Question 2: Free-Response Question — 15 points

(a)

For selecting "Equal to" with an attempt at a relevant justification — **1 point**

For a correct justification — **1 point**

Example Response

Newton's third law says equal/opposite forces, time intervals are same during same collision, so magnitudes of impulse must be equal.

Total for part (a) 2 points

(b)

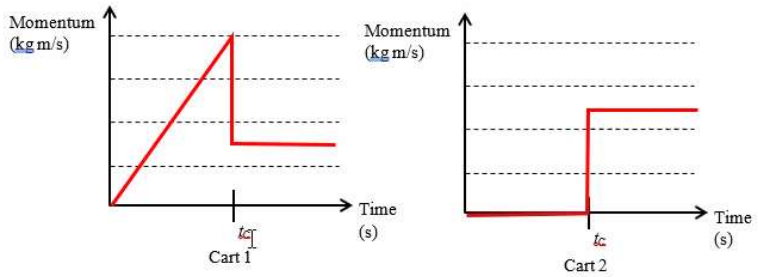
For correctly drawing the momentum for carts 1 and 2 for the time interval $0 < t < t_C$: Linear increasing momentum for Cart 1 and zero for Cart 2 — **1 point**

For drawing a horizontal line for Cart 1 when $t > t_C$ that is smaller in magnitude than the momentum of Cart 1 at time $t = t_C$ — **1 point**

For drawing a horizontal line for Cart 2 when $t > t_C$ that is greater in magnitude than the momentum of Cart 1 after time $t = t_C$ — **1 point**

For carts 1 and 2 having a change in momentum that is equal in magnitude, such that Cart 1 loses momentum and Cart 2 gains momentum or a response with changes in momentum consistent with the response in part (a) — **1 point**

Example Response



Total for part (b) 4 points

(c)

For using conservation of energy to find the speed of Cart 1 at the bottom of the incline — **1 point**

OR

For a correct substitution of acceleration and displacement in a kinematics equation to find the speed of Cart 1 at the bottom of the incline

For using conservation of momentum to find the speed of the two-cart system after the collision — **1 point**

For combining correct equations from above — **1 point**

Example Response

Conservation of energy:

$$m_1 gH = \frac{1}{2} m_1 v_1^2$$

$$v_1 = \sqrt{2gH}$$

OR

$$v_f^2 = v_i^2 + 2g\sin(\theta)L$$

$$H = L\sin(\theta)$$

$$v_f^2 = v_i^2 + 2g\sin(\theta)\left(\frac{H}{\sin\theta}\right)$$

$$v_f^2 = 2gH$$

Conservation of momentum:

$$m_1 v_1 = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1}{m_1 + m_2} v_1$$

Combining:

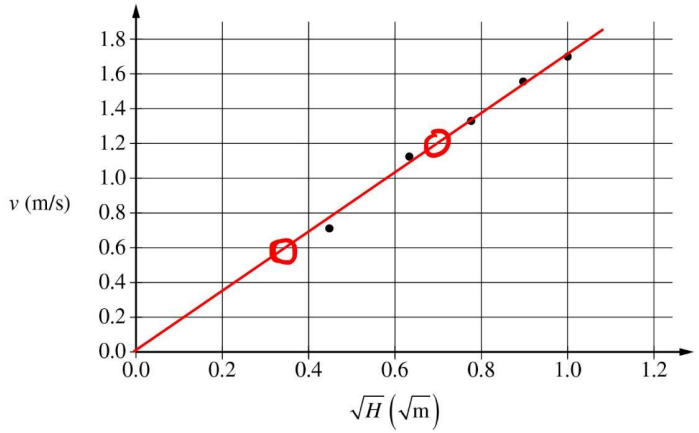
$$v_f = \frac{m_1}{m_1 + m_2}\sqrt{2gH}$$

$$v_f = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}$$

Total for part (c) 3 points

(d)(i) For drawing an appropriate best-fit line including approximately the same number of points above and below the line **1 point**

Example Response



(d)(ii) For calculating the slope using two points on the best-fit line **1 point**

