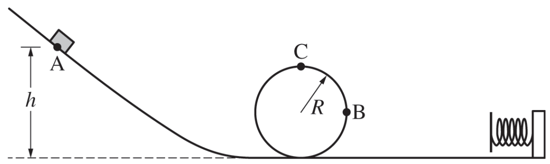
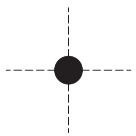


Note: Figure not drawn to scale.

2. A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \frac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

(a) On the dot below, which represents the block, draw an arrow that represents the direction of the acceleration of the block at point B in the figure above. The arrow must start on and point away from the dot.



Justify your answer.

(b)

i. Derive an expression for the speed v of the block at point B.

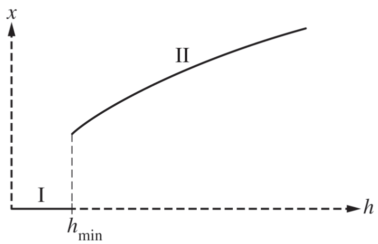
ii. Derive an expression for the magnitude of the net force F on the block at point B.

GO ON TO THE NEXT PAGE.

(c) In terms of R, derive an expression for the minimum height h_{min} necessary for the block to maintain contact with the track through point C.

(d) It is determined that $h = 0.30\,\text{m}$ and $R = 0.10\,\text{m}$. If the block is released from a height greater than that found in part (c), what would be the maximum compression x_{MAX} of the spring?

(e) A graph of the maximum compression of the spring as a function of height is shown below. The height h_{min} is the height calculated in part (c).

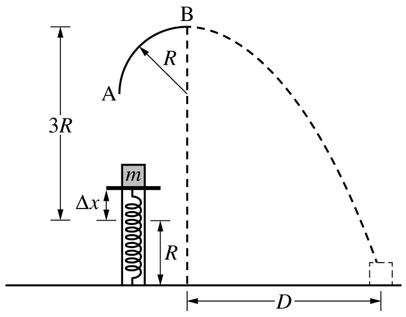


i. Explain why section I appears as a horizontal line segment on the horizontal axis.

ii. Explain the reason for the shape of section II on the graph.

GO ON TO THE NEXT PAGE.

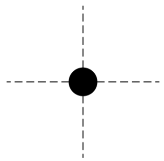
Begin your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

3. A block of mass m is placed on top of an ideal spring of spring constant k. The block is pushed against the spring, compressing the spring a distance Δx. The block is released from rest, leaves the spring at the position shown in the figure, travels upward, and enters a track with a constant radius of curvature R that has negligible friction. The block enters the track at point A, maintains contact with the track, and exits horizontally at point B, a distance $3R$ above the point the block was released. The block then falls to the ground and lands a horizontal distance D from the end of the track. Express all algebraic answers in terms of m, k, Δx, R, and physical constants, as appropriate. The size of the block is much smaller than the radius of curvature of the track.

(a) On the dot below, which represents the block, draw and label the forces (not components) that act on the block while still in contact with the track at point B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



Justify your choice of vectors.

GO ON TO THE NEXT PAGE.

(b)

i. Derive an expression for the speed v of the block at point B.

ii. Derive an expression for the magnitude of the net force F on the block at point B.

(c) Derive an expression for the minimum value of Δx_{min} required in order for the block to maintain contact with the track through point B.

The procedure is repeated several times with the distance $\Delta x > \Delta x_{min}$.

(d) Calculate the distance D that the block travels.

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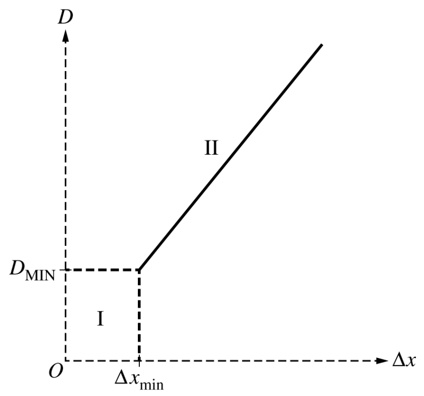
(e) The graph below shows the best-fit line drawn by the students through their data of D as a function of Δx.



