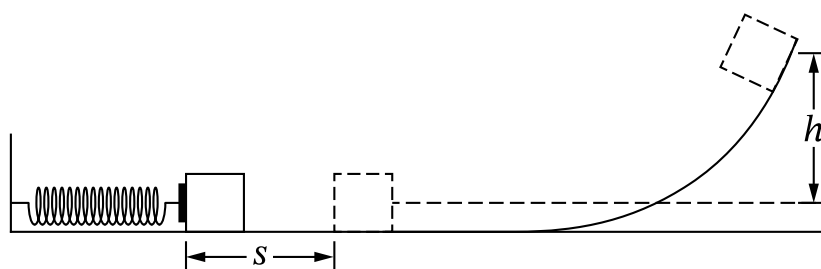
- Indicate** what quantities the student could graph on the horizontal and vertical axes to create a graph that can be used to determine μ_k .
- Briefly **describe** the relationship between μ_k and a feature of the graph from part B (i). Your answer may include an equation that relates μ_k and the chosen feature of the graph.

The following information applies to parts C and D.

In a different experiment, the student is asked to determine the spring constant k_{new} of a new spring that is attached to a wall. A block of mass 2.0 kg is placed next to the spring on a surface where frictional forces are negligible. The surface is horizontal near the spring and slopes upward into a ramp to the right of the initial position of the block, as shown in Figure 2.

Figure 2

The student pushes the block, compressing the spring a distance s , as shown in Figure 3. The block is released from rest.

Figure 3

The student uses a meterstick to measure the maximum height h of the block on the ramp. The experiment is repeated several times with different compression distances s . The student's measurements of s and h are shown in Table 1.

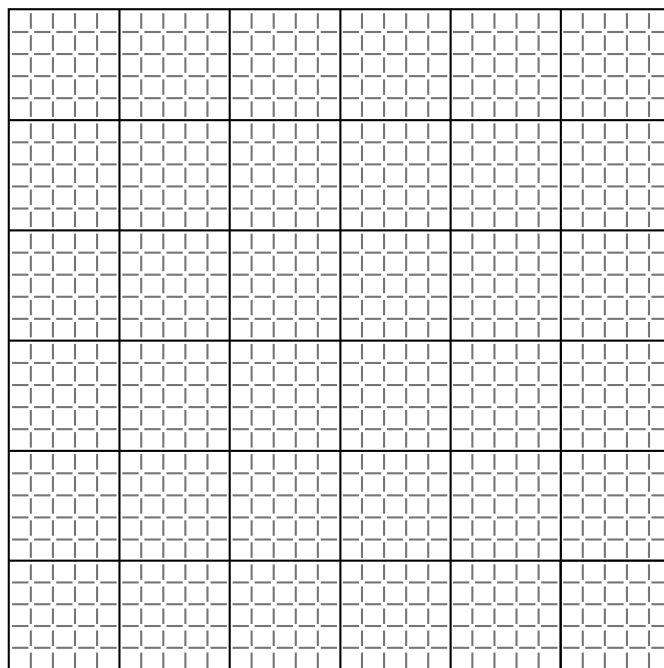
Table 1

s (m)	h (m)
0.020	0.02
0.040	0.04
0.060	0.13
0.080	0.18
0.100	0.33

Part C

- i. **Label** the axes of the grid provided with measured or calculated quantities. Include units, as appropriate. The graphed quantities should yield a linear graph that can be used to determine k_{new} .
- ii. On the grid provided, create a graph of the quantities indicated in part C (i).
- Clearly **label** the vertical and horizontal axes with numerical scales.
 - Plot** the corresponding data points on the grid.
 - Table 2 is provided in your booklet for scratch work and will not be scored.

Quantity (units, if appropriate)



Quantity (units, if appropriate)

- iii. **Draw** a best-fit line for the data graphed in part C (ii).

Part D

Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for k_{new} .

Question 2: Version J

2. In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

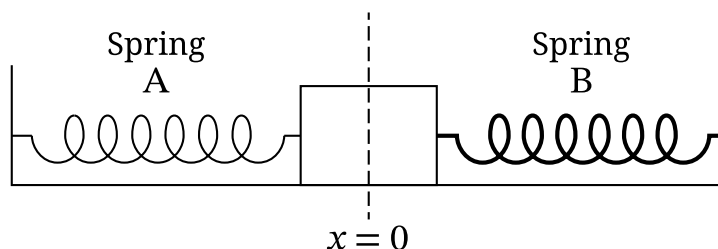


Figure 1

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

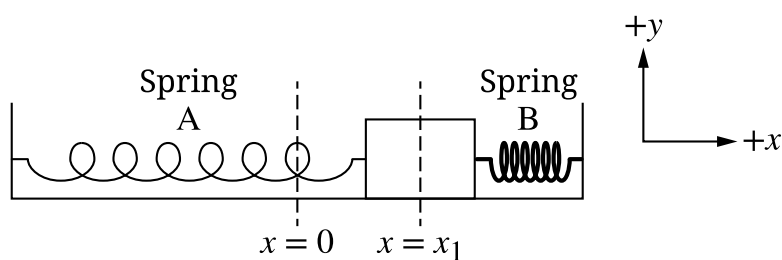


Figure 2

- A. An energy bar chart can be used to represent the elastic potential energy U_A of Spring A, the elastic potential energy U_B of Spring B, and the kinetic energy K_{block} of the block. On the energy bar chart in Figure 3, **draw** shaded bars to represent the energy of the system for when the block is at $x = x_1$.
- The height of the shaded bars should be proportional to the relative values of U_A , U_B , and K_{block} .
 - Any energy that is equal to zero should be represented by a distinct line on the zero-energy line.

