

Question 1: Version J

1. Two blocks, 1 and 2, slide toward each other on a horizontal surface. Block 1 has mass m and slides in the $+x$ -direction with constant speed $2v_0$. Block 2 has mass $6m$ and slides in the $-x$ -direction with constant speed v_0 , as shown in Figure 1. The blocks then collide and stick together. The collision occurs from time $t = 0$ to $t = t_c$. After the collision, where $t > t_c$, the blocks move together with the same constant speed.

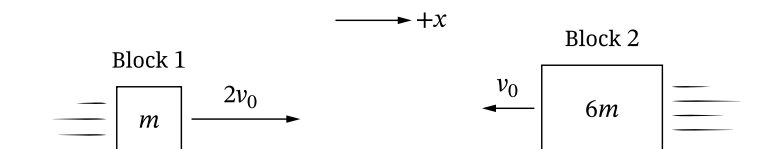


Figure 1

- A. The diagrams in Figure 2 can be used to represent the momentum of blocks 1 and 2 before and after the collision. The momentum vector diagram for Block 1 before the collision is shown.

- i. **Draw** arrows on the grids to represent the momentum vectors of Block 2 before the collision and the two-block system before and after the collision.
- Arrows should start at the zero-momentum line.
 - The length of the arrows should be proportional to the relative magnitudes of the vectors.
 - Represent an arrow of zero length by drawing a dot at zero.

	Momentum Before Collision	Momentum After Collision
Block 1		
Block 2		
Two-Block System		

Figure 2

- ii. During the time interval $0 \leq t \leq t_c$, a force F is exerted on Block 2 by Block 1 along the x -direction as a function of t that is modeled by $F(t) = F_{\max} \sin(At)$, where A is a positive constant and $F_{\max}$ is the magnitude of the maximum force exerted on Block 2 by Block 1 during the collision.

Derive an expression for $F_{\max}$. Express your answer in terms of m , v_0 , A , t_c , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

- B. Consider a new scenario where Block 1 initially slides in the $+x$ -direction with a new constant speed v_1 and Block 2 again initially slides in the $-x$ -direction with constant speed v_0 . The blocks collide and stick together. In this new scenario, the two-block system has constant speed v_0 after the collision.

Derive an expression for v_1 in terms of v_0 . Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 2: Version J

2. In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

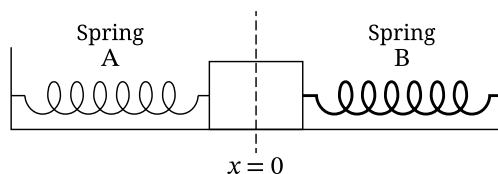


Figure 1

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

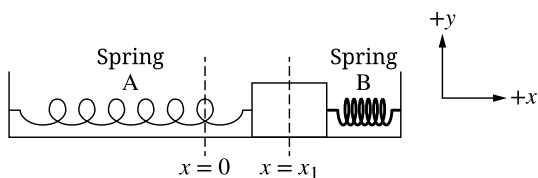


Figure 2

- A. An energy bar chart can be used to represent the elastic potential energy U_A of Spring A, the elastic potential energy U_B of Spring B, and the kinetic energy K_{block} of the block. On the energy bar chart in Figure 3, **draw** shaded bars to represent the energy of the system for when the block is at $x = x_1$.

- The height of the shaded bars should be proportional to the relative values of U_A , U_B , and K_{block} .
- Any energy that is equal to zero should be represented by a distinct line on the zero-energy line.

