Question 1: Mathematical Routines (MR) — 10 points

A **(i)** For drawing only one arrow for the momentum of Block 2 before the collision that points leftward and is 6 units long — **Point A1**

For drawing only identical arrows for the momentum of the two-block system before and after the collision that are equal to the sum of the momentums drawn for blocks 1 and 2 before the collision — **Point A2**

Example Response

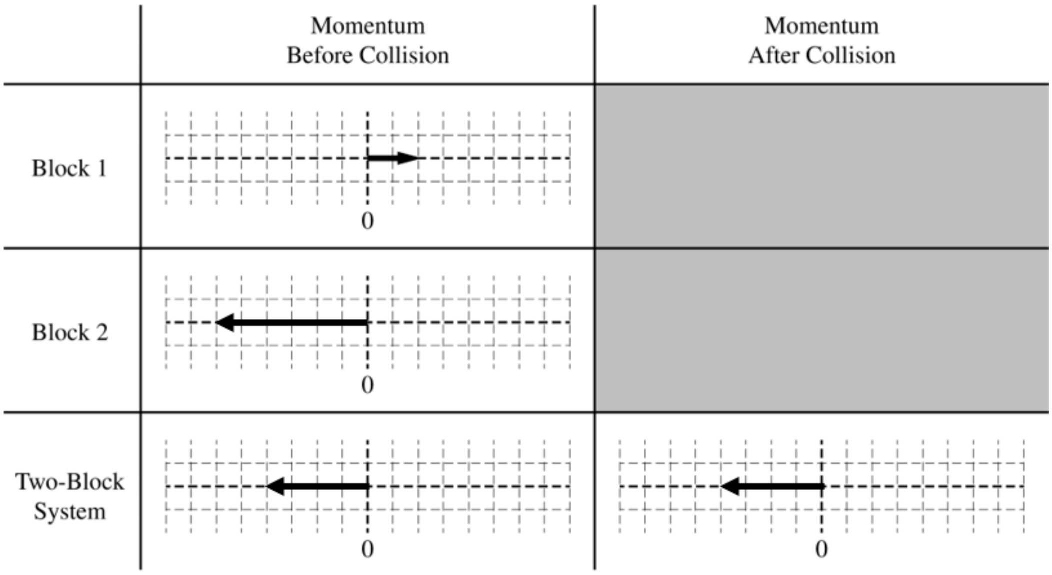


Figure 2

(ii) For a multistep derivation that includes the integral form of impulse or the differential form of Newton’s second law — **Point A3**

For indicating **one** of the following: — **Point A4**

- The final speed of Block 1 or Block 2 is $\frac{4}{7}v_0$.
- The change in the momentum of Block 2 is $+\frac{18}{7}mv_0$.
- The change in the momentum of Block 1 is $-\frac{18}{7}mv_0$.
- The change in the velocity of Block 2 is $+\frac{3}{7}v_0$.
- The change in the velocity of Block 1 is $-\frac{18}{7}v_0$.

For substituting the given expression for $F(t)$ into an expression for the impulse exerted on either block or the differential form of Newton’s second law — **Point A5**

For an integral with appropriate limits or a constant of integration — **Point A6**

For a correct expression for F_{max} in terms of given quantities — **Point A7**

Example Responses

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{\text{net}}(t)\,dt = \Delta\vec{p} \qquad \text{OR} \qquad \begin{aligned} \vec{F}_{\text{net}} &= \frac{d\vec{p}}{dt} \\ \int \vec{F}_{\text{net}}\,dt &= \Delta\vec{p} \end{aligned}$$

$$\sum \vec{p}_0 = \sum \vec{p}_f$$
$$(m)(2v_0) + (6m)(-v_0) = (m + 6m)(v_f)$$
$$v_f = -\frac{4}{7}v_0$$

$$\Delta p_{\text{Block 2}} = -\Delta p_{\text{Block 1}} = 6m\left(-\frac{4}{7}v_0 - (-v_0)\right)$$

$$\Delta p_{\text{Block 2}} = \int_0^{t_c} F_{\max} \sin(At)\,dt$$

$$\frac{18}{7}mv_0 = \frac{F_{\max}}{A}\left[-\cos(At)\right]\Big|_0^{t_c}$$
$$\frac{18}{7}mv_0 = \frac{F_{\max}}{A}\left[1 - \cos(At_c)\right]$$

$$F_{\max} = \frac{18}{7}\left(\frac{Amv_0}{1 - \cos(At_c)}\right)$$

B	For a multistep derivation that includes conservation of momentum	**Point B1**
	For indicating that the magnitude of the final momentum of the two-block system is $7mv_0$	**Point B2**
	For a correct expression for v_1 in terms of v_0	**Point B3**

Example Response

$$\sum \vec{p}_0 = \sum \vec{p}_f$$

$$mv_1 + 6m(-v_0) = 7m(v_0)$$
$$mv_1 = 13mv_0$$
$$v_1 = 13v_0$$

Question 2: Translation Between Representations (TBR) — 12 points

A	For indicating that K_{block} is zero	**Point A1**
	For drawing bars with positive heights for U_A and U_B	**Point A2**
	For drawing a bar for U_B with a height that is twice the height of the bar drawn for U_A **Scoring Note:** This point may be earned regardless of the signs of either bar.	**Point A3**