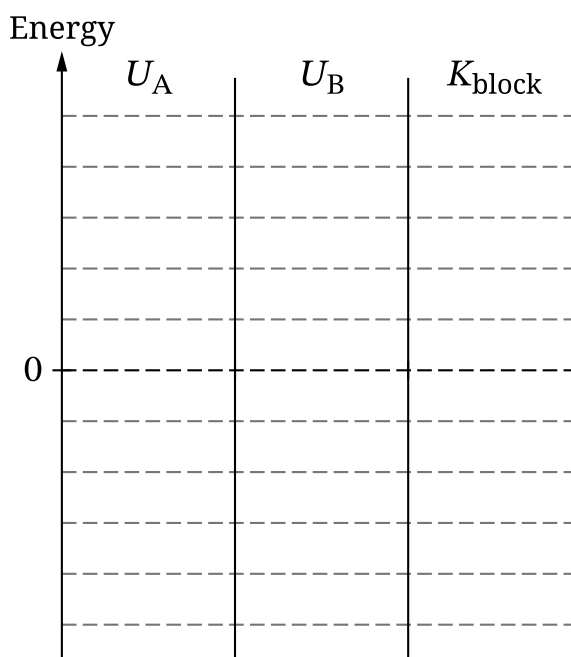
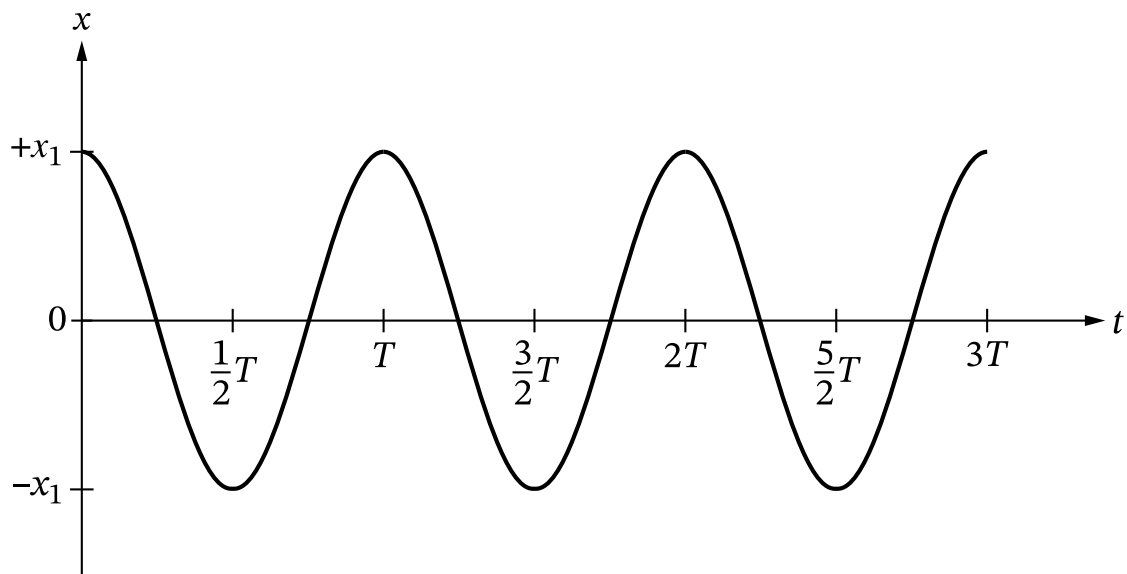


Figure 3

- B. The block is released from rest at $x = x_1$ and begins to oscillate. **Derive** an expression for the speed v of the block as the block passes through $x = \frac{1}{2}x_1$. Express your answer in terms of m , k , x_1 , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

- C. In Scenario 1, the block oscillates with period T . The position x of the block in Scenario 1 as a function of time t is shown in Figure 4.