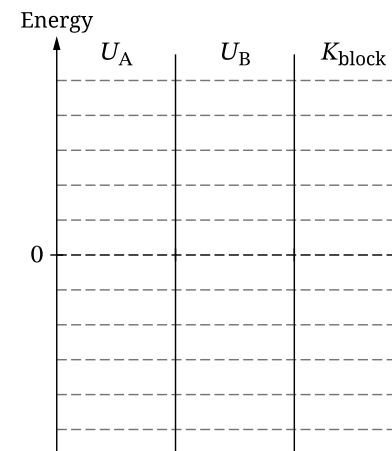
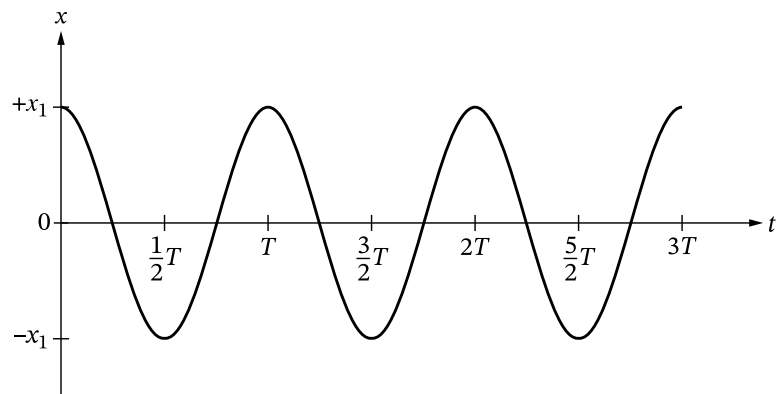


Figure 3

- B. The block is released from rest at $x = x_1$ and begins to oscillate. **Derive** an expression for the speed v of the block as the block passes through $x = \frac{1}{2}x_1$. Express your answer in terms of m , k , x_1 , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

- C. In Scenario 1, the block oscillates with period T . The position x of the block in Scenario 1 as a function of time t is shown in Figure 4.

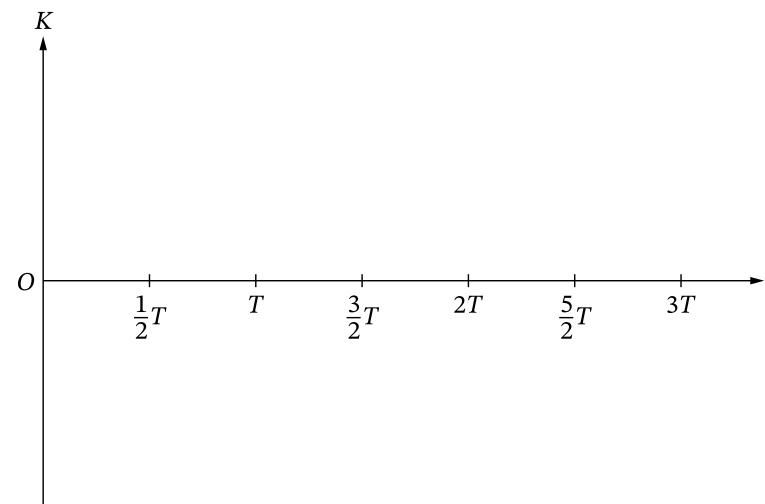


Scenario 1

Figure 4

In Scenario 2, the block-springs system is placed on a new surface. There is friction between the block and the new surface. The block is again moved to the same position $x = x_1$ and released from rest. The block completes multiple oscillations with the same period as in Scenario 1 before coming to rest.

On the axes shown in Figure 5, **sketch** a graph of the kinetic energy K of the block as a function of t for Scenario 2.



Scenario 2

Figure 5

- D. In Scenario 3, the block is replaced with a new block of larger mass. The coefficient of kinetic friction between the new block and the surface in Scenario 3 is the same as the coefficient of kinetic friction between the original block and the surface in Scenario 2.

The new block is moved to position $x = x_1$ and released from rest. The kinetic energy of the new block is plotted as a function of time.

Describe how one feature of the graph of K as a function of t in Scenario 3 would differ from the graph you drew in Figure 5 for Scenario 2.

Briefly **justify** your answer.

Question 3

3. A box is connected to one end of a rigid rod. Both the box and the rod have negligible mass. The other end of the rod is connected to a pivot. The box is open on one side, and a block is placed inside the box.

The center of mass of the block is displaced a vertical distance h , as shown in Figure 1. The block-box system is then released from rest and swings downward. There is negligible friction about the pivot. When the system is at the lowest point of its swing, the rod collides with a rigid stopper, as shown in Figure 2. The box comes to rest, and the block is launched horizontally out of the box. The block moves across a horizontal surface toward a motion sensor that measures the speed of the block. All frictional forces are negligible.

