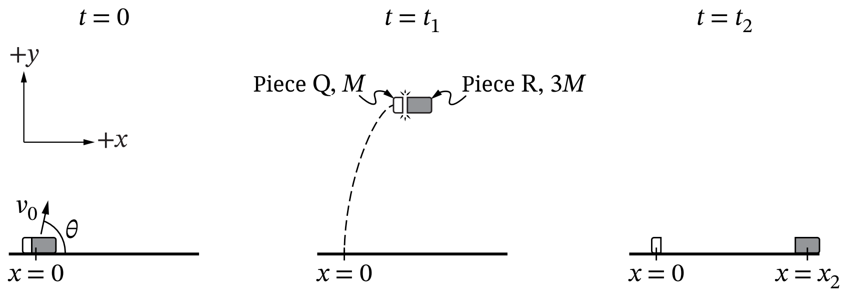
Question 2: Version J

2. A projectile of total mass $4M$ is launched from the ground at position $x = 0$ and time $t = 0$. The projectile is launched with an initial speed v_0 at an angle θ above the horizontal. When the projectile is at the highest point in its trajectory, it breaks into Pieces Q and R of masses M and $3M$, respectively.

The motion of the projectile is described for the following times.

- At $t = t_1$, immediately after the projectile breaks apart, the two pieces are moving away from each other horizontally.
- At $t = t_2$, Piece Q reaches the ground at $x = 0$ and Piece R reaches the ground at $x = x_2$, as shown in Figure 1.

Figure 1



Note: Figure not drawn to scale.

AP Physics C: **Mechanics: 2026**

Part A

The horizontal and vertical components of a momentum vector are represented by p_x and p_y, respectively. The shaded bars in Figure 2 represent p_x and p_y of the projectile immediately after $t = 0$.

Figure 2

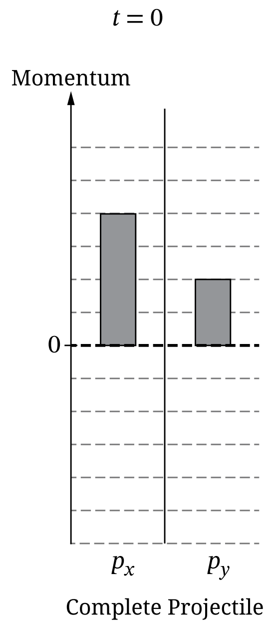
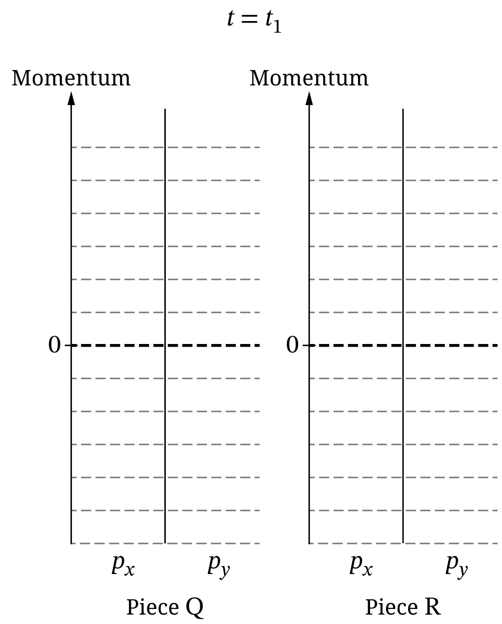


Figure 3



On Figure 3, **draw** shaded bars to represent p_x and p_y of Pieces Q and R at $t = t_1$.

- Start the shaded bars at the dashed line that represents zero momentum.
- Represent the magnitudes of the components of the momentum by using the heights of the shaded bars, consistent with the scale used in Figure 2.
- Represent any momentum component that is equal to zero by drawing a distinct line on the zero-momentum line.