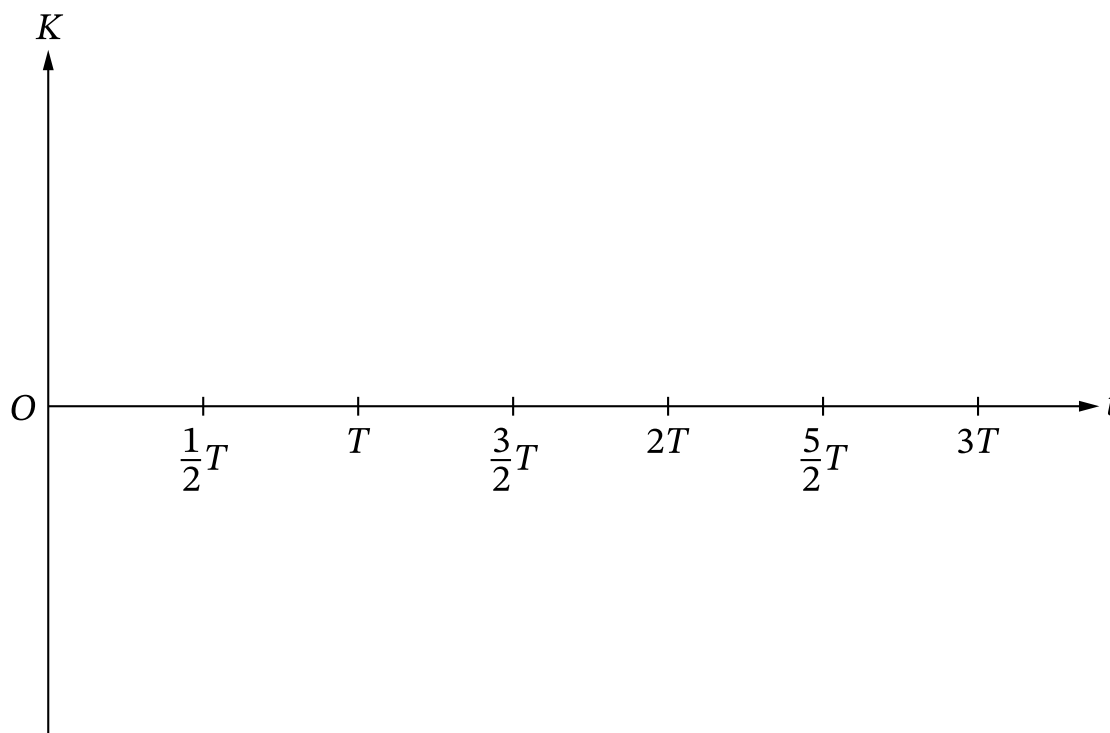


Scenario 1

Figure 4

In Scenario 2, the block-springs system is placed on a new surface. There is friction between the block and the new surface. The block is again moved to the same position $x = x_1$ and released from rest. The block completes multiple oscillations with the same period as in Scenario 1 before coming to rest.

On the axes shown in Figure 5, **sketch** a graph of the kinetic energy K of the block as a function of t for Scenario 2.



Scenario 2

Figure 5

- D.** In Scenario 3, the block is replaced with a new block of larger mass. The coefficient of kinetic friction between the new block and the surface in Scenario 3 is the same as the coefficient of kinetic friction between the original block and the surface in Scenario 2.

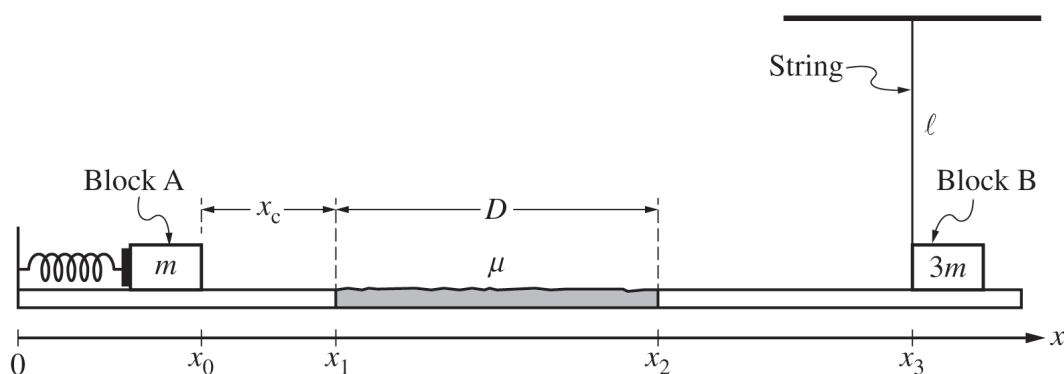
The new block is moved to position $x = x_1$ and released from rest. The kinetic energy of the new block is plotted as a function of time.

Describe how one feature of the graph of K as a function of t in Scenario 3 would differ from the graph you drew in Figure 5 for Scenario 2.

Briefly **justify** your answer.

Begin your response to **QUESTION 1** on this page.**PHYSICS C: MECHANICS****SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

Figure 1

1. Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D , where the coefficient of kinetic friction between Block A and the surface is μ . Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c .

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c . Block A moves with speed v , and the spring is at its equilibrium position.
- At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction.
- At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m , k , D , μ , x_c , and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1 .

ii. **Derive** an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3 .

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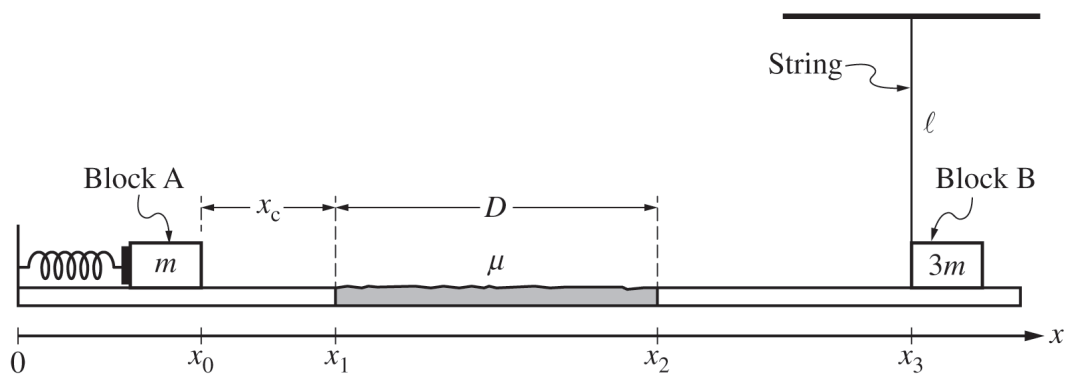
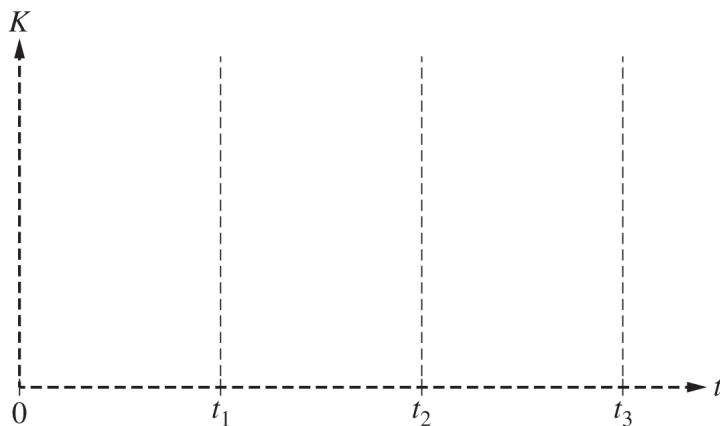
Continue your response to **QUESTION 1** on this page.Note: Figure not drawn to scale.