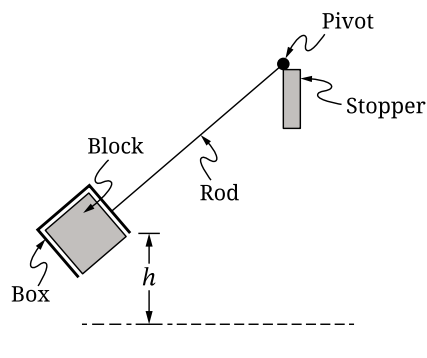


Figure 1

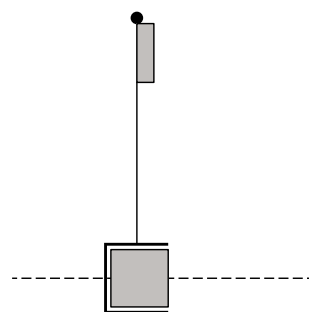


Figure 2

- A. Students are asked to experimentally determine the acceleration due to gravity g using a linear graph. To determine g , the students are permitted to use measurements from only a meterstick and the motion sensor.

Describe an experimental procedure using the described setup to collect data that would allow the students to determine an experimental value of g using a linear graph. Include any steps necessary to reduce experimental uncertainty.

- B. **Describe** how the data collected in part A could be graphed and how that graph would be analyzed to determine the value of g .

- C. The experiment is repeated, but the horizontal surface on which the block slides is replaced with a new rough surface, as shown in Figure 3. The coefficient of kinetic friction between the block and the new surface is μ .

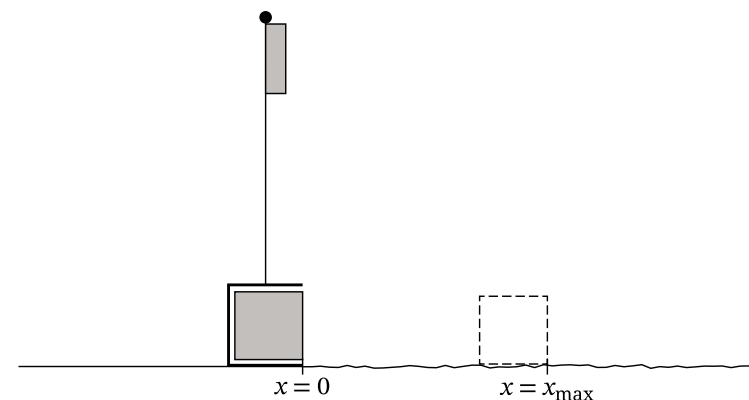


Figure 3

The block-box system is pulled aside so that the center of mass of the block is displaced various vertical distances h and then released from rest. For each vertical distance, students measure the position $x = x_{\max}$ at which the block comes to rest.

The students' measurements of h and $x_{\max}$ are shown in Table 1.

Table 1

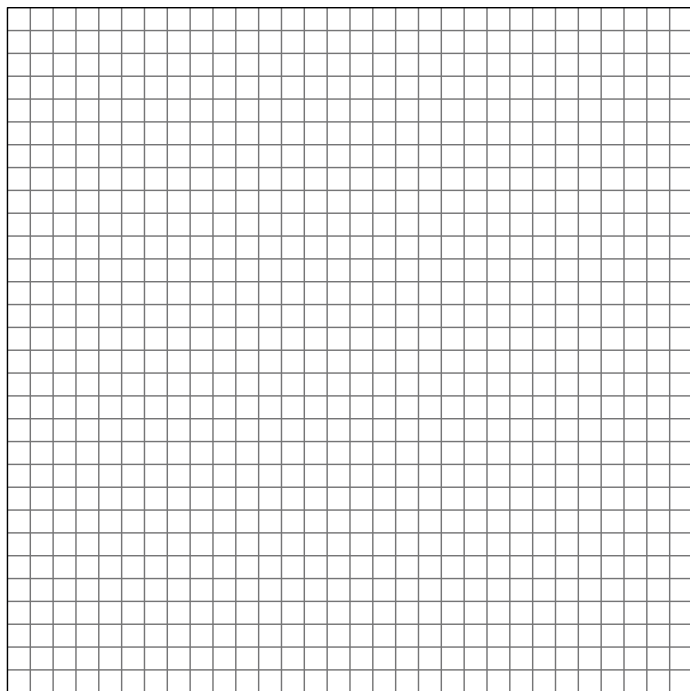
h (m)	$x_{\max}$ (m)
0.30	0.76
0.45	1.10
0.60	1.40
0.75	1.90
0.90	2.30

- i. **Indicate** two quantities, either measured quantities from Table 1 or additional calculated quantities, that could be graphed to produce a straight line that could be used to determine μ .