Example Response

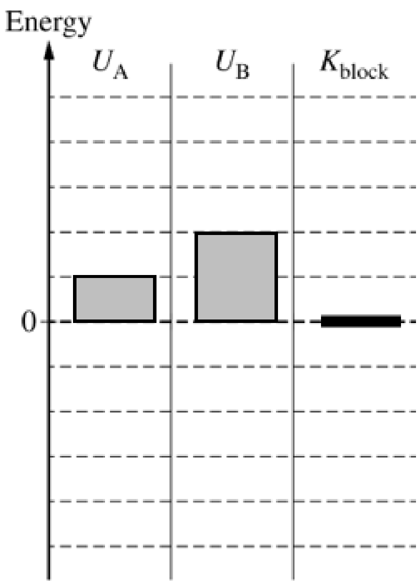


Figure 3

B	For a multistep derivation that includes energy conservation or simple harmonic motion	**Point B1**
	For relating the presence of both springs to the behavior of the system	**Point B2**
	For relating positions $x = x_1$ and $x = \frac{1}{2}x_1$ to the oscillation of the block	**Point B3**
	For a correct expression for v in terms of given quantities	**Point B4**

Example Responses

$$E_{\text{tot},0} = U_A(x_1) + U_B(x_1) + K_{\text{block}}(x_1)$$
$$E_{\text{tot},f} = U_A\left(\frac{1}{2}x_1\right) + U_B\left(\frac{1}{2}x_1\right) + K_{\text{block}}\left(\frac{1}{2}x_1\right)$$

$$E_{\text{tot},0} = \frac{1}{2}(k)(x_1)^2 + \frac{1}{2}(2k)(x_1)^2 + 0$$
$$E_{\text{tot},f} = \frac{1}{2}(k)\left(\frac{1}{2}x_1\right)^2 + \frac{1}{2}(2k)\left(\frac{1}{2}x_1\right)^2 + K_{\text{block},\,\frac{1}{2}x_1}$$

$$E_{\text{tot},0} = E_{\text{tot},f}$$
$$\frac{1}{2}kx_1^2 + kx_1^2 = \frac{1}{8}kx_1^2 + \frac{1}{4}kx_1^2 + K_{\text{block},\,\frac{1}{2}x_1}$$
$$\frac{3}{2}kx_1^2 = \frac{3}{8}kx_1^2 + K_{\text{block},\,\frac{1}{2}x_1}$$

$$K_{\text{block},\,\frac{1}{2}x_1} = \frac{9}{8}kx_1^2$$
$$\frac{1}{2}mv^2 = \frac{9}{8}kx_1^2$$

$$v = \frac{3}{2}x_1\sqrt{\frac{k}{m}}$$

OR

$$x = x_{\max}\cos(\omega t + \phi)$$

$$x(t) = x_1\cos\omega t$$
$$v(t) = -x_1\omega\sin\omega t$$

$$k_{\text{eff}} = k + 2k = 3k$$
$$\omega = \sqrt{\frac{3k}{m}}$$

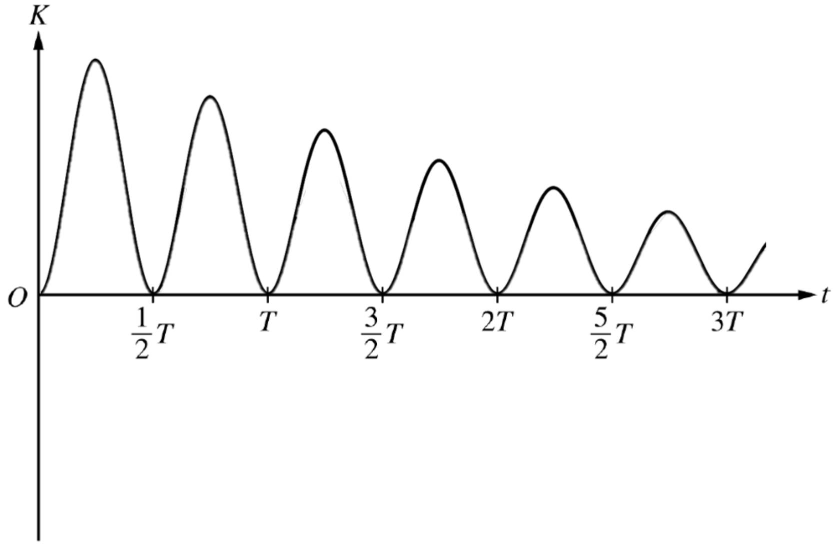
$$\frac{x_1}{2} = x_1\cos\omega t_{\frac{1}{2}x_1}$$
$$\cos\omega t_{\frac{1}{2}x_1} = \frac{1}{2}$$
$$t_{\frac{1}{2}x_1} = \frac{\pi}{3\omega}$$

$$v_{\frac{1}{2}x_1} = -x_1\omega\sin\omega t_{\frac{1}{2}x_1}$$
$$v_{\frac{1}{2}x_1} = -x_1\sqrt{\frac{3k}{m}}\sin\frac{\pi}{3}$$

$$v = \left|v_{\frac{1}{2}x_1}\right| = \frac{3}{2}x_1\sqrt{\frac{k}{m}}$$

C	For sketching a curve that starts at zero and is always positive or zero	**Point C1**
	For sketching a periodic curve with zeros that have a period of $\frac{1}{2}T$	**Point C2**
	For sketching a periodic curve with a decreasing amplitude	**Point C3**

Example Response



Scenario 2

Figure 5

D	For indicating **one** of the following: • The period increases. • The rate at which the graph approaches zero increases. • The maximum value of K over each period is less than the graph drawn.	**Point D1**
	For a correct justification relevant to the feature indicated	**Point D2**

Example Responses

The period of the graph increases. The period of a simple harmonic oscillator increases with increasing mass.

OR

The rate at which the graph approaches zero amplitude increases. Because the mass of the block is larger, the force of friction is greater, which causes the block to lose energy at a greater rate.

Question 3: Experimental Design and Analysis (LAB) — 10 points

A

For describing a procedure that includes measuring h and the speed of the block as the block moves across the surface — **Point A1**

For a procedure that indicates a reasonable method of reducing experimental uncertainty — **Point A2**

Examples of acceptable responses may include the following:

- Repeating the experiment multiple times for the same value of h
- Repeating the experiment for multiple values of h

Example Response

Measure the height h at which the block-box system is released. Measure the speed of the block as the block slides across the horizontal surface. Repeat the measurement of the speed of the block for varying release heights.

