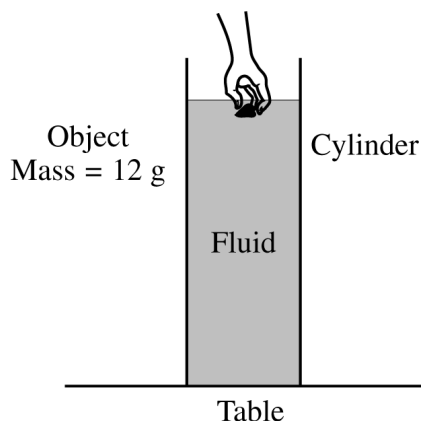
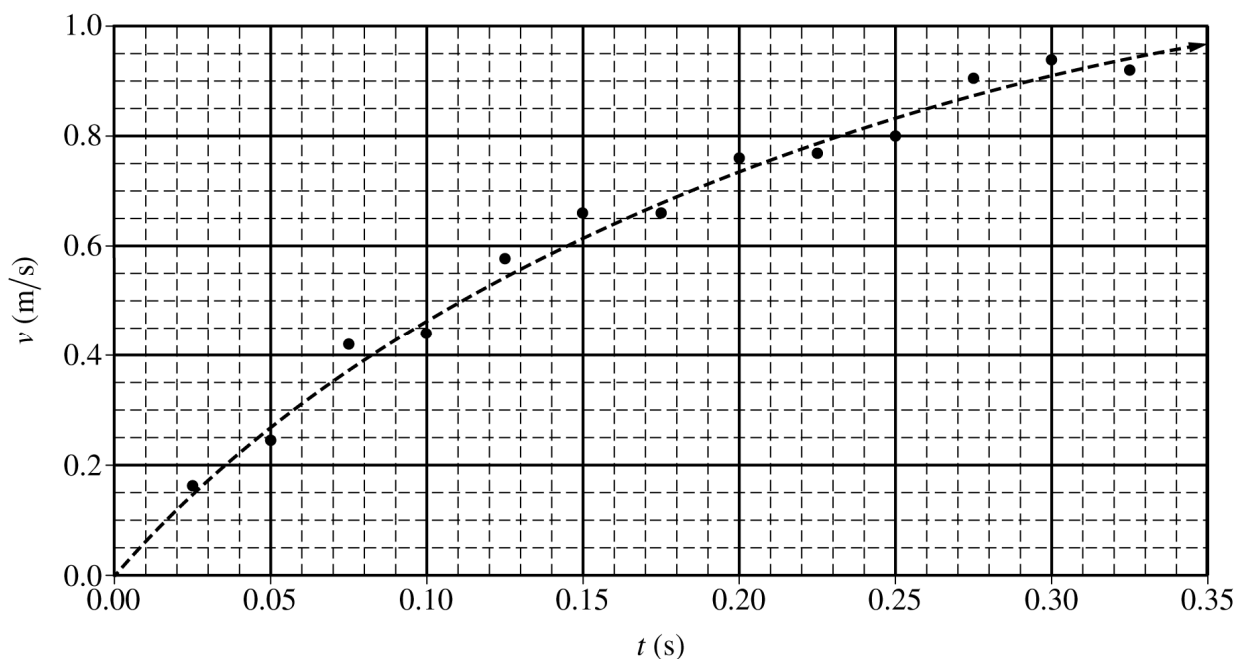


PHYSICS C: MECHANICS**SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of $m = 12\text{ g}$ is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.



(a)

- i. Does the speed of the object increase, decrease, or remain the same?

_____ Increase _____ Decrease _____ Remain the same

- ii. In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.
- iii. Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20$ s.

The students use the equation $v = A(1 - e^{-Bt})$ to model the speed of the falling object and find the best fit coefficients to be $A = 1.18$ m/s and $B = 5$ s⁻¹.

(b) Use the above equation to:

- i. Derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time t .
- ii. Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time t .

The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed.

(c)

- i. Determine the constant speed of the object.

Justify your answer.