Vertical axis: _____ Horizontal axis: _____

ii. On the grid provided, create a graph of the quantities indicated in part C (i).

- Use Table 2 to record the measured or calculated quantities that you will plot.
- Clearly **label** the axes, including units as appropriate.
- **Plot** the points you recorded in Table 2.



iii. **Draw** a best-fit line to the data graphed in part C (ii).

D. Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for μ .

Question 4

4. A uniform disk and ring, each of mass M and radius R , roll without slipping along a horizontal surface, as shown in Figure 1. The outer edges of the disk and ring are made of the same material. The center of mass of the disk and the center of mass of the ring each initially move with the same constant speed v .

The disk and the ring then smoothly transition to a ramp that is inclined at an angle θ above the horizontal. Both the disk and the ring continue to roll without slipping as they move up the ramp, as shown in Figure 2.

The ring travels a greater distance along the ramp than the disk travels before each momentarily comes to rest.

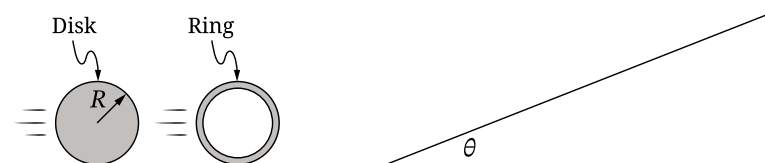


Figure 1

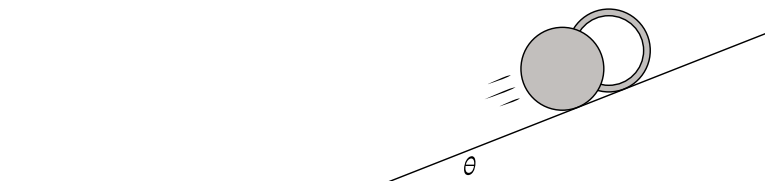


Figure 2

- A. While the disk and the ring are rolling on the ramp without slipping, the magnitudes of the static frictional force exerted on the disk and on the ring by the ramp are f_D and f_R , respectively.

Indicate whether f_D is greater than, less than, or equal to f_R by writing one of the following.

- $f_D > f_R$
- $f_D < f_R$
- $f_D = f_R$

Justify your answer using qualitative reasoning beyond referencing equations.

- B. A cylinder has mass M , radius R , and rotational inertia I about its central axis. The cylinder rolls without slipping up a ramp that is inclined at an angle θ above the horizontal.

Derive an expression for the magnitude of the static frictional force f exerted on the cylinder by the ramp. Express your answer in terms of M , R , I , θ , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

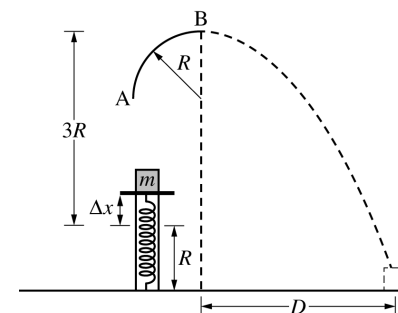
- C. In a different scenario, the centers of mass of the original disk and ring each have the same initial speed v as they did in the original scenario. The ramp is replaced by a new ramp on which the disk and the ring initially slip as they roll up the new ramp.

Indicate whether the magnitude of the kinetic frictional force exerted on the disk by the new ramp is greater than, less than, or equal to the magnitude of the kinetic frictional force exerted on the ring by the new ramp while both are slipping.

Briefly **justify** your answer.

STOP
END OF EXAM

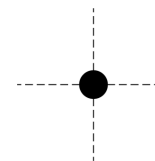
Begin your response to QUESTION 3 on this page.



Note: Figure not drawn to scale.