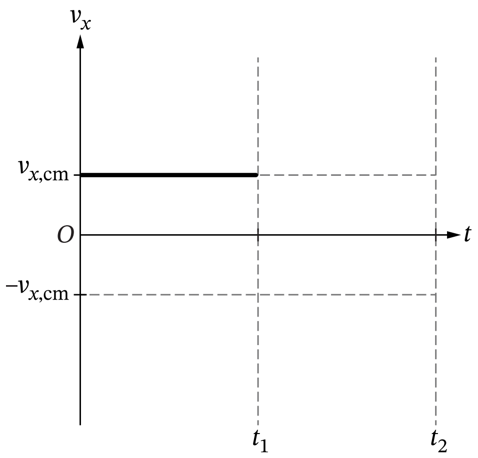
Part B

Derive an expression for x_2 in terms of v_0, θ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

The horizontal component of a velocity vector is represented by v_x. Figure 4 shows the horizontal component $v_{x,\mathrm{cm}}$ of the velocity of the center of mass of the projectile as a function of t during the time interval $0 < t < t_1$.

Figure 4



On Figure 4, **sketch** a line or curve to represent v_x as a function of t for the time interval $t_1 < t < t_2$ for each of the following.

- Piece Q
- Piece R
- The center of mass of the two-piece system

Clearly **label** all lines or curves.

Part D

Consider a case in which the projectile is launched at the same angle and initial speed as initially described. When the projectile breaks into Pieces Q and R, Piece Q falls straight down. In this case, Piece R reaches the ground at $x = x_{\mathrm{new}}$.

Indicate whether x_{new} is greater than, less than, or equal to x_2 by writing one of the following.

- $x_{\mathrm{new}} > x_2$
- $x_{\mathrm{new}} < x_2$
- $x_{\mathrm{new}} = x_2$

Briefly **justify** your answer either by referencing a feature of the representations you drew in part A or C or by using conceptual reasoning beyond algebraic solutions.

Begin your response to **QUESTION 3** on this page.

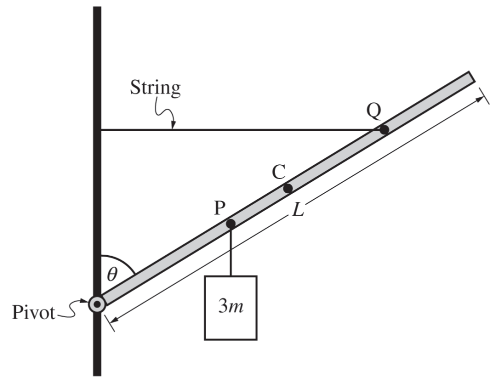
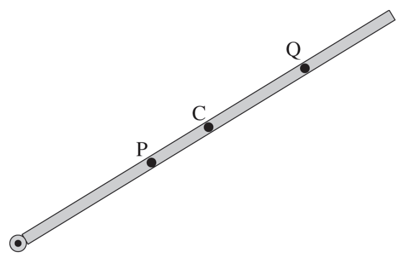


Figure 1

Note: Figure not drawn to scale.

3. A uniform rod of length L and mass m is attached to a pivot on a vertical pole, as shown in Figure 1. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

(a) On the following representation of the rod, **draw** and **label** the forces (not components) that are exerted on the rod. Each force must be represented by a distinct arrow that starts on and points away from the point at which the force is exerted on the rod.



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Continue your response to **QUESTION 3** on this page.

(b) In Figure 1, Point P is located $\frac{3}{8}L$ from the pivot and Point Q is located $\frac{6}{8}L$ from the pivot. **Derive** an equation for the tension F_T in the horizontal string in terms of L, m, θ, and physical constants, as appropriate.

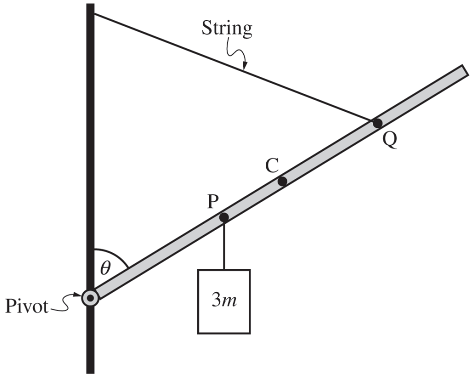

Figure 2

Note: Figure not drawn to scale.

