Week 17

AP Physics C: Mechanics

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

AP Physics C: Mechanics Name: ____________ Score: ______

1. A 4 kg cart moves at 5 $\frac{m}{s}$. What is its momentum (in kg · m/s)?

   ____________ kg · m/s

2. A force of 50 N acts for 0.4 s. What impulse does it deliver (in N · s)?

   ____________ N · s

3. A 50 kg skater at rest throws a 5 kg ball at 8 $\frac{m}{s}$. The skater's recoil speed (in m/s)?

   ____________ m/s

4. A 3 kg cart at 4 $\frac{m}{s}$ sticks to a stationary 1 kg cart. Their common speed afterward (in m/s)?

   ____________ m/s

5. Two objects moving toward each other can have a total momentum of zero.

   ☐ True ☐ False

6. An impulse of 12 N · s acts on a 3 kg cart at rest. Its final speed (in m/s)?

   ____________ m/s

7. An internal explosion inside a floating system changes the system's total momentum.

   ☐ True ☐ False

8. In which collision is kinetic energy conserved?

   ☐ an elastic collision

   ☐ an inelastic collision

   ☐ a perfectly inelastic collision

   ☐ all collisions

9. An airbag reduces injury by...

   ☐ reducing the change in momentum

   ☐ increasing the stopping time, which lowers the average force

   ☐ increasing the force

   ☐ removing all momentum instantly

10. Momentum is conserved only when...

    ☐ no forces act at all

    ☐ the net external force is zero

☐ kinetic energy is also conserved

☐ the objects are identical

AP Physics C: Mechanics

1. **20 kg · m/s**

   $p = mv = 4 \times 5 = 20\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$.

2. **20 N · s**

   $J = F\Delta t = 50 \times 0.4 = 20\ \text{N}\cdot\text{s}$.

3. **0.8 m/s**

   $0 = 5(8) + 50v$, so $v = -40/50 = -0.8\ \frac{\text{m}}{\text{s}}$ – the skater recoils at $0.8\ \frac{\text{m}}{\text{s}}$.

4. **3 m/s**

   $3(4) = (3+1)v_f$, so $v_f = 12/4 = 3\ \frac{\text{m}}{\text{s}}$.

5. **True**

   Momentum is a vector – opposite directions carry opposite signs, which can cancel to zero.

6. **4 m/s**

   $J = \Delta p = m\Delta v$, so $\Delta v = 12/3 = 4\ \frac{\text{m}}{\text{s}}$ from rest.

7. **False**

   Explosion forces are internal (third-law pairs), so the total momentum is unchanged.

8. **an elastic collision**

   Only an elastic collision conserves kinetic energy. Momentum, though, is conserved in all of them.

9. **increasing the stopping time, which lowers the average force**

   The momentum change is fixed; the airbag stretches the time, so the average force $F = \Delta p/\Delta t$ drops.

10. **the net external force is zero**

    An isolated system – zero *net external* force – conserves momentum. Internal forces are fine.

4.1 Linear Momentum

Name: ____________ Class: ________ Date: __________ **Total: 15 marks**

KEY WORDS momentum 动量 • vector 矢量 • kinetic energy 动能

Objective

Build the skills to answer exam questions on **linear momentum**.

You must be able to:

- calculate **momentum 动量**: $\vec{p} = m\vec{v}$
- treat momentum as a vector (direction matters)
- find momentum from mass and velocity, or velocity from momentum
- state the unit of momentum (kg m s^{-1})

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Momentum is mass times velocity

$$\vec{p} = m\vec{v}.$$

A 3 kg ball at 4 m s^{-1} has $p = 3 \times 4 = 12$ kg m s^{-1}.

■ Momentum is a vector

Its direction is the direction of the velocity. Taking right as positive, a 2 kg ball moving left at 5 m s^{-1} has $p = -10$ kg m s^{-1}.

■ Comparing momenta

A heavy slow object can have the same momentum as a light fast one: 10 kg at 2 m s^{-1} and 2 kg at 10 m s^{-1} both have $p = 20$ kg m s^{-1}.

2 Practice

Now apply the methods above.

2.1 A 5 kg object moves at 3 m s^{-1}. Find its momentum. [2]

2.2 State the unit of momentum. [1]

2.3 Taking right as positive, find the momentum of a 2 kg ball moving left at 6 m s^{-1}. [2]

2.4 An object has momentum 24 kg m s^{-1} and mass 4 kg. Find its speed. [2]

3 Exam-style questions

3.1 Momentum is calculated as [1]

- **A** $\frac{1}{2}mv^2$
- **B** mv
- **C** ma
- **D** Fd

3.2 Which quantity is a vector? [1]

- **A** mass
- **B** speed
- **C** momentum
- **D** kinetic energy

3.3 A 1200 kg car moves east at 15 m s^{-1}.

(a) Find its momentum. [2]

(b) State the direction of the momentum. [1]

3.4 Object A: 8 kg at 3 m s^{-1}. Object B: 2 kg at 12 m s^{-1}.

(a) Find each momentum. [2]

(b) State which has the greater momentum. [1]

Solutions

2.1 $p = mv = 5 \times 3 = 15$ kg m s^{-1}.

2.2 kg m s^{-1}.

2.3 $p = 2 \times (-6) = -12$ kg m s^{-1}.

2.4 $v = p/m = 24/4 = 6$ m s^{-1}.

3.1 B — $p = mv$.

3.2 C — momentum is a vector.

3.3 (a) $p = 1200 \times 15 = 1.8 \times 10^4$ kg m s^{-1}. (b) East.

3.4 (a) A: 24 kg m s^{-1}; B: 24 kg m s^{-1}. (b) Equal.

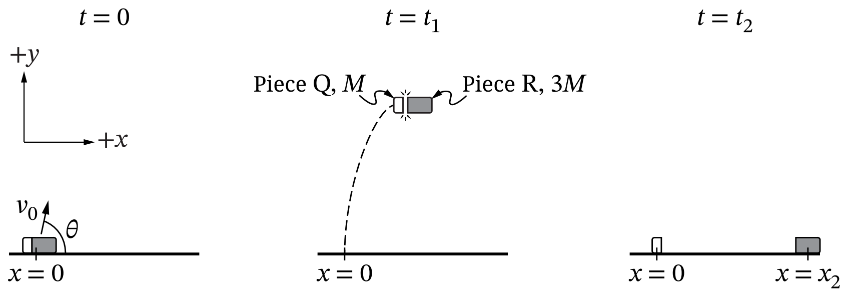
Question 2: Version J

2. A projectile of total mass $4M$ is launched from the ground at position $x = 0$ and time $t = 0$. The projectile is launched with an initial speed v_0 at an angle θ above the horizontal. When the projectile is at the highest point in its trajectory, it breaks into Pieces Q and R of masses M and $3M$, respectively.

The motion of the projectile is described for the following times.

- At $t = t_1$, immediately after the projectile breaks apart, the two pieces are moving away from each other horizontally.
- At $t = t_2$, Piece Q reaches the ground at $x = 0$ and Piece R reaches the ground at $x = x_2$, as shown in Figure 1.

Figure 1



Note: Figure not drawn to scale.

Name: AP Physics C: **Mechanics: 2026**

Part A

The horizontal and vertical components of a momentum vector are represented by p_x and p_y, respectively. The shaded bars in Figure 2 represent p_x and p_y of the projectile immediately after $t = 0$.

Figure 2

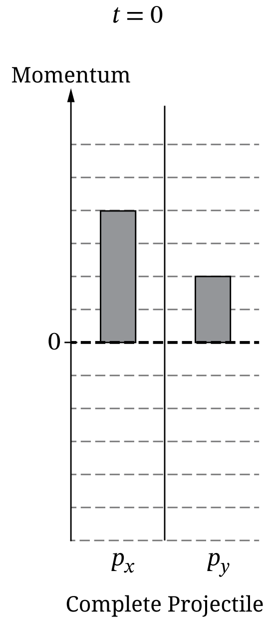
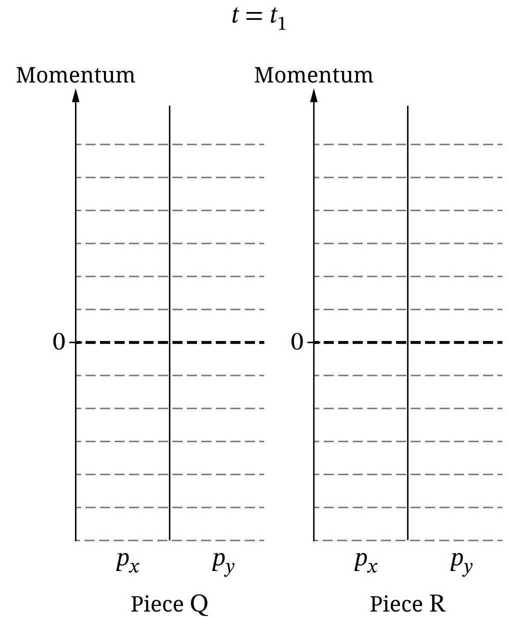


Figure 3



On Figure 3, **draw** shaded bars to represent p_x and p_y of Pieces Q and R at $t = t_1$.

- Start the shaded bars at the dashed line that represents zero momentum.
- Represent the magnitudes of the components of the momentum by using the heights of the shaded bars, consistent with the scale used in Figure 2.
- Represent any momentum component that is equal to zero by drawing a distinct line on the zero-momentum line.

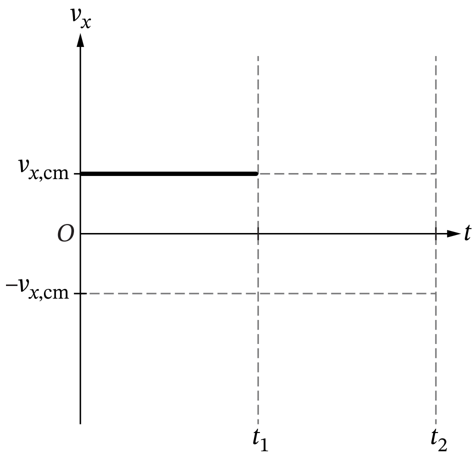
Part B

Derive an expression for x_2 in terms of v_0, θ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

The horizontal component of a velocity vector is represented by v_x. Figure 4 shows the horizontal component $v_{x,\text{cm}}$ of the velocity of the center of mass of the projectile as a function of t during the time interval $0 < t < t_1$.

Figure 4



On Figure 4, **sketch** a line or curve to represent v_x as a function of t for the time interval $t_1 < t < t_2$ for each of the following.

- Piece Q
- Piece R
- The center of mass of the two-piece system

Clearly **label** all lines or curves.

Part D

Consider a case in which the projectile is launched at the same angle and initial speed as initially described. When the projectile breaks into Pieces Q and R, Piece Q falls straight down. In this case, Piece R reaches the ground at $x = x_{\text{new}}$.

Indicate whether x_{new} is greater than, less than, or equal to x_2 by writing one of the following.

