

Question 2: Free-Response Question

15 points

- (a) For a multi-step derivation that includes Newton's second law of motion 1 point

For indicating that the net force exerted on the cylinder includes only the gravitational force and a drag force 1 point

Example Response

$$F_{\text{net}} = F_g - F_{\text{drag}}$$

For a correct differential equation that is in terms of the given variables 1 point

Scoring Note: Variables do not have to be separated for this point to be earned.

Example Response

$$m \frac{dv}{dt} = mg - bv^2$$

Example Solution

$$\Sigma F = ma$$

$$F_g - F_{\text{drag}} = ma_y$$

$$mg - bv^2 = ma_y$$

$$m \frac{dv}{dt} = mg - bv^2$$

Total for part (a) 3 points

- (b)(i) For a vertical line labeled t_1 at the approximate location at which the line becomes horizontal 1 point

Example Response

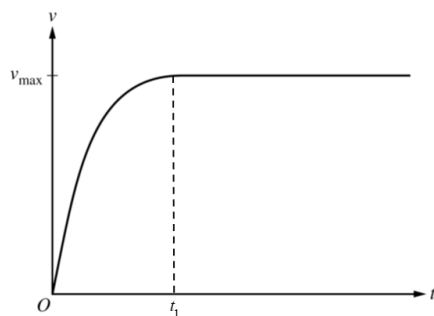


Figure 2

- (b)(ii) For relating t_1 to the time at which the velocity versus time graph is constant or the slope of the line is zero 1 point

For indicating that a constant velocity indicates that the net force is zero 1 point

Example Response

Because the sketched line is horizontal after t_1 , the velocity is constant. If the velocity is constant, then the acceleration is zero. Therefore, the net force is zero, which means that the gravitational and drag forces are equal in magnitude.

Total for part (b) 3 points

- (c) For selecting "Equal to" with an attempt at a relevant justification 1 point

For a correct justification 1 point

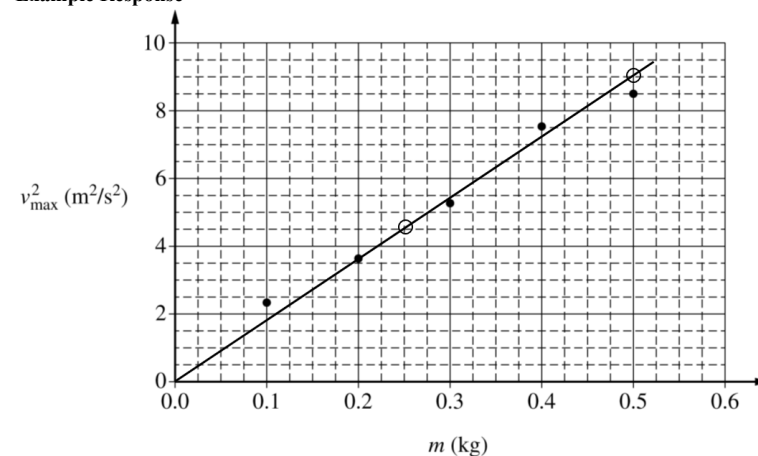
Example Response

At its peak, the cylinder will have a speed of 0 m/s. Therefore, the cylinder would reach the same v_{max} as if the student had dropped the cylinder from rest at that height. Because the cylinder reached v_{max} from the initial drop, the two max speeds are equal.

Total for part (c) 2 points

- (d)(i) For drawing an appropriate line of best fit that approximates the data 1 point

Example Response



m (kg)

- (d)(ii) For calculating a value for the slope of the line using two points on the best-fit line **1 point**

Scoring Note: Using data points that fall on the best-fit line is acceptable.

Example Response

$$\text{slope} = \frac{9 \text{ m}^2/\text{s}^2 - 4.5 \text{ m}^2/\text{s}^2}{0.5 \text{ kg} - 0.25 \text{ kg}}$$

For using the correct relationship between the slope of the best-fit line and the value of b **1 point**

Example Response

$$\text{slope} = \frac{g}{b}$$

For a calculated value of b that is $0.45 \text{ kg/m} \leq b \leq 0.75 \text{ kg/m}$ **1 point**

Example Response

$$b = 0.544 \text{ kg/m}$$

Example Solution

$$mg - bv^2 = 0$$

$$bv^2 = mg$$

$$\frac{v^2}{m} = \frac{g}{b}$$

$$\text{slope} = \frac{g}{b}$$

$$b = \frac{g}{\text{slope}}$$

$$b = \frac{9.8 \text{ m/s}^2}{\frac{9 \text{ m}^2/\text{s}^2 - 4.5 \text{ m}^2/\text{s}^2}{0.5 \text{ kg} - 0.25 \text{ kg}}}$$

$$b = 0.544 \text{ kg/m}$$

Total for part (d) 4 points

- (e)(i) For indicating that the length of the cylinder should be graphed **1 point**

For indicating that the maximum velocity of the cylinder should be graphed **1 point**

- (e)(ii) For describing how the quantities graphed are related to the conclusions of the experiment **1 point**

Example Response

The slope of the length vs. maximum velocity graph can be used to determine if length affects terminal velocity.

Total for part (e) 3 points

Total for question 2 15 points

Question 1: Free-Response Question

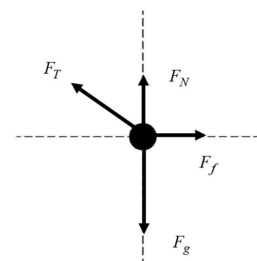
15 points

- (a) For correctly drawing and labeling the force of gravity and the normal force on the block of mass m **1 point**

For correctly drawing and labeling the force of friction on the block of mass m **1 point**

For correctly drawing and labeling the force of tension on the block of mass m **1 point**

Example Response



Scoring Notes:

- Examples of appropriate labels for the force due to gravity include: F_G , F_g , F_{grav} , W , mg , Mg , “grav force,” “F Earth on block,” “F on block by Earth” $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity. F_n , F_N , N , “normal force,” “ground force,” or similar labels may be used for the normal force. F_{string} , F_s , F_T , F_{Tension} , T , “string force,” “tension force,” or similar labels may be used for the tension force exerted by the string.
- A response with extraneous forces or vectors can earn a maximum of two points.

Total for part (a) 3 points

- (b) For a trigonometric expression relating the angle to the horizontal distance of the block from the left corner of the table, x , **1 point**

For any correct trigonometric expression for θ in terms of the given quantities **1 point**

Example Responses

First Point	Second Point
$\sin \theta = \frac{H}{\sqrt{H^2 + x^2}}$	$\theta = \sin^{-1} \left(\frac{H}{\sqrt{H^2 + x^2}} \right)$
$\cos \theta = \frac{x}{\sqrt{H^2 + x^2}}$	$\theta = \cos^{-1} \left(\frac{x}{\sqrt{H^2 + x^2}} \right)$
$\tan \theta = \frac{H}{x}$	$\theta = \tan^{-1} \left(\frac{H}{x} \right)$

Total for part (b) 2 points

- (c)(i) For using Newton's second law to sum the forces in the vertical direction and write an equation that is consistent with part (a) **1 point**

$$\Sigma F_y = ma$$

$$F_{net,y} = F_N + F_{T,y} - F_g = 0$$

$$F_N = F_g - F_{T,y}$$

For correctly substituting the vertical component of the tension in terms of the given variables consistent with part (b) **1 point**

$$F_N = mg - F_T \sin \theta$$

$$F_N = mg - F_T \left(\frac{H}{\sqrt{H^2 + x^2}} \right)$$

- (c)(ii) For using Newton's second law to sum the forces in the horizontal direction and write an equation that is consistent with part (a) **1 point**

$$F_{net,x} = F_{T,x} - F_f$$

For correctly substituting the horizontal component of the force of tension in terms of the given variables consistent with part (b) **1 point**

$$F_{net,x} = F_T \cos \theta - F_f$$

$$F_{net,x} = F_T \left(\frac{x}{\sqrt{H^2 + x^2}} \right) - F_f$$

For correctly substituting the expression for the normal force from part (c)(i) into the expression for the force of friction **1 point**

$$F_{net,x} = F_T \left(\frac{x}{\sqrt{H^2 + x^2}} \right) - \mu_k F_N$$

$$F_{net,x} = F_T \left(\frac{x}{\sqrt{H^2 + x^2}} \right) - \mu_k \left(mg - F_T \left(\frac{H}{\sqrt{H^2 + x^2}} \right) \right)$$

Total for part (c) 5 points

- (d) For any indication that the work done on the block by the string is due only to the horizontal component of the tension in the string **1 point**

$$W = \int F_{T,x} dx$$

For using the horizontal component of the force of tension consistent with part (c) **1 point**

For indicating that work is the integral of the force with respect to x , including limits or a constant of integration **1 point**

$$W = \int_{x=L}^{x=0} -F_T \left(\frac{x}{\sqrt{H^2 + x^2}} \right) dx$$

Total for part (d) 3 points

- (e) For selecting "More work ..." with an attempt at a relevant justification **1 point**

For a correct justification relating the smaller angle to a larger component of the force of tension, thus resulting in greater work. **1 point**

Example Response

F_T stays the same for both halves, the displacement is the same in both halves, but from $x = L$ to $x = L/2$ the angle is smaller, resulting in a larger component of the tension force that aligns with the displacement.