(c) The original string is replaced with a longer string that connects Point Q to a higher location on the vertical pole, as shown in Figure 2. The angle θ remains the same. How does the new tension $F_{T,\text{ new}}$ compare with the original tension F_T from part (b) ? **Justify** your reasoning.

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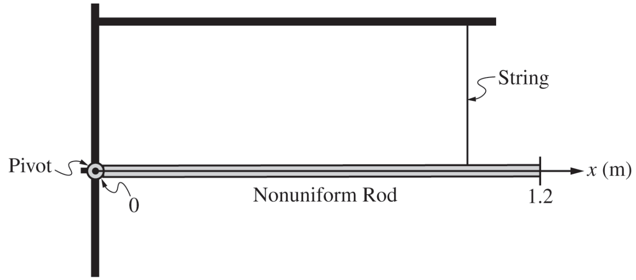

Figure 3

Note: Figure not drawn to scale.

(d) A nonuniform rod is now attached to the pivot, as shown in Figure 3. There is negligible friction between the nonuniform rod and the pivot. The rod has a length of 1.2 m and a linear mass density $\lambda(x) = A + Bx$, where x is the distance from the pivot, $A = 6.0$ kg/m, and $B = 10.0$ kg/m^2.

i. **Calculate** the mass of the rod.

ii. **Calculate** the rotational inertia of the rod about the pivot.

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END OF EXAM

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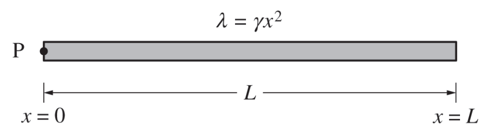

3. A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(a) Using integral calculus, show that the rotational inertia I of the rod about an axis perpendicular to the page and through point P is $\dfrac{3}{5}ML^2$.

(b) Determine the horizontal location of the center of mass of the rod relative to point P. Express your answer in terms of L.

(c) For an axis perpendicular to the page, is the value of the rotational inertia of the rod around point P greater than, less than, or equal to the value of the rotational inertia of the rod around the rod's center of mass?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

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The rod is released from rest in the position shown, and the rod begins to rotate about a horizontal axis perpendicular to the page and through point P.

(d) On the axes below, sketch graphs of the magnitude of the net torque τ on the rod and the angular speed ω of the rod as functions of time t from the time the rod is released until the time its center of mass reaches its lowest point.

(e) As the rod rotates from the horizontal position down through vertical, is the magnitude of the angular acceleration on the rod increasing, decreasing, or not changing?

_____ Increasing　　_____ Decreasing　　_____ Not changing

Justify your answer.

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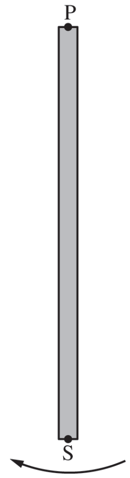

(f) The mass of the rod is $3.0\,\text{kg}$, and the length of the rod is $1.0\,\text{m}$. Calculate the linear speed v of point S as the rod swings through the vertical position shown.

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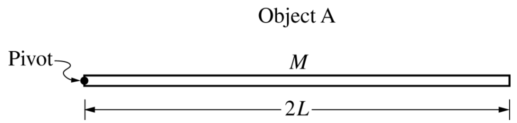
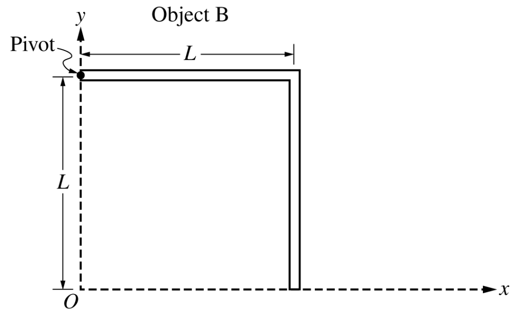
END OF EXAM



2. Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(a) Using integral calculus, derive an expression to show that the rotational inertia I_A of object A about the pivot is given by $\frac{4}{3}ML^2$.