Figure 1

(b)

- i. On the following axes, **sketch** a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to time t_3 .



- ii. Use principles of work and energy to **justify** the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

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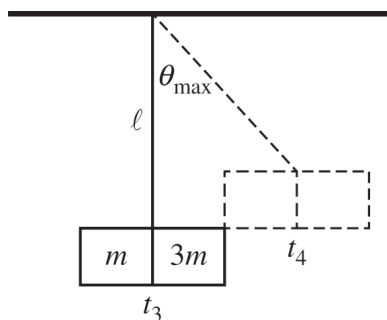
Continue your response to **QUESTION 1** on this page.Note: Figure not drawn to scale.

Figure 2

After the collision, the two-block system instantaneously comes to rest at time t_4 , which occurs when the string makes a small angle $\theta_{\max}$ with the vertical, as shown in Figure 2. For times $t > t_4$, the system oscillates with frequency f_ℓ . The support holding the string is raised, and the procedure is then repeated using a new string of length 2ℓ .

(c) **Indicate** how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length 2ℓ will compare to the frequency of oscillation f_ℓ from the original procedure.

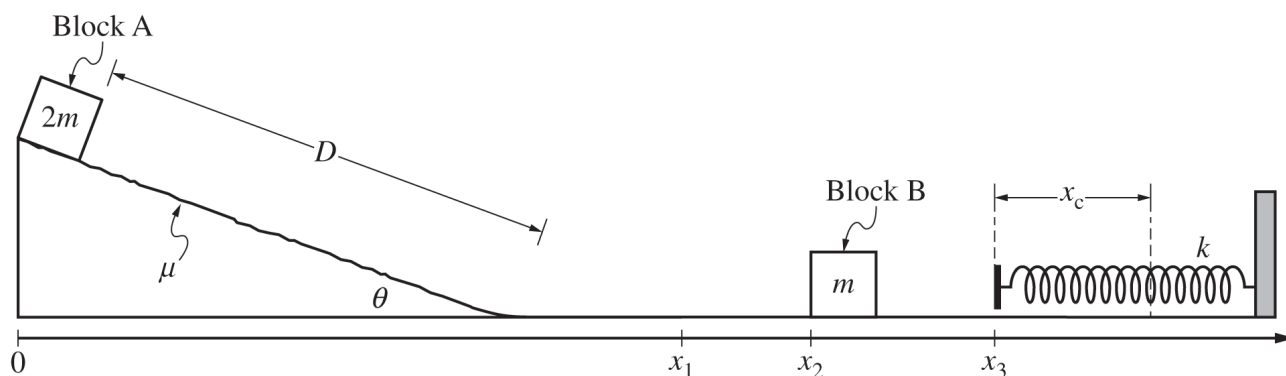
_____ $f_{2\ell} > f_\ell$ _____ $f_{2\ell} < f_\ell$ _____ $f_{2\ell} = f_\ell$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 1** on this page.**PHYSICS C: MECHANICS****SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

1. Blocks A and B of masses $2m$ and m , respectively, are arranged in a setup consisting of a ramp that makes an angle θ with a smooth horizontal table and an ideal spring of spring constant k fixed to a wall, as shown. Block A is held at rest a distance D up the ramp, and Block B is at rest on the horizontal table. The coefficient of kinetic friction between Block A and the rough ramp is μ in the region of length D , and there is negligible friction between the blocks and the smooth table.

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Continue your response to **QUESTION 1** on this page.

At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.

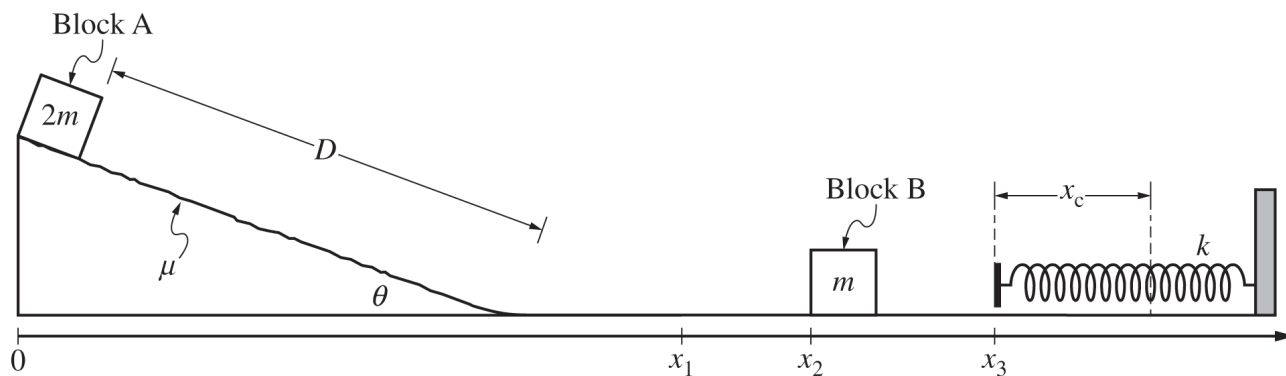
- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$.
- At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B.
- At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position.
- At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m , θ , D , μ , x_c , and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1 .