3. A block of mass m is placed on top of an ideal spring of spring constant k . The block is pushed against the spring, compressing the spring a distance Δx . The block is released from rest, leaves the spring at the position shown in the figure, travels upward, and enters a track with a constant radius of curvature R that has negligible friction. The block enters the track at point A, maintains contact with the track, and exits horizontally at point B, a distance $3R$ above the point the block was released. The block then falls to the ground and lands a horizontal distance D from the end of the track. Express all algebraic answers in terms of m , k , Δx , R , and physical constants, as appropriate. The size of the block is much smaller than the radius of curvature of the track.

- (a) On the dot below, which represents the block, draw and label the forces (not components) that act on the block while still in contact with the track at point B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



Justify your choice of vectors.

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Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.

(b)

- i. Derive an expression for the speed v of the block at point B.
- ii. Derive an expression for the magnitude of the net force F on the block at point B.

(c) Derive an expression for the minimum value of $\Delta x_{\min}$ required in order for the block to maintain contact with the track through point B.

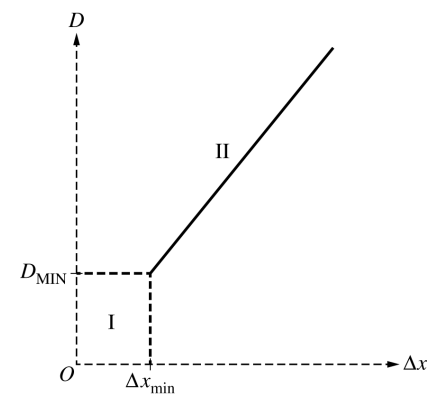
The procedure is repeated several times with the distance $\Delta x > \Delta x_{\min}$.

- (d) Calculate the distance D that the block travels.

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(e) The graph below shows the best-fit line drawn by the students through their data of D as a function of Δx .



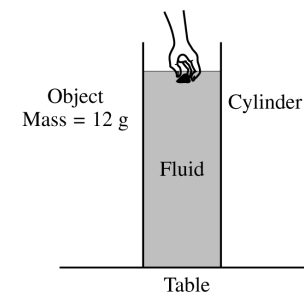
- i. Explain why there are no data for section I of the graph.
- ii. Explain the reason for the shape and minimum value of section II on the graph.

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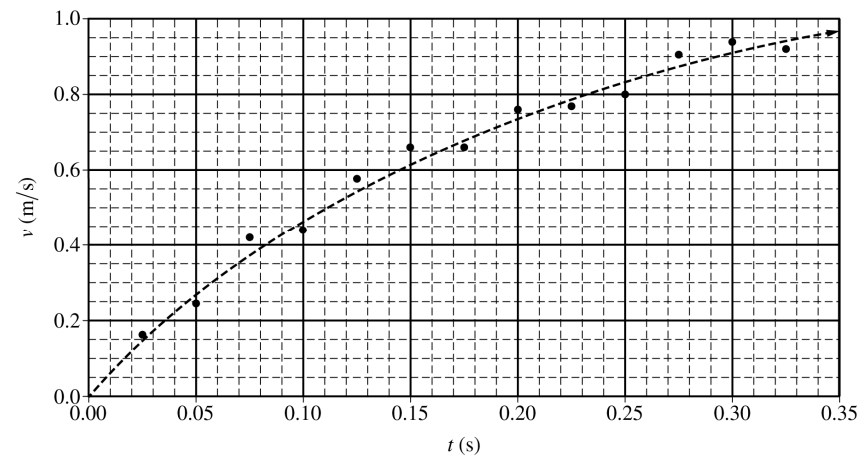
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STOP**END OF EXAM****PHYSICS C: MECHANICS****SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of $m = 12\text{ g}$ is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.



(a)

- i. Does the speed of the object increase, decrease, or remain the same?

____ Increase ____ Decrease ____ Remain the same

- ii. In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.

- iii. Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20$ s.

The students use the equation $v = A(1 - e^{-Bt})$ to model the speed of the falling object and find the best fit coefficients to be $A = 1.18$ m/s and $B = 5$ s⁻¹.

(b) Use the above equation to:

- i. Derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time t .
- ii. Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time t .

The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed.

(c)

- i. Determine the constant speed of the object.

Justify your answer.

- ii. Determine the force exerted by the fluid on the object at this time.

Justify your answer.