Total for part (e) 2 points

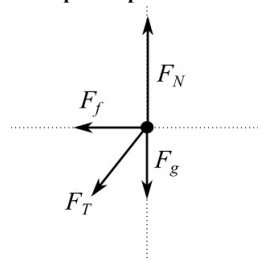
Total for question 1 15 points

Question 1: Free-Response Question

15 points

- (a) For correctly drawing and labeling the force of gravity and the normal force on the sled 1 point
- For correctly drawing and labeling the force of friction on the on the sled 1 point
- For correctly drawing and labeling the force of tension on the on the sled 1 point

Example Response



Scoring Notes:

- Examples of appropriate labels for the force due to gravity include: F_G , F_g , F_{grav} , W , mg , Mg , “grav force,” “F Earth on block,” “F on block by Earth,” $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity. F_n , F_N , N , “normal force,” “ground force,” or similar labels may be used for the normal force. F_{string} , F_s , F_T , F_{Tension} , T , “string force,” “tension force,” or similar labels may be used for the tension force exerted by the string.
- A response with extraneous forces or vectors can earn a maximum of two points.

Total for part (a) 3 points

- (b) For any correct trigonometric expression for θ in terms of the given quantities 1 point

$$\sin \theta = \frac{x}{\sqrt{y^2 + x^2}} \quad \therefore \theta = \sin^{-1} \left(\frac{x}{\sqrt{y^2 + x^2}} \right)$$

OR

$$\cos \theta = \frac{y}{\sqrt{y^2 + x^2}} \quad \therefore \theta = \cos^{-1} \frac{y}{\sqrt{y^2 + x^2}}$$

OR

$$\tan \theta = \frac{x}{y} \quad \therefore \theta = \tan^{-1} \frac{x}{y}$$

Total for part (b) 1 point

- (c)(i) For beginning the derivation with Newton’s second law to write an equation that is consistent with part (a). 1 point

$$\Sigma F = ma$$

$$F_N - F_{T,y} - F_g = 0$$

$$F_N = F_{T,y} + F_g$$

Scoring Note: Derivation must start with a statement of Newton’s second law to earn this point.

- For correctly substituting the vertical component of the tension in terms of the given variables 1 point

$$F_N = mg + F_T \cos \theta$$

$$F_N = mg + F_T \left(\frac{y}{\sqrt{y^2 + x^2}} \right)$$

- (c)(ii) For summing the forces in the horizontal direction, consistent with parts (a) and (b) 1 point

$$F_{\text{net}} = -F_{T,x} - F_f$$

- For correctly substituting the horizontal component of the force of tension in terms of the given variables 1 point

$$F_{\text{net}} = -F_T \sin \theta - F_f$$

$$F_{\text{net}} = -F_T \left(\frac{x}{\sqrt{y^2 + x^2}} \right) - F_f$$

- For correctly substituting the expression for the normal force from part (c)(i) into the expression for the force of friction 1 point

$$F_{\text{net}} = -F_T \left(\frac{x}{\sqrt{y^2 + x^2}} \right) - \mu_k F_N$$

$$F_{\text{net}} = -F_T \left(\frac{x}{\sqrt{y^2 + x^2}} \right) - \mu_k \left(mg + F_T \left(\frac{y}{\sqrt{y^2 + x^2}} \right) \right)$$

Total for part (c) 5 points

- (d) For using the integral definition of work to derive an expression for the work done by the force of tension **1 point**

$$W = \int F \cdot dx$$

- For any indication that the work done on the sled by the string is due only to the horizontal component of the tension in the string **1 point**

$$W = \int F_{T,x} dx$$

- For using the horizontal component of the force of tension consistent with part (b) or (c) **1 point**

$$W = \int_{x=0}^{x=L} -F_T \left(\frac{x}{\sqrt{y^2 + x^2}} \right) dx$$

- For correct limits of integration, $x=0$ to $x=L$, and indicating that the work done is negative let $u = y^2 + x^2$, then $du = 2x dx$ **1 point**

$$W = -F_T \int_{x=0}^{x=L} \frac{1}{2} \left(\frac{1}{\sqrt{u}} \right) du$$

$$W = -F_T \left(\sqrt{y^2 + x^2} \right) \Big|_{x=0}^{x=L}$$

$$W = -F_T \left(\sqrt{y^2 + L^2} - \sqrt{y^2} \right)$$

Total for part (d) 4 points

- (e) For selecting “ $E_1 > E_2$ ” with an attempt at a relevant justification **1 point**

- For a justification that correctly relates the force of friction to the normal force, and the normal force to the position or the angle **1 point**

Example Response

The vertical component of the string force is the largest when the string is more vertical ($0 < x < L$). A larger vertical component of the string tension leads to a larger normal force and, hence, a larger friction force.

Total for part (e) 2 points

Total for question 1 15 points

Question 1: Free-Response Question

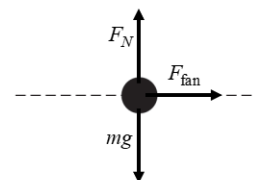
15 points

- (a) For correctly drawing and labeling all the forces on the cart on the flat surface **1 point**
- For correctly drawing and labeling the weight of the cart on the incline **1 point**
- For correctly drawing and labeling the normal force on the cart on the incline **1 point**
- For correctly drawing and labeling the force of the fan on the cart on the incline **1 point**

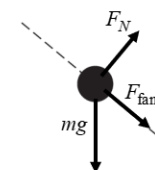
Scoring note: A maximum of three points can be earned if there are any extraneous vectors.

Example responses for part (a)

Cart on Horizontal Track



Cart on Incline



Scoring note: Examples of appropriate labels for the force due to gravity include: F_G , F_g , F_{grav} , W , mg , Mg , “grav force,” “F Earth on cart,” “F on cart by Earth,” $F_{\text{Earth on cart}}$, $F_{E,\text{Cart}}$, $F_{\text{Cart},E}$. The labels G or g are not appropriate labels for the force due to gravity. F_n , F_N , N , “normal force,” “ground force,” or similar labels may be used for the normal force.

Total for part (a) 4 points

- (b) For correctly applying Newton’s second law for the cart on the flat surface **1 point**

$$F_{\text{fan}} = m_{\text{cart}} a_1$$

- For the correct answer with units **1 point**

$$F_{\text{fan}} = (0.50 \text{ kg})(0.8 \text{ m/s}^2) = 0.40 \text{ N}$$

Total for part (b) 2 points

- (c) For including the correct component of weight in a Newton's second law equation for the cart **1 point**

$$F_{\text{fan}} + mg \sin \theta = ma_2$$

For a correct Newton's second law equation **1 point**

$$F_{\text{fan}} + mg \sin \theta = ma_2$$

For correct substitutions consistent with part (b) into the above equation **1 point**

$$mg \sin \theta = ma_2 - F_{\text{fan}}$$

$$\theta = \sin^{-1} \left(\frac{ma_2 - F_{\text{fan}}}{mg} \right)$$

$$\theta = \sin^{-1} \left(\frac{(0.50 \text{ kg})(2.4 \text{ m/s}^2) - (0.50 \text{ kg})(0.8 \text{ m/s}^2)}{(0.50 \text{ kg})(9.8 \text{ m/s}^2)} \right)$$

$$\theta = 9.4^\circ$$

Total for part (c) 3 points

- (d) For selecting "No" and an attempted justification **1 point**

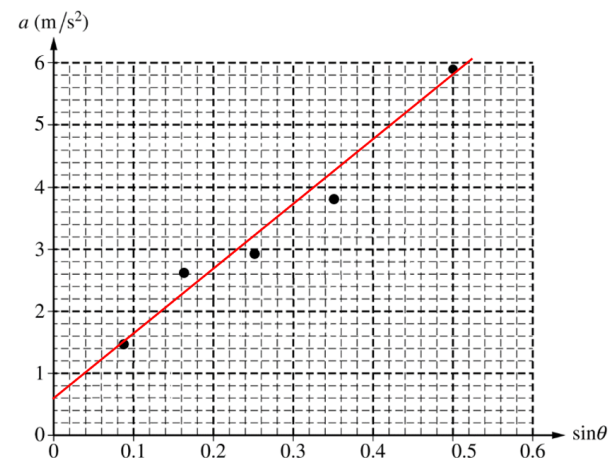
For a correct justification **1 point**

Example response for part (d)

The mass of the cart cancels out in the equation used to find the angle of the incline.

Total for part (d) 2 points

- (e) i. For drawing an appropriate best-fit line **1 point**



- ii. For correctly calculating slope using two points from the best-fit line **1 point**

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{(5 - 1)(\text{m/s}^2)}{(0.42 - 0.04)} = 10.52 \text{ m/s}^2$$

For correctly using an expression that relates the slope to the acceleration due to gravity **1 point**

$$F_{\text{net}} = F_{\text{fan}} + mg \sin \theta = ma$$

$$\therefore a = g \sin \theta + \frac{F_{\text{fan}}}{m}$$

$$\text{from } y = mx + b$$

$$a = (\text{slope}) \sin \theta + (\text{y-intercept})$$

$$\therefore \text{slope} = g = 10.52 \text{ m/s}^2$$

Total for part (e) 3 points

- (f) For a correct explanation **1 point**

Example response for part (f)

The mass of the cart is in the denominator of the y-intercept, so increasing the mass decreases the y-intercept without changing the rest of the graph. So the new line of data is predicted to be parallel to and below the original line.