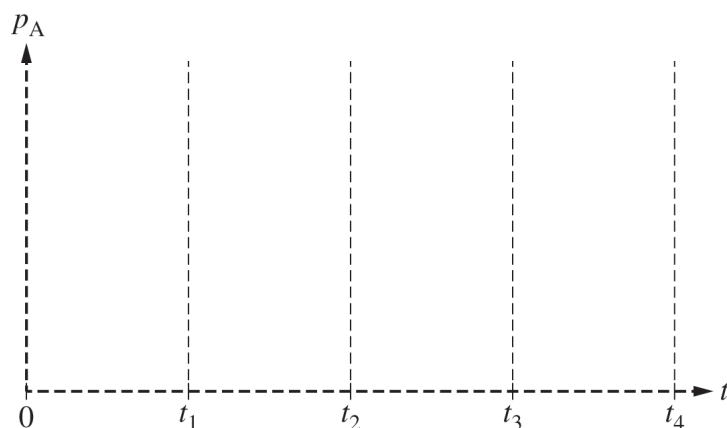
ii. **Derive** an expression for the spring constant k of the spring.

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Continue your response to **QUESTION 1** on this page.Note: Figure not drawn to scale.

(b)

- i. On the following axes, **sketch** a graph of the magnitude of the momentum p_A of Block A as a function of time t from $t = 0$ to t_4 .



- ii. Use principles of forces to **justify** the graph drawn in part (b)(i) for the time interval $t = t_3$ to $t = t_4$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

For times $t > t_4$, the two-block-spring system oscillates with period T_O . The procedure is then repeated using a new ramp, where there is negligible friction between Block A and the ramp.

(c) **Indicate** how the new period of oscillation T_N in the procedure that uses the new ramp compares with the period of oscillation T_O from the original procedure.

_____ $T_N > T_O$ _____ $T_N < T_O$ _____ $T_N = T_O$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

PHYSICS C: MECHANICS**SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

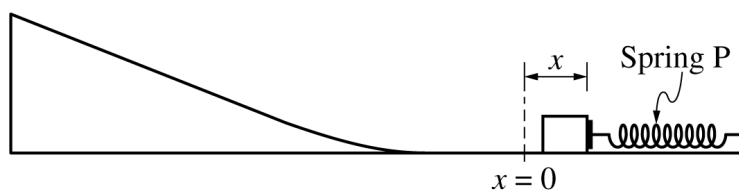


Figure 1

A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x , as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\text{max,P}}$. Frictional forces between the block and all surfaces are negligible.

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Continue your response to **QUESTION 1** on this page.

A student compares $h_{\max,P}$ for Spring P with the maximum height $h_{\max,Q}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x that the spring is compressed, as shown in Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0 \text{ N/m}$, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0 \text{ N/m}^{1/2}$.

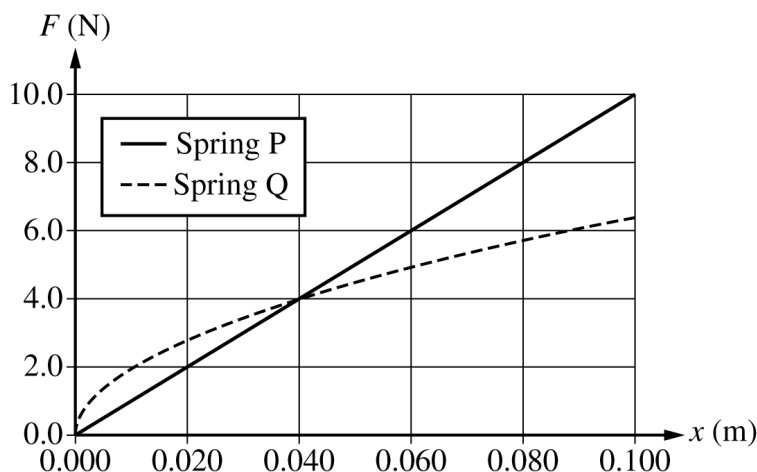
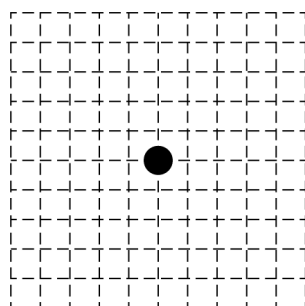


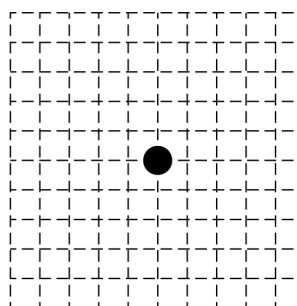
Figure 2

- (a) For the experiment, the block is pushed against one of the springs, compressing the spring a distance $x = 0.010 \text{ m}$. The block is then released from rest. In Trial 1, Spring P is used, and in Trial 2, Spring Q is used. On the following dots, representing the block in Trials 1 and 2, draw and label the forces (not components) that are exerted on the block at the instant the block is released. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. The lengths of the horizontal vectors should represent the relative magnitude of the horizontal forces and the lengths of the vertical vectors should represent the relative magnitude of the vertical forces.