B

For describing a graph that has linear trend that can be used to find g — **Point B1**

Examples of acceptable responses include the following:

- v^2 vs. $2h$
- $\frac{1}{2}v^2$ vs. h
- v^2 vs. h
- v vs. $\sqrt{h}$

Scoring Notes:

- Reponses that include the reciprocals of the preceding examples, in addition to other equivalent graphs, also earn this point.
- This point may be earned independently of the response in part A.

For correctly relating the slope of the best-fit line to g — **Point B2**

Examples of acceptable responses include the following:

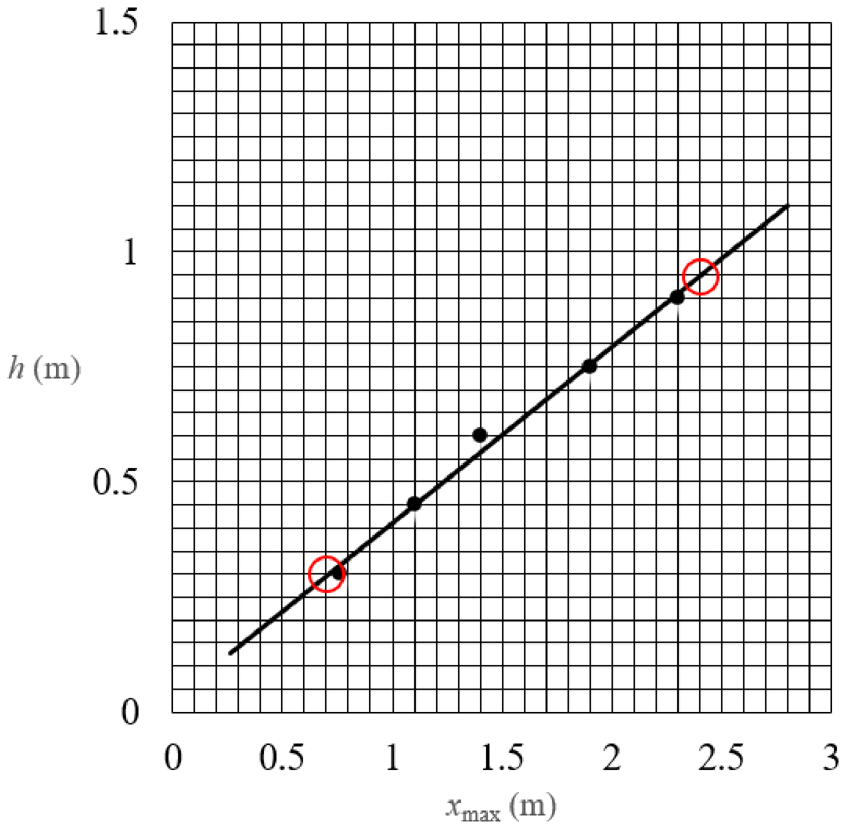
Graph	Analysis
v^2 vs. $2h$	slope $= g$
$\frac{1}{2}v^2$ vs. h	slope $= g$
v^2 vs. h	slope $= 2g$
v vs. $\sqrt{h}$	slope $= \sqrt{2g}$

Example Response

Plot v^2 on the vertical axis and $2h$ on the horizontal axis. The slope of the best-fit line is equal to g.

C	**(i)**	For indicating appropriate quantities that could be plotted to produce a linear graph that can be used to determine μ, such as h **vs.** x_{max} **Scoring Note:** Reponses that include the reciprocal of the preceding example, in addition to other equivalent graphs, also earn this point.	**Point C1**
	(ii)	For labeling the axes (including units) with a linear scale	**Point C2**
		For plotting data points consistent with **one** of the following: • The quantities indicated in part C (i) • The quantities provided in Table 2 • The axes indicated on the grid	**Point C3**
	(iii)	For drawing a line or curve that approximates the trend of the plotted data	**Point C4**

Example Response



D	For correctly relating the slope of the best-fit line to the value of μ	**Point D1**

Examples of acceptable responses includes the following:

Graph	Analysis
h vs. x_{max}	slope $= \mu$

For a value of μ within a range of 0.30 to 0.50	**Point D2**

Example Response

$E_0 + W = E_f$
$mgh - F_f x_{max} = 0$
$mgh = \mu mgx_{max}$
$h = \mu x_{max}$

$y = mx + b$
$h = \mu x_{max}$
slope $= \mu$
$$\text{slope} = \frac{0.95\text{ m} - 0.30\text{ m}}{2.4\text{ m} - 0.70\text{ m}}$$
slope $= 0.38$
$\mu = \text{slope} = 0.38$

Question 4: Qualitative Quantitative Translation (QQT) — 8 points

A

For indicating $f_D < f_R$ — **Point A1**

For a justification that compares **one** of the following: — **Point A2**

- The motions of the disk and the ring using translational kinematics
- The motions of the disk and the ring using rotational kinematics
- The rotational inertias of the disk and the ring

For a justification that includes **one** of the following: — **Point A3**

- Reasoning that attempts Newton's second law in translational form
- Reasoning that attempts Newton's second law in rotational form
- Reasoning that attempts conservation of energy

Example Response

The ring has a greater rotational inertia because it has more mass distributed towards the edge. So the ring travels farther and has less acceleration down the ramp. Because the ring and the disk have the same mass, the gravitational forces exerted on the shapes are the same. From Newton's second law, the ring must have less net force down the ramp and more friction up the ramp.