For correctly relating the slope of the best-fit line to the mass of Cart 2 **1 point**

For a correct mass of Cart 2 **1 point**

Example Response

$$slope = \frac{(1.20 - 0.60)\,\text{m/s}}{(0.70 - 0.35)\sqrt{\text{m}}} = 1.72\,\frac{\sqrt{\text{m}}}{\text{s}}$$

$$slope = \frac{m_1}{m_1 + m_2}\sqrt{2g}$$

$$m_2 = \frac{m_1\sqrt{2g}}{slope} - m_1$$

$$m_2 = \frac{(0.25\,\text{kg})\sqrt{2\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)}}{1.72\,\frac{\sqrt{\text{m}}}{\text{s}}} - (0.25\,\text{kg})$$

$$\therefore m_2 = 0.39\,\text{kg}$$

Scoring Note: Acceptable responses for mass are 0.30 to 0.60 kg

Total for part (d) 4 points

(e) For selecting "$m_1' < 0.250$ kg" with an attempt at a relevant justification **1 point**

For a correct justification **1 point**

Example Response

A smaller m_2 indicates that the initial energy and momentum was smaller. With identical slope and height H this means that the mass m_1' must be smaller.

Total for part (e) 2 points

Total for question 2 15 points

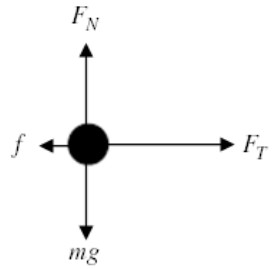
AP® Physics C: Mechanics
2016 Scoring Guidelines

2016 SCORING GUIDELINES

Question

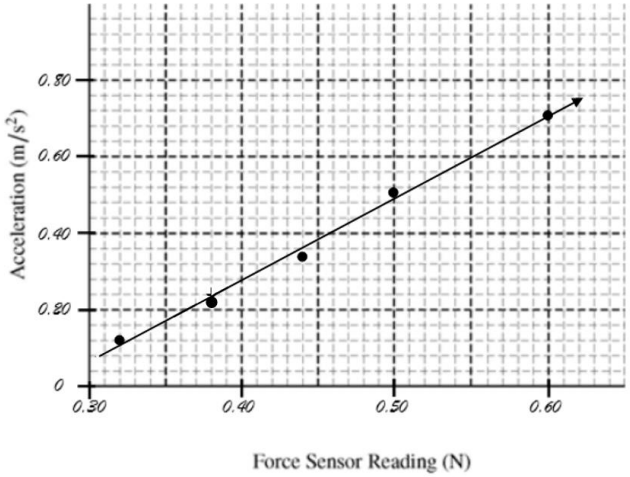
15 points total **Distribution of points**

3 points



For correctly drawing and labeling the force of tension — 1 point
For correctly drawing and labeling the force of friction — 1 point
For correctly drawing and labeling both forces in the vertical direction — 1 point
Note: A maximum of two points may be earned if there are any extraneous vectors.

3 points



For a correct scale that uses more than half the grid — 1 point
For correctly plotting the given data — 1 point
For drawing a straight line consistent with the given data — 1 point
Note: Full credit can be earned if the axes are switched.

SCORING GUIDELINES

Question 1 (continued)

Distribution of points

2 points

For correctly calculating slope using the best-fit straight line and not data points — 1 point

$$\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(0.70 - 0.16)}{(0.60 - 0.35)}\,\text{kg}^{-1} = 2.16\ \text{kg}^{-1}$$

Note: Linear regression gives slope $= 2.12\ \text{kg}^{-1}$.