Trial 1
(Spring P)



Trial 2
(Spring Q)

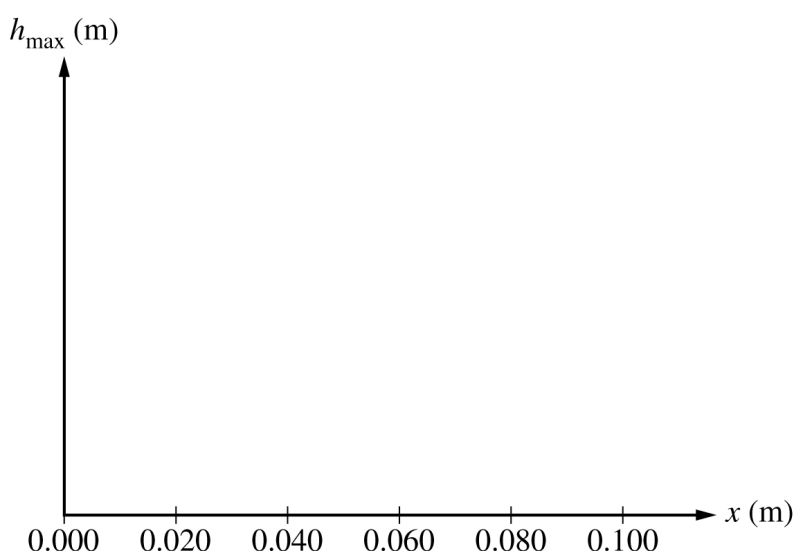
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(b)

i. What feature(s) of the graph in Figure 2 could be used to estimate the work done on the block by each spring as each spring is compressed?

ii. There is one compression distance x_0 for which the maximum height h_{max} reached by the block is the same regardless of which spring, Spring P or Spring Q, is used. Predict whether the value of x_0 is greater than, less than, or equal to 0.040 m. Use the graph in Figure 2 to justify your answer.

iii. On the axes provided, for both Spring P and Spring Q, sketch a graph of the maximum height h_{max} reached by the block as a function of the distance each spring is compressed for values of x ranging from 0 to 0.100 m. Clearly label the curve for Spring P and Spring Q.



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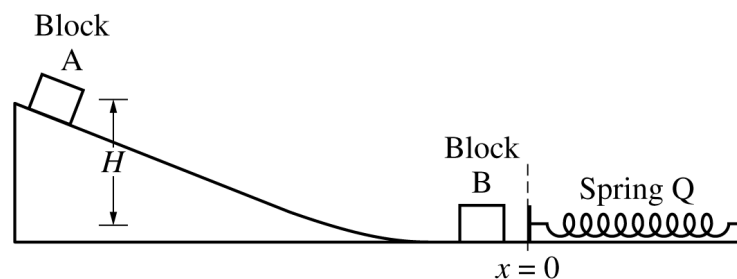
Continue your response to **QUESTION 1** on this page.

Figure 3

(c) Spring Q is attached to the wall and is at equilibrium, as shown in Figure 3. Block A has a mass of 0.120 kg and Block B has a mass of 0.070 kg. Block A is held at rest at the top of the ramp and Block B is at rest on the horizontal surface. The student releases Block A, and it moves down the ramp and collides with Block B. The change in vertical height of Block A is $H = 0.75$ m. After the collision, the blocks stick together and move to the right, compressing the spring. Frictional forces between the blocks and all surfaces are negligible.

i. Calculate the velocity of the two-block system immediately after the collision between Blocks A and B.

ii. Calculate the maximum compression of the spring.

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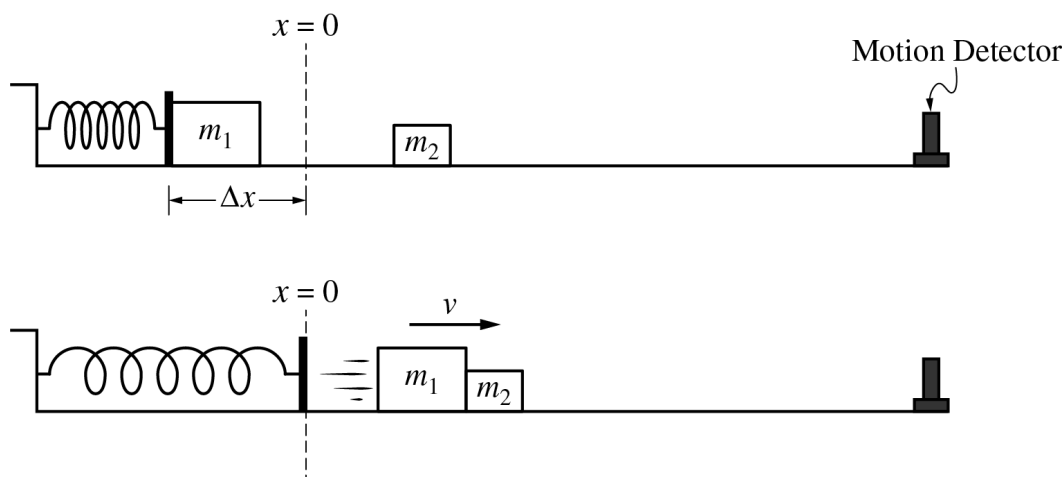
- (d) Spring Q where $F_Q(x) = Cx^{1/2}$ is replaced by a different nonlinear spring, Spring R, and the procedure described in part (c) is repeated. For Spring R, $F_R(x) = Dx^{1/2}$. The maximum compression of Spring R is greater than the maximum compression of Spring Q. Which of the following correctly compares the constants C and D ?