B

For a multistep derivation that includes Newton's second law in both translational and rotational forms — **Point B1**

For indicating opposite signs for the gravitational force and frictional force in an expression for Newton's second law — **Point B2**

For using the relationship $a = r\alpha$ in an attempt to solve a system of equations — **Point B3**

Scoring Note: Responses that include conservation of energy may earn full credit.

Example Response

$$\alpha_{sys} = \frac{\sum \tau}{I_{sys}}$$

$$\left(\frac{a}{R}\right) = \frac{fR}{I}$$

$$a = \frac{fR^2}{I}$$

$$\vec{a}_{sys} = \frac{\sum \vec{F}}{m_{sys}}$$

$$f - Mg\sin\theta = -Ma$$

$$f - Mg\sin\theta = -M\frac{fR^2}{I}$$

$$f + M\frac{fR^2}{I} = Mg\sin\theta$$

$$f\left(1 + \frac{MR^2}{I}\right) = Mg\sin\theta$$

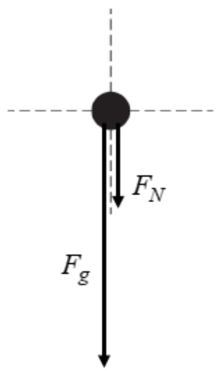
$$f = \frac{Mg\sin\theta}{1 + \frac{MR^2}{I}}$$

C	For indicating "Equal to"	**Point C1**
	For a justification that includes that kinetic friction is only dependent on the surfaces (or equivalently, the coefficient of kinetic friction between those surfaces) and normal force	**Point C2**
	Example Response *The forces of kinetic friction are equal. If slipping, the friction is kinetic, and the force of kinetic friction on the hoop and the disk are the same because the coefficient of kinetic friction is only dependent on the surfaces, and the normal forces on both are the same.*	

Question 3: Free-Response Question — 15 points

(a)	For correctly drawing and labeling the weight of the block	**1 point**
	For correctly drawing and labeling the force exerted by the track on the block	**1 point**
	For a correct justification consistent with the diagram	**1 point**

Example response for part (a)



The force F_g represents the weight of the block and always points downward. The force F_N represents the force the track exerts on the block to keep it moving in a circular path and points perpendicular to the surface of the track.

Scoring Note: Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$, $F_{\text{Block,E}}$ .The labels G or g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force.

Scoring Note: If extraneous forces are present, a maximum of 2 points can be earned.

Total for part (a) — 3 points

(b)	**i.**	For using conservation of energy $U_1 + K_1 = U_2 + K_2 \therefore U_1 + 0 = U_2 + K_2$	**1 point**
		For correctly relating the elastic potential energy at maximum spring compression to the gravitational potential energy at point B $U_{s1} + 0 = U_{g2} + K_2 \therefore K_2 = U_{s1} - U_{g2}$	**1 point**
		For a correct substitution into the equation above $\frac{1}{2}mv_B^2 = \frac{1}{2}k(\Delta x)^2 - mgh_2$ $v_B^2 = \frac{k}{m}(\Delta x)^2 - 2g(3R) \therefore v_B = \sqrt{\frac{k}{m}(\Delta x)^2 - 6gR}$	**1 point**

	ii.	For correctly relating the centripetal force to speed from part (b)(i) $F_C = \frac{mv^2}{r} = \frac{mv_B^2}{R}$	**1 point**
		For an answer consistent with part (b)(i) $F_C = \frac{m}{R}\left(\sqrt{\frac{k}{m}(\Delta x)^2 - 6gR}\right)^2 = \frac{k(\Delta x)^2}{R} - 6mg$	**1 point**
		Total for part (b)	**5 points**
(c)		For correctly relating the net force to the diagram from part (a) and setting the normal force equal to zero	**1 point**
		For correctly substituting into the equation above $\frac{k(\Delta x)^2}{R} - 6mg = mg \therefore \frac{k(\Delta x)^2}{R} = 7mg \therefore \Delta x = \sqrt{\frac{7mgR}{k}}$	**1 point**
		Total for part (c)	**2 points**
(d)		For correctly relating the height of fall to the time of fall $y = y_0 + v_{oy}t + \frac{1}{2}a_y t^2 \therefore H = 0 + 0 + \frac{1}{2}gt^2 \therefore t = \sqrt{\frac{(2)(4R)}{g}} = \sqrt{\frac{8R}{g}}$	**1 point**
		For correctly substituting into the equation for constant velocity consistent with part (b)(i) $D = v_x t = \left(\sqrt{\frac{k}{m}(\Delta x)^2 - 6gR}\right)\left(\sqrt{\frac{8R}{g}}\right)$	**1 point**
		Total for part (d)	**2 points**
(e)	**i.**	For a correct justification	**1 point**
	ii.	For indicating that as the maximum compression of the spring increases, the distance D increases	**1 point**
		For indicating that the minimum value is due to the minimum speed needed to get through the track.	**1 point**
		Example responses for part (e) *The block needs a minimum speed to make it through point B on the track; thus, the horizontal line segment represents compressions of the spring for which the block does not make it to point B.* OR *From the equation in part (d), the compression of the spring is directly proportional to the horizontal distance traveled by the block; thus, the graph would be a straight line. The minimum value is the distance traveled when the compression of the spring generates the minimum speed needed to reach point B on the track.*	
		Total for part (e)	**3 points**
		Total for question 3	**15 points**

2019

AP® Physics C: Mechanics

Scoring Guidelines
Set 1

2019 SCORING GUIDELINES

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

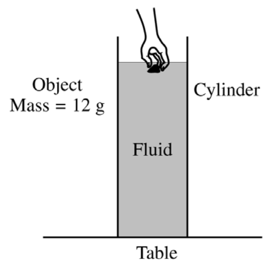
5. The scoring guidelines typically show numerical results using the value $g = 9.8\ \mathrm{m/s^2}$, but the use of $10\ \mathrm{m/s^2}$ is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