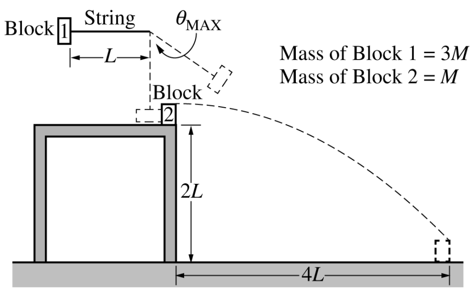
i. Explain why there are no data for section I of the graph.

ii. Explain the reason for the shape and minimum value of section II on the graph.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

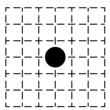


Note: Figure not drawn to scale.

2. A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(a) Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

(b) On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



(c) Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

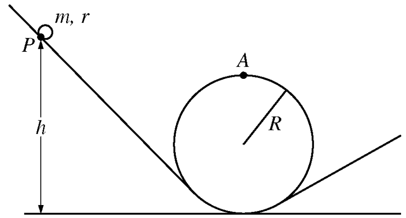
For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

(d) Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

(e) Calculate the speed of block 2 as it leaves the table.

(f) Calculate the speed of block 1 just after it collides with block 2.

(g) Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.



Note: Figure not drawn to scale.

3. The rotational inertia of a rolling object may be written in terms of its mass m and radius r as $I = bmr^2$, where b is a numerical value based on the distribution of mass within the rolling object. Students wish to conduct an experiment to determine the value of b for a partially hollowed sphere. The students use a looped track of radius $R >> r$, as shown in the figure above. The sphere is released from rest a height h above the floor and rolls around the loop.

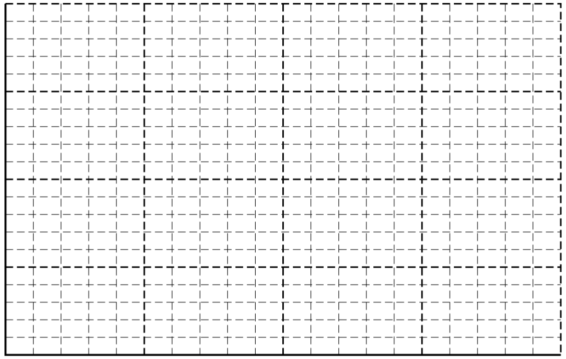
(a) Derive an expression for the minimum speed of the sphere's center of mass that will allow the sphere to just pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

(b) Suppose the sphere is released from rest at some point P and rolls without slipping. Derive an equation for the minimum release height h that will allow the sphere to pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

The students perform an experiment by determining the minimum release height h for various other objects of radius r and known values of b. They collect the following data.

Object	b	h (m)
Solid sphere	0.40	1.08
Hollow sphere	0.67	1.13
Solid cylinder	0.50	1.10
Hollow cylinder	1.0	1.20

(c) On the grid below, plot the release height h as a function of b. Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



(d) The students repeat the experiment with the partially hollowed sphere and determine the minimum release height to be 1.16 m. Using the straight line from part (c), determine the value of b for the partially hollowed sphere.

(e) Calculate R, the radius of the loop.

(f) In part (b), the radius r of the rolling sphere was assumed to be much smaller than the radius R of the loop. If the radius r of the rolling sphere was not negligible, would the value of the minimum release height h be greater, less, or the same?

____ Greater ____ Less ____ The same

Justify your answer.

STOP

END OF EXAM

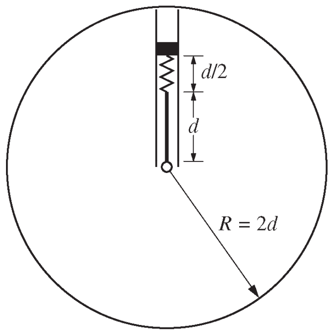


Figure 1

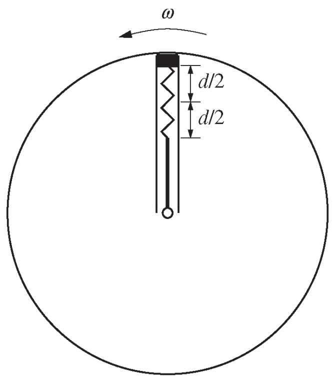


Figure 2

Mech.3.