_____ $C < D$ _____ $C > D$ _____ $C = D$

Briefly justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to Question 2 on this page.



Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx . The spring constant of the spring is k . Block 2 has mass m_2 , where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C , the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v , as shown. Frictional effects are negligible.

- (a) The impulse on Block 1 from the spring during the time interval $0 < t < t_C$ is J_S . The impulse on Block 1 from Block 2 during the collision is J_2 . Which of the following expressions correctly compares the magnitudes of J_S and J_2 ?

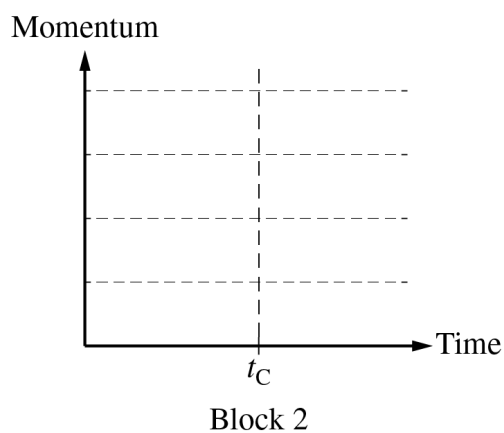
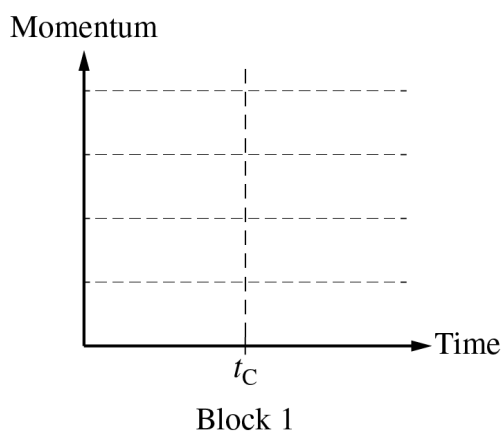
☐ $J_S > J_2$ ☐ $J_S < J_2$ ☐ $J_S = J_2$

Justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

- (b) On the following axes, draw graphs of the magnitude of the momentum of each block as a function of time, before and after t_C . The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.

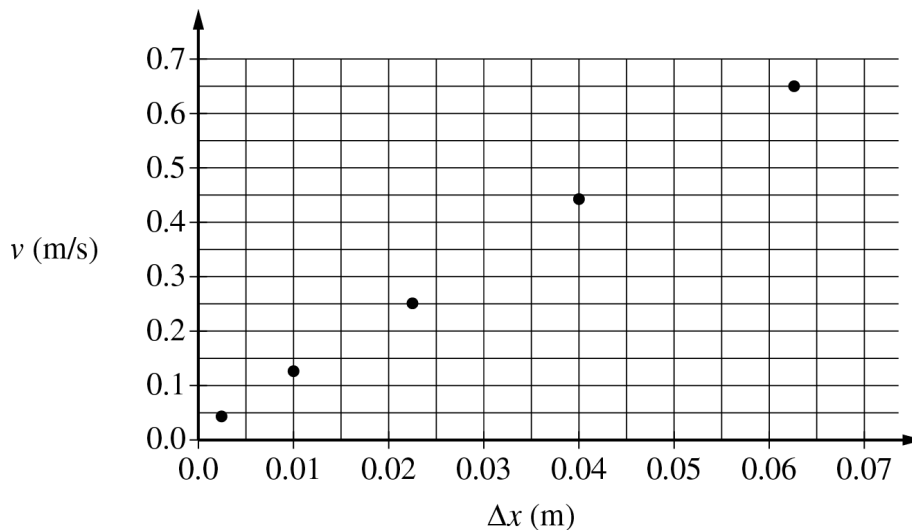


- (c) Show that the velocity v of the two-block system after the collision is given by the equation $v = \frac{\sqrt{km_1}}{m_1 + m_2} \Delta x$.

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Continue your response to **QUESTION 2** on this page.

- (d) A group of students use the setup to perform an experiment. They measure the mass of Block 1 to be $m_1 = 0.500$ kg, and the spring constant k of the spring to be 150 N/m. The mass of Block 2 is unknown. They perform several trials and in each trial the spring is compressed a different distance Δx and the final velocity v of the two-block system is measured. They graph v as a function of Δx , as shown below.



- Draw a line that represents the best fit to the data points shown.
- Use the best-fit line to calculate the mass of Block 2.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

- (e) After the experiment, the students use a balance to measure the mass of Block 2 and find it to be greater than what was determined in part (d). To explain this discrepancy, one of the students proposes that the spring constant was incorrectly measured at the beginning of the experiment. The students measure the spring constant again and record a new value, k' .

Should the students expect that k' be greater than 150 N/m , less than 150 N/m , or equal to 150 N/m ?

$k' > 150 \text{ N/m}$ $k' < 150 \text{ N/m}$ $k' = 150 \text{ N/m}$

Justify your answer.

GO ON TO THE NEXT PAGE.



Mech.2.