- ii. Determine the force exerted by the fluid on the object at this time.

Justify your answer.

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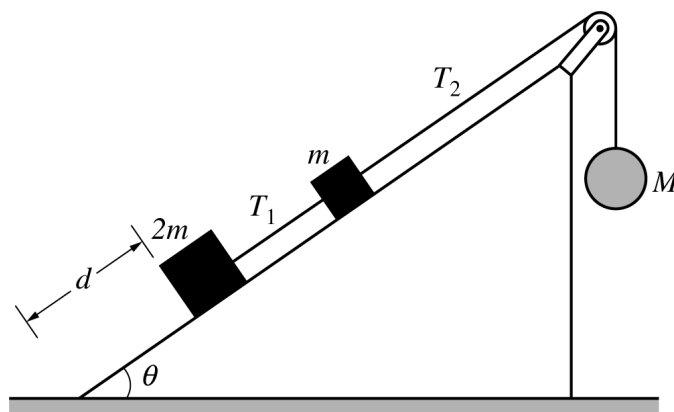


Figure 1

1. Blocks of mass m and $2m$ are connected by a light string and placed on a frictionless inclined plane that makes an angle θ with the horizontal, as shown in Figure 1 above. Another light string connecting the block of mass m to a hanging sphere of mass M passes over a pulley of negligible mass and negligible friction. The entire system is initially at rest and in equilibrium.
- (a) On the dots below that represent the block of mass m and the sphere of mass M , draw and label the forces (not components) that act on each of the objects shown. Each force must be represented by a distinct arrow starting on and pointing away from the dot.



- (b) Derive expressions for the magnitude of each of the following. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figures in part (a).
- The force T_2 exerted on the block of mass m by the string. Express your answers in terms of m , θ , and physical constants, as appropriate.
 - The mass M for which the system can remain in equilibrium. Express your answers in terms of m , θ , and physical constants, as appropriate.
- (c) Now suppose that mass M is large enough to descend and that the sphere reaches the floor before the blocks reach the pulley. Answer the following for the moment immediately after the sphere reaches the floor.
- Does the tension T_1 increase, decrease to a nonzero value, decrease to zero, or stay the same?

☐ Increase

☐ Decrease to a nonzero value

☐ Decrease to zero

☐ Stay the same
 - Is the velocity of the block of mass m up the ramp, down the ramp, or zero?

☐ Up the ramp

☐ Down the ramp

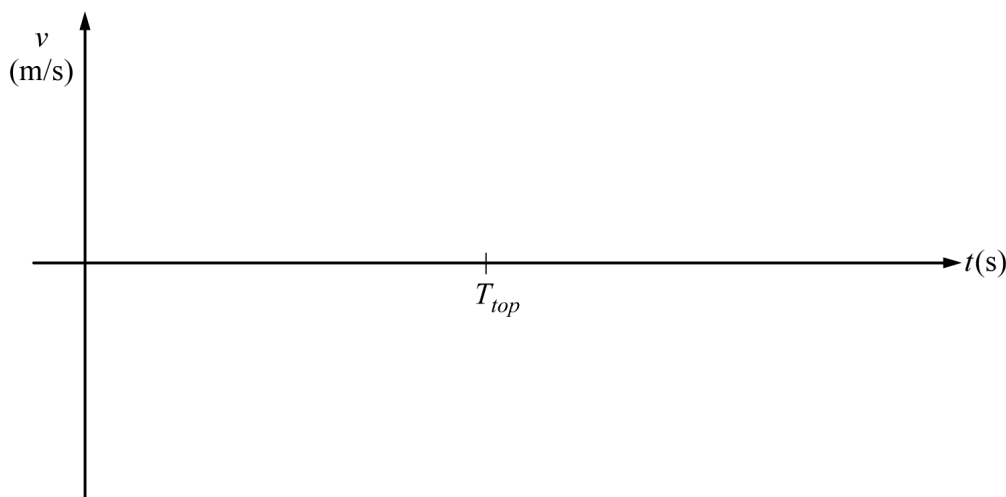
☐ Zero
 - Is the acceleration of the block of mass m up the ramp, down the ramp, or zero?