- $x_{\text{new}} > x_2$
- $x_{\text{new}} < x_2$
- $x_{\text{new}} = x_2$

Briefly **justify** your answer either by referencing a feature of the representations you drew in part A or C or by using conceptual reasoning beyond algebraic solutions.

Question 1: Version J

1. Two blocks, 1 and 2, slide toward each other on a horizontal surface. Block 1 has mass m and slides in the $+x$-direction with constant speed $2v_0$. Block 2 has mass $6m$ and slides in the $-x$-direction with constant speed v_0, as shown in Figure 1. The blocks then collide and stick together. The collision occurs from time $t = 0$ to $t = t_c$. After the collision, where $t > t_c$, the blocks move together with the same constant speed.

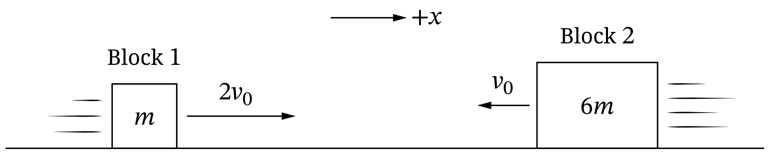


Figure 1

A. The diagrams in Figure 2 can be used to represent the momentum of blocks 1 and 2 before and after the collision. The momentum vector diagram for Block 1 before the collision is shown.

i. **Draw** arrows on the grids to represent the momentum vectors of Block 2 before the collision and the two-block system before and after the collision.

- Arrows should start at the zero-momentum line.
- The length of the arrows should be proportional to the relative magnitudes of the vectors.
- Represent an arrow of zero length by drawing a dot at zero.

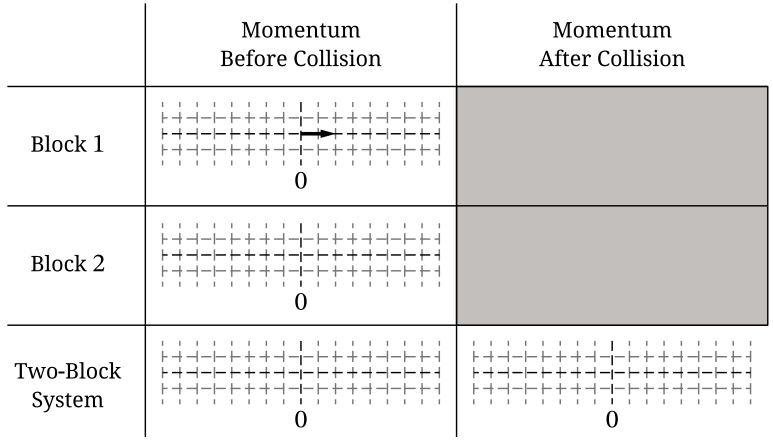


Figure 2

ii. During the time interval $0 \le t \le t_c$, a force F is exerted on Block 2 by Block 1 along the x-direction as a function of t that is modeled by $F(t) = F_{max}\sin(At)$, where A is a positive constant and F_{max} is the magnitude of the maximum force exerted on Block 2 by Block 1 during the collision.

Derive an expression for F_{max}. Express your answer in terms of m, v_0, A, t_c, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

B. Consider a new scenario where Block 1 initially slides in the $+x$-direction with a new constant speed v_1 and Block 2 again initially slides in the $-x$-direction with constant speed v_0. The blocks collide and stick together. In this new scenario, the two-block system has constant speed v_0 after the collision.

Derive an expression for v_1 in terms of v_0. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Begin your response to **QUESTION 1** on this page.

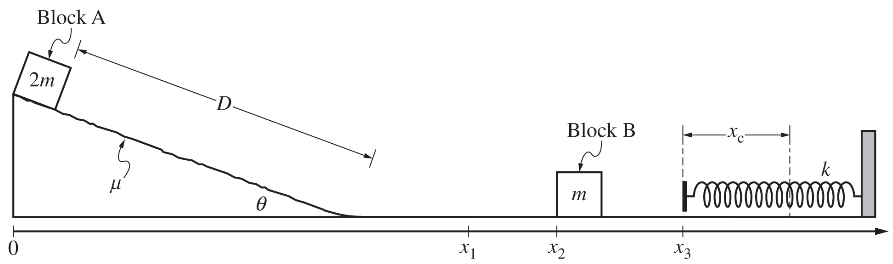
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

1. Blocks A and B of masses $2m$ and m, respectively, are arranged in a setup consisting of a ramp that makes an angle θ with a smooth horizontal table and an ideal spring of spring constant k fixed to a wall, as shown. Block A is held at rest a distance D up the ramp, and Block B is at rest on the horizontal table. The coefficient of kinetic friction between Block A and the rough ramp is μ in the region of length D, and there is negligible friction between the blocks and the smooth table.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$.
- At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B.
- At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position.
- At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, θ, D, μ, x_c, and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1.

ii. **Derive** an expression for the spring constant k of the spring.

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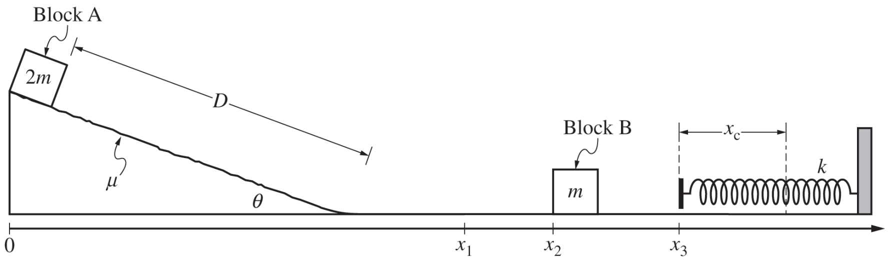
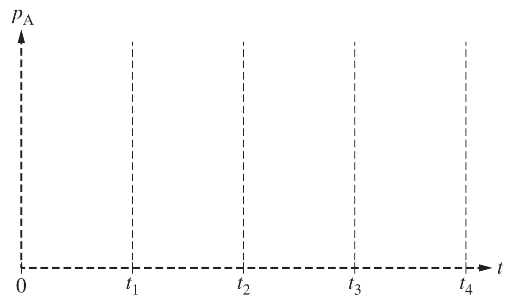
Continue your response to **QUESTION 1** on this page.



Note: Figure not drawn to scale.

(b)

i. On the following axes, **sketch** a graph of the magnitude of the momentum p_A of Block A as a function of time t from $t = 0$ to t_4.



ii. Use principles of forces to **justify** the graph drawn in part (b)(i) for the time interval $t = t_3$ to $t = t_4$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

For times $t > t_4$, the two-block-spring system oscillates with period T_O. The procedure is then repeated using a new ramp, where there is negligible friction between Block A and the ramp.

(c) **Indicate** how the new period of oscillation T_N in the procedure that uses the new ramp compares with the period of oscillation T_O from the original procedure.

_____ $T_N > T_O$ _____ $T_N < T_O$ _____ $T_N = T_O$

Briefly **justify** your answer.

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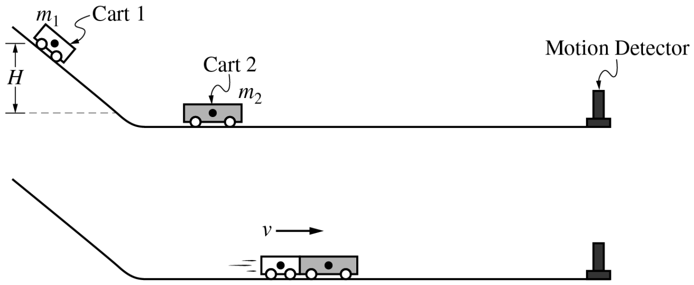
Begin your response to **QUESTION 2** on this page.



. Cart 1 of mass m_1 is held at rest above the bottom of an incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and smoothly transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance of H, as shown. At time t_C, Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(a) During the collision, is the impulse on Cart 1 from Cart 2 greater than, less than, or equal to the magnitude of the impulse on Cart 2 from Cart 1 ?

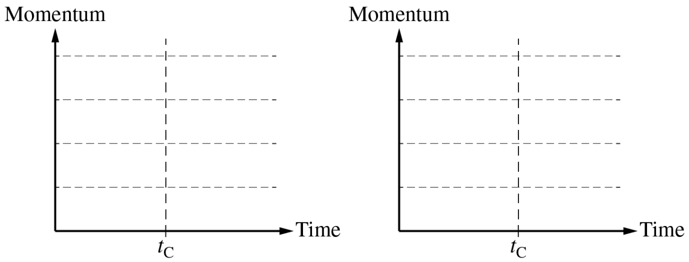
___ Greater than ___ Less than ___ Equal to

Justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b) On the following axes, draw graphs of the magnitude of the momentum of each cart as a function of time t, before and after t_C. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.



Cart 1 Cart 2

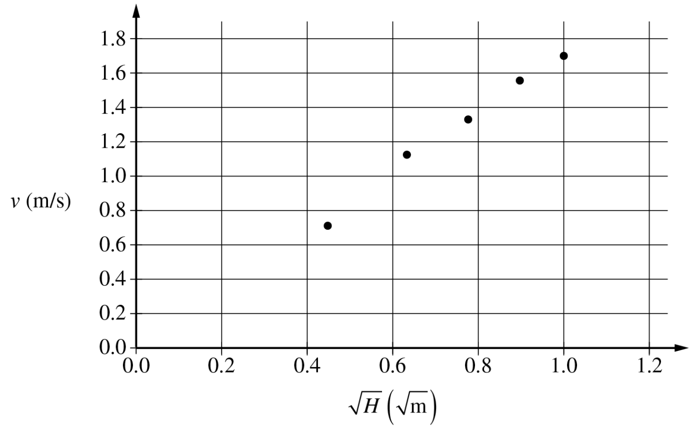
(c) Show that the velocity v of the two-cart system after the collision is given by the equation

$$v = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}.$$

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) A group of students use the setup to perform an experiment. They measure the mass of Cart 1 to be $m_1 = 0.250\text{ kg}$. The mass of Cart 2 is unknown. The students perform several trials and in each trial, Cart 1 is released from a different height H and the final velocity of the two-cart system is measured. The students graph v as a function of $\sqrt{H}$, as shown below.



i. Draw a line that represents the best fit to the data points shown.

ii. Use the best-fit line to calculate the mass of Cart 2.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(e) After the experiment, the students use a balance to measure the mass of Cart 2 and find it to be less than what was determined in part (d). To explain this discrepancy, one of the students proposes that the mass of Cart 1 was incorrectly measured at the beginning of the experiment. The students measure the mass of Cart 1 again and record a new value, m_1'.

Should the students expect that m_1' will be greater than 0.250 kg, less than 0.250 kg, or equal to 0.250 kg ?

_____ $m_1' > 0.250$ kg   _____ $m_1' < 0.250$ kg   _____ $m_1' = 0.250$ kg

Justify your answer.