Object B of total mass M is formed by attaching two thin, uniform, identical rods of length L at a right angle to each other. Object B is held in place, as shown above. Express your answers in part (b) in terms of L .

(b) Determine the following for the given coordinate system shown in the figure.

i. The x-coordinate of the center of mass of object B

ii. The y-coordinate of the center of mass of object B

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Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.

Object B has a rotational inertia of I_B about its pivot.

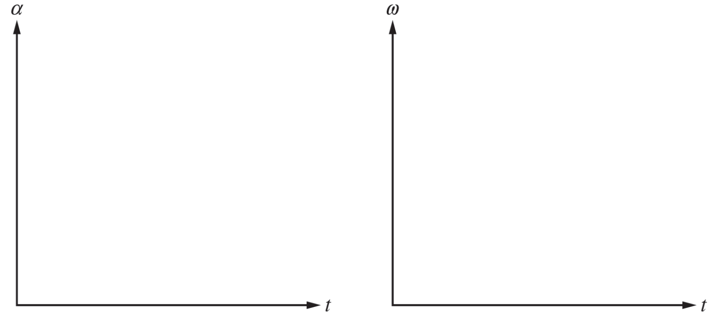
(c) Is the value of I_B greater than, less than, or equal to I_A ?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Object B is released from rest and begins to rotate about its pivot.

(d) On the axes below, sketch graphs of the magnitude of the angular acceleration α and the angular speed ω of object B as functions of time t from the time it is released to the time its center of mass reaches its lowest point.



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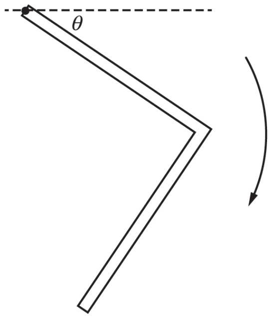
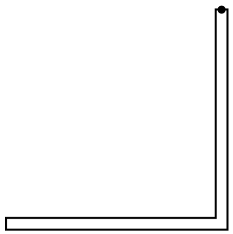


(e) While object B rotates from the horizontal position down through the angle θ shown above, is the magnitude of its angular acceleration increasing, decreasing, or not changing?

_____ Increasing _____ Decreasing _____ Not changing

Justify your answer.



Object B rotates through the position shown above.

(f) Derive an expression for the angular speed of object B when it is in the position shown above. Express your answer in terms of M, L , I_B , and physical constants, as appropriate.

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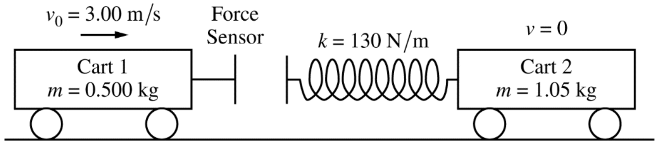
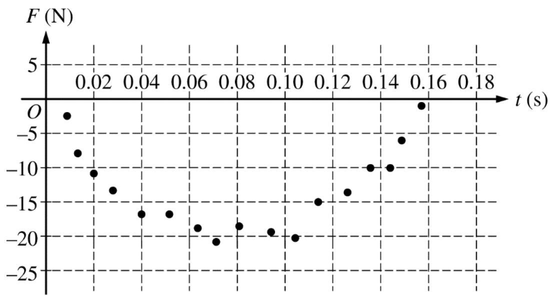


2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00\ \text{m/s}$ toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130\ \text{N/m}$. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200\ \text{N/s}^2\right)t^2 - (500\ \text{N/s})t$.



(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)

i. Calculate the speed of cart 1 after the collision.

ii. In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)

i. Calculate the speed of cart 2 after the collision.

ii. Show that the collision between the two carts is elastic.

(d)

i. Calculate the speed of the center of mass of the two-cart–spring system.

ii. Calculate the maximum elastic potential energy stored in the spring.