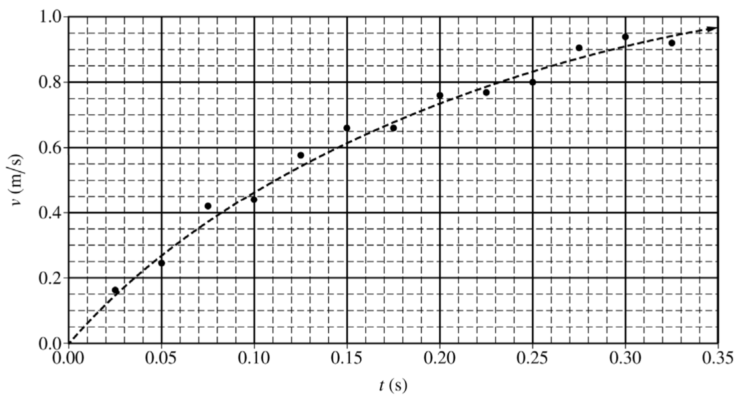
2019 SCORING GUIDELINES

Question 1

15 points



In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of $m = 12$ g is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.



(a)

i. LO CHA-1.C, SP 7.A
1 point

Does the speed of the object increase, decrease, or remain the same?

____ Increase ____ Decrease ____ Remain the same

For selecting “Increase”		1 point

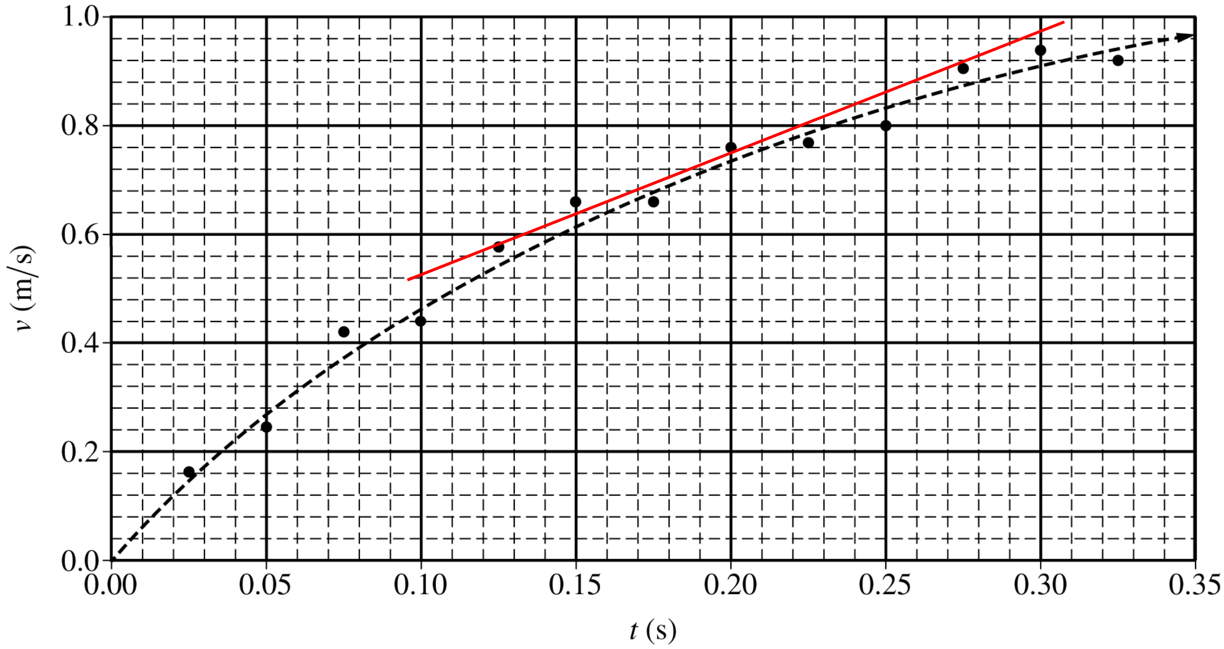
ii. LO CHA-1.C, SP 4.D
2 points

In a brief statement, describe the direction of the object’s acceleration and how the magnitude of this acceleration changed as the object fell.

For a description that includes the direction of the acceleration as being “downwards”		1 point
For a description that includes the decrease in the magnitude of the acceleration		1 point
Example: Because the object is moving downwards and speeding up, the acceleration must be downwards. Because the slope of the graph of speed as a function of time is decreasing, the magnitude of the acceleration must be decreasing.		

Question 1 (continued)

(a) continued



iii. LO CHA-1.C, SP 4.D, 6.C
2 points

Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20$ s.

For calculating the slope of a trend line at $t = 0.20$ s		1 point
$\text{slope} = a = \frac{\Delta v}{\Delta t} = \frac{(0.8 - 0.6)\ \text{m/s}}{(0.226 - 0.136)\ \text{s}}$		
For a correct answer using points from a tangent line		1 point
$a = 2.22\ \text{m/s}^2$		

The students use the equation $v = A\left(1 - e^{-Bt}\right)$ to model the speed of the falling object and find the best-fit coefficients to be $A = 1.18\ \text{m/s}$ and $B = 5\ \text{s}^{-1}$.

2019 SCORING GUIDELINES

Question 1 (continued)

(b) Use the above equation to:

i. LO CHA-1.B, SP 6.B, 6.C
3 points

Derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time *t*.