☐ Up the ramp

☐ Down the ramp

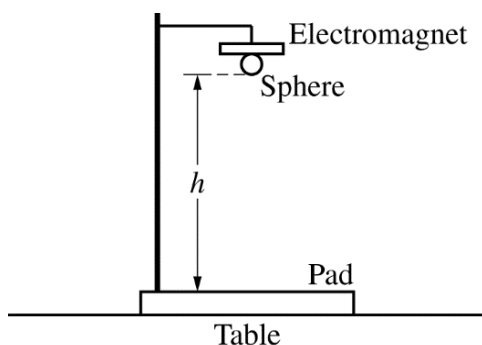
☐ Zero
- (d) Consider the initial setup in Figure 1. Now suppose the surface of the incline is rough and the coefficient of static friction between the blocks and the inclined plane is μ_s . Derive an expression for the minimum possible value of M that will keep the blocks from moving down the incline. Express your answer in terms of m , μ_s , θ , and fundamental constants, as appropriate.
- (e) The string connecting block m and the sphere of mass M then breaks, and the blocks begin to move from rest down the incline. The lower block starts a distance d from the bottom of the incline, as shown in Figure 1. The coefficient of kinetic friction between the blocks and the inclined plane is μ_k . Derive an expression for the speed of the blocks when the lower block reaches the bottom of the incline. Express your answer in terms of m , d , μ_k , θ , and fundamental constants, as appropriate.

2. A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0\text{ m/s}^2$, $L = 0.25\text{ m/s}^4$, and t is the time in seconds. At $t = 6.0\text{ s}$, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.
- (a) Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0\text{ s}$.
- (b) Calculate the speed of the rocket at $t = 6.0\text{ s}$.
- (c)
- Calculate the kinetic energy of the rocket at $t = 6.0\text{ s}$.
 - Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0\text{ s}$.
- (d) Calculate the maximum height reached by the rocket relative to its launching point.
- (e) On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



PHYSICS C: MECHANICS**SECTION II****Time—45 minutes****3 Questions**

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1. A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.
- (a) While taking the first data point, the student notices that the electromagnet actually releases the sphere after the timer begins. Would the value of g calculated from this one measurement be greater than, less than, or equal to the actual value of g at the student's location?
- ____ Greater than ____ Less than ____ Equal to

Justify your answer.

The electromagnet is replaced so that the timer begins when the sphere is released. The student varies the distance h . The student measures and records the time Δt of the fall for each particular height, resulting in the following data table.

h (m)	0.10	0.20	0.60	0.80	1.00
Δt (s)	0.105	0.213	0.342	0.401	0.451

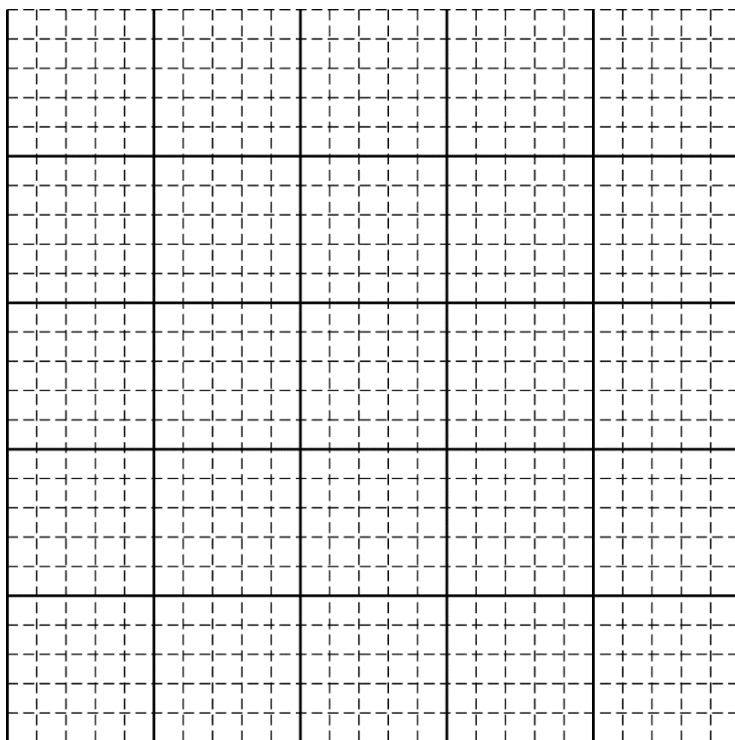
- (b) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for g .