Total for question 1 15 points

Question 2: Free-Response Question

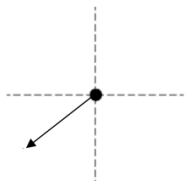
15 points

- (a) For a single acceleration vector pointing down and to the left and attempting a justification 1 point

For a correct justification 1 point

Example response for part (a)

The block is changing direction with a centripetal component of the acceleration toward the center of the circle and a gravitational acceleration downward. Therefore, the acceleration of the block will be down and to the left.



Total for part (a) 2 points

- (b) i. For correctly using conservation of energy for the block at point B 1 point

$$K_A + U_{gA} = K_B + U_{gB}$$

$$\frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B$$

For correctly substituting into the above equation 1 point

$$0 + mgh = \frac{1}{2}mv_B^2 + mgR$$

$$v_B = \sqrt{2g(h - R)}$$

- ii. For correctly substituting the expression for v_B from part (b)(i) into an expression for centripetal force 1 point

$$F_c = \frac{mv^2}{r} = \frac{mv_B^2}{R} = \frac{m(\sqrt{2g(h - R)})^2}{R}$$

For correctly substituting into an equation for vector addition to derive an expression for the net force at point B 1 point

$$F_{\text{net}} = \sqrt{F_c^2 + (mg)^2}$$

$$F_{\text{net}} = \sqrt{\left(\frac{mv_B^2}{R}\right)^2 + (mg)^2} = \sqrt{\left(\frac{2mg}{R}(h - R)\right)^2 + (mg)^2}$$

Total for part (b) 4 points

- (c) For correctly using conservation of energy for the speed of the block at point C 1 point

$$K_A + U_{gA} = K_C + U_{gC}$$

$$\frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_C^2 + mgh_C$$

$$0 + mgh = \frac{1}{2}mv_C^2 + mg(2R)$$

$$v_C = \sqrt{2g(h - 2R)}$$

For correctly applying Newton's second law at point C 1 point

$$F_c = \frac{mv_C^2}{r}$$

$$F_N + mg = \frac{mv_C^2}{r}$$

Set the normal force equal to zero

$$0 + mg = \frac{mv_C^2}{R} \therefore v_C = \sqrt{Rg}$$

For combining the two equations above 1 point

$$\sqrt{Rg} = \sqrt{2g(h - 2R)} \therefore R = 2(h - 2R) \therefore R/2 = h - 2R$$

$$h = 2.5R$$

Total for part (c) 3 points

- (d) For correctly using conservation of energy for the block compressing the spring 1 point

For correctly substituting the gravitational and elastic potential energies into an equation for conservation of energy 1 point

$$mgh = \frac{1}{2}kx_{\text{MAX}}^2$$

For correctly substituting for the spring constant k into the equation above 1 point

$$x_{\text{MAX}} = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2mgh}{mg/(2R)}} = \sqrt{4hR} = \sqrt{(4)(0.30 \text{ m})(0.10 \text{ m})} = 0.35 \text{ m}$$

Total for part (d) 3 points

- (e) i. For a correct justification **1 point**

Example response for part (e)(i)

Because the block does not make it through the loop at this height, it will not compress the spring.

- ii. For indicating that as the height increases, the compression of the spring increases **1 point**

For indicating that the height is proportional to the square of the compression of the spring **1 point**

Example response for part (e)(ii)

From the equation in part (c), the compression of the spring is directly proportional to the square root of the height that the block is released. Thus, the graph would be an x-axis parabola, as shown.

Total for part (e) 3 points

Total for question 2 15 points

Question 1: Free-Response Question**15 points**

- (a) For correctly evaluating Newton's second law equations for the system at rest **1 point**
 $m_2g - f = (m_1 + m_2)a = 0 \therefore m_2g = f$

For correctly substituting for static friction into above equation: **1 point**

$$m_2g = f = \mu_s F_N = \mu_s m_1g$$

$$\mu_s = \frac{m_2}{m_1} = \frac{(0.20 \text{ kg})}{(0.44 \text{ kg})} = 0.45$$

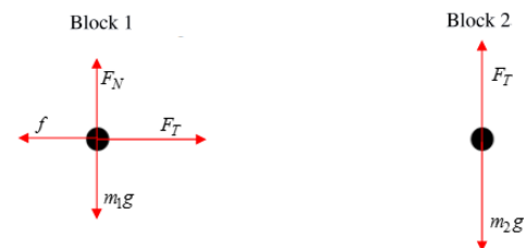
Total for part (a) 2 points

- (b) For correctly drawing and labeling the horizontal forces of friction and tension on block of mass m_1 **1 point**

For correctly drawing and labeling the vertical forces of weight and normal force on block of mass m_1 **1 point**

For correctly drawing and labeling forces of weight and tension on block of mass m_2 **1 point**

For indicating that the gravitational forces on each block are different **1 point**

Example responses for part (b)

Scoring note: Examples of appropriate labels for the force due to gravity include: F_G , F_g , F_{grav} , W , mg , Mg , “grav force,” “F Earth on block,” “F on block by Earth,” $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$, $F_{\text{Block,E}}$. The labels G or g are not appropriate labels for the force due to gravity. F_n , F_N , N , “normal force,” “ground force,” or similar labels may be used for the normal force.

Total for part (b) 4 points

(c)	For correctly evaluating Newton's second law equation for block 1: $T - f = m_1 a$	1 point
	For correctly evaluating Newton's second law equation for block 2: $m_2 g - T = m_2 a$	1 point
	Combining the two equations $m_2 g - f = (m_1 + m_2) a \therefore f = m_2 g - (m_1 + m_2) a$	
	Scoring note: Both points are earned for a single correct Newton's second law equation for the two-block system.	
	For correctly substituting for kinetic friction into above equation	1 point
	$f = \mu_k F_N = \mu_k m_1 g = m_2 g - (m_1 + m_2) a \therefore \mu_k = \frac{m_2 g - (m_1 + m_2) a}{m_1 g}$	
	$\mu_k = \frac{(0.20 \text{ kg})(9.8 \text{ m/s}^2) - (0.44 \text{ kg} + 0.20 \text{ kg})(2.3 \text{ m/s}^2)}{(0.44 \text{ kg})(9.8 \text{ m/s}^2)} = 0.11$	
	Total for part (c)	3 points
(d)	For selecting "Yes" and attempting a relevant justification	1 point
	For a correct justification	1 point
	Example response for part (d) <i>If the track is not level, the angle of the track must be incorporated into the equation for acceleration, and this could account for the larger coefficient of kinetic friction.</i>	
	Total for part (d)	2 points

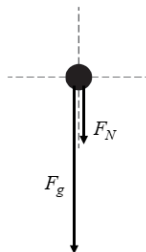
(e)	i. For drawing an appropriate best-fit line	1 point
	ii. For calculating slope using two points from the best-fit line	1 point
	$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{(6 - 2)(\text{m/s}^2)}{(0.45 - 0.15)(\text{kg})} = 13.3 \text{ m/kg}\cdot\text{s}^2$	
	For correctly using an expression that relates the slope to the acceleration due to gravity	1 point
	From $y = mx + b$ $a = (\text{slope})m_2 + (\text{y-intercept})$ $a = \frac{m_2 g}{(m_1 + m_2)} \therefore \text{slope} = \frac{g}{(m_1 + m_2)}$ $g = \text{slope} \times (m_1 + m_2) = (13.3 \text{ m/kg}\cdot\text{s}^2)(0.44 \text{ kg} + 0.20 \text{ kg}) = 8.5 \text{ m/s}^2$	
	Total for part (e)	3 points
(f)	For a correct justification	1 point
	Example response for part (f) <i>The acceleration would be greater because there would be a component of the gravitational force on block 1 along the surface, which would be in the same direction as the tension force.</i>	
	Total for question 1	15 points

Question 3: Free-Response Question

15 points

- (a) For correctly drawing and labeling the weight of the block 1 point
 For correctly drawing and labeling the force exerted by the track on the block 1 point
 For a correct justification consistent with the diagram 1 point

Example response for part (a)



The force F_g represents the weight of the block and always points downward. The force F_N represents the force the track exerts on the block to keep it moving in a circular path and points perpendicular to the surface of the track.

Scoring Note: Examples of appropriate labels for the force due to gravity include: F_G , F_g , F_{grav} , W , mg , Mg , “grav force,” “F Earth on block,” “F on block by Earth,” $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$, $F_{\text{Block,E}}$. The labels G or g are not appropriate labels for the force due to gravity. F_n , F_N , N , “normal force,” “ground force,” or similar labels may be used for the normal force.

Scoring Note: If extraneous forces are present, a maximum of 2 points can be earned.