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

	Mass	Rotational Inertia
Block	m	
Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)
Platform	$m_P = 5m$	$\frac{m_P R^2}{2}$ (about the central axis)

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(a) Derive an expression for the spring constant of the spring.

(b)

i. Determine an expression for the rotational inertia of the block around the axis of the platform.

ii. Derive an expression for the rotational inertia of the entire system about the axis of the platform.

(c) Determine an expression for the angular momentum of the entire system about the axis of the platform.

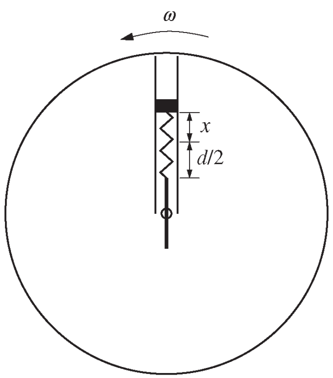


Figure 3

While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed ω is maintained by the motor driving the platform.

(d) Derive an expression for the distance x that the spring is stretched when the rod reaches the position shown in Figure 3 above.

For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

(e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

_____ Increasing _____ Decreasing _____ Staying the same

Justify your answer.

(f) In order to keep the system rotating with constant angular speed ω, is the motor doing positive work, negative work, or no work on the rotating system?

_____ Positive _____ Negative _____ No work

Justify your answer.

(g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.

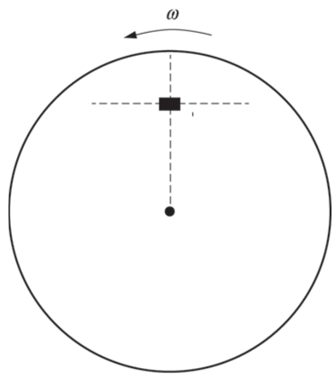
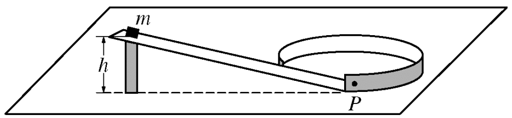


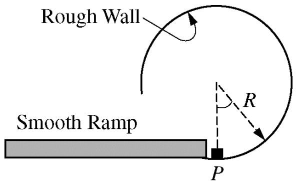
Figure 4

STOP

END OF EXAM



Side View



Top View

Mech. 2.

A small block of mass m starts from rest at the top of a frictionless ramp, which is at a height h above a horizontal tabletop, as shown in the side view above. The block slides down the smooth ramp and reaches point P with a speed v_0 . After the block reaches point P at the bottom of the ramp, it slides on the tabletop guided by a circular vertical wall with radius R, as shown in the top view. The tabletop has negligible friction, and the coefficient of kinetic friction between the block and the circular wall is μ .

(a) Derive an expression for the height of the ramp h. Express your answer in terms of v_0 , m, and fundamental constants, as appropriate.

A short time after passing point P, the block is in contact with the wall and moves with a speed of v .

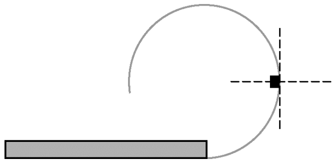
(b)

i. Is the vertical component of the net force on the block upward, downward, or zero?

____ Upward ____ Downward ____ Zero

Justify your answer.

ii. On the figure below, draw an arrow starting on the block to indicate the direction of the horizontal component of the net force on the moving block when it is at the position shown.



Top View

Justify your answer.

Express your answers to the following in terms of v_0 , v , m, R, μ , and fundamental constants, as appropriate.

(c) Determine an expression for the magnitude of the normal force N exerted on the block by the circular wall as a function of v .

(d) Derive an expression for the magnitude of the tangential acceleration of the block at the instant the block has attained a speed of v .

(e) Derive an expression for $v(t)$, the speed of the block as a function of time t after passing point P on the track.