Vertical axis: _____

Horizontal axis: _____

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given in the table. Label each row you use and include units.

- (c) Plot the data points for the quantities indicated in part (b) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



- (d) Using the straight line, calculate an experimental value for g .

Another student fits the data in the table to a quadratic equation. The student's equation for the distance fallen y as a function of time t is $y = At^2 + Bt + C$, where $A = 5.75 \text{ m/s}^2$, $B = -0.524 \text{ m/s}$, and $C = +0.080 \text{ m}$. Vertically down is the positive direction.

- (e) Using the student's equation above, do the following.

- i. Derive an expression for the velocity of the sphere as a function of time.
- ii. Calculate the new experimental value for g .
- iii. Using 9.81 m/s^2 as the accepted value for g at this location, calculate the percent error for the value found in part (e)ii.
- iv. Assuming the sphere is at a height of 1.40 m at $t = 0$, calculate the velocity of the sphere just before it strikes the pad.



Note: Figure not drawn to scale.

2. A block of mass m starts at rest at the top of an inclined plane of height h , as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k . There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m , h , k , and physical constants, as appropriate.

- (a)
- Derive an expression for the speed of the block just before it collides with the spring.
 - Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

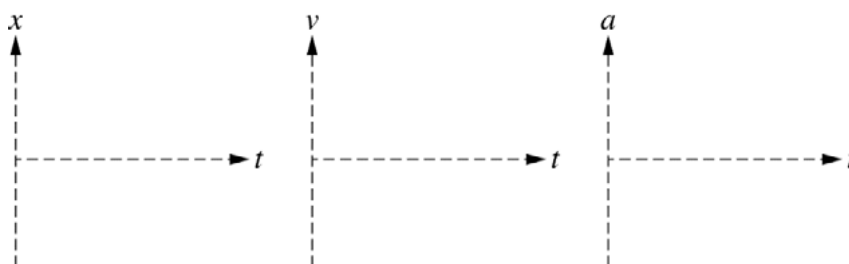
- (b) Derive an expression for the maximum compression of the spring.
- (c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0 . Now suppose the block experiences a resistive force as it slides on the horizontal surface.

The magnitude of the resistive force F is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of kg/m .

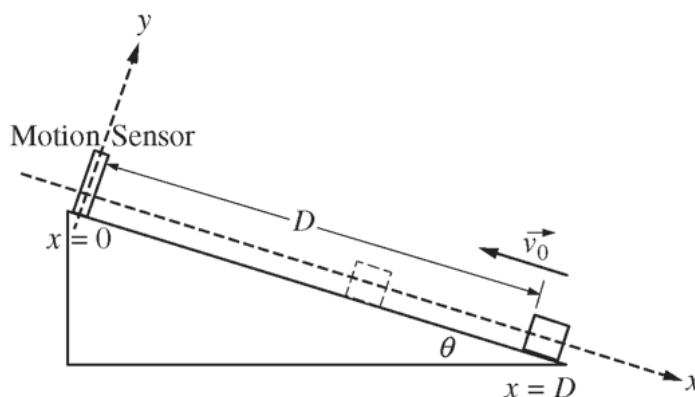
- (d)
- Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m , h , k , β , v , and physical constants, as appropriate.
 - Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time t can be written in the form $\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$, where v_0 is the speed at $t = 0$.

- (e) Sketch graphs of position x as a function of time t , velocity v as a function of time t , and acceleration a as a function of time t for the block as it is moving on the horizontal surface before it reaches the spring.



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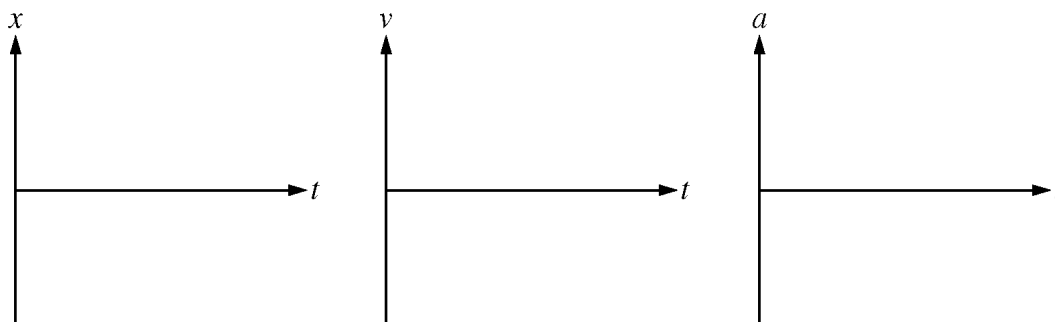


Mech.1.

A block of mass m is projected up from the bottom of an inclined ramp with an initial velocity of magnitude v_0 . The ramp has negligible friction and makes an angle θ with the horizontal. A motion sensor aimed down the ramp is mounted at the top of the incline so that the positive direction is down the ramp. The block starts a distance D from the motion sensor, as shown above. The block slides partway up the ramp, stops before reaching the sensor, and then slides back down.

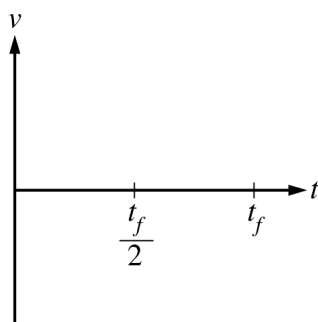
- (a) Consider the motion of the block at some time t after it has been projected up the ramp. Express your answers in terms of m , D , v_0 , t , θ and physical constants, as appropriate.
- Determine the acceleration a of the block.
 - Determine an expression for the velocity v of the block.
 - Determine an expression for the position x of the block.
- (b) Derive an expression for the position x_{min} of the block when it is closest to the motion sensor. Express your answer in terms of m , D , v_0 , θ , and physical constants, as appropriate.

- (c) On the axes provided below, sketch graphs of position x , velocity v , and acceleration a as functions of time t for the motion of the block while it goes up and back down the ramp. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



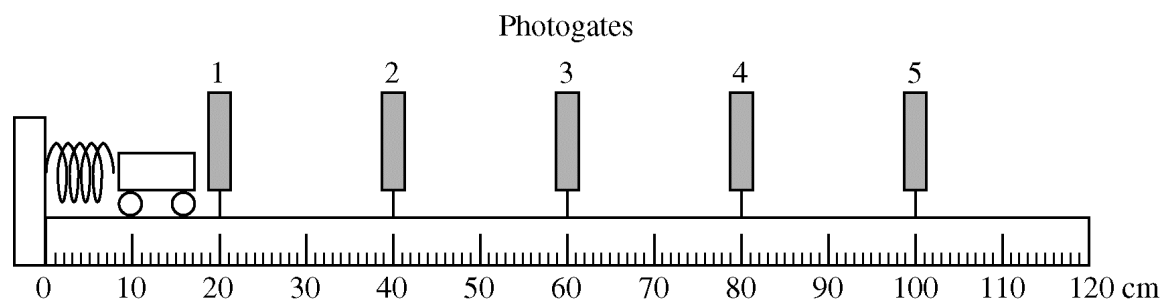
- (d) After the block slides back down and leaves the bottom of the ramp, it slides on a horizontal surface with a coefficient of friction given by μ_k . Derive an expression for the distance the block slides before stopping. Express your answer in terms of m , D , v_0 , θ , μ_k , and physical constants, as appropriate.

- (e) Suppose the ramp now has friction. The same block is projected up with the same initial speed v_0 and comes back down the ramp. On the axes provided below, sketch a graph of the velocity v as a function of time t for the motion of the block while it goes up and back down the ramp, arriving at the bottom of the ramp at time t_f . Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



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**Mech. 1.**

In an experiment, a student wishes to use a spring to accelerate a cart along a horizontal, level track. The spring is attached to the left end of the track, as shown in the figure above, and produces a nonlinear restoring force of magnitude $F_s = As^2 + Bs$, where s is the distance the spring is compressed, in meters. A measuring tape, marked in centimeters, is attached to the side of the track. The student places five photogates on the track at the locations shown.

- (a) Derive an expression for the potential energy U as a function of the compression s . Express your answer in terms of A , B , s , and fundamental constants, as appropriate.

In a preliminary experiment, the student pushes the cart of mass 0.30 kg into the spring, compressing the spring 0.040 m. For this spring, $A = 200 \text{ N/m}^2$ and $B = 150 \text{ N/m}$. The cart is released from rest. Assume friction and air resistance are negligible only during the short time interval when the spring is accelerating the cart.

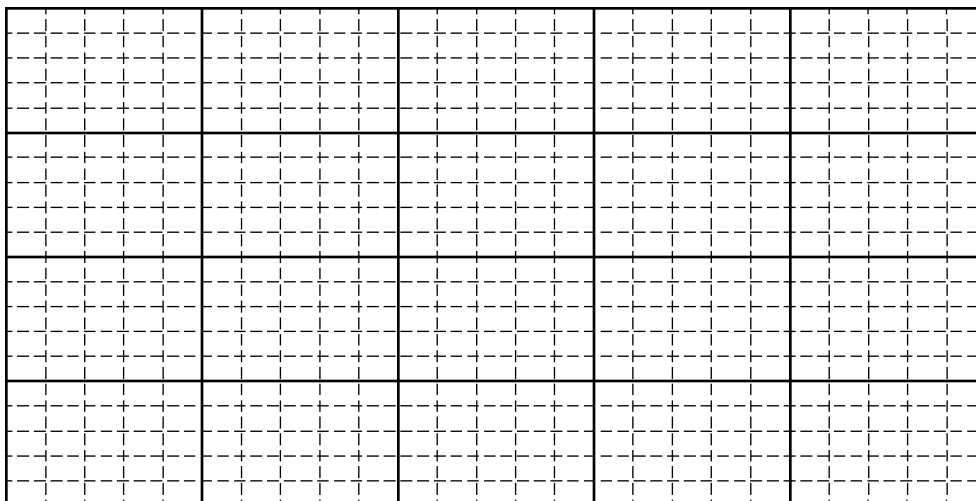
(b) Calculate the following:

- i. The speed of the cart immediately after it loses contact with the spring
- ii. The impulse given to the cart by the spring

In a second experiment, the student collects data using the photogates. Each photogate measures the speed of the cart as it passes through the gate. The student calculates a spring compression that should give the cart a speed of 0.320 m/s after the cart loses contact with the spring. The student runs the experiment by pushing the cart into the spring, compressing the spring the calculated distance, and releasing the cart. The speeds are measured with a precision of $\pm 0.002 \text{ m/s}$. The positions are measured with a precision of $\pm 0.005 \text{ m}$.

Photogate	1	2	3	4	5
Cart speed (m/s)	0.412	0.407	0.399	0.374	0.338
Photogate position (m)	0.20	0.40	0.60	0.80	1.00

(c) On the axes below, plot the data points for the speed v of the cart as a function of position x . Clearly scale and label all axes, as appropriate.



- (d)
- i. Compare the speed of the cart measured by photogate 1 to the predicted value of the speed of the cart just after it loses contact with the spring. List a physical source of error that could account for the difference.
 - ii. From the measured speed values of the cart as it rolls down the track, give a physical explanation for any trend you observe.