For correctly calculating the mass of the cart using the slope — 1 point

$$m = \frac{1}{\text{slope}} = \frac{1}{\left(2.16\ \text{kg}^{-1}\right)}$$

Correct answer:

$m = 0.463\ \text{kg}$ (Note: linear regression gives $m = 0.472\ \text{kg}$)

1 point

For an answer with correct units consistent with the x-intercept of the graph from (b) i. — 1 point

$f = 0.272\ \text{N}$

(

1 point

lying Newton's second law and substituting the values from part (b)

$$\sum F = ma = F_a - f$$

$$a = \frac{F_a - f}{m} = \frac{0.45\ \text{N} - 0.272\ \text{N}}{0.463\ \text{kg}}$$

For an answer with correct units consistent with part (b), either from the graph or calculated using the mass and frictional force. — 1 point

$a = 0.376\ \text{m/s}^2$

SCORING GUIDELINES

Question 1 (continued)

Distribution of points

3 points

For using a correct equation to solve for the speed of the cart when the string breaks, with an acceleration consistent with part (c) i. — 1 point

$v_2 = v_1 + at$

$v_2 = 0 + \left(0.376\ \text{m/s}^2\right)(2.0\ \text{s})$

$v_2 = 0.752\ \text{m/s}$

For recognizing that the acceleration of the cart after the string breaks is due to the frictional force determined in part (b) — 1 point

Use a correct equation to solve for the time for the cart to stop after the string breaks

$v_2 = v_1 + at$

$0 = v_1 - \frac{f}{m}t$

For using the final velocity before the string breaks as the initial velocity for the cart stopping in the correct equation for time — 1 point

$t = \frac{mv_1}{f}$

$t = \frac{(0.463\ \text{kg})(0.752\ \text{m/s})}{(0.272\ \text{N})}$

Correct answer

$t = 1.28\ \text{s}$

Alternate solution — *Alternate points*

For setting the magnitude of the impulse before the string breaks equal to the magnitude of the impulse after the string breaks — *1 point*

$F_1 t_1 = F_2 t_2$

For correctly using the proper force (e.g., the tension force minus the friction force) for F_1 — *1 point*

For correctly using the proper force (e.g., the friction force) for F_2 — *1 point*

Correct answer

$t = 1.28\ \text{s}$

1 point

For selecting "Equal to" — 1 point

1 point

For selecting "Greater than" — 1 point

SCORING GUIDELINES

Question

15 points total **Distribution of points**

2 points

For correctly drawing and labeling vectors on the block of mass $2M$ 1 point
For correctly drawing and labeling vectors on the block of mass $3M$ using symbols for the vectors that are physically correct and different from those on the block of mass $2M$ 1 point
Note: A maximum of one point can be earned if there are any extraneous vectors.

2 points

Using a proper expression for conservation of momentum
$p_1 = p_2$
For correctly substituting into the above equation 1 point
$3Mv_0 = (3M + 2M)v_f$
For a correct answer 1 point
$v_f = \frac{3}{5}v_0$

(1 point

Using a proper expression for kinetic energy of the two-block system
$K = \frac{1}{2}mv^2$
$K = \frac{1}{2}(5M)\left(\frac{3}{5}v_0\right)^2$
For an answer consistent with part (b) 1 point
$K = \frac{9}{10}Mv_0^2$

SCORING GUIDELINES

Question continued)

Distribution of points

4 points

For a correct expression of the conservation of energy 1 point
$\Delta K_{\text{system}} + \Delta U_{\text{system}} = 0$
$K_0 = U_{\text{final}}$
For attempting to integrate the spring force equation 1 point
$\frac{9}{10}Mv_0^2 = -\int_{x_1}^{x_2} -Bx^3 dx$
$\frac{9}{10}Mv_0^2 = \int_{x_1}^{x_2} Bx^3 dx$
For using the correct limits of integration or an appropriate constant of integration 1 point
$\frac{9}{10}Mv_0^2 = \int_0^D Bx^3 dx$
$\frac{9}{10}Mv_0^2 = \left[\frac{Bx^4}{4}\right]_0^D$
For an answer consistent with the speed from (b) or the kinetic energy from part (c) 1 point
$D = \sqrt[4]{\frac{18Mv_0^2}{5B}}$

2 points

For selecting the correct answer "Right" with a reasonable attempt at a justification 1 point
If the incorrect selection is made, no points are earned for the justification.
For an indication that at maximum compression the block of mass $2M$ has an acceleration to the right due to the forces acting on the block of mass $2M$ or an acceleration to the right due to the external spring force acting on the system of blocks 1 point
Example: At maximum compression the two-block system is instantaneously at rest. The only horizontal external force acting on the system is due to the spring. This force is directed to the right. The system and therefore the block of mass $2M$ is accelerated to the right, which implies that the net force acting on the block of mass $2M$ is also to the right.