For indicating that the vertical displacement is the integration of the velocity		1 point
$\Delta y = \int v dt = \int A\left(1 - e^{-Bt}\right)dt$		
For the equation for speed using appropriate limits or constant of integration		1 point
$\Delta y = \int_{t'=0}^{t'=t} A\left(1 - e^{-Bt'}\right)dt' = A\left[t' - \frac{1}{-B}e^{-Bt'}\right]_{t'=0}^{t'=t} A\left[\left(t + \frac{1}{B}e^{-Bt}\right) - \left(0 + \frac{1}{B}e^{0}\right)\right]$		
For a correct answer		1 point
$\Delta y = A\left(t + \frac{1}{B}\left(e^{-Bt} - 1\right)\right) = (1.18)\left(t + \frac{1}{5}\left(e^{-5t} - 1\right)\right)$		
Note: Credit given for using *A* and *B* or plugging in the given values		

ii. LO INT-1.C.d, SP 6.B, 6.C
3 points

Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time *t*.

For attempting the derivative of the equation for speed		1 point
$a = \frac{dv}{dt} = \frac{d}{dt}\left[A\left(1 - e^{-Bt}\right)\right]$		
For a correct equation for the acceleration		1 point
$a = ABe^{-Bt}$		
For multiplying the equation above by the mass of the object		1 point
$F = ma = m\left(ABe^{-Bt}\right) = mABe^{-Bt}$		
$F = (.012)(1.18)(5)e^{-5t} = 0.071e^{-5t}$		
Note: Credit given for using *A*, *B*, and *m* or plugging in the given values		

Question 1 (continued)

(c)

The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed.

i. LO INT-1.I, SP 7.A, 7.C
2 points

Determine the constant speed of the object.
Justify your answer.

For stating the constant speed is $v = A$		1 point
For indicating that the constant speed can be determined by setting the time equal to infinity		1 point
Example: After a long time, the falling object will reach a terminal constant speed in the fluid. This can be determined by setting the time t in the equation for speed equal to infinity. By doing this, the constant speed is determined to be $v = A$.		
<u>Note:</u> Credit given for solving mathematically		

ii. LO INT-1.H.b, SP 7.A, 7.C
2 points

Determine the force exerted by the fluid on the object at this time. Justify your answer.

For indicating that the net force is equal to zero when the object moves with constant speed		1 point
For indicating the resistive force is equal to the weight of the object at this time		1 point
Example: When the falling object reaches a constant speed in the fluid, the net force must be zero. Because the only vertical forces acting on the object are Earth's gravitational pull and the resistive force of the fluid, these two forces must be equal. So, the resistive force must be equal to the weight of the object or 0.12 N.		
<u>Note:</u> Credit given for solving mathematically		

2019 SCORING GUIDELINES

Question 1 (continued)

Learning Objectives

CHA-1.B: Determine functions of position, velocity, and acceleration that are consistent with each other, for the motion of an object with a nonuniform acceleration.
CHA-1.C: Describe the motion of an object in terms of the consistency that exists between position and time, velocity and time, and acceleration and time.
INT-1.C.d: Derive an expression for the net force on an object in translational motion.
INT-1.H.b: Describe the acceleration, velocity, or position in relation to time for an object subject to a resistive force (with different initial conditions, i.e., falling from rest or projected vertically).
INT-1.I: Calculate the terminal velocity of an object moving vertically under the influence of a resistive force of a given relationship.

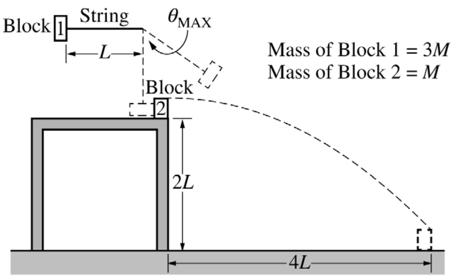
Science Practices

4.D: Select relevant features of a graph to describe a physical situation or solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

2019 SCORING GUIDELINES

Question 2

15 points



Note: Figure not drawn to scale.

A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

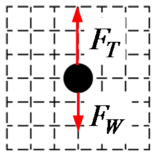
(a) LO CON-2.C.a, SP 5.E
1 point

Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

For correctly calculating the speed of block 1		1 point
$U_{g1} = K_2 \therefore mgh = \frac{1}{2}mv^2 \therefore gL = \frac{1}{2}v^2$		
$v = \sqrt{2gL}$		
Note: Credit is earned even if no work is shown.		

(b) LO INT-2.B.b, SP 3.D
2 points

On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



For correctly drawing and labeling the weight of the block and the tension in the string		1 point
For drawing the tension longer than the weight of the block		1 point
Note: A maximum of one point can be earned if there are any extraneous vectors.		

2019 SCORING GUIDELINES

Question 2 (continued)

(c) LO INT-2.B.b, SP 5.A, 5.E
2 points

Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

For using an appropriate equation to calculate the tension in the string		1 point
$F_T = mg + ma_C = mg + \dfrac{mv^2}{r} = 3Mg + \dfrac{(3M)\left(\sqrt{2gL}\right)^2}{L}$		
For a correct answer		1 point
$F_T = 9Mg$		

For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

(d) LO CHA-2.C, SP 6.A, 6.C
2 points

Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

For using a correct kinematic equation to calculate the time		1 point
$\Delta y = v_1 t + \dfrac{1}{2}at^2 = \dfrac{1}{2}at^2 \therefore t = \sqrt{\dfrac{2\Delta y}{a}} = \sqrt{\dfrac{2(2L)}{g}} = \sqrt{\dfrac{(4)(0.75\text{ m})}{\left(9.8\text{ m/s}^2\right)}}$		
For a correct answer		1 point
$t = 0.55$ s		

(e) LO CHA-2.C, SP 6.A, 6.C
2 points

Calculate the speed of block 2 as it leaves the table.

For using a correct kinematic equation to calculate the speed		1 point
$\Delta x = vt \therefore v = \dfrac{\Delta x}{t} = \dfrac{4L}{t}$		
For substituting the time from part (d) into equation above		1 point
$v = \dfrac{(4)(0.75\text{ m})}{(0.55\text{ s})} = 5.45\text{ m/s}$		

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Question 2 (continued)

(f) LO CON-4.E.a, SP 6.B, 6.C
3 points

Calculate the speed of block 1 just after it collides with block 2.