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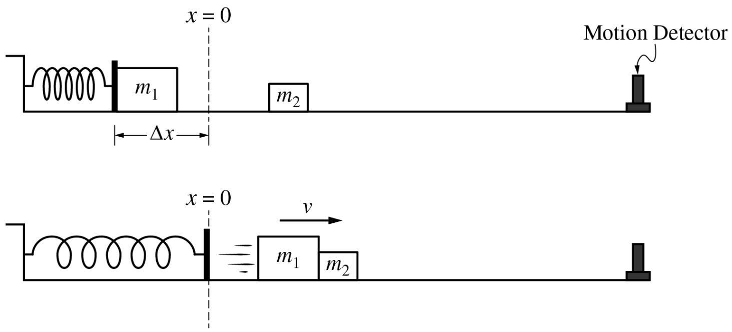
Begin your response to **QUESTION 2** on this page.



. Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v, as shown. Frictional effects are negligible.

(a) The impulse on Block 1 from the spring during the time interval $0 < t < t_C$ is J_S. The impulse on Block 1 from Block 2 during the collision is J_2. Which of the following expressions correctly compares the magnitudes of J_S and J_2 ?

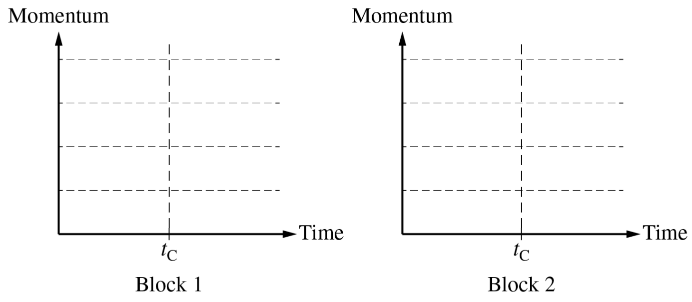
___ $J_S > J_2$ ___ $J_S < J_2$ ___ $J_S = J_2$

Justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b) On the following axes, draw graphs of the magnitude of the momentum of each block as a function of time, before and after t_C. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.

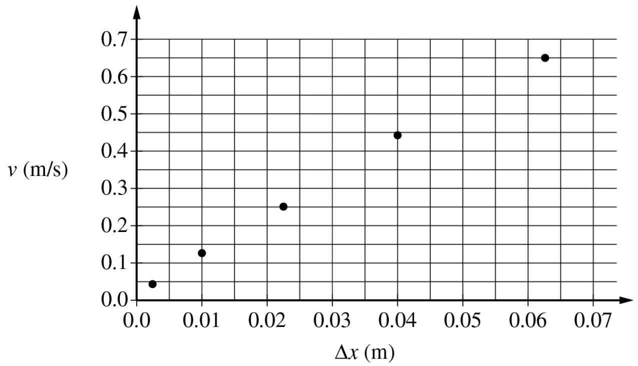


(c) Show that the velocity v of the two-block system after the collision is given by the equation $v = \frac{\sqrt{km_1}}{m_1 + m_2}\Delta x$.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) A group of students use the setup to perform an experiment. They measure the mass of Block 1 to be $m_1 = 0.500$ kg, and the spring constant k of the spring to be 150 N/m. The mass of Block 2 is unknown. They perform several trials and in each trial the spring is compressed a different distance Δx and the final velocity v of the two-block system is measured. They graph v as a function of Δx, as shown below.



i. Draw a line that represents the best fit to the data points shown.

ii. Use the best-fit line to calculate the mass of Block 2.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(e) After the experiment, the students use a balance to measure the mass of Block 2 and find it to be greater than what was determined in part (d). To explain this discrepancy, one of the students proposes that the spring constant was incorrectly measured at the beginning of the experiment. The students measure the spring constant again and record a new value, k'.

Should the students expect that k' be greater than 150 N / m, less than 150 N / m, or equal to 150 N / m ?

_____ $k' > 150$ N / m   _____ $k' < 150$ N / m   _____ $k' = 150$ N / m

Justify your answer.

GO ON TO THE NEXT PAGE.

AP PHYSICS C: MECHANICS • EXERCISE SHEET

4.2 Change in Momentum and Impulse

Name: ________ Class: ________ Date: ________ **Total: 16 marks**

KEY WORDS impulse 冲量 • kinetic energy 动能

Objective

Build the skills to answer exam questions on **change in momentum and impulse**.

You must be able to:

- calculate **impulse 冲量**: $J = F\,\Delta t$
- use the impulse-momentum theorem: $J = \Delta p$
- explain why a longer contact time reduces the force for a given change in momentum
- find force, time, or change in momentum given the other two

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Impulse equals change in momentum

$$J = F\,\Delta t = \Delta p = m\Delta v.$$

A 50 N force acting for 0.2 s delivers $J = 50 \times 0.2 = 10$ kg m s^{-1}, changing the momentum by 10 kg m s^{-1}.

Longer time, smaller force

For a fixed change in momentum, a longer contact time means a smaller force:

$$F = \frac{\Delta p}{\Delta t}.$$

This is why crumple zones and airbags help – they stretch Δt, cutting the force.

Finding the change in velocity

A 2 kg ball receives an impulse of 12 kg m s^{-1}: $\Delta v = J/m = 12/2 = 6$ m s^{-1}.

2 Practice

Now apply the methods above.

2.1 A 20 N force acts for 3 s. Find the impulse. [2]

2.2 State what impulse equals (the theorem). [1]

2.3 A 4 kg object receives an impulse of 16 kg m s^{-1}. Find its change in velocity. [2]

2.4 Explain why a longer collision time reduces the force. [2]

3 Exam-style questions

3.1 The impulse of a force equals the change in [1]

- **A** kinetic energy
- **B** momentum
- **C** mass
- **D** acceleration

3.2 For a fixed change in momentum, increasing the contact time [1]

- **A** increases the force
- **B** decreases the force
- **C** leaves the force unchanged
- **D** reverses the force

3.3 A 0.15 kg ball hits a wall at 10 m s^{-1} and bounces back at 8 m s^{-1} (opposite direction).

(a) Find the change in momentum. [2]

(b) If the contact lasts 0.02 s, find the average force. [2]

3.4 A 60 kg person jumps down and lands, changing momentum by 300 kg m s^{-1}.

(a) Find the force if they stop in 0.1 s (stiff legs). [1]

(b) Find the force if they stop in 0.5 s (bent knees). [2]

Solutions

2.1 $J = F\Delta t = 20 \times 3 = 60$ kg m s^{-1}.

2.2 The change in momentum, Δp.

2.3 $\Delta v = J/m = 16/4 = 4$ m s^{-1}.

2.4 For a fixed Δp, $F = \Delta p/\Delta t$; a larger Δt gives a smaller F.

3.1 B — impulse equals the change in momentum.

3.2 B — a longer time reduces the force.

3.3 (a) Taking the incoming direction positive: $\Delta p = m(v_f - v_i) = 0.15(-8 - 10) = -2.7$ kg m s^{-1} (magnitude 2.7). (b) $F = \Delta p/\Delta t = 2.7/0.02 = 135$ N.

3.4 (a) $F = 300/0.1 = 3000$ N. (b) $F = 300/0.5 = 600$ N.

Question 1: Version J

1. Two blocks, 1 and 2, slide toward each other on a horizontal surface. Block 1 has mass m and slides in the $+x$-direction with constant speed $2v_0$. Block 2 has mass $6m$ and slides in the $-x$-direction with constant speed v_0, as shown in Figure 1. The blocks then collide and stick together. The collision occurs from time $t = 0$ to $t = t_c$. After the collision, where $t > t_c$, the blocks move together with the same constant speed.

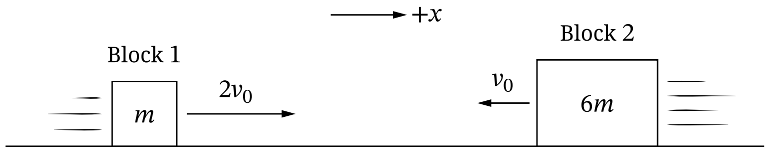


Figure 1

A. The diagrams in Figure 2 can be used to represent the momentum of blocks 1 and 2 before and after the collision. The momentum vector diagram for Block 1 before the collision is shown.

i. **Draw** arrows on the grids to represent the momentum vectors of Block 2 before the collision and the two-block system before and after the collision.

- Arrows should start at the zero-momentum line.
- The length of the arrows should be proportional to the relative magnitudes of the vectors.
- Represent an arrow of zero length by drawing a dot at zero.

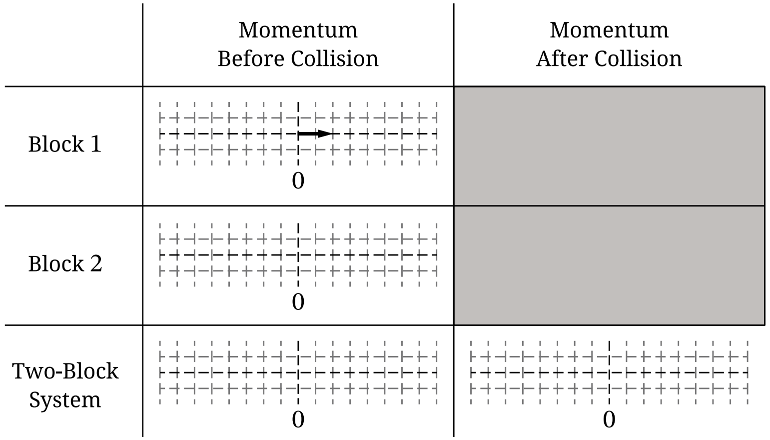


Figure 2

ii. During the time interval $0 \le t \le t_c$, a force F is exerted on Block 2 by Block 1 along the x-direction as a function of t that is modeled by $F(t) = F_{max}\sin(At)$, where A is a positive constant and F_{max} is the magnitude of the maximum force exerted on Block 2 by Block 1 during the collision.

Derive an expression for F_{max}. Express your answer in terms of m, v_0, A, t_c, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

B. Consider a new scenario where Block 1 initially slides in the $+x$-direction with a new constant speed v_1 and Block 2 again initially slides in the $-x$-direction with constant speed v_0. The blocks collide and stick together. In this new scenario, the two-block system has constant speed v_0 after the collision.

Derive an expression for v_1 in terms of v_0. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

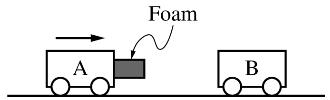
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

irections: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, hich are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this ooklet in the spaces provided after each part.



. Scientists have created a new type of lightweight foam and are performing experiments to investigate the properties of the foam. The mass of Cart A is 1000 kg and the mass of Cart B is 2000 kg. A piece of foam with negligible mass is attached to the front of Cart A, as shown. Cart A moves with a constant speed toward Cart B, which is initially at rest. At time $t = 0$ s, the foam connected to Cart A makes contact with Cart B. The foam remains in contact with Cart B for 0.5 s, after which the carts separate and both carts move with constant velocities.

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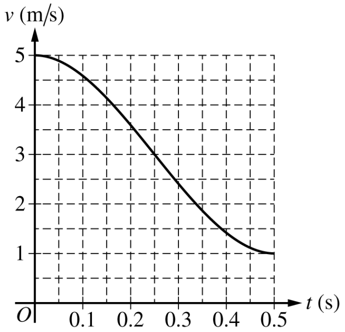
Continue your response to **QUESTION 1** on this page.

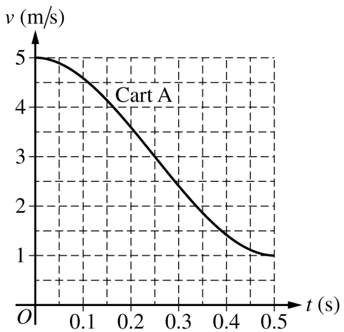


(a) The graph shows the velocity v of Cart A as a function of time t for the time interval when the foam and Cart B are in contact.

i. What feature(s) of the graph could be used to estimate the displacement of Cart A during the collision?

ii. Using the information shown in the graph, determine the speed of Cart B at $t = 0.5$ s.

iii. On the following grid, draw a smooth curve of the velocity of Cart B as a function of time.



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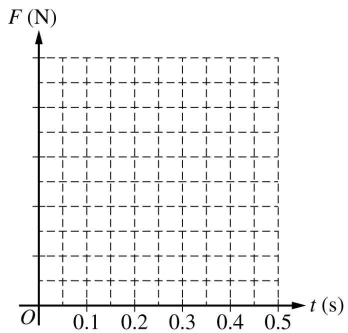
Continue your response to **QUESTION 1** on this page.

(b) For $0 \leq t \leq 0.50$ s, the velocity v of Cart A can be described by the function $v(t) = 64t^3 - 48t^2 + 5$.

i. Calculate the magnitude of the maximum net force acting on Cart A during this interval.

ii. On the following grid, draw a smooth curve of the magnitude of the force acting on Cart A as a function of time. Clearly indicate the value of the maximum force on the vertical axis.



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Continue your response to **QUESTION 1** on this page.

The foam is removed from the front of Cart A and the experiment is repeated. The carts collide, with both Cart A and Cart B having the same initial and final velocities as in the original collision. The time intervals during which the carts are in contact are different in the collision with the foam and the collision without the foam. In the collision without the foam, Cart A is in contact with Cart B for a shorter duration than in the original collision, when the foam was present.

For the original collision when the foam is present, the magnitude of the average net force exerted on Cart B is F_1. For the collision without the foam, the magnitude of the average net force exerted on Cart B is F_2.

(c) What is the relationship between the magnitude of the average net force F_1 exerted on Cart B for the collision with the foam and the magnitude of the average net force F_2 exerted on Cart B for the collision without the foam?

_____ $F_1 > F_2$ _____ $F_1 < F_2$ _____ $F_1 = F_2$

Justify your answer.

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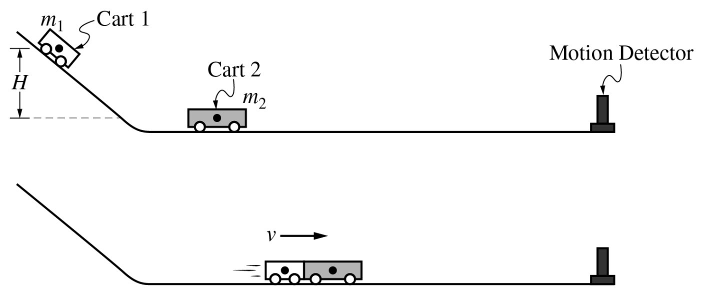
Begin your response to **QUESTION 2** on this page.



. Cart 1 of mass m_1 is held at rest above the bottom of an incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and smoothly transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance of H, as shown. At time t_C, Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(a) During the collision, is the impulse on Cart 1 from Cart 2 greater than, less than, or equal to the magnitude of the impulse on Cart 2 from Cart 1 ?

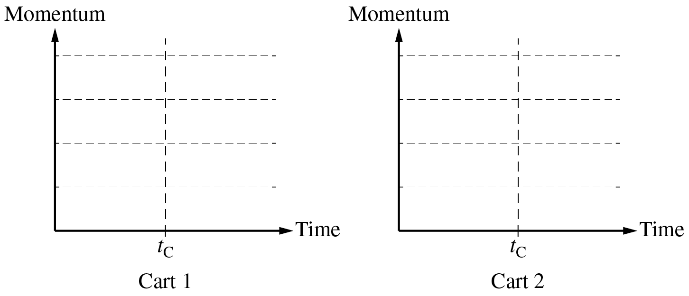
___ Greater than ___ Less than ___ Equal to

Justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b) On the following axes, draw graphs of the magnitude of the momentum of each cart as a function of time t, before and after t_C. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.



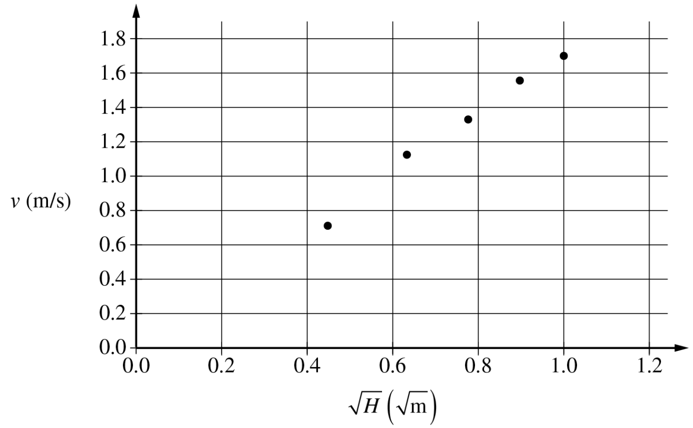
(c) Show that the velocity v of the two-cart system after the collision is given by the equation

$$v = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}.$$

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) A group of students use the setup to perform an experiment. They measure the mass of Cart 1 to be $m_1 = 0.250\text{ kg}$. The mass of Cart 2 is unknown. The students perform several trials and in each trial, Cart 1 is released from a different height H and the final velocity of the two-cart system is measured. The students graph v as a function of $\sqrt{H}$, as shown below.



i. Draw a line that represents the best fit to the data points shown.

ii. Use the best-fit line to calculate the mass of Cart 2.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(e) After the experiment, the students use a balance to measure the mass of Cart 2 and find it to be less than what was determined in part (d). To explain this discrepancy, one of the students proposes that the mass of Cart 1 was incorrectly measured at the beginning of the experiment. The students measure the mass of Cart 1 again and record a new value, m_1'.

Should the students expect that m_1' will be greater than 0.250 kg, less than 0.250 kg, or equal to 0.250 kg ?

_____ $m_1' > 0.250$ kg   _____ $m_1' < 0.250$ kg   _____ $m_1' = 0.250$ kg

Justify your answer.

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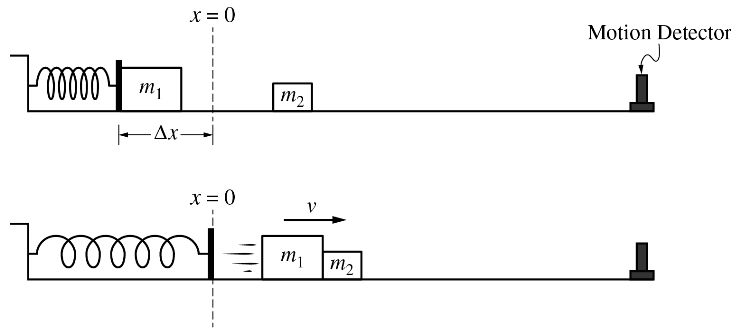
Begin your response to **QUESTION 2** on this page.



. Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v, as shown. Frictional effects are negligible.

(a) The impulse on Block 1 from the spring during the time interval $0 < t < t_C$ is J_S. The impulse on Block 1 from Block 2 during the collision is J_2. Which of the following expressions correctly compares the magnitudes of J_S and J_2 ?

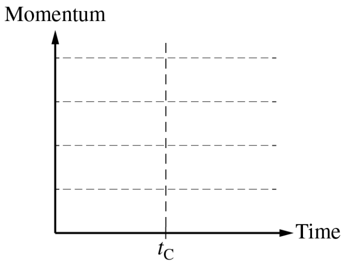
___ $J_S > J_2$ ___ $J_S < J_2$ ___ $J_S = J_2$

Justify your answer.

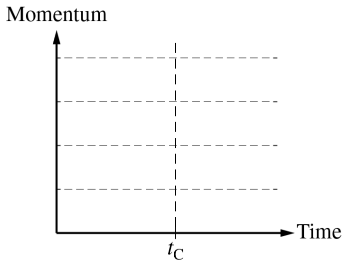
GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b) On the following axes, draw graphs of the magnitude of the momentum of each block as a function of time, before and after t_C. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.



Block 1



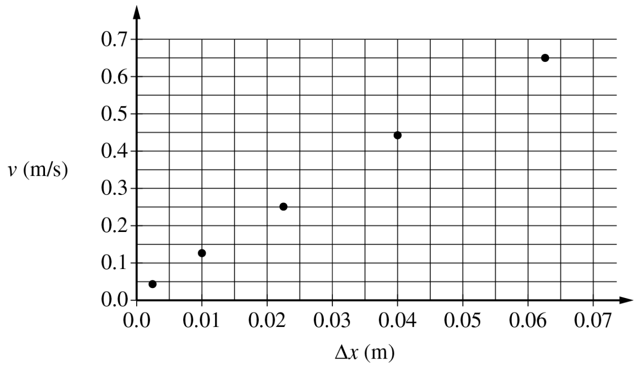
Block 2

(c) Show that the velocity v of the two-block system after the collision is given by the equation $v = \frac{\sqrt{km_1}}{m_1 + m_2}\Delta x$.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) A group of students use the setup to perform an experiment. They measure the mass of Block 1 to be $m_1 = 0.500$ kg, and the spring constant k of the spring to be 150 N/m. The mass of Block 2 is unknown. They perform several trials and in each trial the spring is compressed a different distance Δx and the final velocity v of the two-block system is measured. They graph v as a function of Δx, as shown below.



i. Draw a line that represents the best fit to the data points shown.

ii. Use the best-fit line to calculate the mass of Block 2.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(e) After the experiment, the students use a balance to measure the mass of Block 2 and find it to be greater than what was determined in part (d). To explain this discrepancy, one of the students proposes that the spring constant was incorrectly measured at the beginning of the experiment. The students measure the spring constant again and record a new value, k'.

Should the students expect that k' be greater than 150 N / m, less than 150 N / m, or equal to 150 N / m ?

_____ $k' > 150$ N / m _____ $k' < 150$ N / m _____ $k' = 150$ N / m

Justify your answer.

GO ON TO THE NEXT PAGE.

2. A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0\ \text{m/s}^2$, $L = 0.25\ \text{m/s}^4$, and t is the time in seconds. At $t = 6.0$ s, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

(a) Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0$ s.

(b) Calculate the speed of the rocket at $t = 6.0$ s.

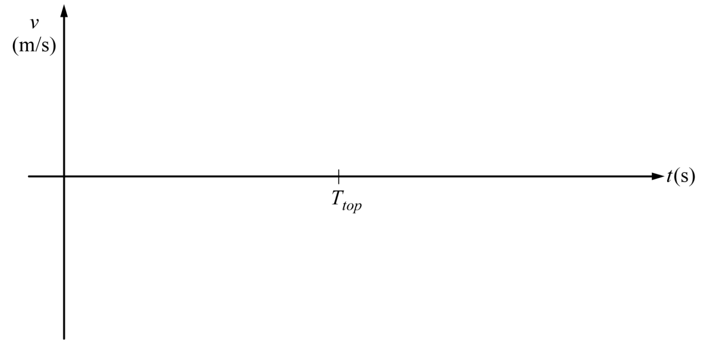
(c)

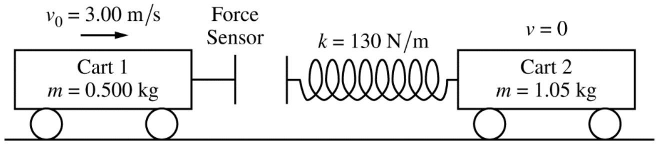
i. Calculate the kinetic energy of the rocket at $t = 6.0$ s.

ii. Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.

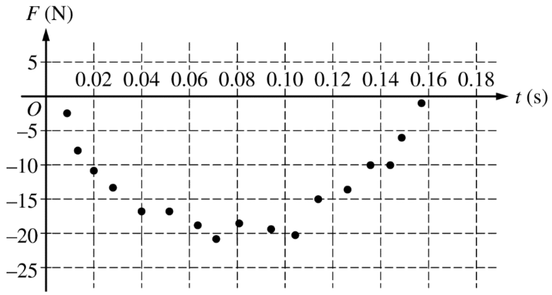
(d) Calculate the maximum height reached by the rocket relative to its launching point.

(e) On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.





2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00$ m/s toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130$ N/m. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200 \text{ N/s}^2\right)t^2 - (500 \text{ N/s})t$.



(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)

i. Calculate the speed of cart 1 after the collision.

ii. In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)

i. Calculate the speed of cart 2 after the collision.

ii. Show that the collision between the two carts is elastic.

(d)

i. Calculate the speed of the center of mass of the two-cart–spring system.

ii. Calculate the maximum elastic potential energy stored in the spring.

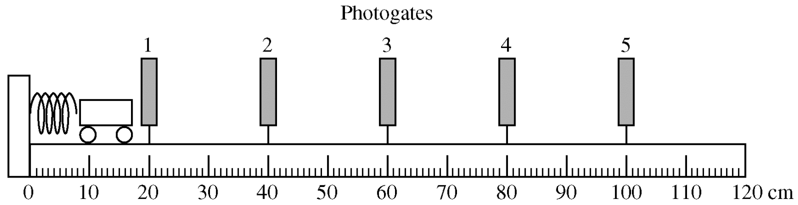
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Mech. 1.

In an experiment, a student wishes to use a spring to accelerate a cart along a horizontal, level track. The spring is attached to the left end of the track, as shown in the figure above, and produces a nonlinear restoring force of magnitude $F_s = As^2 + Bs$, where s is the distance the spring is compressed, in meters. A measuring tape, marked in centimeters, is attached to the side of the track. The student places five photogates on the track at the locations shown.

(a) Derive an expression for the potential energy U as a function of the compression s. Express your answer in terms of A, B, s, and fundamental constants, as appropriate.

In a preliminary experiment, the student pushes the cart of mass 0.30 kg into the spring, compressing the spring 0.040 m. For this spring, $A = 200\ \mathrm{N/m^2}$ and $B = 150\ \mathrm{N/m}$. The cart is released from rest. Assume friction and air resistance are negligible only during the short time interval when the spring is accelerating the cart.

(b) Calculate the following:

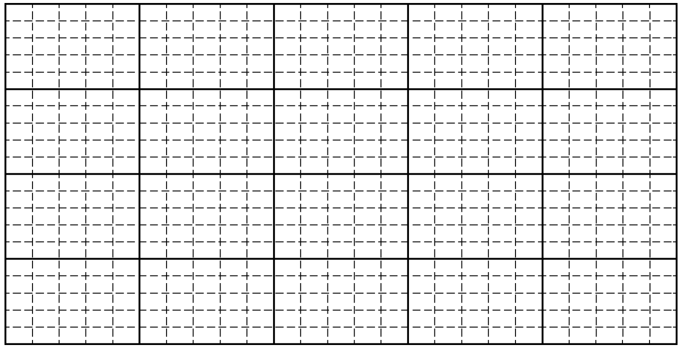
i. The speed of the cart immediately after it loses contact with the spring

ii. The impulse given to the cart by the spring

In a second experiment, the student collects data using the photogates. Each photogate measures the speed of the cart as it passes through the gate. The student calculates a spring compression that should give the cart a speed of $0.320\ \mathrm{m/s}$ after the cart loses contact with the spring. The student runs the experiment by pushing the cart into the spring, compressing the spring the calculated distance, and releasing the cart. The speeds are measured with a precision of $\pm 0.002\ \mathrm{m/s}$. The positions are measured with a precision of ± 0.005 m.

Photogate	1	2	3	4	5
Cart speed (m/s)	0.412	0.407	0.399	0.374	0.338
Photogate position (m)	0.20	0.40	0.60	0.80	1.00

(c) On the axes below, plot the data points for the speed v of the cart as a function of position x. Clearly scale and label all axes, as appropriate.



(d)

i. Compare the speed of the cart measured by photogate 1 to the predicted value of the speed of the cart just after it loses contact with the spring. List a physical source of error that could account for the difference.

ii. From the measured speed values of the cart as it rolls down the track, give a physical explanation for any trend you observe.

AP PHYSICS C: MECHANICS • EXERCISE SHEET

4.3 Conservation of Linear Momentum

Name: ____________ Class: ________ Date: __________ **Total: 13 marks**

KEY WORDS momentum 动量 • conserved 守恒 • kinetic energy 动能 • collisions 碰撞 • explosions 爆炸

Objective

Build the skills to answer exam questions on **conservation of linear momentum**.

You must be able to:

- state that total **momentum 动量** is conserved when no net external force acts
- apply $p_{\text{before}} = p_{\text{after}}$ to collisions and explosions
- solve for an unknown velocity after an interaction
- use signs for direction in one dimension

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Total momentum is conserved

If no net external force acts, the total momentum before equals the total after:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2.$$

A collision

A 2 kg cart at 3 m s^{-1} hits a stationary 1 kg cart and they stick. Total before $= 2(3) + 1(0) = 6$. After, combined mass 3 kg at speed v:

$$6 = 3v \Rightarrow v = 2\ \mathrm{m\ s^{-1}}.$$

An explosion

A stationary object splits. Total momentum stays zero, so the pieces fly apart with equal and opposite momenta.

2 Practice

Now apply the methods above.

2.1 State the condition for total momentum to be conserved. [1]

2.2 A 4 kg cart at 2 m s^{-1} collides and sticks to a stationary 4 kg cart. Find their common speed. [2]

2.3 A stationary object explodes into two pieces. State the total momentum afterward. [1]

2.4 Before a collision the total momentum is 30 kg m s^{-1}. State the total momentum after. [1]

3 Exam-style questions

3.1 Momentum is conserved in a collision provided [1]

- **A** kinetic energy is conserved
- **B** no net external force acts
- **C** the objects stick together
- **D** the objects are identical

3.2 A 3 kg ball at 4 m s^{-1} strikes a stationary 1 kg ball and they stick. Their speed after is [1]

- **A** 1 m s^{-1}
- **B** 3 m s^{-1}
- **C** 4 m s^{-1}
- **D** 12 m s^{-1}

3.3 A 60 kg skater at rest throws a 2 kg ball at 10 m s^{-1}.

(a) State the total momentum before the throw. [1]

(b) Find the skater's recoil speed. [2]

3.4 A 2 kg trolley at 5 m s^{-1} collides with a 3 kg trolley at rest; they couple together.

(a) Find the total momentum before. [1]

(b) Find the speed of the coupled trolleys. [2]

Solutions

2.1 When no net external force acts on the system.

2.2 $4(2) = 8 = 8v \Rightarrow v = 1\ \text{m s}^{-1}$.

2.3 Zero (equal and opposite momenta).

2.4 $30\ \text{kg m s}^{-1}$ (conserved).

3.1 B — momentum is conserved when no net external force acts.

3.2 B — $3(4) = 12 = 4v \Rightarrow v = 3\ \text{m s}^{-1}$.

3.3 (a) Zero (both at rest). (b) $0 = 60v + 2(10) \Rightarrow v = -0.33\ \text{m s}^{-1}$, i.e. $0.33\ \text{m s}^{-1}$ backward.

3.4 (a) $2(5) = 10\ \text{kg m s}^{-1}$. (b) $10 = 5v \Rightarrow v = 2\ \text{m s}^{-1}$.

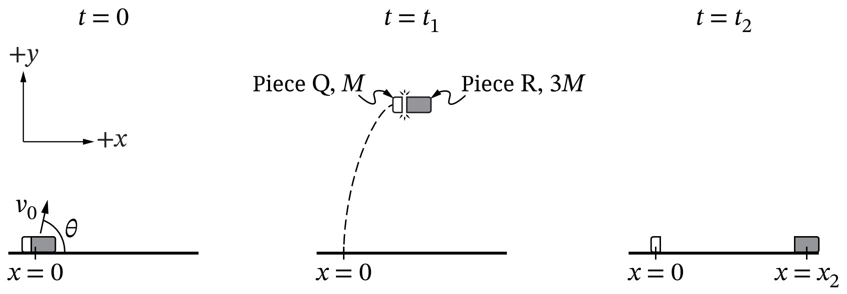
Question 2: Version J

2. A projectile of total mass $4M$ is launched from the ground at position $x = 0$ and time $t = 0$. The projectile is launched with an initial speed v_0 at an angle θ above the horizontal. When the projectile is at the highest point in its trajectory, it breaks into Pieces Q and R of masses M and $3M$, respectively.

The motion of the projectile is described for the following times.

- At $t = t_1$, immediately after the projectile breaks apart, the two pieces are moving away from each other horizontally.
- At $t = t_2$, Piece Q reaches the ground at $x = 0$ and Piece R reaches the ground at $x = x_2$, as shown in Figure 1.

Figure 1



Note: Figure not drawn to scale.

Part A

The horizontal and vertical components of a momentum vector are represented by p_x and p_y, respectively. The shaded bars in Figure 2 represent p_x and p_y of the projectile immediately after $t = 0$.

Figure 2

$t = 0$

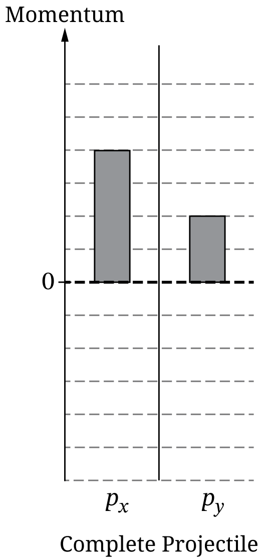
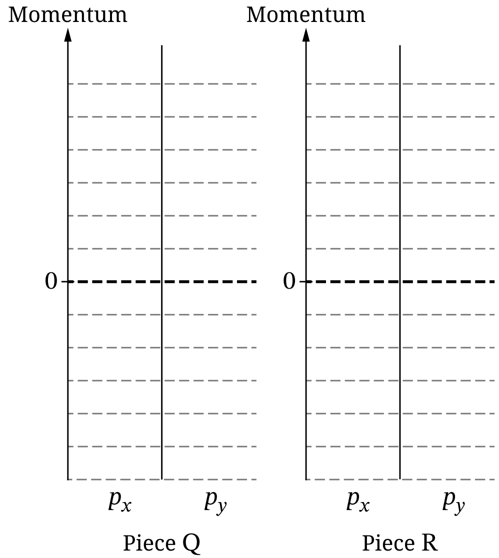


Figure 3

$t = t_1$



On Figure 3, **draw** shaded bars to represent p_x and p_y of Pieces Q and R at $t = t_1$.

- Start the shaded bars at the dashed line that represents zero momentum.
- Represent the magnitudes of the components of the momentum by using the heights of the shaded bars, consistent with the scale used in Figure 2.
- Represent any momentum component that is equal to zero by drawing a distinct line on the zero-momentum line.

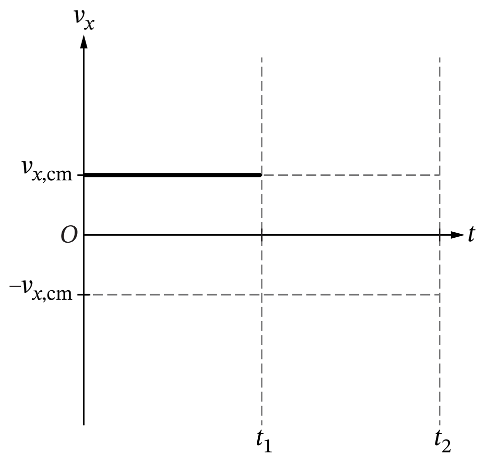
Part B

Derive an expression for x_2 in terms of v_0, θ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

The horizontal component of a velocity vector is represented by v_x. Figure 4 shows the horizontal component $v_{x,\text{cm}}$ of the velocity of the center of mass of the projectile as a function of t during the time interval $0 < t < t_1$.

Figure 4



On Figure 4, **sketch** a line or curve to represent v_x as a function of t for the time interval $t_1 < t < t_2$ for each of the following.

- Piece Q
- Piece R
- The center of mass of the two-piece system

Clearly **label** all lines or curves.

Part D

Consider a case in which the projectile is launched at the same angle and initial speed as initially described. When the projectile breaks into Pieces Q and R, Piece Q falls straight down. In this case, Piece R reaches the ground at $x = x_{\text{new}}$.

Indicate whether x_{new} is greater than, less than, or equal to x_2 by writing one of the following.

- $x_{\text{new}} > x_2$
- $x_{\text{new}} < x_2$
- $x_{\text{new}} = x_2$

Briefly **justify** your answer either by referencing a feature of the representations you drew in part A or C or by using conceptual reasoning beyond algebraic solutions.

Question 1: Version J

1. Two blocks, 1 and 2, slide toward each other on a horizontal surface. Block 1 has mass m and slides in the $+x$-direction with constant speed $2v_0$. Block 2 has mass $6m$ and slides in the $-x$-direction with constant speed v_0, as shown in Figure 1. The blocks then collide and stick together. The collision occurs from time $t = 0$ to $t = t_c$. After the collision, where $t > t_c$, the blocks move together with the same constant speed.

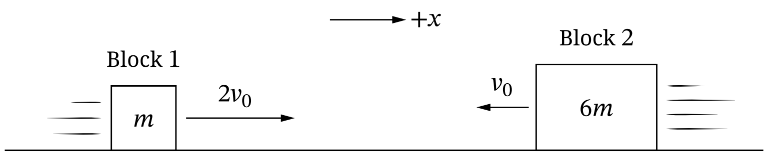


Figure 1

A. The diagrams in Figure 2 can be used to represent the momentum of blocks 1 and 2 before and after the collision. The momentum vector diagram for Block 1 before the collision is shown.

i. **Draw** arrows on the grids to represent the momentum vectors of Block 2 before the collision and the two-block system before and after the collision.

- Arrows should start at the zero-momentum line.
- The length of the arrows should be proportional to the relative magnitudes of the vectors.
- Represent an arrow of zero length by drawing a dot at zero.

Name: AP Physics C: **Mechanics: 2025**

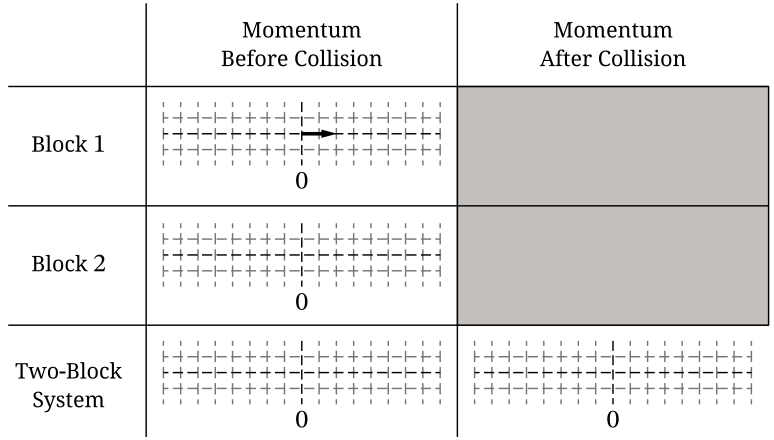


Figure 2

ii. During the time interval $0 \leq t \leq t_c$, a force F is exerted on Block 2 by Block 1 along the x-direction as a function of t that is modeled by $F(t) = F_{max}\sin(At)$, where A is a positive constant and F_{max} is the magnitude of the maximum force exerted on Block 2 by Block 1 during the collision.

Derive an expression for F_{max}. Express your answer in terms of m, v_0, A, t_c, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

B. Consider a new scenario where Block 1 initially slides in the $+x$-direction with a new constant speed v_1 and Block 2 again initially slides in the $-x$-direction with constant speed v_0. The blocks collide and stick together. In this new scenario, the two-block system has constant speed v_0 after the collision.

Derive an expression for v_1 in terms of v_0. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Begin your response to **QUESTION 1** on this page.

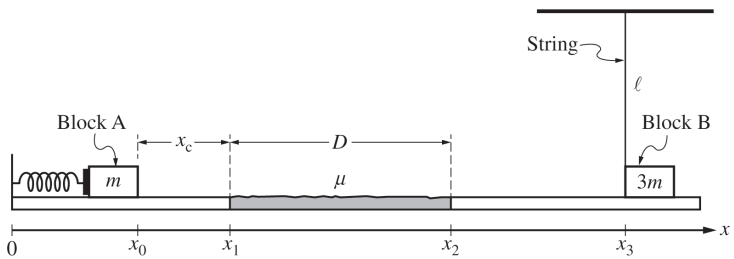
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

Figure 1

1. Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

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Continue your response to **QUESTION 1** on this page.

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position.
- At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction.
- At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1.

ii. **Derive** an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3.

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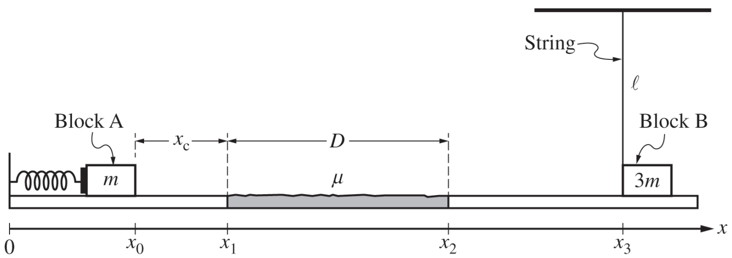
Continue your response to **QUESTION 1** on this page.

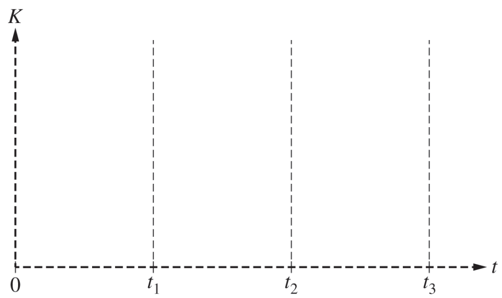


Note: Figure not drawn to scale.

Figure 1

(b)

i. On the following axes, **sketch** a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to time t_3.



ii. Use principles of work and energy to **justify** the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

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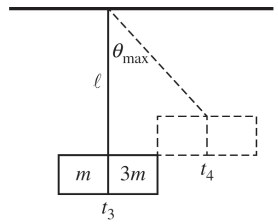
Continue your response to **QUESTION 1** on this page.



Note: Figure not drawn to scale.

Figure 2

After the collision, the two-block system instantaneously comes to rest at time t_4, which occurs when the string makes a small angle θ_{max} with the vertical, as shown in Figure 2. For times $t > t_4$, the system oscillates with frequency f_ℓ. The support holding the string is raised, and the procedure is then repeated using a new string of length 2ℓ.

(c) **Indicate** how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length 2ℓ will compare to the frequency of oscillation f_ℓ from the original procedure.

____ $f_{2\ell} > f_\ell$ ____ $f_{2\ell} < f_\ell$ ____ $f_{2\ell} = f_\ell$

Briefly **justify** your answer.

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PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

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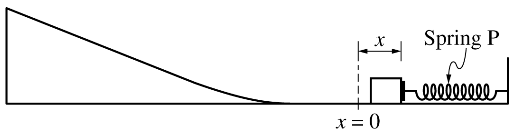

Figure 1

1. A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{max,P}$. Frictional forces between the block and all surfaces are negligible.

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Continue your response to **QUESTION 1** on this page.

A student compares $h_{max,P}$ for Spring P with the maximum height $h_{max,Q}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x that the spring is compressed, as shown in Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0$ N/m, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0$ N/m$^{1/2}$.

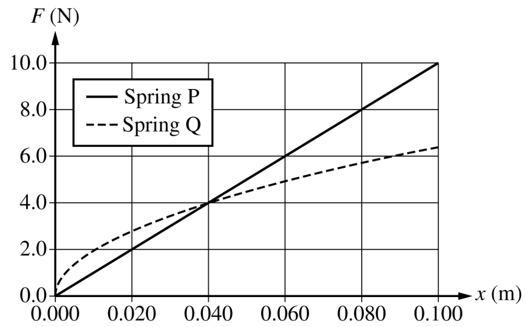
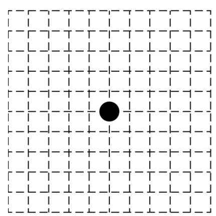


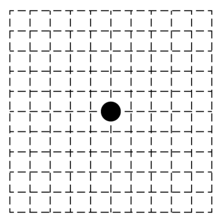
Figure 2

(a) For the experiment, the block is pushed against one of the springs, compressing the spring a distance $x = 0.010$ m. The block is then released from rest. In Trial 1, Spring P is used, and in Trial 2, Spring Q is used. On the following dots, representing the block in Trials 1 and 2, draw and label the forces (not components) that are exerted on the block at the instant the block is released. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. The lengths of the horizontal vectors should represent the relative magnitude of the horizontal forces and the lengths of the vertical vectors should represent the relative magnitude of the vertical forces.

Trial 1
(Spring P)



Trial 2
(Spring Q)



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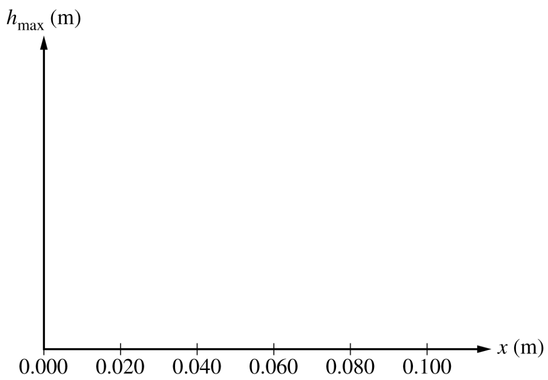
Continue your response to **QUESTION 1** on this page.

(b)

i. What feature(s) of the graph in Figure 2 could be used to estimate the work done on the block by each spring as each spring is compressed?

ii. There is one compression distance x_0 for which the maximum height h_{max} reached by the block is the same regardless of which spring, Spring P or Spring Q, is used. Predict whether the value of x_0 is greater than, less than, or equal to 0.040 m. Use the graph in Figure 2 to justify your answer.

iii. On the axes provided, for both Spring P and Spring Q, sketch a graph of the maximum height h_{max} reached by the block as a function of the distance each spring is compressed for values of x ranging from 0 to 0.100 m. Clearly label the curve for Spring P and Spring Q.



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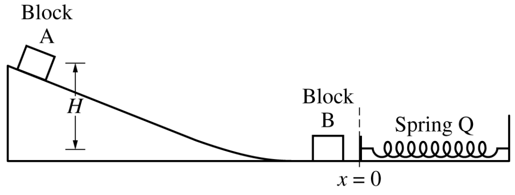

Figure 3

(c) Spring Q is attached to the wall and is at equilibrium, as shown in Figure 3. Block A has a mass of 0.120 kg and Block B has a mass of 0.070 kg. Block A is held at rest at the top of the ramp and Block B is at rest on the horizontal surface. The student releases Block A, and it moves down the ramp and collides with Block B. The change in vertical height of Block A is $H = 0.75$ m. After the collision, the blocks stick together and move to the right, compressing the spring. Frictional forces between the blocks and all surfaces are negligible.

i. Calculate the velocity of the two-block system immediately after the collision between Blocks A and B.

ii. Calculate the maximum compression of the spring.

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Continue your response to **QUESTION 1** on this page.

(d) Spring Q where $F_Q(x) = Cx^{1/2}$ is replaced by a different nonlinear spring, Spring R, and the procedure described in part (c) is repeated. For Spring R, $F_R(x) = Dx^{1/2}$. The maximum compression of Spring R is greater than the maximum compression of Spring Q. Which of the following correctly compares the constants C and D ?

__________ $C < D$ __________ $C > D$ __________ $C = D$

Briefly justify your answer.

GO ON TO THE NEXT PAGE.

PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

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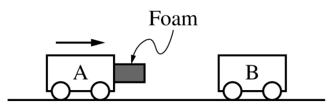

. Scientists have created a new type of lightweight foam and are performing experiments to investigate the properties of the foam. The mass of Cart A is 1000 kg and the mass of Cart B is 2000 kg. A piece of foam with negligible mass is attached to the front of Cart A, as shown. Cart A moves with a constant speed toward Cart B, which is initially at rest. At time $t = 0$ s, the foam connected to Cart A makes contact with Cart B. The foam remains in contact with Cart B for 0.5 s, after which the carts separate and both carts move with constant velocities.

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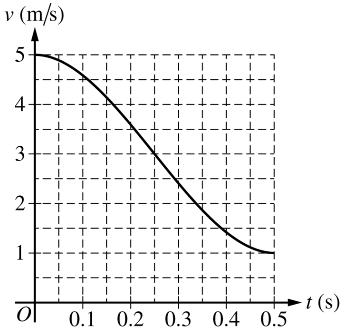
Continue your response to **QUESTION 1** on this page.

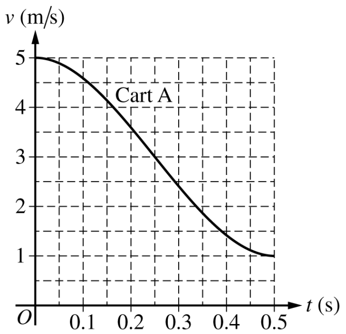


(a) The graph shows the velocity v of Cart A as a function of time t for the time interval when the foam and Cart B are in contact.

i. What feature(s) of the graph could be used to estimate the displacement of Cart A during the collision?

ii. Using the information shown in the graph, determine the speed of Cart B at $t = 0.5$ s.

iii. On the following grid, draw a smooth curve of the velocity of Cart B as a function of time.



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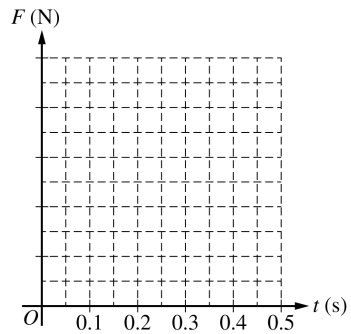
Continue your response to **QUESTION 1** on this page.

(b) For $0 \le t \le 0.50$ s, the velocity v of Cart A can be described by the function $v(t) = 64t^3 - 48t^2 + 5$.

i. Calculate the magnitude of the maximum net force acting on Cart A during this interval.

ii. On the following grid, draw a smooth curve of the magnitude of the force acting on Cart A as a function of time. Clearly indicate the value of the maximum force on the vertical axis.



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Continue your response to **QUESTION 1** on this page.

The foam is removed from the front of Cart A and the experiment is repeated. The carts collide, with both Cart A and Cart B having the same initial and final velocities as in the original collision. The time intervals during which the carts are in contact are different in the collision with the foam and the collision without the foam. In the collision without the foam, Cart A is in contact with Cart B for a shorter duration than in the original collision, when the foam was present.

For the original collision when the foam is present, the magnitude of the average net force exerted on Cart B is F_1. For the collision without the foam, the magnitude of the average net force exerted on Cart B is F_2.

(c) What is the relationship between the magnitude of the average net force F_1 exerted on Cart B for the collision with the foam and the magnitude of the average net force F_2 exerted on Cart B for the collision without the foam?

_____ $F_1 > F_2$ _____ $F_1 < F_2$ _____ $F_1 = F_2$

Justify your answer.

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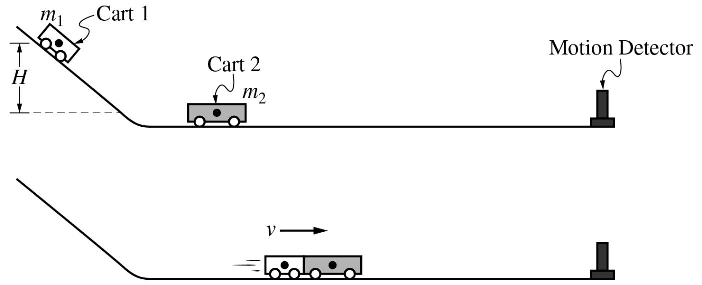
Begin your response to **QUESTION 2** on this page.



. Cart 1 of mass m_1 is held at rest above the bottom of an incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and smoothly transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance of H, as shown. At time t_C, Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(a) During the collision, is the impulse on Cart 1 from Cart 2 greater than, less than, or equal to the magnitude of the impulse on Cart 2 from Cart 1 ?

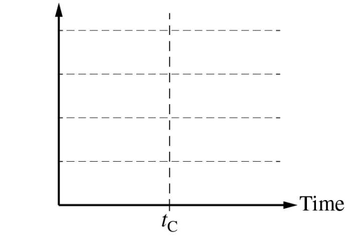
___ Greater than ___ Less than ___ Equal to

Justify your answer.

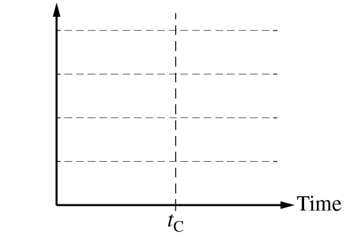
GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b) On the following axes, draw graphs of the magnitude of the momentum of each cart as a function of time t, before and after t_C. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.



Cart 1



Cart 2

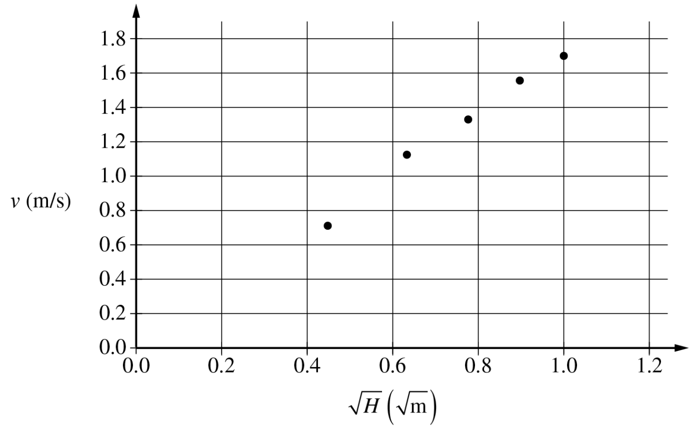
(c) Show that the velocity v of the two-cart system after the collision is given by the equation

$$v = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}.$$

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) A group of students use the setup to perform an experiment. They measure the mass of Cart 1 to be $m_1 = 0.250$ kg. The mass of Cart 2 is unknown. The students perform several trials and in each trial, Cart 1 is released from a different height H and the final velocity of the two-cart system is measured. The students graph v as a function of $\sqrt{H}$, as shown below.



i. Draw a line that represents the best fit to the data points shown.

ii. Use the best-fit line to calculate the mass of Cart 2.

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Continue your response to **QUESTION 2** on this page.

(e) After the experiment, the students use a balance to measure the mass of Cart 2 and find it to be less than what was determined in part (d). To explain this discrepancy, one of the students proposes that the mass of Cart 1 was incorrectly measured at the beginning of the experiment. The students measure the mass of Cart 1 again and record a new value, m_1'.

Should the students expect that m_1' will be greater than 0.250 kg, less than 0.250 kg, or equal to 0.250 kg ?

_____ $m_1' > 0.250$ kg   _____ $m_1' < 0.250$ kg   _____ $m_1' = 0.250$ kg

Justify your answer.

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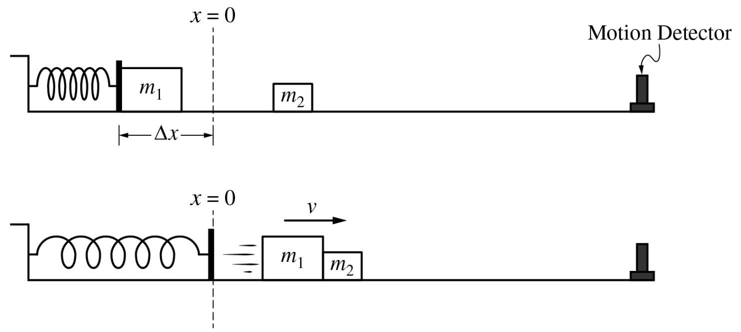


. Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v, as shown. Frictional effects are negligible.

(a) The impulse on Block 1 from the spring during the time interval $0 < t < t_C$ is J_S. The impulse on Block 1 from Block 2 during the collision is J_2. Which of the following expressions correctly compares the magnitudes of J_S and J_2 ?

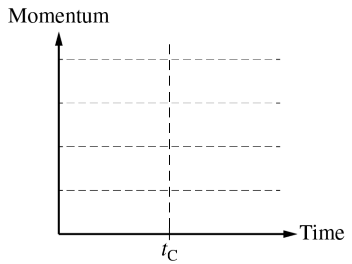
___ $J_S > J_2$ ___ $J_S < J_2$ ___ $J_S = J_2$

Justify your answer.

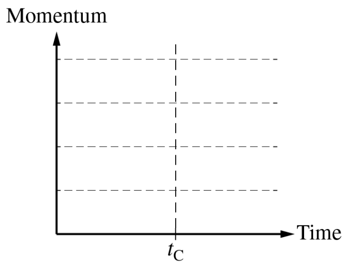
GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b) On the following axes, draw graphs of the magnitude of the momentum of each block as a function of time, before and after t_C. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.



Block 1



Block 2

(c) Show that the velocity v of the two-block system after the collision is given by the equation $v = \dfrac{\sqrt{km_1}}{m_1 + m_2}\Delta x$.

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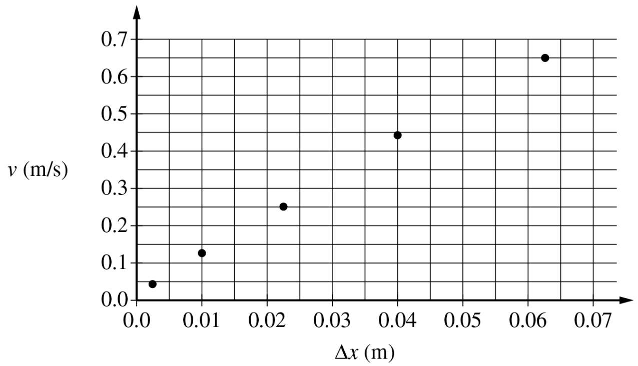
Continue your response to **QUESTION 2** on this page.

(d) A group of students use the setup to perform an experiment. They measure the mass of Block 1 to be $m_1 = 0.500$ kg, and the spring constant k of the spring to be 150 N/m. The mass of Block 2 is unknown. They perform several trials and in each trial the spring is compressed a different distance Δx and the final velocity v of the two-block system is measured. They graph v as a function of Δx, as shown below.



i. Draw a line that represents the best fit to the data points shown.

ii. Use the best-fit line to calculate the mass of Block 2.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(e) After the experiment, the students use a balance to measure the mass of Block 2 and find it to be greater than what was determined in part (d). To explain this discrepancy, one of the students proposes that the spring constant was incorrectly measured at the beginning of the experiment. The students measure the spring constant again and record a new value, k'.

Should the students expect that k' be greater than 150 N / m, less than 150 N / m, or equal to 150 N / m ?

_____ $k' > 150\text{ N/m}$ _____ $k' < 150\text{ N/m}$ _____ $k' = 150\text{ N/m}$

Justify your answer.

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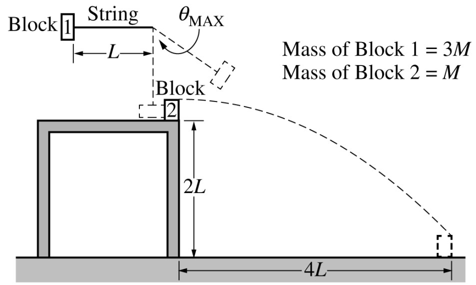
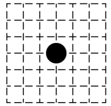


Note: Figure not drawn to scale.

2. A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(a) Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

(b) On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



(c) Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

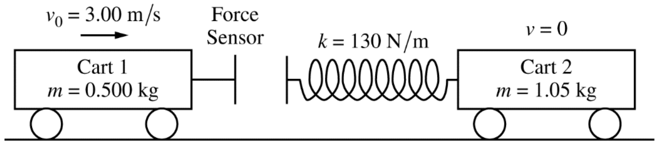
For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

(d) Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

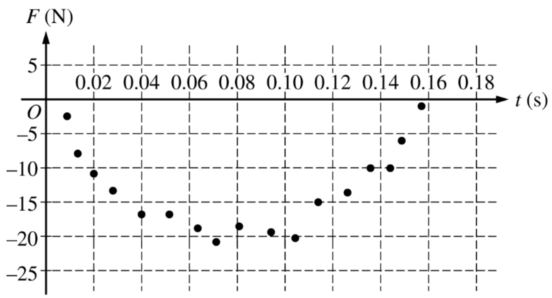
(e) Calculate the speed of block 2 as it leaves the table.

(f) Calculate the speed of block 1 just after it collides with block 2.

(g) Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.



2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00$ m/s toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130$ N/m. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200\ \text{N/s}^2\right)t^2 - (500\ \text{N/s})t$.



(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)

i. Calculate the speed of cart 1 after the collision.

ii. In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)

i. Calculate the speed of cart 2 after the collision.

ii. Show that the collision between the two carts is elastic.

(d)

i. Calculate the speed of the center of mass of the two-cart–spring system.

ii. Calculate the maximum elastic potential energy stored in the spring.

Mech.2.

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(a) On the dots below, which represent the blocks of mass $2M$ and $3M$, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

Block of Mass $2M$ Block of Mass $3M$

The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance D, as shown above. Express your answers to parts (b), (c), and (d) in terms of M, B, v_0, and physical constants, as appropriate.

(b) Derive an expression for the speed of the blocks immediately after the collision.

(c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.

(d) Derive an expression for the maximum distance D that the spring is compressed.

(e)

i. In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

____ Left ____ Right ____ The net force on the block of mass $2M$ is zero.

Justify your answer.

ii. Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

____ The magnitude of the net force is greater on the block of mass $2M$.

____ The magnitude of the net force is greater on the block of mass $3M$.

____ The magnitude of the net force on each block has the same nonzero value.

____ The magnitude of the net force on each block is zero.

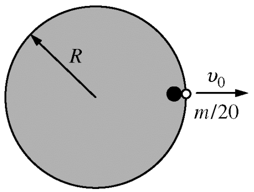
Justify your answer.

(f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

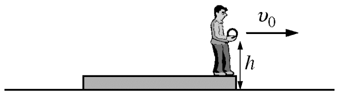
____ Yes ____ No

Justify your answer.

AP Physics C: **Mechanics: 2014**



Top View



Side View

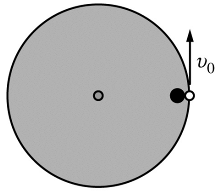
Mech. 3.

A large circular disk of mass m and radius R is initially stationary on a horizontal icy surface. A person of mass $m/2$ stands on the edge of the disk. Without slipping on the disk, the person throws a large stone of mass $m/20$ horizontally at initial speed v_0 from a height h above the ice in a radial direction, as shown in the figures above. The coefficient of friction between the disk and the ice is μ. All velocities are measured relative to the ground. The time it takes to throw the stone is negligible. Express all algebraic answers in terms of m, R, v_0, h, μ, and fundamental constants, as appropriate.

(a) Derive an expression for the length of time it will take the stone to strike the ice.

(b) Assuming that the disk is free to slide on the ice, derive an expression for the speed of the disk and person immediately after the stone is thrown.

(c) Derive an expression for the time it will take the disk to stop sliding.



Top View

The person now stands on a similar disk of mass m and radius R that has a fixed pole through its center so that it can only rotate on the ice. The person throws the same stone horizontally in a tangential direction at initial speed v_0, as shown in the figure above. The rotational inertia of the disk is $mR^2/2$.

(d) Derive an expression for the angular speed ω of the disk immediately after the stone is thrown.

(e) The person now stands on the disk at rest $R/2$ from the center of the disk. The person now throws the stone horizontally with a speed v_0 in the same direction as in part (d). Is the angular speed of the disk immediately after throwing the stone from this new position greater than, less than, or equal to the angular speed found in part (d) ?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

STOP

END OF EXAM

4.4 Elastic and Inelastic Collisions

Name: ____________ Class: ________ Date: __________ **Total: 13 marks**

KEY WORDS inelastic 非弹性 • elastic collision 弹性碰撞 • inelastic collision 非弹性碰撞 • deformation 形变 • perfectly inelastic collision 完全非弹性碰撞

Objective

Build the skills to answer exam questions on **elastic and inelastic collisions**.

You must be able to:

- distinguish an **elastic collision 弹性碰撞** (kinetic energy conserved) from an **inelastic 非弹性** one
- state that momentum is conserved in **both** kinds of collision
- identify a **perfectly inelastic** collision (objects stick together)
- reason about where the lost kinetic energy goes

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Momentum always, kinetic energy sometimes

- **Elastic**: momentum **and** kinetic energy are both conserved.
- **Inelastic**: momentum is conserved, but some kinetic energy is lost (to heat, sound, deformation).

■ Perfectly inelastic

The objects stick together and move as one. This loses the most kinetic energy of any collision that conserves momentum.

■ Checking the type

Compute the total kinetic energy before and after. If it is the same, the collision is elastic; if less, it is inelastic.

2 Practice

Now apply the methods above.

2.1 State what is conserved in **every** collision. [1]

2.2 State the extra quantity conserved in an elastic collision. [1]

2.3 In a perfectly inelastic collision, what do the objects do? [1]

2.4 In an inelastic collision, where does the lost kinetic energy go? [1]

3 Exam-style questions

3.1 In an inelastic collision, which is conserved? [1]

- **A** kinetic energy only
- **B** momentum only
- **C** both momentum and kinetic energy
- **D** neither

3.2 Two balls collide and stick together. This collision is [1]

- **A** elastic
- **B** perfectly inelastic
- **C** frictionless
- **D** impossible

3.3 Before a collision the total KE is 50 J; after, it is 32 J.

(a) State whether the collision is elastic or inelastic. [1]

(b) State how much kinetic energy was lost, and where it went. [2]

3.4 A 2 kg ball at 6 m s^{-1} hits a stationary 2 kg ball and they stick.

(a) