Week 27

AP Physics C: Mechanics

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

AP Physics C: Mechanics Name: ____________ Score: ______

1. A spring with $k = 20$ N/m is stretched $x = 0.15$ m. The magnitude of the restoring force (in N)?

   ____________ N

2. An oscillator has a period of 0.25 s. Its frequency (in Hz)?

   ____________ Hz

3. An oscillator has $A = 0.2$ m and $\omega = 4$ rad/s. Its maximum speed (in m/s)?

   ____________ m/s

4. A spring with $k = 100$ N/m oscillates with amplitude $A = 0.2$ m. Its total energy (in J)?

   ____________ J

5. A simple pendulum has $L = 0.25$ m and $g = 9.8$ m/s^2. Its period (in s, to 1 decimal)?

   ____________ s

6. A motion is simple harmonic when the restoring force is...
   - ☐ proportional to displacement and opposite in direction
   - ☐ constant in size and direction
   - ☐ proportional to speed

7. You replace a mass-spring oscillator's mass with a heavier one. The period...
   - ☐ increases
   - ☐ decreases
   - ☐ stays the same

8. At which point is the oscillator's energy entirely kinetic?
   - ☐ at the equilibrium position
   - ☐ at the extremes
   - ☐ nowhere – it is always shared

9. In the physical pendulum period $T = 2\pi\sqrt{I/(mgd)}$, what is d?
   - ☐ the distance from the pivot to the centre of mass
   - ☐ the full length of the body
   - ☐ the radius of the pivot

10. The minus sign in $F = -kx$ means the force always points back toward equilibrium.

☐ True ☐ False

AP Physics C: Mechanics

1. **3 N**

   $|F| = kx = 20 \times 0.15 = 3$ N.

2. **4 Hz**

   $f = 1/T = 1/0.25 = 4$ Hz.

3. **0.8 m/s**

   $v_{max} = A\omega = 0.2 \times 4 = 0.8$ m/s.

4. **2 J**

   $E = \frac{1}{2}kA^2 = \frac{1}{2}(100)(0.2)^2 = 2$ J.

5. **1.0 s**

   $T = 2\pi\sqrt{L/g} = 2\pi\sqrt{0.25/9.8} \approx 1.0$ s.

6. **proportional to displacement and opposite in direction**

   $F = -kx$ – proportional to x and pointing back toward equilibrium.

7. **increases**

   $T = 2\pi\sqrt{m/k}$, so a bigger m gives a longer T.

8. **at the equilibrium position**

   At equilibrium $x = 0$ so $U = 0$ and all the energy is kinetic.

9. **the distance from the pivot to the centre of mass**

   d is the pivot-to-centre-of-mass distance – the lever arm of gravity.

10. **True**

    The force opposes the displacement, restoring the mass toward $x = 0$.

7.1 Defining Simple Harmonic Motion (SHM)

Name: ______________ Class: __________ Date: ____________ **Total: 14 marks**

KEY WORDS equilibrium 平衡位置 • simple harmonic motion 简谐运动 • restoring force 回复力 • amplitude 振幅

Objective

Build the skills to answer exam questions on **defining simple harmonic motion (SHM)**.

You must be able to:

- state the defining condition of **simple harmonic motion 简谐运动**: $F = -kx$
- explain the minus sign (the **restoring force 回复力** points back toward equilibrium)
- use $a = -\omega^2 x$ with $\omega^2 = k/m$
- identify where the acceleration is largest and where it is zero

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ The defining condition

SHM happens when the restoring force is proportional to the displacement and points back toward equilibrium:

$$F = -kx.$$

■ The acceleration form

Dividing by mass,

$$a = -\frac{k}{m}x = -\omega^2 x, \qquad \omega^2 = \frac{k}{m}.$$

Acceleration is proportional to displacement but opposite in direction.

■ Where it is largest

The acceleration is greatest at the extremes (largest x) and zero at equilibrium ($x = 0$) – the opposite of the speed.

2 Practice

Now apply the methods above.

2.1 A spring with $k = 20$ N m^{-1} is stretched 0.15 m. Find the restoring force. [2]

2.2 State the meaning of the minus sign in $F = -kx$. [1]

2.3 For SHM with $\omega^2 = 25$ s^{-2} at $x = 0.2$ m, find the acceleration magnitude. [2]

2.4 Where in the motion is the acceleration largest? [1]

3 Exam-style questions

3.1 A motion is simple harmonic when the restoring force is [1]

- **A** constant
- **B** proportional to displacement and opposite in direction
- **C** proportional to speed
- **D** zero

3.2 In SHM, the acceleration is zero at the [1]

- **A** extremes
- **B** equilibrium position
- **C** turning points
- **D** it is never zero

3.3 A 0.5 kg mass on a spring ($k = 8$ N m^{-1}) is pulled 0.1 m and released.

(a) Find the restoring force at release. [2]

(b) Find the acceleration at release. [2]

3.4 A pendulum swings with a small amplitude.

(a) State whether its motion is SHM. [1]

(b) State the condition (angle) required for this to hold. [1]

Solutions

2.1 $|F| = kx = 20 \times 0.15 = 3$ N.

2.2 The force opposes the displacement (points back to equilibrium).

2.3 $|a| = \omega^2 x = 25 \times 0.2 = 5$ m s^{-2}.

2.4 At the extremes (largest x).

3.1 B — proportional to displacement and opposite in direction.

3.2 B — at equilibrium, $x = 0$ so $a = 0$.

3.3 (a) $F = -kx = -8 \times 0.1 = -0.8$ N (magnitude 0.8 N). (b) $\omega^2 = k/m = 16$; $a = -\omega^2 x = -1.6$ m s^{-2}.

3.4 (a) Yes (approximately). (b) Small angles (roughly under 15°).

Note: Figure not drawn to scale.

2. A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

(a)

i. Derive an expression for the speed of the block just before it collides with the spring.

ii. Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

____ Greater than ______ Less than ____ Equal to

Justify your answer.

(b) Derive an expression for the maximum compression of the spring.

(c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

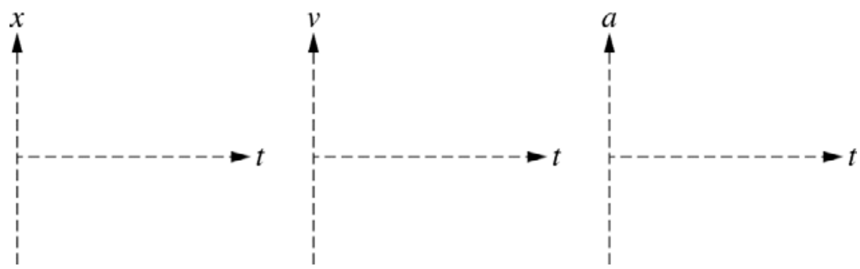
The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0. Now suppose the block experiences a resistive force as it slides on the horizontal surface. The magnitude of the resistive force F is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of kg/m.

(d)

i. Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m, h, k, β, v, and physical constants, as appropriate.

ii. Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time t can be written in the form $\dfrac{1}{v(t)} = \dfrac{1}{v_0} + \dfrac{\beta t}{m}$, where v_0 is the speed at $t = 0$.

(e) Sketch graphs of position x as a function of time t, velocity v as a function of time t, and acceleration a as a function of time t for the block as it is moving on the horizontal surface before it reaches the spring.



Mech.2.

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(a) On the dots below, which represent the blocks of mass $2M$ and $3M$, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance D, as shown above. Express your answers to parts (b), (c), and (d) in terms of M, B, v_0, and physical constants, as appropriate.

(b) Derive an expression for the speed of the blocks immediately after the collision.

(c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.

(d) Derive an expression for the maximum distance D that the spring is compressed.

(e)

i. In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

____ Left ____ Right ____ The net force on the block of mass $2M$ is zero.

Justify your answer.

ii. Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

____ The magnitude of the net force is greater on the block of mass $2M$.

____ The magnitude of the net force is greater on the block of mass $3M$.

____ The magnitude of the net force on each block has the same nonzero value.

____ The magnitude of the net force on each block is zero.

Justify your answer.

(f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

____ Yes ____ No

Justify your answer.

7.2 Frequency and Period of SHM

Name: ______________ Class: __________ Date: ____________ **Total: 14 marks**

KEY WORDS period 周期 • frequency 频率 • amplitude 振幅 • angular frequency 角频率 • simple pendulum 单摆

Objective

Build the skills to answer exam questions on **frequency and period of SHM**.

You must be able to:

- use the **period** 周期 T, **frequency** 频率 $f = 1/T$, and **angular frequency** 角频率 $\omega = 2\pi/T$
- use $T = 2\pi\sqrt{m/k}$ for a mass-spring system
- use $T = 2\pi\sqrt{L/g}$ for a simple pendulum
- state that the period is independent of amplitude

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Period, frequency, angular frequency

$$T = \frac{1}{f}, \qquad \omega = 2\pi f = \frac{2\pi}{T}.$$

A period of 0.25 s gives $f = 4$ Hz.

Mass on a spring

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

Heavier mass or a softer spring means a longer period.

A simple pendulum

$$T = 2\pi\sqrt{\frac{L}{g}}.$$

The mass of the bob does not appear – only the length matters.

2 Practice

Now apply the methods above.

2.1 An oscillator has a period of 0.5 s. Find its frequency. [2]

2.2 State how the period of a mass-spring system changes if the mass increases. [1]

2.3 A pendulum's length is quadrupled. By what factor does its period change? [2]

2.4 Does a larger swing amplitude change an ideal pendulum's period? Answer yes or no. [1]

3 Exam-style questions

3.1 The period of a simple pendulum depends on [1]

- **A** the bob's mass
- **B** the length and g
- **C** the amplitude
- **D** the colour

3.2 Quadrupling the mass on a spring changes the period by a factor of [1]

- **A** 2
- **B** 4
- **C** 16
- **D** 1/2

3.3 A 0.2 kg mass hangs on a spring with $k = 50$ N m^{-1}.

(a) Find the period. [2]

(b) Find the frequency. [1]

3.4 A pendulum of length 1.0 m swings ($g = 9.8$ m s^{-2}).

(a) Find its period. [2]

(b) State the effect on the period of doubling the bob's mass. [1]

Solutions

2.1 $f = 1/T = 1/0.5 = 2$ Hz.

2.2 The period increases.

2.3 $T \propto \sqrt{L}$, so $\sqrt{4} = 2$ times longer.

2.4 No.

3.1 **B** — the length and g.

3.2 **A** — $T \propto \sqrt{m}$, so $\sqrt{4} = 2$.

3.3 (a) $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.2/50} = 0.40$ s. (b) $f = 1/T = 2.5$ Hz.

3.4 (a) $T = 2\pi\sqrt{1/9.8} \approx 2.0$ s. (b) No change (mass does not appear).

Begin your response to **QUESTION 1** on this page.

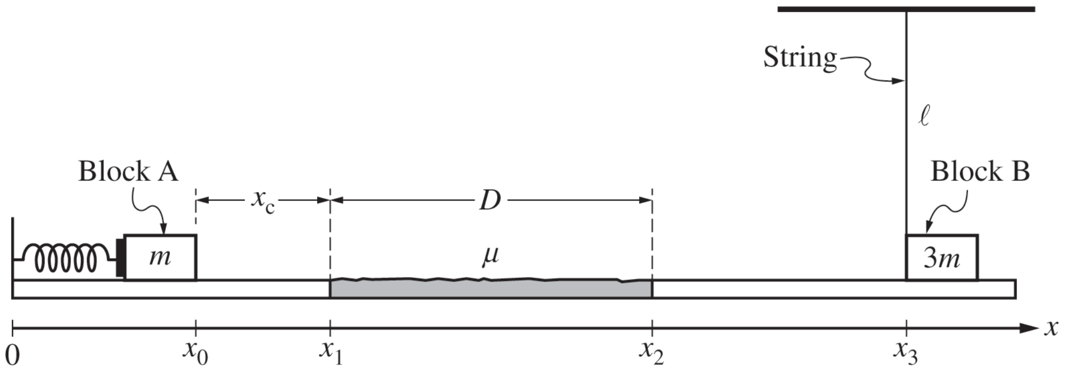
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

Figure 1

1. Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position.
- At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction.
- At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1.

ii. **Derive** an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3.

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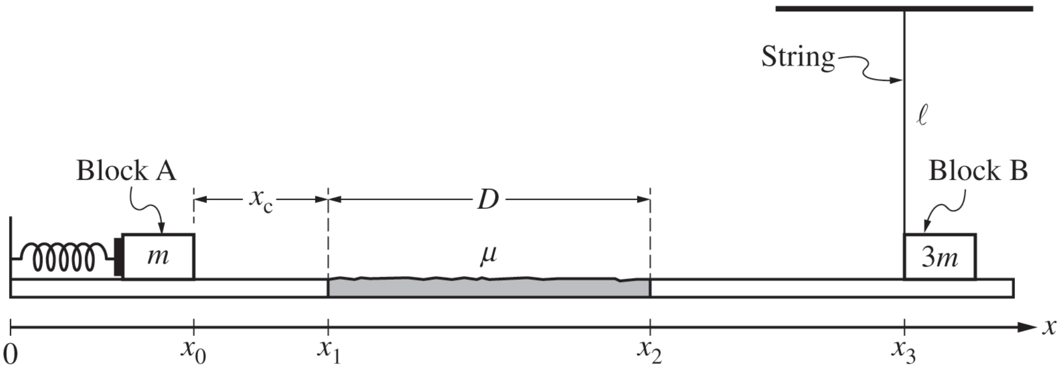
Continue your response to **QUESTION 1** on this page.

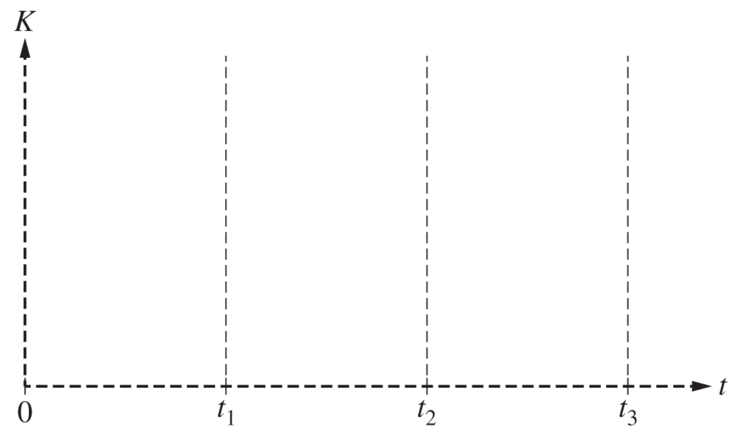


Note: Figure not drawn to scale.

Figure 1

(b)

i. On the following axes, **sketch** a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to time t_3.



ii. Use principles of work and energy to **justify** the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

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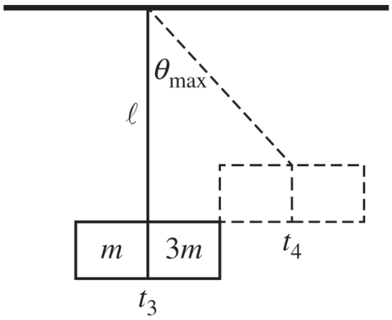
Continue your response to **QUESTION 1** on this page.



Note: Figure not drawn to scale.

Figure 2

After the collision, the two-block system instantaneously comes to rest at time t_4, which occurs when the string makes a small angle θ_{max} with the vertical, as shown in Figure 2. For times $t > t_4$, the system oscillates with frequency f_ℓ. The support holding the string is raised, and the procedure is then repeated using a new string of length 2ℓ.

(c) **Indicate** how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length 2ℓ will compare to the frequency of oscillation f_ℓ from the original procedure.

____ $f_{2\ell} > f_\ell$ ____ $f_{2\ell} < f_\ell$ ____ $f_{2\ell} = f_\ell$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 1** on this page.

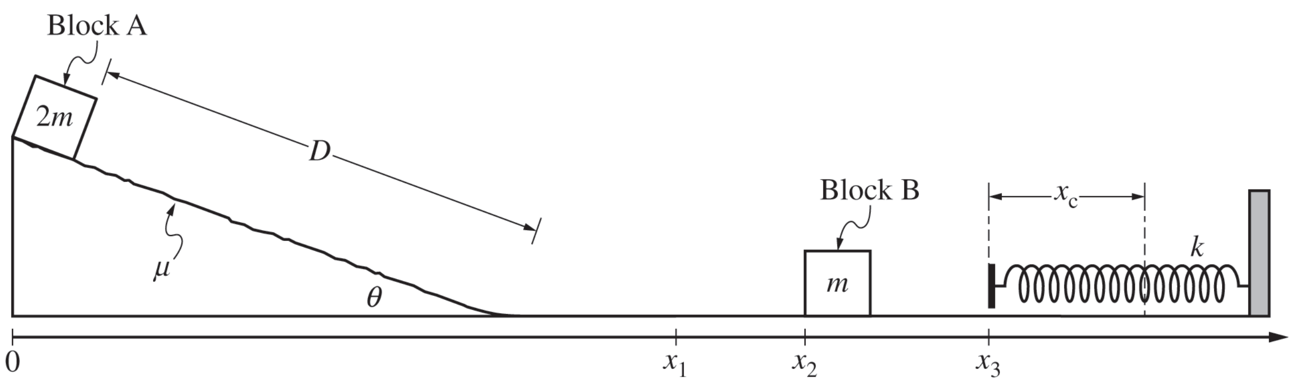
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

1. Blocks A and B of masses $2m$ and m, respectively, are arranged in a setup consisting of a ramp that makes an angle θ with a smooth horizontal table and an ideal spring of spring constant k fixed to a wall, as shown. Block A is held at rest a distance D up the ramp, and Block B is at rest on the horizontal table. The coefficient of kinetic friction between Block A and the rough ramp is μ in the region of length D, and there is negligible friction between the blocks and the smooth table.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$.
- At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B.
- At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position.
- At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, θ, D, μ, x_c, and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1.

ii. **Derive** an expression for the spring constant k of the spring.

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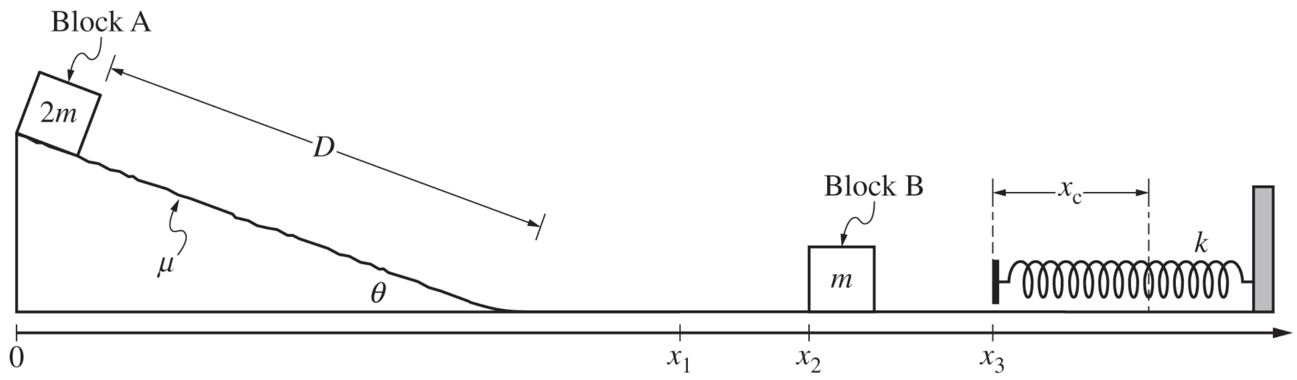
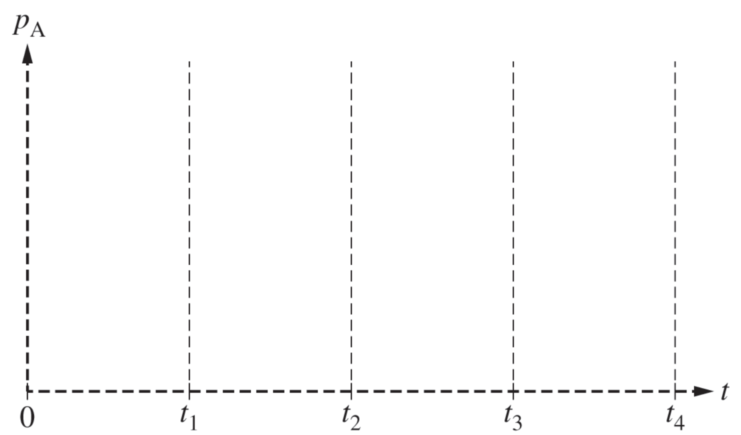


Note: Figure not drawn to scale.

(b)

i. On the following axes, **sketch** a graph of the magnitude of the momentum p_A of Block A as a function of time t from $t = 0$ to t_4.



ii. Use principles of forces to **justify** the graph drawn in part (b)(i) for the time interval $t = t_3$ to $t = t_4$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

For times $t > t_4$, the two-block-spring system oscillates with period T_O. The procedure is then repeated using a new ramp, where there is negligible friction between Block A and the ramp.

(c) **Indicate** how the new period of oscillation T_N in the procedure that uses the new ramp compares with the period of oscillation T_O from the original procedure.

_____ $T_N > T_O$ _____ $T_N < T_O$ _____ $T_N = T_O$

Briefly **justify** your answer.

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Begin your response to **QUESTION 2** on this page.

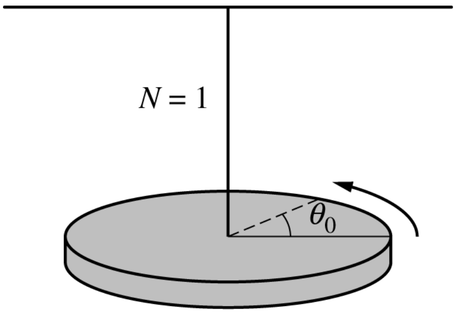

Figure 1

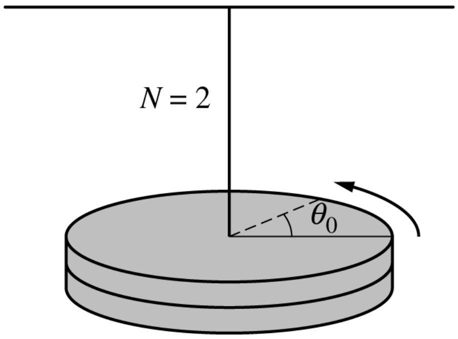

Figure 2

2. A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

(a) Using $T = 2\pi\sqrt{\frac{I}{\kappa}}$, derive an expression for T as a function of N. Express your answer in terms of M, R, κ, N, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b) The potential energy stored in the torsional pendulum when the disks are displaced is $U = \frac{1}{2}\kappa(\Delta\theta)^2$. On the following axes, sketch a graph of the maximum kinetic energy K_{max} of the torsional pendulum as a function of N for $N \geq 1$.

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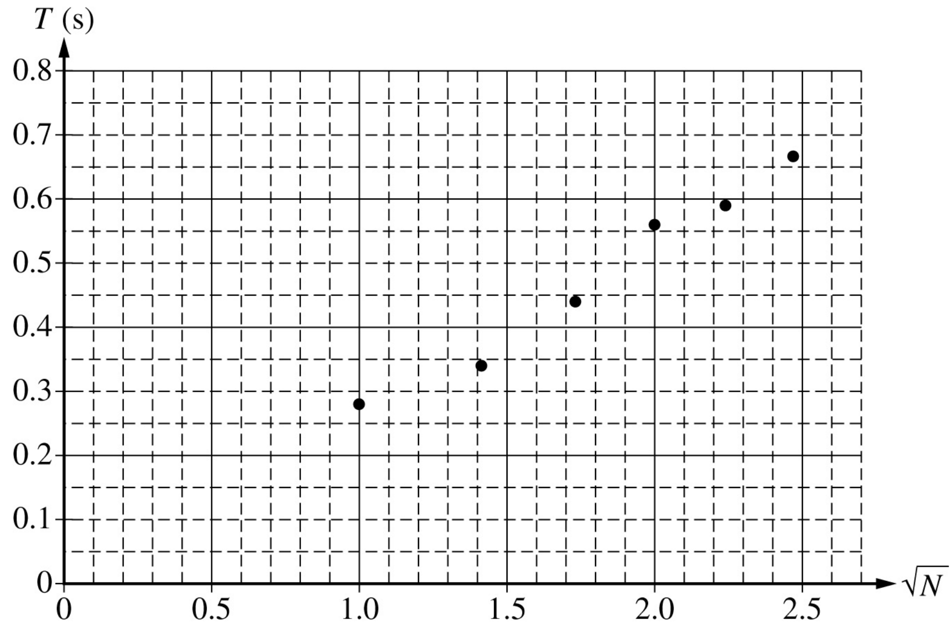
(c) The student plots the data for T as a function of $\sqrt{N}$, as shown.



i. Draw the best-fit line for the data.

ii. The student previously determined that the radius of a disk is $R = 0.2$ m and found that $\kappa = 1.6$ N·m. Using the graph, calculate the mass M of a single disk.

iii. The student finds that the value given by the manufacturer for the mass of the disk is less than the value determined experimentally in part (c)(ii). Determine a single source of experimental error that could result in the observed difference in the value of M. Justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) The student repeats the experiment, but now the disks have a density that varies as a function of the radius of the disk according to $\rho = 0.3r$.

i. Would the slope of the best-fit line for this new data be greater than, less than, or the same as the slope of the best-fit line in part (c)(i) ?

___ greater than ___less than ___the same as

Justify your answer.

ii. When $N = 1$, the maximum angular speed of the torsional pendulum with a uniform disk is found to be ω_U. When $N = 1$, the maximum angular speed of the torsional pendulum with a nonuniform disk is $\omega_{\text{non-U}}$. Which of the following correctly compares ω_U and $\omega_{\text{non-U}}$?

____$\omega_U > \omega_{\text{non-U}}$ ____$\omega_U < \omega_{\text{non-U}}$ ____ $\omega_U = \omega_{\text{non-U}}$

Briefly justify your answer.

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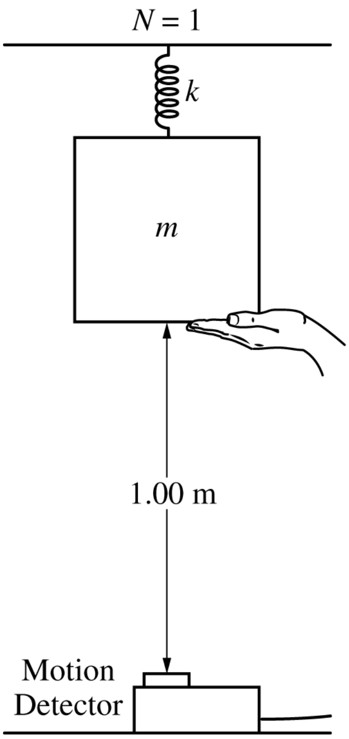

Figure 1

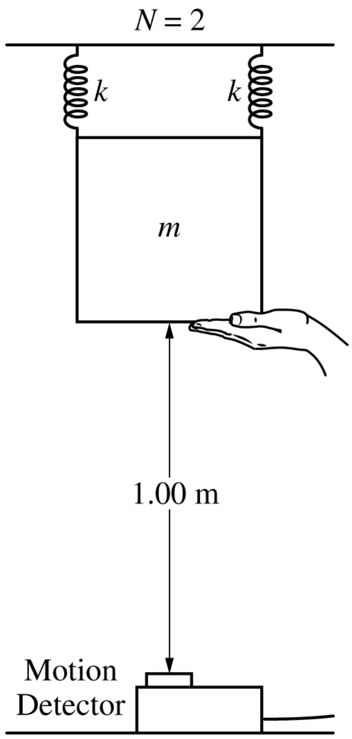

Figure 2

. A student conducts an investigation to determine the relationship between the period of oscillation T of a system consisting of a block and N attached springs. The student starts with a block of mass m attached to a single ideal spring of spring constant k, as shown in Figure 1. The student holds the block so that the spring is neither stretched nor compressed at a vertical height 1.00 m above a motion detector. The student releases the block from rest and records the period of oscillation for the system consisting of the single spring and block. An additional identical spring is attached in parallel, as shown in Figure 2, and the procedure is repeated for $N = 2$ springs. This procedure is repeated through $N = 10$ springs.

(a) Derive an expression for T as a function of N. Express your answer in terms of m, k, N, and physical constants as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b) On the following axes, sketch a graph of T as a function of N for $N \geq 1$.

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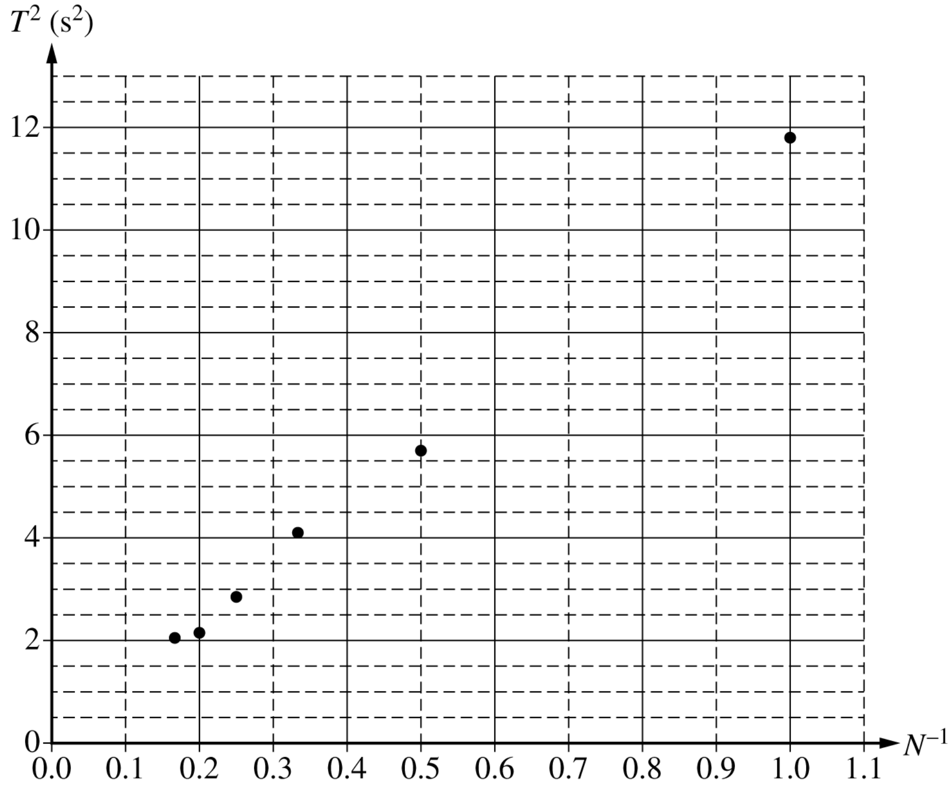
(c) The student plots the data for T^2 as a function of N^{-1}, as shown.



i. Draw the best-fit line for the data.

ii. The mass of the block is measured to be $m = 1.5$ kg. Using the graph, calculate an experimental value for the spring constant k for a single spring.

iii. The student finds that the value given by the manufacturer for the spring constant is larger than the value determined experimentally in part (c)(ii). Determine a single source of experimental error that could result in the observed difference in the value for k. Briefly justify your answer.

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Continue your response to **QUESTION 2** on this page.

$N = 2$

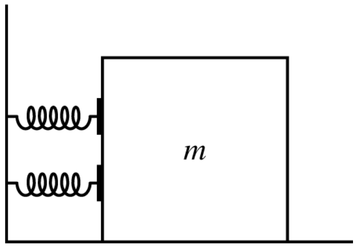


Figure 3

(d) The student conducts a similar investigation to determine the relationship between the period of oscillation T of a system consisting of a block and a number N of identical springs but arranges the block-spring system horizontally on a table, as shown in Figure 3. Frictional forces between the table and the block are negligible. In each trial, the block is displaced the same horizontal distance from equilibrium and released from rest.

i. The student plots T^2 as a function of N^{-1} for this new data. Would the slope of the best-fit line from this new investigation be greater than, less than, or the same as the slope of the best-fit line in part (c)(i) ?

___ greater than   ___ less than   ___the same as

Briefly justify your answer.

ii. When $N = 1$, the maximum speed of the block is found to be v_{max}. When N increases, will v_{max} increase, decrease, or stay the same?

____ increase   ____ decrease   ____ stay the same

Justify your answer.

GO ON TO THE NEXT PAGE.

Note: Figure not drawn to scale.

2. A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

(a)

i. Derive an expression for the speed of the block just before it collides with the spring.

ii. Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

____ Greater than ______ Less than ____ Equal to

Justify your answer.

(b) Derive an expression for the maximum compression of the spring.

(c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

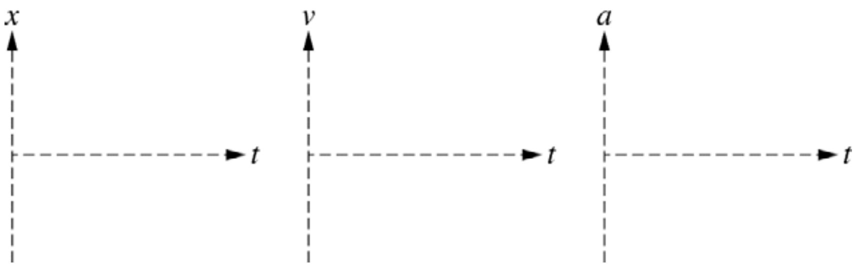
The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0. Now suppose the block experiences a resistive force as it slides on the horizontal surface. The magnitude of the resistive force F is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of kg/m.

(d)

i. Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m, h, k, β, v, and physical constants, as appropriate.

ii. Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time t can be written in the form $\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$, where v_0 is the speed at $t = 0$.

(e) Sketch graphs of position x as a function of time t, velocity v as a function of time t, and acceleration a as a function of time t for the block as it is moving on the horizontal surface before it reaches the spring.



7.3 Representing and Analyzing SHM

Name: ____________ Class: ________ Date: __________ **Total: 14 marks**

KEY WORDS phase 相位 • amplitude 振幅 • equilibrium 平衡位置

Objective

Build the skills to answer exam questions on **representing and analyzing SHM**.

You must be able to:

- use the position equation $x(t) = A\cos(\omega t + \varphi)$ with **amplitude** 振幅 A and **phase** 相位 φ
- find the maximum speed $v_{max} = A\omega$ (at equilibrium)
- find the maximum acceleration $a_{max} = A\omega^2$ (at the extremes)
- state that displacement and acceleration are out of phase

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ The position equation

$$x(t) = A\cos(\omega t + \varphi).$$

A is the amplitude (maximum displacement) and φ the phase (starting point).

■ Maximum speed and acceleration

Differentiating gives

$$v_{max} = A\omega \text{ (at equilibrium)}, \qquad a_{max} = A\omega^2 \text{ (at the extremes)}.$$

■ Out of step

Speed is greatest at the middle, where the displacement and acceleration are zero; acceleration is greatest at the ends, where the speed is zero.

2 Practice

Now apply the methods above.

2.1 An oscillator has $A = 0.2$ m and $\omega = 4$ rad s^{-1}. Find its maximum speed. [2]

2.2 For the same oscillator, find the maximum acceleration. [2]

2.3 Where in the motion is the speed greatest? [1]

2.4 In $x = A\cos(\omega t + \varphi)$, what does A represent? [1]

3 Exam-style questions

3.1 In SHM, the maximum speed occurs at the [1]

- **A** extremes
- **B** equilibrium position
- **C** turning points
- **D** it is constant

3.2 The maximum acceleration in SHM equals [1]

- **A** $A\omega$
- **B** $A\omega^2$
- **C** A/ω
- **D** ω^2/A

3.3 An oscillator has $A = 0.05$ m and $\omega = 10$ rad s^{-1}.

(a) Find the maximum speed. [2]

(b) Find the maximum acceleration. [2]

3.4 A mass in SHM starts from rest at its far extreme.

(a) State whether to model it with cosine or sine (phase). [1]

(b) State where its acceleration is largest. [1]

Solutions

2.1 $v_{max} = A\omega = 0.2 \times 4 = 0.8$ m s^{-1}.

2.2 $a_{max} = A\omega^2 = 0.2 \times 16 = 3.2$ m s^{-2}.

2.3 At the equilibrium position.

2.4 The amplitude (the maximum displacement).

3.1 B — maximum speed is at equilibrium.

3.2 B — $a_{max} = A\omega^2$.

3.3 (a) $v_{max} = 0.05 \times 10 = 0.5$ m s^{-1}. (b) $a_{max} = 0.05 \times 100 = 5$ m s^{-2}.

3.4 (a) Cosine, with $\varphi = 0$. (b) At the extremes.

7.4 Energy of Simple Harmonic Oscillators

Name: ____________ Class: ________ Date: __________ **Total: 13 marks**

KEY WORDS kinetic energy 动能 • potential energy 势能 • amplitude 振幅 • equilibrium 平衡位置

Objective

Build the skills to answer exam questions on **the energy of simple harmonic oscillators**.

You must be able to:

- use the total mechanical energy $E = \frac{1}{2}kA^2$
- describe the trade between **potential energy** 势能 $\frac{1}{2}kx^2$ and **kinetic energy** 动能 $\frac{1}{2}mv^2$
- state that E is all potential at the extremes and all kinetic at equilibrium
- explain that $E \propto A^2$

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Total energy

$$E = \frac{1}{2}kA^2.$$

A spring with $k = 200$ N m^{-1} and amplitude 0.05 m stores $E = \frac{1}{2}(200)(0.05)^2 = 0.25$ J.

The energy trade

At the extremes it is all potential; at equilibrium it is all kinetic. In between it is shared, but the total is constant.

Amplitude squared

Because $E \propto A^2$, doubling the amplitude quadruples the energy.

2 Practice

Now apply the methods above.

2.1 A spring with $k = 100$ N m^{-1} oscillates with amplitude 0.2 m. Find the total energy. [2]

2.2 At which point is the energy entirely kinetic? [1]

2.3 If the amplitude doubles, by what factor does the total energy change? [1]

2.4 A spring with $k = 100$ N m^{-1} is displaced 0.1 m. Find the stored potential energy. [2]

3 Exam-style questions

3.1 The total energy of a simple harmonic oscillator is [1]

- **A** $\frac{1}{2}kA^2$
- **B** $\frac{1}{2}kA$
- **C** kA^2
- **D** $\frac{1}{2}mA^2$

3.2 Doubling the amplitude changes the total energy by a factor of [1]

- **A** 2
- **B** 4
- **C** 1/2
- **D** 1

3.3 A spring with $k = 400$ N m^{-1} oscillates with amplitude 0.1 m.

(a) Find the total energy. [2]

(b) State the kinetic energy at the equilibrium position. [1]

3.4 A mass oscillates on a spring.

(a) State where the potential energy is greatest. [1]

(b) State whether the kinetic and potential energies can both be maximum at the same instant. [1]

Solutions

2.1 $E = \frac{1}{2}kA^2 = \frac{1}{2}(100)(0.2)^2 = 2$ J.

2.2 At the equilibrium position.

2.3 Four times.

2.4 $U = \frac{1}{2}kx^2 = \frac{1}{2}(100)(0.1)^2 = 0.5$ J.

3.1 **A** — $E = \frac{1}{2}kA^2$.

3.2 **B** — $E \propto A^2$, so $2^2 = 4$.

3.3 (a) $E = \frac{1}{2}(400)(0.1)^2 = 2$ J. (b) 2 J (all kinetic there).

3.4 (a) At the extremes (maximum x). (b) No — each is greatest where the other is zero.

Question 2: Version J

2. In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

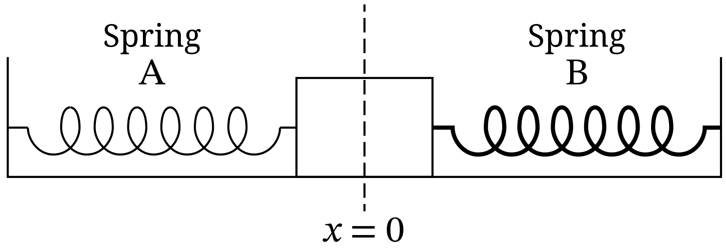


Figure 1

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

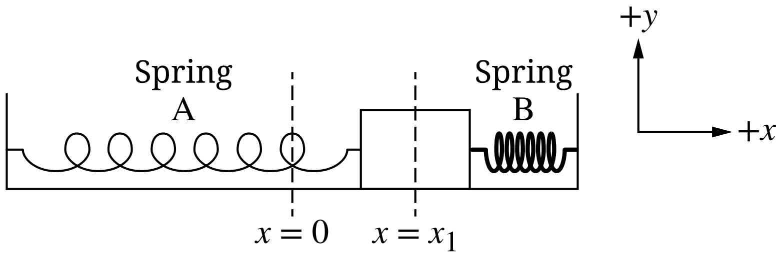


Figure 2

A. An energy bar chart can be used to represent the elastic potential energy U_A of Spring A, the elastic potential energy U_B of Spring B, and the kinetic energy K_{block} of the block. On the energy bar chart in Figure 3, **draw** shaded bars to represent the energy of the system for when the block is at $x = x_1$.

- The height of the shaded bars should be proportional to the relative values of U_A, U_B, and K_{block}.
- Any energy that is equal to zero should be represented by a distinct line on the zero-energy line.

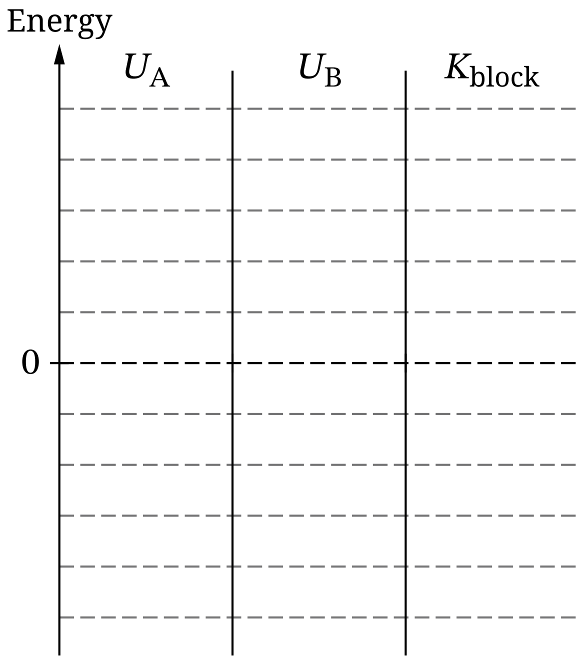
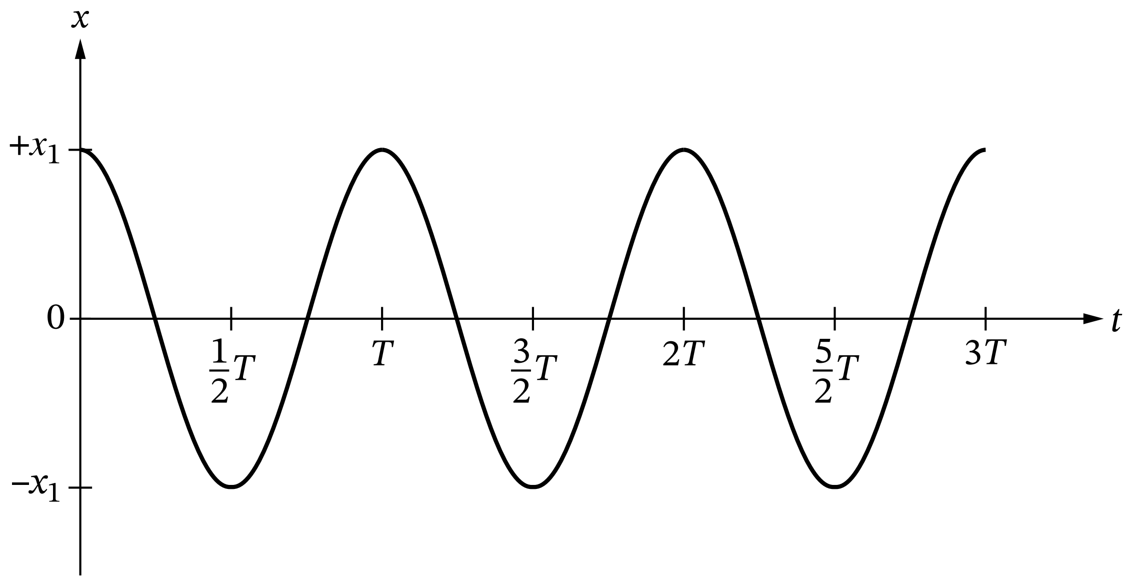


Figure 3

B. The block is released from rest at $x = x_1$ and begins to oscillate. **Derive** an expression for the speed v of the block as the block passes through $x = \frac{1}{2}x_1$. Express your answer in terms of m, k, x_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

C. In Scenario 1, the block oscillates with period T. The position x of the block in Scenario 1 as a function of time t is shown in Figure 4.

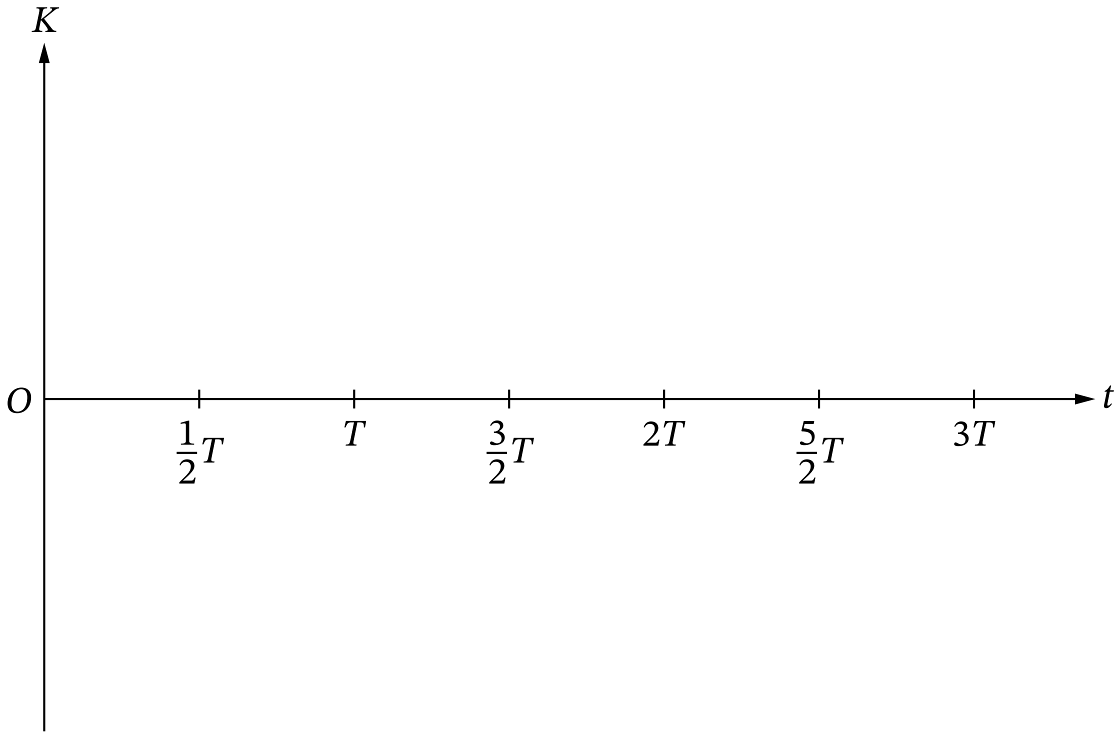


Scenario 1

Figure 4

In Scenario 2, the block-springs system is placed on a new surface. There is friction between the block and the new surface. The block is again moved to the same position $x = x_1$ and released from rest. The block completes multiple oscillations with the same period as in Scenario 1 before coming to rest.

On the axes shown in Figure 5, **sketch** a graph of the kinetic energy K of the block as a function of t for Scenario 2.



Scenario 2

Figure 5

D. In Scenario 3, the block is replaced with a new block of larger mass. The coefficient of kinetic friction between the new block and the surface in Scenario 3 is the same as the coefficient of kinetic friction between the original block and the surface in Scenario 2.

The new block is moved to position $x = x_1$ and released from rest. The kinetic energy of the new block is plotted as a function of time.

Describe how one feature of the graph of K as a function of t in Scenario 3 would differ from the graph you drew in Figure 5 for Scenario 2.

Briefly **justify** your answer.

Begin your response to **QUESTION 2** on this page.

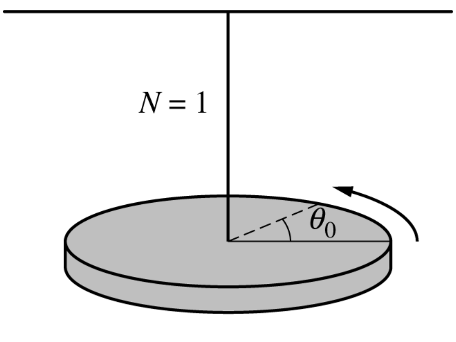


Figure 1

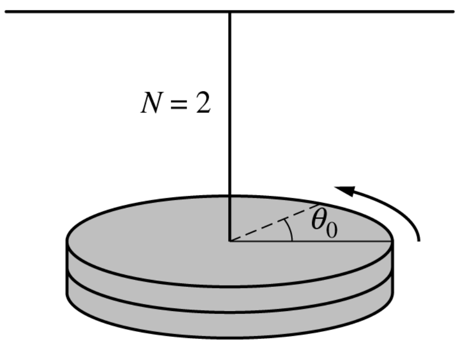


Figure 2

2. A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

(a) Using $T = 2\pi\sqrt{\frac{I}{\kappa}}$, derive an expression for T as a function of N. Express your answer in terms of M, R, κ, N, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b) The potential energy stored in the torsional pendulum when the disks are displaced is $U = \frac{1}{2}\kappa(\Delta\theta)^2$. On the following axes, sketch a graph of the maximum kinetic energy K_{max} of the torsional pendulum as a function of N for $N \geq 1$.

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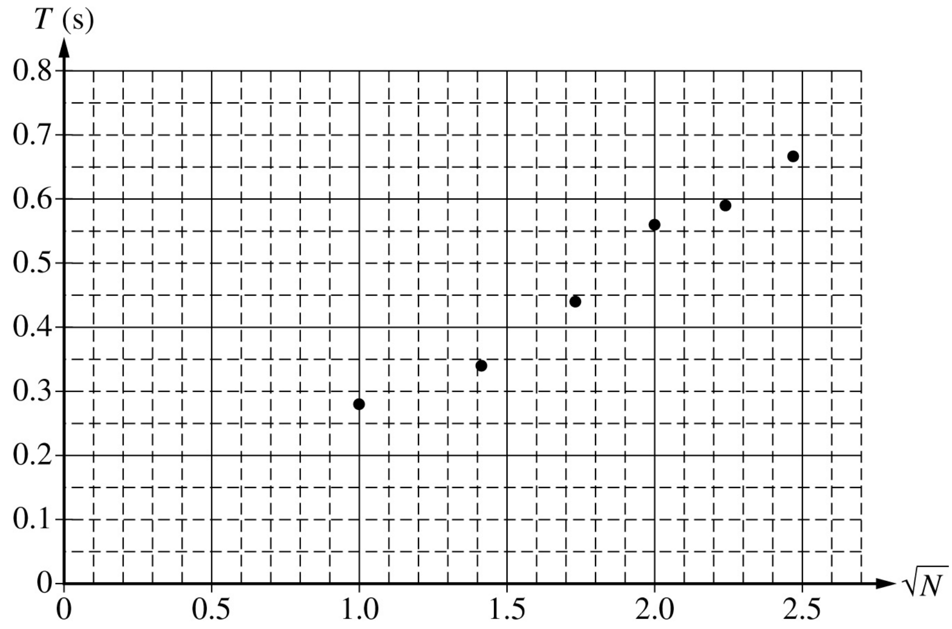
Continue your response to **QUESTION 2** on this page.

(c) The student plots the data for T as a function of $\sqrt{N}$, as shown.



i. Draw the best-fit line for the data.

ii. The student previously determined that the radius of a disk is $R = 0.2\text{ m}$ and found that $\kappa = 1.6\text{ N·m}$. Using the graph, calculate the mass M of a single disk.

iii. The student finds that the value given by the manufacturer for the mass of the disk is less than the value determined experimentally in part (c)(ii). Determine a single source of experimental error that could result in the observed difference in the value of M. Justify your answer.

GO ON TO THE NEXT PAGE.

(d) The student repeats the experiment, but now the disks have a density that varies as a function of the radius of the disk according to $\rho = 0.3r$.

i. Would the slope of the best-fit line for this new data be greater than, less than, or the same as the slope of the best-fit line in part (c)(i) ?

___ greater than ___less than ___the same as

Justify your answer.

ii. When $N = 1$, the maximum angular speed of the torsional pendulum with a uniform disk is found to be ω_{U}. When $N = 1$, the maximum angular speed of the torsional pendulum with a nonuniform disk is $\omega_{\text{non-U}}$. Which of the following correctly compares ω_{U} and $\omega_{\text{non-U}}$?

_____$\omega_{\text{U}} > \omega_{\text{non-U}}$ _____$\omega_{\text{U}} < \omega_{\text{non-U}}$ _____ $\omega_{\text{U}} = \omega_{\text{non-U}}$

Briefly justify your answer.