SCORING GUIDELINES

Question continued)

	Distribution of points
2 points	
e magnitude of the net force is greater on the block of mass $3M$.	
If the incorrect selection is made, no points are earned for the justification.	
For an indication that both blocks will have the same acceleration	1 point
For a correct justification for why the net force is greater on the block of mass $3M$	1 point
Example:	
Because the blocks stick together, both blocks must have the same acceleration. Because the block of mass $3M$ has more mass, the net force on it must be greater than the net force on the block of mass $2M$.	
2 points	
For selecting the correct answer "No," with a reasonable attempt at a justification	1 point
If the incorrect selection is made, no points are earned for the justification.	
If the correct answer is selected without any justification, the point is not earned for the selection.	
For an indication that the spring does not apply a linear force and that simple harmonic motion is the resulting motion of a linear restoring force $\left(a = -\frac{k}{m}\Delta x\right)$	1 point
Example:	
Because the blocks are sticking together and are attached to the spring, the spring will apply a restoring force to the blocks. However, because the restoring force exerted by a nonlinear spring is not proportional to the blocks' displacement from equilibrium, the blocks do not exhibit simple harmonic motion.	

2016 SCORING GUIDELINES

Question

15 points total	Distribution of points
3 points	
For an indication that the spring force equals the mass times the centripetal acceleration $F_S = F_C = ma_C$	1 point
$kx = mr\omega^2$ correct substitution for x in the equation	1 point
For a correct substitution for r in the equation that can be used to solve for k $k\left(\frac{d}{2}\right) = m(2d)\omega^2$	1 point
rect answer: $k = 4m\omega^2$	
1 point	
Substitute values for the block into the equation for rotational inertia $I = \sum mr^2 = m(2d)^2$	
For a correct answer or an answer consistent with the radius used in part (a) $I = 4md^2$	1 point
2 points	
For an indication that the rotational inertia of the system must include the platform, the rod, and the object $I = I_P + I_R + I_O$ $I = \frac{m_P R^2}{2} + \frac{m_R d^2}{3} + mr^2$	1 point
For correctly substituting into the equation the rotational inertia of the platform, the rod, and the object consistent with (b) i. $I = \frac{5m(2d)^2}{2} + \frac{3m\,d^2}{3} + 4md^2$ $I = (10 + 1 + 4)md^2$	1 point
rect answer: $I = 15md^2$	
1 point	
Using a correct expression of angular momentum $L = I\omega$	
For an answer consistent with the answer from part (b) ii. $L = 15md^2\omega$	1 point

SCORING GUIDELINES

Question continued)

Distribution of points

3 points

For indicating that the spring force equals the mass times the centripetal acceleration — 1 point

$F_S = F_C = ma_C$

$kx = mr\omega^2$

For a correct substitution for k, or a substitution consistent with part (a), in the equation — 1 point

For a correct substitution for r in the equation that can be used to solve for x — 1 point

$(4m\omega^2)x = m(d + x)\omega^2$

$4x = d + x$

$3x = d$

Correct answer:

$x = d/3$

2 points

electing "Decreasing" — 1 point

the wrong selection is made, no points are earned for the justification.

For a correct justification — 1 point

Example: The rotational inertia I of the system decreases while the angular velocity w stays the same. Because the angular momentum is Iw, it decreases.

2 points

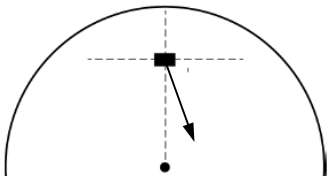
For selecting "Negative" — 1 point

If the wrong selection is made, no points are earned for the justification.

For a correct justification — 1 point

Example: Because the angular velocity is constant and the rotational inertia has decreased, the rotational kinetic energy has decreased. Therefore, work must be negative.

1 point



n arrow starting on the box and pointing down and to the right — 1 point