Total for part (a) 3 points

- (b) i. For using conservation of energy 1 point
 $U_1 + K_1 = U_2 + K_2 \therefore U_1 + 0 = U_2 + K_2$
 For correctly relating the elastic potential energy at maximum spring compression to the gravitational potential energy at point B 1 point
 $U_{s1} + 0 = U_{g2} + K_2 \therefore K_2 = U_{s1} - U_{g2}$
 For a correct substitution into the equation above 1 point
 $\frac{1}{2}mv_B^2 = \frac{1}{2}k(\Delta x)^2 - mgh_2$
 $v_B^2 = \frac{k}{m}(\Delta x)^2 - 2g(3R) \therefore v_B = \sqrt{\frac{k}{m}(\Delta x)^2 - 6gR}$

- ii. For correctly relating the centripetal force to speed from part (b)(i) 1 point

$$F_C = \frac{mv^2}{r} = \frac{mv_B^2}{R}$$

- For an answer consistent with part (b)(i) 1 point

$$F_C = \frac{m}{R} \left(\sqrt{\frac{k}{m}(\Delta x)^2 - 6gR} \right)^2 = \frac{k(\Delta x)^2}{R} - 6mg$$

Total for part (b) 5 points

- (c) For correctly relating the net force to the diagram from part (a) and setting the normal force equal to zero 1 point

- For correctly substituting into the equation above 1 point

$$\frac{k(\Delta x)^2}{R} - 6mg = mg \therefore \frac{k(\Delta x)^2}{R} = 7mg \therefore \Delta x = \sqrt{\frac{7mgR}{k}}$$

Total for part (c) 2 points

- (d) For correctly relating the height of fall to the time of fall 1 point

$$y = y_0 + v_{oy}t + \frac{1}{2}a_yt^2 \therefore H = 0 + 0 + \frac{1}{2}gt^2 \therefore t = \sqrt{\frac{(2)(4R)}{g}} = \sqrt{\frac{8R}{g}}$$

- For correctly substituting into the equation for constant velocity consistent with part (b)(i) 1 point

$$D = v_x t = \left(\sqrt{\frac{k}{m}(\Delta x)^2 - 6gR} \right) \left(\sqrt{\frac{8R}{g}} \right)$$

Total for part (d) 2 points

- (e) i. For a correct justification 1 point

- ii. For indicating that as the maximum compression of the spring increases, the distance D increases 1 point

- For indicating that the minimum value is due to the minimum speed needed to get through the track. 1 point

Example responses for part (e)

The block needs a minimum speed to make it through point B on the track; thus, the horizontal line segment represents compressions of the spring for which the block does not make it to point B.

OR

From the equation in part (d), the compression of the spring is directly proportional to the horizontal distance traveled by the block; thus, the graph would be a straight line. The minimum value is the distance traveled when the compression of the spring generates the minimum speed needed to reach point B on the track.

Total for part (e) 3 points

Total for question 3 15 points

AP[®] Physics C: Mechanics

Scoring Guidelines Set 1

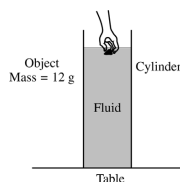
2019 SCORING GUIDELINES

General Notes About 2019 AP Physics Scoring Guidelines

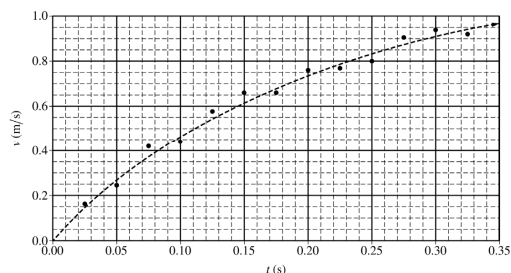
1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.
2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at <https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf>.
3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as “derive” and “calculate” on the exams, and what is expected for each, see “The Free-Response Sections — Student Presentation” in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or “Terms Defined” in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 1

15 points



In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of $m = 12 \text{ g}$ is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.



- (a)
i. LO CHA-1.C, SP 7.A
1 point

Does the speed of the object increase, decrease, or remain the same?

☐ Increase ☐ Decrease ☐ Remain the same

For selecting "Increase"

1 point

- ii. LO CHA-1.C, SP 4.D
2 points

In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.

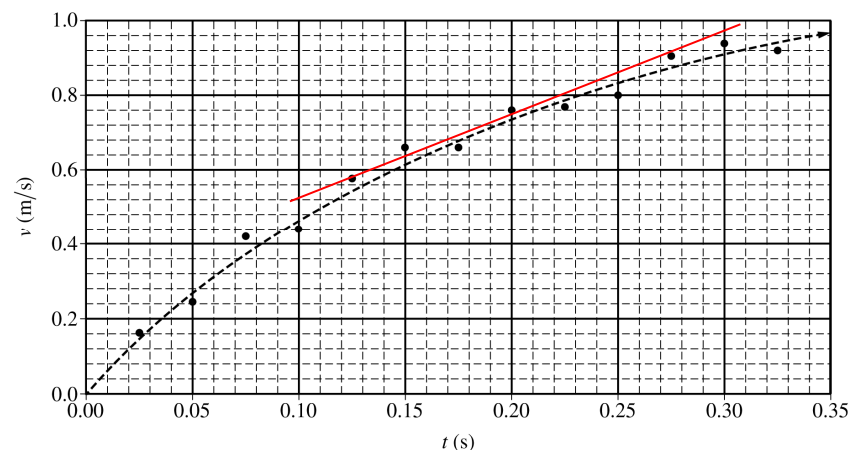
For a description that includes the direction of the acceleration as being "downwards" 1 point

For a description that includes the decrease in the magnitude of the acceleration 1 point

Example: Because the object is moving downwards and speeding up, the acceleration must be downwards. Because the slope of the graph of speed as a function of time is decreasing, the magnitude of the acceleration must be decreasing.

Question 1 (continued)

(a) continued



- iii. LO CHA-1.C, SP 4.D, 6.C
2 points

Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20 \text{ s}$.

For calculating the slope of a trend line at $t = 0.20 \text{ s}$	1 point
$\text{slope} = a = \frac{\Delta v}{\Delta t} = \frac{(0.8 - 0.6) \text{ m/s}}{(0.226 - 0.136) \text{ s}}$	
For a correct answer using points from a tangent line	1 point
$a = 2.22 \text{ m/s}^2$	

The students use the equation $v = A(1 - e^{-Bt})$ to model the speed of the falling object and find the best-fit coefficients to be $A = 1.18 \text{ m/s}$ and $B = 5 \text{ s}^{-1}$.

2019 SCORING GUIDELINES

Question 1 (continued)

(b) Use the above equation to:

- i. LO CHA-1.B, SP 6.B, 6.C
3 points

Derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time t .

For indicating that the vertical displacement is the integration of the velocity	1 point
$\Delta y = \int v dt = \int A(1 - e^{-Bt}) dt$	
For the equation for speed using appropriate limits or constant of integration	1 point
$\Delta y = \int_{t'=0}^{t'=t} A(1 - e^{-Bt'}) dt' = A \left[t' - \frac{1}{B} e^{-Bt'} \right]_{t'=0}^{t'=t} = A \left[\left(t + \frac{1}{B} e^{-Bt} \right) - \left(0 + \frac{1}{B} e^0 \right) \right]$	
For a correct answer	1 point
$\Delta y = A \left(t + \frac{1}{B} (e^{-Bt} - 1) \right) = (1.18) \left(t + \frac{1}{5} (e^{-5t} - 1) \right)$	
<u>Note:</u> Credit given for using A and B or plugging in the given values	

- ii. LO INT-1.C.d, SP 6.B, 6.C
3 points

Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time t .

For attempting the derivative of the equation for speed	1 point
$a = \frac{dv}{dt} = \frac{d}{dt} [A(1 - e^{-Bt})]$	
For a correct equation for the acceleration	1 point
$a = AB e^{-Bt}$	
For multiplying the equation above by the mass of the object	1 point
$F = ma = m(AB e^{-Bt}) = mAB e^{-Bt}$	
$F = (.012)(1.18)(5) e^{-5t} = 0.071 e^{-5t}$	
<u>Note:</u> Credit given for using A , B , and m or plugging in the given values	

2019 SCORING GUIDELINES

Question 1 (continued)

(c)

The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed.

- i. LO INT-1.I, SP 7.A, 7.C
2 points

Determine the constant speed of the object.
Justify your answer.

For stating the constant speed is $v = A$	1 point
For indicating that the constant speed can be determined by setting the time equal to infinity	1 point
Example: After a long time, the falling object will reach a terminal constant speed in the fluid. This can be determined by setting the time t in the equation for speed equal to infinity. By doing this, the constant speed is determined to be $v = A$.	
<u>Note:</u> Credit given for solving mathematically	

- ii. LO INT-1.H.b, SP 7.A, 7.C
2 points

Determine the force exerted by the fluid on the object at this time. Justify your answer.

For indicating that the net force is equal to zero when the object moves with constant speed	1 point
For indicating the resistive force is equal to the weight of the object at this time	1 point
Example: When the falling object reaches a constant speed in the fluid, the net force must be zero. Because the only vertical forces acting on the object are Earth's gravitational pull and the resistive force of the fluid, these two forces must be equal. So, the resistive force must be equal to the weight of the object or 0.12 N.	
<u>Note:</u> Credit given for solving mathematically	

2019 SCORING GUIDELINES

Question 1 (continued)

Learning Objectives

CHA-1.B: Determine functions of position, velocity, and acceleration that are consistent with each other, for the motion of an object with a nonuniform acceleration.

CHA-1.C: Describe the motion of an object in terms of the consistency that exists between position and time, velocity and time, and acceleration and time.

INT-1.C.d: Derive an expression for the net force on an object in translational motion.

INT-1.H.b: Describe the acceleration, velocity, or position in relation to time for an object subject to a resistive force (with different initial conditions, i.e., falling from rest or projected vertically).

INT-1.I: Calculate the terminal velocity of an object moving vertically under the influence of a resistive force of a given relationship.

Science Practices

4.D: Select relevant features of a graph to describe a physical situation or solve problems.

6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.