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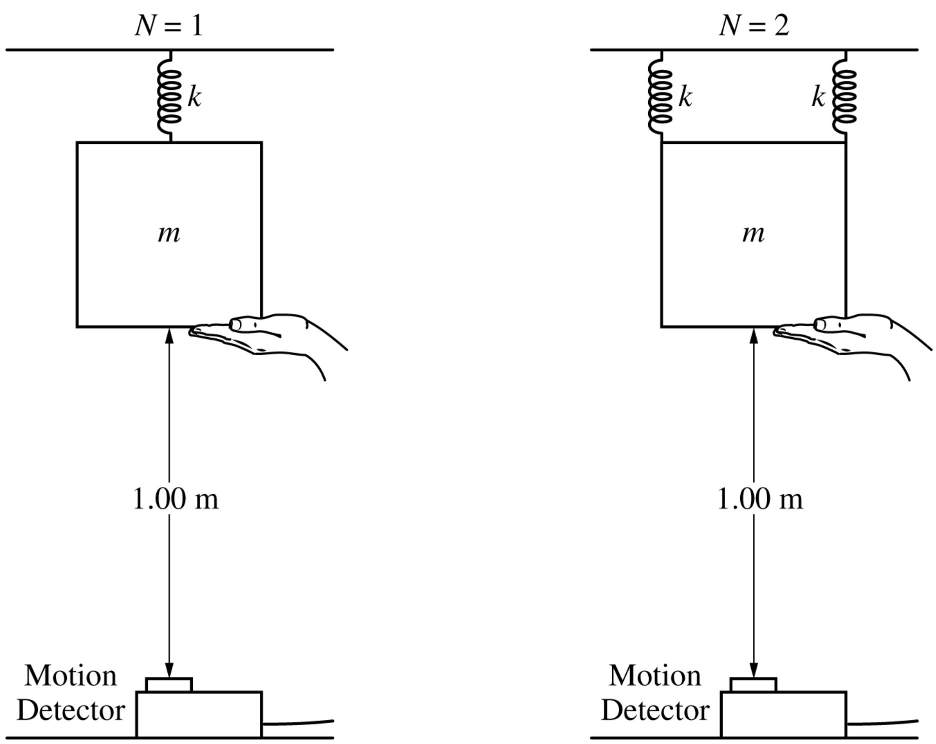

Figure 1 Figure 2

. A student conducts an investigation to determine the relationship between the period of oscillation T of a system consisting of a block and N attached springs. The student starts with a block of mass m attached to a single ideal spring of spring constant k, as shown in Figure 1. The student holds the block so that the spring is neither stretched nor compressed at a vertical height 1.00 m above a motion detector. The student releases the block from rest and records the period of oscillation for the system consisting of the single spring and block. An additional identical spring is attached in parallel, as shown in Figure 2, and the procedure is repeated for $N = 2$ springs. This procedure is repeated through $N = 10$ springs.

(a) Derive an expression for T as a function of N. Express your answer in terms of m, k, N, and physical constants as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b) On the following axes, sketch a graph of T as a function of N for $N \geq 1$.

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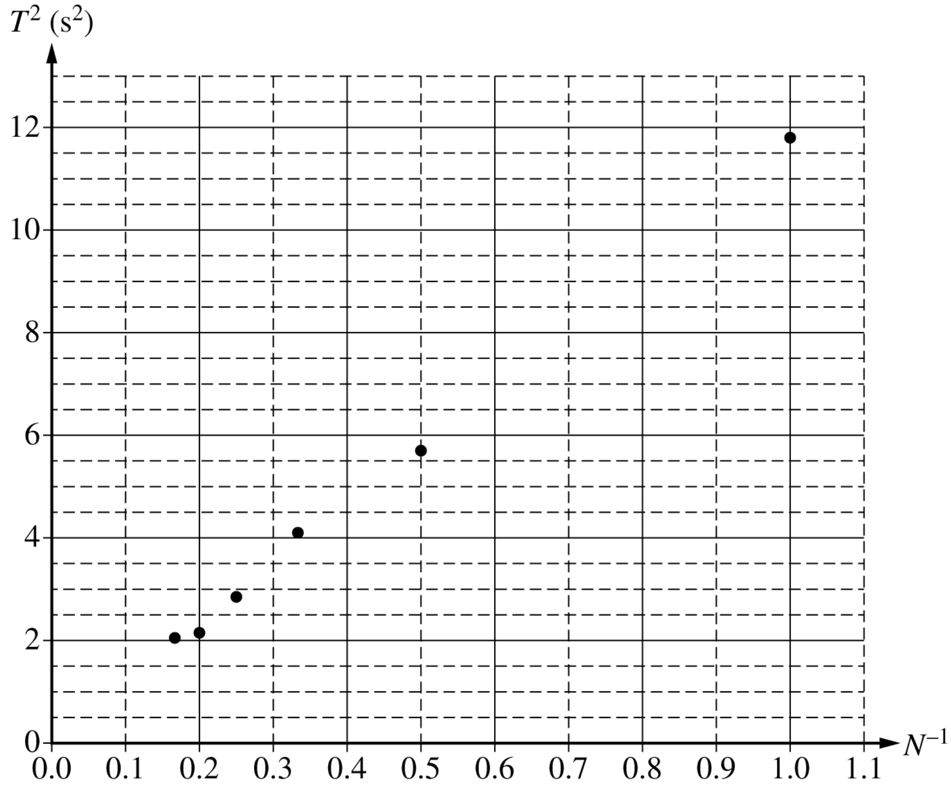
Continue your response to **QUESTION 2** on this page.

(c) The student plots the data for T^2 as a function of N^{-1}, as shown.



i. Draw the best-fit line for the data.

ii. The mass of the block is measured to be $m = 1.5$ kg. Using the graph, calculate an experimental value for the spring constant k for a single spring.

iii. The student finds that the value given by the manufacturer for the spring constant is larger than the value determined experimentally in part (c)(ii). Determine a single source of experimental error that could result in the observed difference in the value for k. Briefly justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

$N = 2$

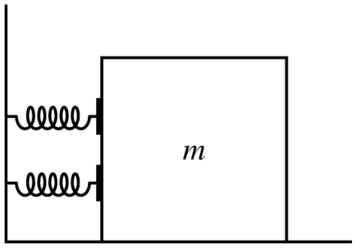


Figure 3

(d) The student conducts a similar investigation to determine the relationship between the period of oscillation T of a system consisting of a block and a number N of identical springs but arranges the block-spring system horizontally on a table, as shown in Figure 3. Frictional forces between the table and the block are negligible. In each trial, the block is displaced the same horizontal distance from equilibrium and released from rest.

i. The student plots T^2 as a function of N^{-1} for this new data. Would the slope of the best-fit line from this new investigation be greater than, less than, or the same as the slope of the best-fit line in part (c)(i) ?

___ greater than ___ less than ___the same as

Briefly justify your answer.

ii. When $N = 1$, the maximum speed of the block is found to be v_{max}. When N increases, will v_{max} increase, decrease, or stay the same?

____ increase ____ decrease ____ stay the same

Justify your answer.

GO ON TO THE NEXT PAGE.

AP PHYSICS C: MECHANICS • EXERCISE SHEET

7.5 Simple and Physical Pendulums

Name: ______________ Class: __________ Date: ____________ **Total: 13 marks**

KEY WORDS simple pendulum 单摆 • physical pendulum 物理摆 • period 周期 • amplitude 振幅

Objective

Build the skills to answer exam questions on **simple and physical pendulums**.

You must be able to:

- use the **simple pendulum 单摆** period: $T = 2\pi\sqrt{L/g}$
- use the **physical pendulum 物理摆** period: $T = 2\pi\sqrt{I/(mgd)}$
- identify d as the pivot-to-centre-of-mass distance and I as the inertia about the pivot
- recognise the simple pendulum as a special case of the physical one

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

The simple pendulum

A point mass on a light string of length L:

$$T = 2\pi\sqrt{\frac{L}{g}}.$$

The physical pendulum

Any rigid body swinging about a pivot:

$$T = 2\pi\sqrt{\frac{I}{mgd}},$$

where I is about the pivot and d reaches the centre of mass.

One is a special case of the other

For a point mass at distance L, $I = mL^2$ and $d = L$, so $T = 2\pi\sqrt{mL^2/(mgL)} = 2\pi\sqrt{L/g}$.

2 Practice

Now apply the methods above.

2.1 A simple pendulum has $L = 0.25$ m ($g = 9.8$ m s^{-2}). Find its period. [2]

2.2 In $T = 2\pi\sqrt{I/(mgd)}$, what is d? [1]

2.3 State whether making a pendulum's bob heavier changes its period. [1]

2.4 For a physical pendulum, about which axis must I be measured? [1]

3 Exam-style questions

3.1 The period of a simple pendulum increases if you [1]

- **A** increase the mass
- **B** increase the length
- **C** increase the amplitude
- **D** paint it black

3.2 The simple-pendulum formula is a special case of the [1]

- **A** mass-spring formula
- **B** physical-pendulum formula
- **C** ideal gas law
- **D** work-energy theorem

3.3 A uniform rod of length L is pivoted at one end, with $I = \frac{1}{3}mL^2$ and $d = L/2$.

(a) Write its period in terms of L and g. [2]

(b) State whether it swings faster or slower than a simple pendulum of the same length.[1]

3.4 A pendulum has $L = 1.0$ m ($g = 9.8$ m s^{-2}).

(a) Find its period. [2]

(b) State the effect of moving it to the Moon (smaller g). [1]

Solutions

2.1 $T = 2\pi\sqrt{L/g} = 2\pi\sqrt{0.25/9.8} \approx 1.0$ s.

2.2 The distance from the pivot to the centre of mass.

2.3 No (mass cancels).

2.4 About the pivot.

3.1 B — increasing the length increases the period.

3.2 B — the physical-pendulum formula.

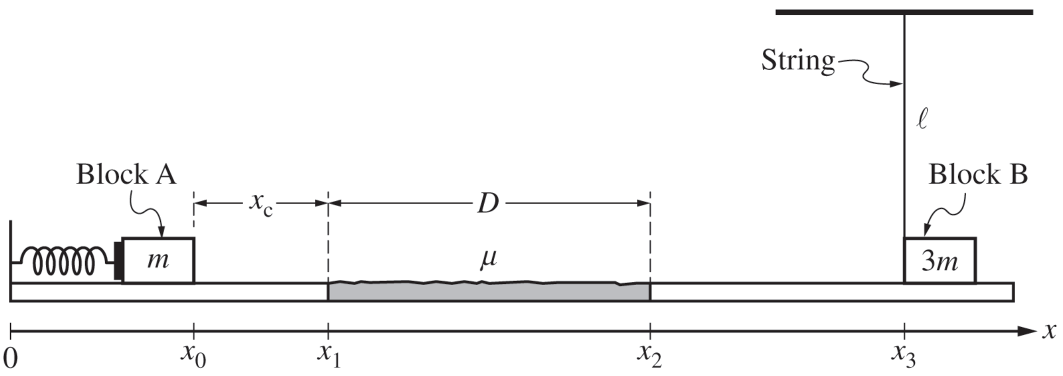
3.3 (a) $T = 2\pi\sqrt{\dfrac{\frac{1}{3}mL^2}{mg(L/2)}} = 2\pi\sqrt{\dfrac{2L}{3g}}$. (b) Slightly faster (shorter effective period).

3.4 (a) $T = 2\pi\sqrt{1/9.8} \approx 2.0$ s. (b) The period gets longer (smaller g).

Begin your response to **QUESTION 1** on this page.

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

Figure 1

1. Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position.
- At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction.
- At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

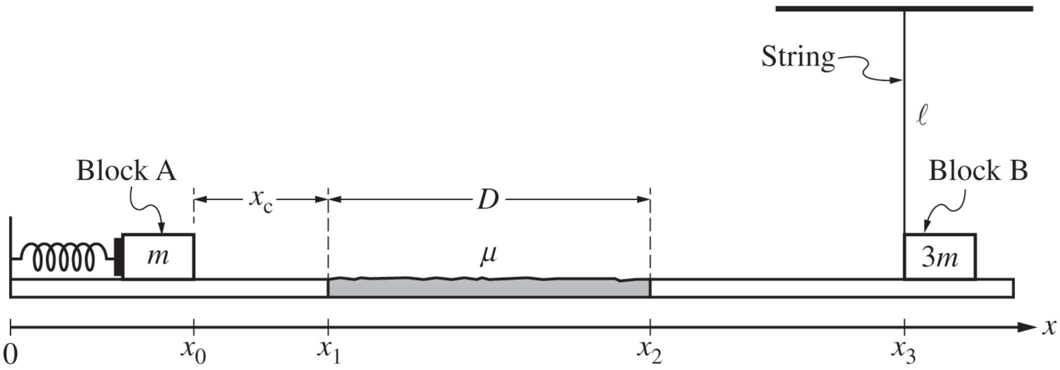
(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1.

ii. **Derive** an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

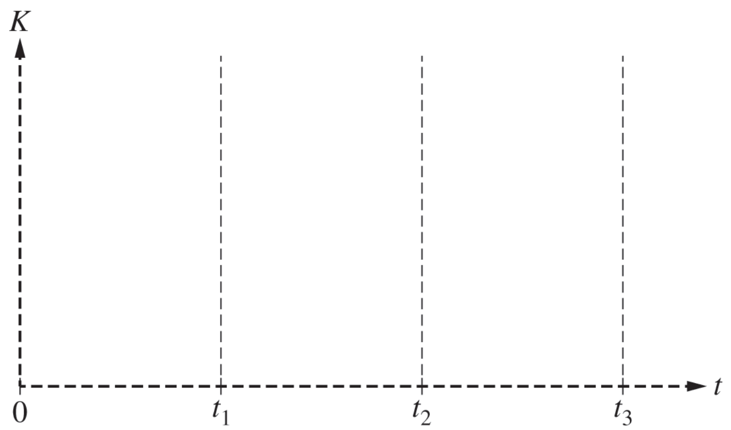


Note: Figure not drawn to scale.

Figure 1

(b)

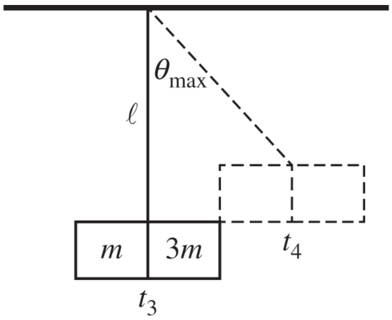
i. On the following axes, **sketch** a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to time t_3.



ii. Use principles of work and energy to **justify** the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.



Note: Figure not drawn to scale.

Figure 2

After the collision, the two-block system instantaneously comes to rest at time t_4, which occurs when the string makes a small angle θ_{max} with the vertical, as shown in Figure 2. For times $t > t_4$, the system oscillates with frequency f_ℓ. The support holding the string is raised, and the procedure is then repeated using a new string of length 2ℓ.

(c) **Indicate** how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length 2ℓ will compare to the frequency of oscillation f_ℓ from the original procedure.

____ $f_{2\ell} > f_\ell$ ____ $f_{2\ell} < f_\ell$ ____ $f_{2\ell} = f_\ell$

Briefly **justify** your answer.

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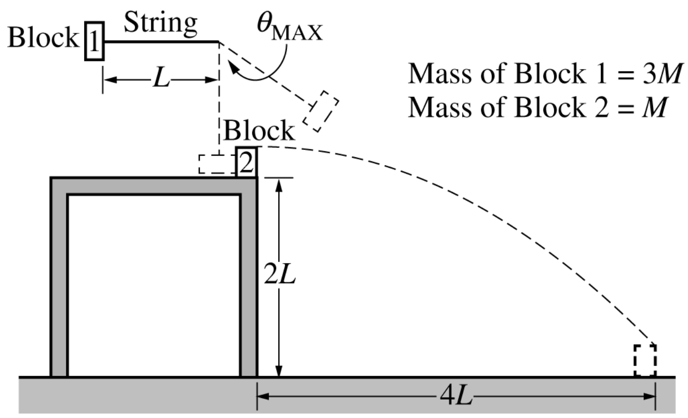
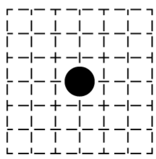


Note: Figure not drawn to scale.

2. A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(a) Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

(b) On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



(c) Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

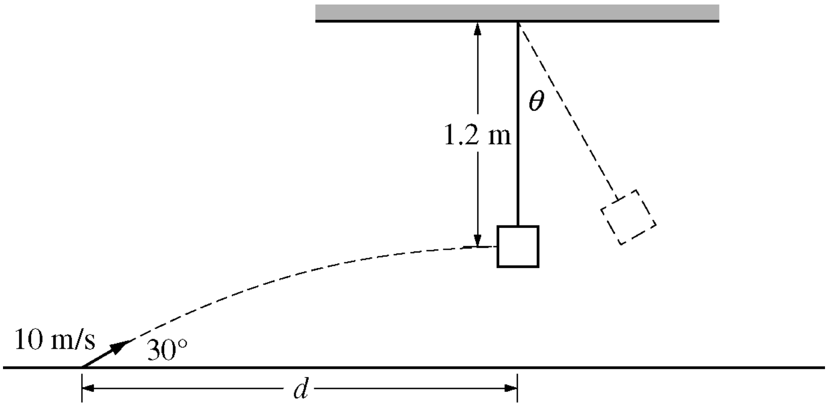
For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

(d) Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

(e) Calculate the speed of block 2 as it leaves the table.

(f) Calculate the speed of block 1 just after it collides with block 2.

(g) Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.



Mech.2.

A small dart of mass 0.020 kg is launched at an angle of 30° above the horizontal with an initial speed of 10 m/s. At the moment it reaches the highest point in its path and is moving horizontally, it collides with and sticks to a wooden block of mass 0.10 kg that is suspended at the end of a massless string. The center of mass of the block is 1.2 m below the pivot point of the string. The block and dart then swing up until the string makes an angle θ with the vertical, as shown above. Air resistance is negligible.

(a) Determine the speed of the dart just before it strikes the block.

(b) Calculate the horizontal distance d between the launching point of the dart and a point on the floor directly below the block.

(c) Calculate the speed of the block just after the dart strikes.

(d) Calculate the angle θ through which the dart and block on the string will rise before coming momentarily to rest.

(e) The block then continues to swing as a simple pendulum. Calculate the time between when the dart collides with the block and when the block first returns to its original position.

(f) In a second experiment, a dart with more mass is launched at the same speed and angle. The dart collides with and sticks to the same wooden block.

i. Would the angle θ that the dart and block swing to increase, decrease, or stay the same?

_____ Increase _____ Decrease _____ Stay the same

Justify your answer.

ii. Would the period of oscillation after the collision increase, decrease, or stay the same?

_____ Increase _____ Decrease _____ Stay the same

Justify your answer.