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

- (a) On the dots below, which represent the blocks of mass $2M$ and $3M$, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

Block of Mass $2M$

Block of Mass $3M$



The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance D , as shown above. Express your answers to parts (b), (c), and (d) in terms of M , B , v_0 , and physical constants, as appropriate.

- (b) Derive an expression for the speed of the blocks immediately after the collision.
(c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.
(d) Derive an expression for the maximum distance D that the spring is compressed.

(e)

- i. In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

☐ Left ☐ Right ☐ The net force on the block of mass $2M$ is zero.

Justify your answer.

- ii. Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

☐ The magnitude of the net force is greater on the block of mass $2M$.
☐ The magnitude of the net force is greater on the block of mass $3M$.
☐ The magnitude of the net force on each block has the same nonzero value.
☐ The magnitude of the net force on each block is zero.

Justify your answer.

- (f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

☐ Yes ☐ No

Justify your answer.

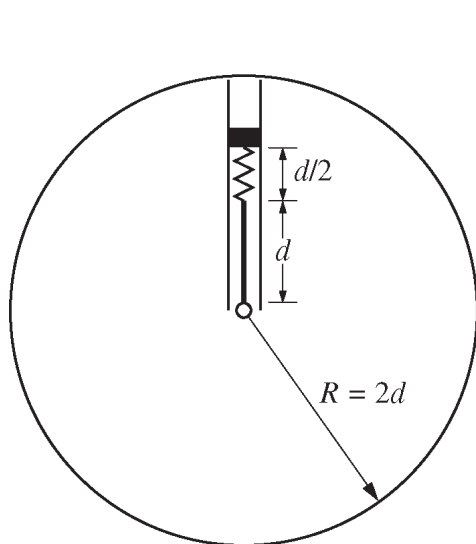


Figure 1

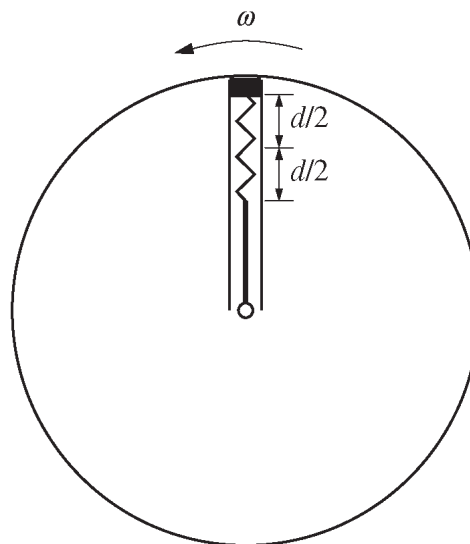


Figure 2

Mech.3.

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

	Mass	Rotational Inertia
Block	m	
Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)
Platform	$m_P = 5m$	$\frac{m_P R^2}{2}$ (about the central axis)

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω . Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω . Express all algebraic answers in terms of m , d , ω , and physical constants, as appropriate.

- (a) Derive an expression for the spring constant of the spring.
- (b)
 - i. Determine an expression for the rotational inertia of the block around the axis of the platform.
 - ii. Derive an expression for the rotational inertia of the entire system about the axis of the platform.
- (c) Determine an expression for the angular momentum of the entire system about the axis of the platform.

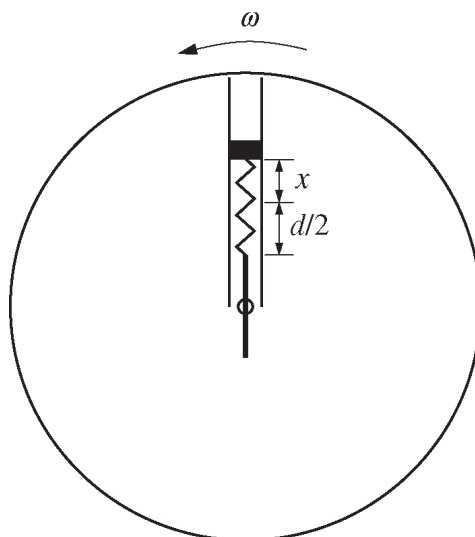


Figure 3

While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed ω is maintained by the motor driving the platform.

- (d) Derive an expression for the distance x that the spring is stretched when the rod reaches the position shown in Figure 3 above.

For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

- (e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

_____ Increasing _____ Decreasing _____ Staying the same

Justify your answer.

- (f) In order to keep the system rotating with constant angular speed ω , is the motor doing positive work, negative work, or no work on the rotating system?

_____ Positive _____ Negative _____ No work

Justify your answer.

- (g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.

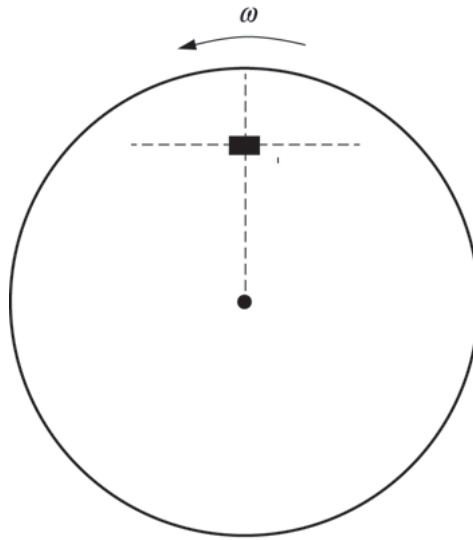


Figure 4

STOP

END OF EXAM