For indicating the use of the correct conservation of momentum to calculate the speed		1 point
$p_i = p_f \therefore m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$		
For correctly substituting the mass into equation above		1 point
$(3M)v_{1i} + 0 = (3M)v_{1f} + (M)v_{2f}$		
For substituting the speeds from parts (a) and (e) into equation above		1 point
$v_{1f} = \dfrac{(3M)v_{1i} - (M)v_{2f}}{(3M)} = \dfrac{(3M)\sqrt{2gL} - (M)v_{2f}}{(3M)}$		
$v_{1f} = \dfrac{(3)\sqrt{(2)\left(9.8 \text{ m/s}^2\right)(0.75 \text{ m})} - (1)(5.45 \text{ m/s})}{(3)} = 2.02 \text{ m/s}$		
($v_{1f} = 2.06$ m/s when using $g = 10 \text{ m/s}^2$)		

(g) LO CON-2.C.a, SP 6.B, 6.C
3 points

Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.

For using conservation of energy to calculate the angle		1 point
$K_1 = U_{g2}$		
$\frac{1}{2}mv^2 = mgh$		
For correctly substituting $h = L(1 - \cos\theta)$ into the equation above		1 point
For correctly substituting the speed from part (f) into the equation above		1 point
$\frac{1}{2}mv^2 = mgL(1 - \cos\theta) \therefore \dfrac{v^2}{2gL} = 1 - \cos\theta$		
$\theta = \cos^{-1}\left(1 - \dfrac{v^2}{2gL}\right) = \cos^{-1}\left(1 - \dfrac{(2.02 \text{ m/s})^2}{(2)\left(9.8 \text{ m/s}^2\right)(0.75 \text{ m})}\right) = 43.5°$		
($\theta = 44.1°$ when using $g = 10 \text{ m/s}^2$)		

2019 SCORING GUIDELINES

Question 2 (continued)

Learning Objectives

CHA-2.C: Calculate kinematic quantities of an object in projectile motion, such as displacement, velocity, speed, acceleration, and time, given initial conditions of various launch angles, including a horizontal launch at some point in its trajectory.
INT-2.B.b: Describe forces that are exerted on objects undergoing horizontal circular motion, vertical circular motion, or horizontal circular motion on a banked curve.
CON-2.C.a: Calculate unknown quantities (e.g., speed or positions of an object) that are in a conservative system of connected objects, such as the masses in an Atwood machine, masses connected with pulley/string combinations, or the masses in a modified Atwood machine.
CON-4.E.a: Calculate the changes in speeds, changes in velocities, changes in kinetic energy, or changes in momenta of objects in all types of collisions (elastic or inelastic) in one dimension, given the initial conditions of the objects.

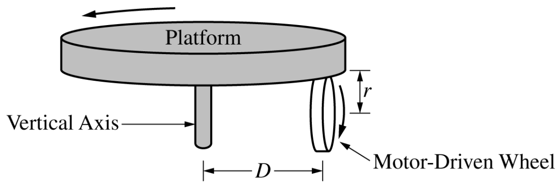
Science Practices

3.D: Create appropriate diagrams to represent physical situations.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

2019 SCORING GUIDELINES

Question 3

15 points



A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the wheel until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

(a) LO INT-7.A.b, CHA-4.A.b, SP 5.A, 5.E
2 points

Derive an expression for the angular speed ω_P of the platform. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.

For correctly substituting into the rotational form of Newton's second law			1 point
$\tau = I\alpha \therefore FD = I_P\alpha$			
$\alpha = \dfrac{FD}{I_P}$			
For correctly substituting into a rotational kinematic equation to calculate the angular speed			1 point
$\omega_2 = \omega_1 + \alpha\Delta t = 0 + \left(\dfrac{FD}{I_P}\right)\Delta t$			
$\omega_P = \dfrac{FD\Delta t}{I_P}$			

(b) LO INT-7.D.a, SP 5.A, 5.E
2 points

Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed ω_P. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.

For using the equation for rotational kinetic energy		1 point
$K = \dfrac{1}{2}I\omega_P^2 = \left(\dfrac{1}{2}\right)(I)\left(\dfrac{FD\Delta t}{I}\right)^2$		
For an answer consistent with part (a)		1 point
$K = \dfrac{(FD\Delta t)^2}{2I}$		

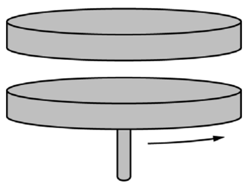
2019 SCORING GUIDELINES

Question 3 (continued)

(c) LO INT-7.C, SP 5.A, 5.E
2 points

Derive an expression for the angular speed of the wheel ω_W when the platform has reached angular speed ω_P. Express your answer in terms of D, r, ω_P, and physical constants, as appropriate.

For indicating that the linear speed of the platform is equal to the linear speed of the wheel		1 point
$v_P = v_W$ OR $(r\omega)_P = (r\omega)_W$		
For correctly relating the linear speeds to the angular speeds in the above equation		1 point
$D\omega_P = r\omega_W$		
$\omega_W = \dfrac{D\omega_P}{r}$		



When the platform is spinning at angular speed ω_P, the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia I_P as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed ω_f.

(d) LO CON-5.D.c, SP 5.A, 5.E
2 points

Derive an expression for ω_f. Express your answer in terms of ω_P, I_P, and physical constants, as appropriate.