6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

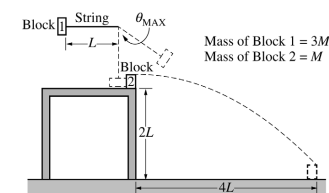
7.A: Make a scientific claim.

7.C: Support a claim with evidence from physical representations.

2019 SCORING GUIDELINES

Question 2

15 points



Note: Figure not drawn to scale.

A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M , which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M , L , and physical constants, as appropriate.

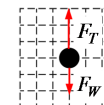
- (a) LO CON-2.C.a, SP 5.E
1 point

Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

For correctly calculating the speed of block 1	1 point
$U_{g1} = K_2 \therefore mgh = \frac{1}{2}mv^2 \therefore gL = \frac{1}{2}v^2$	
$v = \sqrt{2gL}$	
Note: Credit is earned even if no work is shown.	

- (b) LO INT-2.B.b, SP 3.D
2 points

On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



For correctly drawing and labeling the weight of the block and the tension in the string	1 point
For drawing the tension longer than the weight of the block	1 point
Note: A maximum of one point can be earned if there are any extraneous vectors.	

2019 SCORING GUIDELINES

Question 2 (continued)

- (c) LO INT-2.B.b, SP 5.A, 5.E
2 points

Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

For using an appropriate equation to calculate the tension in the string	1 point
$F_T = mg + ma_C = mg + \frac{mv^2}{r} = 3Mg + \frac{(3M)(\sqrt{2gL})^2}{L}$	
For a correct answer	1 point
$F_T = 9Mg$	

For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

- (d) LO CHA-2.C, SP 6.A, 6.C
2 points

Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

For using a correct kinematic equation to calculate the time	1 point
$\Delta y = v_{1i}t + \frac{1}{2}at^2 = \frac{1}{2}at^2 \therefore t = \sqrt{\frac{2\Delta y}{a}} = \sqrt{\frac{2(2L)}{g}} = \sqrt{\frac{(4)(0.75 \text{ m})}{(9.8 \text{ m/s}^2)}}$	
For a correct answer	1 point
$t = 0.55 \text{ s}$	

- (e) LO CHA-2.C, SP 6.A, 6.C
2 points

Calculate the speed of block 2 as it leaves the table.

For using a correct kinematic equation to calculate the speed	1 point
$\Delta x = vt \therefore v = \frac{\Delta x}{t} = \frac{4L}{t}$	
For substituting the time from part (d) into equation above	1 point
$v = \frac{(4)(0.75 \text{ m})}{(0.55 \text{ s})} = 5.45 \text{ m/s}$	

2019 SCORING GUIDELINES

Question 2 (continued)

- (f) LO CON-4.E.a, SP 6.B, 6.C
3 points

Calculate the speed of block 1 just after it collides with block 2.

For indicating the use of the correct conservation of momentum to calculate the speed	1 point
$p_i = p_f \therefore m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$	
For correctly substituting the mass into equation above	1 point
$(3M)v_{1i} + 0 = (3M)v_{1f} + (M)v_{2f}$	
For substituting the speeds from parts (a) and (e) into equation above	1 point
$v_{1f} = \frac{(3M)v_{1i} - (M)v_{2f}}{(3M)} = \frac{(3M)\sqrt{2gL} - (M)v_{2f}}{(3M)}$	
$v_{1f} = \frac{(3)\sqrt{(2)(9.8 \text{ m/s}^2)(0.75 \text{ m})} - (1)(5.45 \text{ m/s})}{(3)} = 2.02 \text{ m/s}$	
$(v_{1f} = 2.06 \text{ m/s when using } g = 10 \text{ m/s}^2)$	

- (g) LO CON-2.C.a, SP 6.B, 6.C
3 points

Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.

For using conservation of energy to calculate the angle	1 point
$K_1 = U_{g2}$	
$\frac{1}{2}mv^2 = mgh$	
For correctly substituting $h = L(1 - \cos\theta)$ into the equation above	1 point
For correctly substituting the speed from part (f) into the equation above	1 point
$\frac{1}{2}mv^2 = mgL(1 - \cos\theta) \therefore \frac{v^2}{2gL} = 1 - \cos\theta$	
$\theta = \cos^{-1}\left(1 - \frac{v^2}{2gL}\right) = \cos^{-1}\left(1 - \frac{(2.02 \text{ m/s})^2}{(2)(9.8 \text{ m/s}^2)(0.75 \text{ m})}\right) = 43.5^\circ$	
$(\theta = 44.1^\circ \text{ when using } g = 10 \text{ m/s}^2)$	

2019 SCORING GUIDELINES

Question 2 (continued)

Learning Objectives

CHA-2.C: Calculate kinematic quantities of an object in projectile motion, such as displacement, velocity, speed, acceleration, and time, given initial conditions of various launch angles, including a horizontal launch at some point in its trajectory.

INT-2.B.b: Describe forces that are exerted on objects undergoing horizontal circular motion, vertical circular motion, or horizontal circular motion on a banked curve.

CON-2.C.a: Calculate unknown quantities (e.g., speed or positions of an object) that are in a conservative system of connected objects, such as the masses in an Atwood machine, masses connected with pulley/string combinations, or the masses in a modified Atwood machine.

CON-4.E.a: Calculate the changes in speeds, changes in velocities, changes in kinetic energy, or changes in momenta of objects in all types of collisions (elastic or inelastic) in one dimension, given the initial conditions of the objects.

Science Practices

3.D: Create appropriate diagrams to represent physical situations.

5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.

5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

6.A: Extract quantities from narratives or mathematical relationships to solve problems.

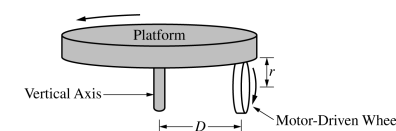
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.

6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

2019 SCORING GUIDELINES

Question 3

15 points



A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the wheel until the platform reaches an angular speed ω_P after time Δt . During time Δt , the wheel stays in contact with the platform without slipping.

- (a) LO INT-7.A.b, CHA-4.A.b, SP 5.A, 5.E
2 points

Derive an expression for the angular speed ω_P of the platform. Express your answer in terms of I_P , r , D , F , Δt , and physical constants, as appropriate.

For correctly substituting into the rotational form of Newton's second law			1 point
$\tau = I\alpha \therefore FD = I_P\alpha$			
$\alpha = \frac{FD}{I_P}$			
For correctly substituting into a rotational kinematic equation to calculate the angular speed			1 point
$\omega_2 = \omega_1 + \alpha\Delta t = 0 + \left(\frac{FD}{I_P}\right)\Delta t$			
$\omega_P = \frac{FD\Delta t}{I_P}$			

- (b) LO INT-7.D.a, SP 5.A, 5.E
2 points

Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed ω_P . Express your answer in terms of I_P , r , D , F , Δt , and physical constants, as appropriate.

For using the equation for rotational kinetic energy		1 point
$K = \frac{1}{2}I\omega_P^2 = \left(\frac{1}{2}\right)(I_P)\left(\frac{FD\Delta t}{I_P}\right)^2$		
For an answer consistent with part (a)		1 point
$K = \frac{(FD\Delta t)^2}{2I_P}$		

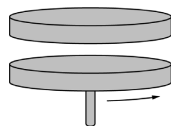
2019 SCORING GUIDELINES

Question 3 (continued)

- (c) LO INT-7.C, SP 5.A, 5.E
2 points

Derive an expression for the angular speed of the wheel ω_W when the platform has reached angular speed ω_P . Express your answer in terms of D , r , ω_P , and physical constants, as appropriate.

For indicating that the linear speed of the platform is equal to the linear speed of the wheel		1 point
$v_P = v_W$ OR $(r\omega)_P = (r\omega)_W$		
For correctly relating the linear speeds to the angular speeds in the above equation		1 point
$D\omega_P = r\omega_W$		
$\omega_W = \frac{D\omega_P}{r}$		



When the platform is spinning at angular speed ω_P , the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia I_P as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed ω_f .

- (d) LO CON-5.D.c, SP 5.A, 5.E
2 points

Derive an expression for ω_f . Express your answer in terms of ω_P , I_P , and physical constants, as appropriate.

For using an expression for the conservation of angular momentum		1 point
$L_1 = L_2 \therefore I_1\omega_1 = I_2\omega_2$		
For correctly substituting into the above equation		1 point
$I\omega_P = (2I)\omega_f$		
$\omega_f = \frac{1}{2}\omega_P$		

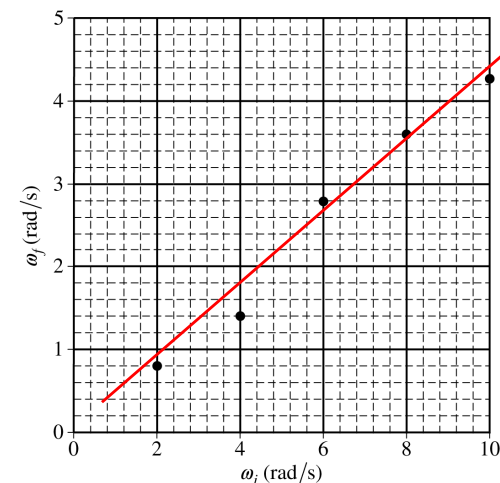
2019 SCORING GUIDELINES

Question 3 (continued)

A student now uses the rotating platform ($I_P = 3.1 \text{ kg} \cdot \text{m}^2$) to determine the rotational inertia I_U of an unknown object about a vertical axis that passes through the object's center of mass. The platform is rotating at an initial angular speed ω_i when the unknown object is dropped with its center of mass directly above the center of the platform. The platform and object are observed to be rotating together at angular speed ω_f . Trials are repeated for different values of ω_i . A graph of ω_f as a function of ω_i is shown on the axes below.

- (e) i. LO CON-5.D.c, SP 4.C
1 points

On the graph on the previous page, draw a best-fit line for the data.



For an appropriate best-fit line for the graph above

1 point

2019 SCORING GUIDELINES

Question 3 (continued)

(e) continued

- ii. LO CON-5.D.c, SP 4.D, 6.C
2 points

Using the straight line, calculate the rotational inertia of the unknown object I_U about a vertical axis passing through its center of mass.

For using conservation of angular momentum to derive an expression that includes I_U	1 point
$L_i = L_f \therefore I_P \omega_i = (I_P + I_U) \omega_f$	
$I_U = I_P \left(\frac{\omega_i}{\omega_f} \right) - I_P$	
For substituting points from the best-fit line into the expression above	1 point
$I_U = (3.1 \text{ kg}\cdot\text{m}^2) \left(\frac{(4.0 - 1.0) \text{ rad/s}}{(9.3 - 2.4) \text{ rad/s}} \right) - (3.1 \text{ kg}\cdot\text{m}^2) = 4.1 \text{ kg}\cdot\text{m}^2$	
Note: The point (0, 0) can be used implicitly if the best-fit line goes through the origin.	

- (f) LO CON-5.D.c, SP 7.A, 7.C
2 points

The kinetic energy of the spinning platform before the object is dropped on it is K_i . The total kinetic energy of the platform-object system when it reaches angular speed ω_f is K_f . Which of the following expressions is true?

___ $K_f < K_i$ ___ $K_f = K_i$ ___ $K_f > K_i$

Justify your answer.

For selecting $K_f < K_i$ with an attempt at a relevant justification	1 point
For a correct justification	1 point
Example: Because the two disks will be rotating with the same final angular speed, this is an inelastic collision, and kinetic energy will be lost during the collision.	

2019 SCORING GUIDELINES

Question 3 (continued)

- (g) LO INT-6.E, SP 7.A, 7.C
2 points

One of the students observes that the center of mass of the object is not actually aligned with the axis of the platform. Is the experimental value of I_U obtained in part (e) greater than, less than, or equal to the actual value of the rotational inertia of the unknown object about a vertical axis that passes through its center of mass?

___ Greater than ___ Less than ___ Equal to

Justify your answer.

For selecting "Greater than" with an attempt at a relevant justification	1 point
For a correct justification	1 point
Example: Because the center of mass of the object is off the axis of the platform, the parallel axis theorem would be used to calculate the total rotational inertia of the platform-object system. Using $I = I_{CM} + Mh^2$, the experimental value will be increased by Mh^2 .	

Learning Objectives

INT-6.E: Derive the moments of inertia of an extended rigid body for different rotational axes (parallel to an axis that goes through the object's center of mass) if the moment of inertia is known about an axis through the object's center of mass.

CHA-4.A.b: Calculate unknown quantities such as angular positions, displacement, angular speeds, or angular acceleration of a rigid body in uniformly accelerated motion, given initial conditions.

INT-7.A.b: Calculate unknown quantities such as net torque, angular acceleration, or moment of inertia for a rigid body undergoing rotational acceleration.

INT-7.C: Derive expressions for physical systems such as Atwood Machines, pulleys with rotational inertia, or strings connecting discs or strings connecting multiple pulleys that relate linear or translational motion characteristics to the angular motion characteristics of rigid bodies in the system that are: **(a)** rolling (or rotating on a fixed axis) without slipping. **(b)** rotating and sliding simultaneously.

INT-7.D.a: Calculate the rotational kinetic energy of a rotating rigid body.

CON-5.D.c: Calculate the changes of angular momentum of each disc in a rotating system of two rotating discs that collide with each other inelastically about a common rotational axis.

Science Practices

4.C: Linearize data and/or determine a best-fit line or curve.

4.D: Select relevant features of a graph to describe a physical situation or solve problems.

5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.

5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

7.A: Make a scientific claim.

7.C: Support a claim with evidence from physical representations.

2019

AP[®] CollegeBoard

AP[®] Physics C: Mechanics

Scoring Guidelines Set 2

2019 SCORING GUIDELINES

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.
2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at <https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf>.
3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as “derive” and “calculate” on the exams, and what is expected for each, see “The Free-Response Sections — Student Presentation” in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or “Terms Defined” in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

2019 SCORING GUIDELINES

Question 1

15 points

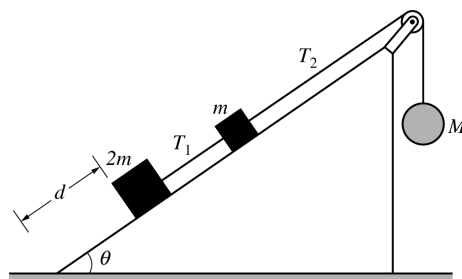
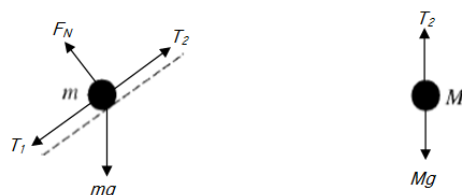


Figure 1

Blocks of mass m and $2m$ are connected by a light string and placed on a frictionless inclined plane that makes an angle θ with the horizontal, as shown in Figure 1 above. Another light string connecting the block of mass m to a hanging sphere of mass M passes over a pulley of negligible mass and negligible friction. The entire system is initially at rest and in equilibrium.

- (a) LO INT-1.A, SP 3.D
3 points

On the dots below that represent the block of mass m and the sphere of mass M , draw and label the forces (not components) that act on each of the objects shown. Each force must be represented by a distinct arrow starting on and pointing away from the dot.



For correctly drawing and labeling vectors representing the normal force and the gravitational force on the block of mass m	1 point
For correctly drawing and labeling vectors representing the forces of tension on the block of mass m	1 point
For correctly drawing and labeling vectors representing the tension force and the gravitational force on the sphere of mass M	1 point
Note: A maximum of two points can be earned if there are any extraneous vectors.	

2019 SCORING GUIDELINES

Question 1 (continued)

- (b)

Derive expressions for the magnitude of each of the following. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figures in part (a).

- i. LO INT-1.C.e, SP 1.D, 5.E
2 points

The force T_2 exerted on the block of mass m by the string. Express your answers in terms of m , θ , and physical constants, as appropriate.

For using an attempt at a correct statement of Newton's second law for the two blocks	1 point
$F_{\text{net}} = (m + 2m)a = 0 \therefore T_2 - (m + 2m)g \sin \theta = 0$	
For a correct answer with supporting work	1 point
$T_2 = 3mg \sin \theta$	

- ii. LO INT-1.D, SP 5.E
1 point

The mass M for which the system can remain in equilibrium. Express your answers in terms of m , θ , and physical constants, as appropriate.

For using a correct statement of Newton's second law for the whole system to derive an expression for M	1 point
$F_{\text{net}} = m_{\text{tot}}a \therefore Mg - 2mg \sin \theta - mg \sin \theta = 0$	
$M = 3m \sin \theta$	
<i>Alternate Solution</i>	<i>Alternate Points</i>
For a correct statement of Newton's second law for the sphere and an answer consistent with part (b)(i)	1 point
$T_2 - Mg = 0 \therefore Mg = T_2 \therefore M = T_2/g$	
$M = 3m \sin \theta$	

2019 SCORING GUIDELINES

Question 1 (continued)

(c)

Now suppose that mass M is large enough to descend and that the sphere reaches the floor before the blocks reach the pulley. Answer the following for the moment immediately after the sphere reaches the floor.

i. LO INT-1.D, SP 7.A

1 point

Does the tension T_1 increase, decrease to a nonzero value, decrease to zero, or stay the same?

____ Increase ____ Decrease to a nonzero value

____ Decrease to zero ____ Stay the same

For correctly stating that the magnitude of T_1 drops to zero

1 point

ii. LO INT-1.D, SP 7.A

1 point

Is the velocity of the block of mass m up the ramp, down the ramp, or zero?

____ Up the ramp ____ Down the ramp ____ Zero

For selecting “Up the ramp”

1 point

iii. LO INT-1.D, SP 7.A

1 point

Is the acceleration of the block of mass m up the ramp, down the ramp, or zero?

____ Up the ramp ____ Down the ramp ____ Zero

For selecting “Down the ramp”

1 point

(d) LO INT-1.D, SP 5.A, 5.E

3 points

Consider the initial setup in Figure 1. Now suppose the surface of the incline is rough and the coefficient of static friction between the blocks and the inclined plane is μ_s . Derive an expression for the minimum possible value of M that will keep the blocks from moving down the incline. Express your answer in terms of m , μ_s , θ , and fundamental constants, as appropriate.

For an attempt at a correct statement of Newton’s second law for the system

1 point

$$F_{\text{net}} = m_{\text{tot}} a \therefore 2mg \sin \theta - f_{s2} + mg \sin \theta - f_{s1} - Mg = 0$$

For attempting to substitute in for the force of friction

1 point

$$3mg \sin \theta - \mu_s F_{N2} - \mu_s F_{N1} = Mg$$

$$3mg \sin \theta - \mu_s (2mg \cos \theta) - \mu_s (mg \cos \theta) = Mg$$

For a correct answer with supporting work

1 point

$$M = 3m(\sin \theta - \mu_s \cos \theta)$$

2019 SCORING GUIDELINES

Question 1 (continued)

(e)

LO INT-1.D, CHA-1.A.b, SP 5.A, 5.E

3 points

The string connecting block m and the sphere of mass M then breaks, and the blocks begin to move from rest down the incline. The lower block starts a distance d from the bottom of the incline, as shown in Figure 1. The coefficient of kinetic friction between the blocks and the inclined plane is μ_k . Derive an expression for the speed of the blocks when the lower block reaches the bottom of the incline. Express your answer in terms of m , d , μ_k , θ , and fundamental constants, as appropriate.

For an attempt at a correct statement of Newton’s second law for the two blocks	1 point
$F_{\text{net}} = m_{\text{tot}} a \therefore 2mg \sin \theta - f_{k2} + mg \sin \theta - f_{k1} = 3ma$	
Solve for the acceleration	
$3mg \sin \theta - \mu_k (2mg \cos \theta) - \mu_k mg \cos \theta = 3ma$	
$a = g(\sin \theta - \mu_k \cos \theta)$	
For using a correct kinematics equation to solve for the final velocity	1 point
$v_2^2 = v_1^2 + 2ad$	
$v_2^2 = 0^2 + 2g(\sin \theta - \mu_k \cos \theta)d$	
For a correct answer with supporting work	1 point
$v = \sqrt{2gd(\sin \theta - \mu_k \cos \theta)}$	
<i>Alternate Solution</i>	<i>Alternate Points</i>
For an attempt at a correct statement of conservation of energy for the two blocks	1 point
$U_1 + K_1 = U_2 + K_2 + E_{\text{lost}}$	
$2mgh + mgh + 0 = 0 + \frac{1}{2}(2m)v^2 + \frac{1}{2}mv^2 + fd$	
For correct substitutions of height and friction	1 point
$3mgd \sin \theta = \frac{1}{2}(3m)v^2 + \mu_k (2mg \cos \theta)d + \mu_k (mg \cos \theta)d$	
$3gd \sin \theta - 3\mu_k g d \cos \theta = \frac{3}{2}v^2$	
For a correct answer with supporting work	1 point
$v = \sqrt{2gd(\sin \theta - \mu_k \cos \theta)}$	

2019 SCORING GUIDELINES

Question 1 (continued)

Learning Objectives

CHA-1.A.b: Calculate unknown variables of motion such as acceleration, velocity, or positions for an object undergoing uniformly accelerated motion in one dimension.

INT-1.A: Describe an object (either in a state of equilibrium or acceleration) in different types of physical situations such as inclines, falling through air resistance, Atwood machines, or circular tracks).

INT-1.C.e: Derive a complete Newton's second law statement (in the appropriate direction) for an object in various physical dynamic situations (e.g., mass on incline, mass in elevator, strings/pulleys, or Atwood machines).

INT-1.D: Calculate a value for an unknown force acting on an object accelerating in a dynamic situation (e.g., inclines, Atwood Machines, falling with air resistance, pulley systems, mass in elevator, etc.)

Science Practices

1.D: Select relevant features of a representation to answer a question or solve a problem.

3.C: Sketch a graph that shows a functional relationship between two quantities.

3.D: Create appropriate diagrams to represent physical situations.

5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.

5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

7.A: Make a scientific claim.

2019 SCORING GUIDELINES

Question 2

15 points

A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0 \text{ m/s}^2$, $L = 0.25 \text{ m/s}^4$, and t is the time in seconds. At $t = 6.0 \text{ s}$, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

- (a) LO INT-5.E, SP 6.B, 6.C
2 points

Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0 \text{ s}$.

For an expression for calculating impulse and correct substitution of $a(t)$ and m into the correct expression.	1 point
$J = \int F(t) dt$	
$J = \int ma(t) dt = (0.50) \int (9.0 - 0.25t^2) dt$	
For integrating with correct limits or including a constant of integration	1 point
$J = (0.50) \int_{t=0}^{t=6} (9.0 - 0.25t^2) dt = (0.50) \left[9.0t - \frac{1}{12}t^3 \right]_{t=0}^{t=6}$	
$J = (0.50) \left[(9.0)(6) - \left(\frac{1}{12} \right) (6)^3 \right] - 0 = 18 \text{ N}\cdot\text{s}$	
Alternate Solution (using an alternate solution from part (b))	Alternate Points
For correctly relating impulse to the speed of the rocket	1 point
$J = \Delta p = m(v_2 - v_1)$ OR $J = m\Delta v = mv_f$	
For correctly substituting answer from part (b) into equation above	1 point
$J = (0.50 \text{ kg})(36 - 0) \therefore J = 18 \text{ N}\cdot\text{s}$	

2019 SCORING GUIDELINES

Question 2 (continued)

- (b) LO INT-5.A.a, SP 6.A, 6.C
2 points

Calculate the speed of the rocket at $t = 6.0$ s.

For correctly relating impulse to the speed of the rocket $J = \Delta p = m(v_2 - v_1)$ OR $J = m\Delta v = mv_f$	1 point
For correctly substituting answer from part (a) into equation above $(18 \text{ N}\cdot\text{s}) = (0.50 \text{ kg})(v_2 - 0) \therefore v_2 = 36 \text{ m/s}$	1 point
<i>Alternate Solution</i>	<i>Alternate Points</i>
<i>Integrate expression for a(t) (This may have already been done in solving part (a).)</i>	1 point
$v = \int a dt = \int (9.0 - 0.25t^2) dt$	
<i>For integrating with correct limits or including a constant of integration</i>	1 point
$v = \int_{t=0}^{t=6} (9.0 - 0.25t^2) dt = \left[9.0t - \frac{1}{12}t^3 \right]_{t=0}^{t=6} = 36 \text{ m/s}$	

- (c) i. LO INT-4.C.c, SP 6.B, 6.C
2 points

Calculate the kinetic energy of the rocket at $t = 6.0$ s.

For substituting the mass of the rocket into the equation for kinetic energy	1 point
For substituting the answer from part (b) into the equation for kinetic energy	1 point
$K = \frac{1}{2}mv^2 = \left(\frac{1}{2}\right)(0.50 \text{ kg})(36 \text{ m/s})^2 = 324 \text{ J}$	

- ii. LO CHA-1.B, CON-1.E, SP 6.A, 6.C
3 points

Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.

For integrating the acceleration twice to derive an expression for position	1 point
For integrating with correct limits or including a constant of integration	1 point
$v(t) = \int a(t) dt = \int (9.0 - 0.25t^2) dt = 9.0t - \frac{1}{12}t^3 + v_0 = 9.0t - \frac{1}{12}t^3$	
$\Delta y = \int v(t) dt = \int_{t=0}^{t=6} \left(9.0t - \frac{1}{12}t^3 \right) dt = \left[\frac{9}{2}t^2 - \frac{1}{48}t^4 \right]_{t=0}^{t=6}$	
$\Delta y = \left[\left(\frac{9}{2} \right)(6)^2 - \left(\frac{1}{48} \right)(6)^4 \right] - 0 = 135 \text{ m}$	
For substituting into equation for potential energy	1 point
$\Delta U_g = mg\Delta y = (0.50 \text{ kg})(9.8 \text{ m/s}^2)(135 \text{ m}) = 660 \text{ J}$	

2019 SCORING GUIDELINES

Question 2 (continued)

- (d) LO CHA-1.B, CON-2.B, SP 6.B, 6.C
3 points

Calculate the maximum height reached by the rocket relative to its launching point.

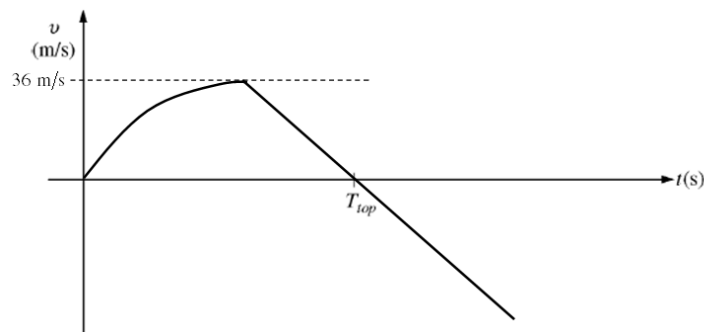
For using $a = g$ in a correct kinematics equation to solve for height $v_2^2 = v_1^2 + 2a(y_2 - y_1) \therefore 0 = v_1^2 + 2a(y_2 - y_1)$	1 point
$y_2 - y_1 = -\frac{v_1^2}{2a} \therefore y_2 = -\frac{v_1^2}{2a} + y_1$	
For substituting the speed from part (b)	1 point
$y_2 = -\frac{(36 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} + y_1$	
For substituting the height from part (c)	1 point
$y_2 = 66 \text{ m} + 135 \text{ m} = 201 \text{ m}$ (199.8 m if $g = 10 \text{ m/s}^2$)	
<i>Alternate Solution 1</i>	<i>Alternate Points</i>
<i>For using energy conservation to find maximum height, consistent with the speed found in part (b).</i>	1 point
$mg\Delta y = \frac{1}{2}mv^2 \therefore \Delta y = \frac{v^2}{2g}$	
<i>For substituting the speed from part (b)</i>	1 point
$\Delta y = \frac{(36 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 66 \text{ m}$	
<i>For substituting the height from part (c)</i>	1 point
$y_2 = 66 \text{ m} + 135 \text{ m} = 201 \text{ m}$ (199.8 m if $g = 10 \text{ m/s}^2$)	
<i>Alternate Solution 2</i>	<i>Alternate Points</i>
<i>For using energy conservation to find maximum height, from kinetic and potential energies found in part (c)</i>	1 point
$K_1 + U_1 = 0 + U_{top}$	
<i>For substituting the kinetic energy from part (c)(i) into the equation above</i>	1 point
<i>For substituting the potential energy from part (c)(ii) into the equation above</i>	1 point
$324 + 660 = (0.5)(9.8)\Delta y$	
$\Delta y = 201 \text{ m}$ (199.8 m if $g = 10 \text{ m/s}^2$)	

2019 SCORING GUIDELINES

Question 2 (continued)

- (e) LO CHA-1.C, SP 3.C
3 points

On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



For an initial concave down curve that starts at the origin.	1 point
For a transition that occurs before T_{top} into a straight line with a negative slope.	1 point
For labeling the maximum value of the velocity, consistent with part (b), and a line that crosses the x-axis at T_{top}	1 point

2019 SCORING GUIDELINES

Question 2 (continued)

Learning Objectives

CHA-1.B: Determine functions of position, velocity, and acceleration that are consistent with each other, for the motion of an object with a nonuniform acceleration.

CHA-1.C: Describe the motion of an object in terms of the consistency that exists between position and time, velocity and time, and acceleration and time.

CON-1.E: Calculate the potential energy of a system consisting of an object in a uniform gravitational field.

CON-2.B: Describe kinetic energy, potential energy, and total energy in relation to time (or position) for a “conservative” mechanical system.

INT-4.C.c: Calculate changes in an object’s kinetic energy or changes in speed that result from the application of specified forces.

INT-5.A.a: Calculate the total momentum of an object or system of objects.

INT-5.E: Calculate the change in momentum of an object given a nonlinear function, $F(t)$, for a net force acting on the object.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.

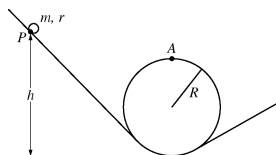
6.A: Extract quantities from narratives or mathematical relationships to solve problems.

6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.

6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

Question 3

15 points



Note: Figure not drawn to scale.

The rotational inertia of a rolling object may be written in terms of its mass m and radius r as $I = bmr^2$, where b is a numerical value based on the distribution of mass within the rolling object. Students wish to conduct an experiment to determine the value of b for a partially hollowed sphere. The students use a looped track of radius $R \gg r$, as shown in the figure above. The sphere is released from rest a height h above the floor and rolls around the loop.

- (a) LO INT-2.D, SP 5.A, 5.E
 2 points

Derive an expression for the minimum speed of the sphere's center of mass that will allow the sphere to just pass point A without losing contact with the track. Express your answer in terms of b , m , R , and fundamental constants, as appropriate.

For an expression relating the gravitational force to the centripetal force	1 point
$F_C = mg + F_N = \frac{mv^2}{R}$	
$mg + 0 = \frac{mv^2}{R}$	
For a correct substitution into a correct expression.	1 point
$v = \sqrt{Rg}$	

Question 3 (continued)

- (b) LO INT-7.E, SP 5.A, 5.E
 3 points

Suppose the sphere is released from rest at some point P and rolls without slipping. Derive an equation for the minimum release height h that will allow the sphere to pass point A without losing contact with the track. Express your answer in terms of b , m , R , and fundamental constants, as appropriate.

For using a correct expression of the conservation of energy for the object	1 point
$K_1 + U_1 = K_2 + U_2$	
For correct substitutions, including rotational kinetic energy	1 point
$0 + mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh_2$	
$mgh = \frac{1}{2}mv^2 + \frac{1}{2}(bmr^2)\left(\frac{v}{r}\right)^2 + mgh_2$	
For correctly substituting for the final height and the answer from part (a) for the velocity	1 point
$gh = \frac{1}{2}(\sqrt{Rg})^2 + \frac{1}{2}(b)(\sqrt{Rg})^2 + g(2R)$	
$h = \frac{1}{2}R + \frac{1}{2}bR + 2R = \frac{1}{2}(b+5)R$	

2019 SCORING GUIDELINES

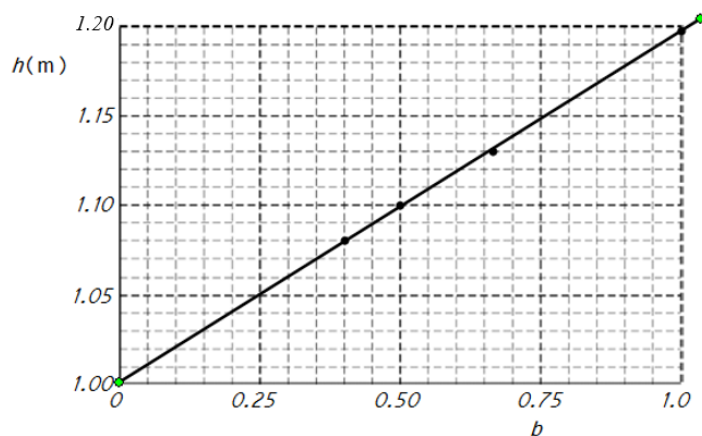
Question 3 (continued)

The students perform an experiment by determining the minimum release height h for various other objects of radius r and known values of b . They collect the following data.

Object	b	h (m)
Solid sphere	0.40	1.08
Hollow sphere	0.67	1.13
Solid cylinder	0.50	1.10
Hollow cylinder	1.0	1.20

- (c) LO INT-7.E, SP 3.A, 4.C
4 points

On the grid below, plot the release height h as a function of b . Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



For correctly labeling both axes with variables, units for h , no units for b	1 point
For correctly using and labeling the scale of the axes so that the data uses at least half the grid	1 point
For correctly plotting the data	1 point
For drawing a best-fit straight line that represents the data	1 point

2019 SCORING GUIDELINES

Question 3 (continued)

- (d) LO INT-7.E, SP 4.D, 6.C
2 points

The students repeat the experiment with the partially hollowed sphere and determine the minimum release height to be 1.16 m. Using the straight line from part (c), determine the value of b for the partially hollowed sphere.

For correctly calculating the slope from the best-fit straight line and substituting height into the line equation	1 point
$\text{slope} = \frac{1.20 - 1.00}{1.0 - 0.0} = 0.20 \text{ m}$	
$y = mx + b \therefore h = 0.20b + 1.00$	
$1.16 = 0.20b + 1.00$	
For a correct answer for b	1 point
$b = 0.80$	
<u>Note:</u> Full credit can be earned for the correct answer if there is an indication that this was read from the graph.	

- (e) LO INT-7.E, SP 6.A, 6.C
2 points

Calculate R , the radius of the loop.

For substituting into the expression from part (b)	1 point
$h = \frac{1}{2}(b + 5)R \therefore R = \frac{2(1.16 \text{ m})}{(0.80 + 5)}$	
For an answer consistent with previous parts	1 point
$R = 0.40 \text{ m}$	
<i>Alternate Solution</i>	<i>Alternate Points</i>
For using an expression for the y-intercept (set $b = 0$)	1 point
$h = \frac{1}{2}(b + 5)R = \frac{1}{2}(0 + 5)R = \frac{5}{2}R$	
For determining the value of the intercept (1.0 m) from the graph	1 point
$1.0 \text{ m} = \frac{5}{2}R \therefore R = \frac{2}{5} \text{ m} = 0.40 \text{ m}$	

2019 SCORING GUIDELINES

Question 3 (continued)

- (f) LO INT-7.E, SP 7.A, 7.C
2 points

In part (b), the radius r of the rolling sphere was assumed to be much smaller than the radius R of the loop. If the radius r of the rolling sphere was not negligible, would the value of the minimum release height h be greater, less, or the same?

____ Greater ____ Less ____ The same

Justify your answer.

For selecting “Less” and any justification		1 point
For a correct justification		1 point
Examples:		
If the radius of the object is not negligible, then the center of mass of the object travels in a radius less than R . If the radius of the circle decreases, the height needed to pass through the loop decreases according to the equation from part (b).		
If the radius of the object is not negligible, then the center of mass of the object travels in a radius less than R . If the radius of the circle decreases, the speed needed to pass through the loop decreases, and thus the height needed also decreases.		

Learning Objectives

INT-2.D: Derive expressions relating centripetal force to the minimum speed or maximum speed of an object moving in a vertical circular path.

INT-7.E: Derive expressions using energy conservation principles for physical systems such as rolling bodies on inclines, Atwood Machines, pendulums, physical pendulums, and systems with massive pulleys that relate linear or angular motion characteristics to initial conditions (such as height or position) or properties of rolling body (such as moment of inertia or mass).

Science Practices

3.A: Select and plot appropriate data.

4.C: Linearize data and/or determine a best-fit line or curve.

4.D: Select relevant features of a graph to describe a physical situation or solve problems.

5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.

5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

6.A: Extract quantities from narratives or mathematical relationships to solve problems.

6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

7.A: Make a scientific claim.

7.C: Support a claim with evidence from physical representations.