For using an expression for the conservation of angular momentum		1 point
$L_1 = L_2 \therefore I_1\omega_1 = I_2\omega_2$		
For correctly substituting into the above equation		1 point
$I\omega_P = (2I)\omega_F$		
$\omega_F = \frac{1}{2}\omega_P$		

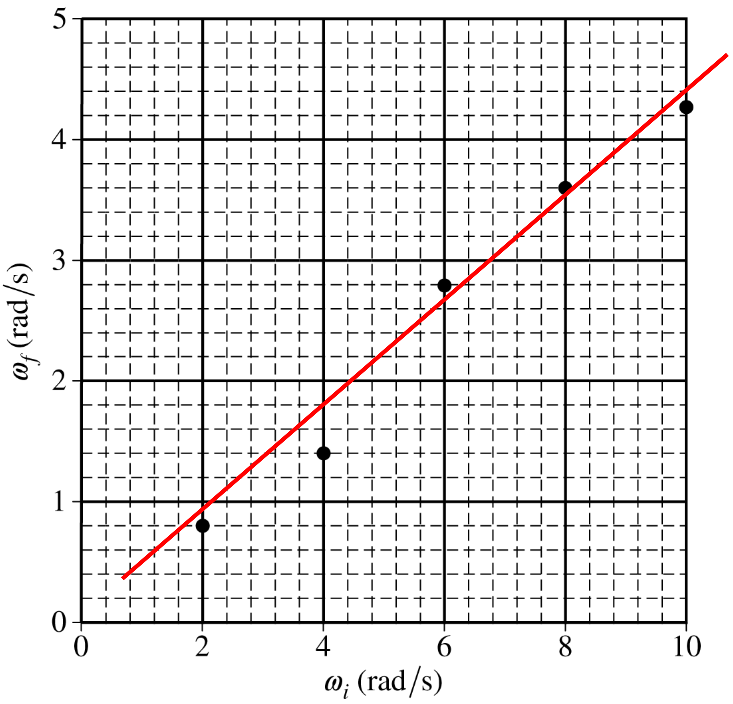
Question 3 (continued)

A student now uses the rotating platform $\left(I_P = 3.1\ \text{kg} \bullet \text{m}^2\right)$ to determine the rotational inertia I_U of an unknown object about a vertical axis that passes through the object's center of mass. The platform is rotating at an initial angular speed ω_i when the unknown object is dropped with its center of mass directly above the center of the platform. The platform and object are observed to be rotating together at angular speed ω_f. Trials are repeated for different values of ω_i. A graph of ω_f as a function of ω_i is shown on the axes below.

(e)

i. LO CON-5.D.c, SP 4.C
1 points

On the graph on the previous page, draw a best-fit line for the data.



For an appropriate best-fit line for the graph above		1 point

Question 3 (continued)

(e) continued

ii. LO CON-5.D.c, SP 4.D, 6.C
2 points

Using the straight line, calculate the rotational inertia of the unknown object I_U about a vertical axis passing through its center of mass.

For using conservation of angular momentum to derive an expression that includes I_U		1 point
$L_i = L_f \;\therefore\; I_P\omega_i = (I_P + I_U)\omega_f$		
$I_U = I_P\left(\dfrac{\omega_i}{\omega_f}\right) - I_P$		
For substituting points from the best-fit line into the expression above		1 point
$I_U = \left(3.1\ \text{kg•m}^2\right)\left(\dfrac{(4.0-1.0)\ \text{rad/s}}{(9.3-2.4)\ \text{rad/s}}\right) - \left(3.1\ \text{kg•m}^2\right) = 4.1\ \text{kg•m}^2$		
Note: The point (0, 0) can be used implicitly if the best-fit line goes through the origin.		

(f) LO CON-5.D.c, SP 7.A, 7.C
2 points

The kinetic energy of the spinning platform before the object is dropped on it is K_i. The total kinetic energy of the platform-object system when it reaches angular speed ω_f is K_f. Which of the following expressions is true?

____ $K_f < K_i$ ____ $K_f = K_i$ ____ $K_f > K_i$

Justify your answer.

For selecting $K_f < K_i$ with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because the two disks will be rotating with the same final angular speed, this is an inelastic collision, and kinetic energy will be lost during the collision.		

Question 3 (continued)

(g) LO INT-6.E, SP 7.A, 7.C
2 points

One of the students observes that the center of mass of the object is not actually aligned with the axis of the platform. Is the experimental value of I_U obtained in part (e) greater than, less than, or equal to the actual value of the rotational inertia of the unknown object about a vertical axis that passes through its center of mass?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

For selecting "Greater than" with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because the center of mass of the object is off the axis of the platform, the parallel axis theorem would be used to calculate the total rotational inertia of the platform-object system. Using $I = I_{CM} + Mh^2$, the experimental value will be increased by Mh^2.		

Learning Objectives

INT-6.E: Derive the moments of inertia of an extended rigid body for different rotational axes (parallel to an axis that goes through the object's center of mass) if the moment of inertia is known about an axis through the object's center of mass.
CHA-4.A.b: Calculate unknown quantities such as angular positions, displacement, angular speeds, or angular acceleration of a rigid body in uniformly accelerated motion, given initial conditions.
INT-7.A.b: Calculate unknown quantities such as net torque, angular acceleration, or moment of inertia for a rigid body undergoing rotational acceleration.
INT-7.C: Derive expressions for physical systems such as Atwood Machines, pulleys with rotational inertia, or strings connecting discs or strings connecting multiple pulleys that relate linear or translational motion characteristics to the angular motion characteristics of rigid bodies in the system that are: **(a)** rolling (or rotating on a fixed axis) without slipping. **(b)** rotating and sliding simultaneously.
INT-7.D.a: Calculate the rotational kinetic energy of a rotating rigid body.
CON-5.D.c: Calculate the changes of angular momentum of each disc in a rotating system of two rotating discs that collide with each other inelastically about a common rotational axis.

Science Practices

4.C: Linearize data and/or determine a best-fit line or curve.
4.D: Select relevant features of a graph to describe a physical situation or solve problems.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.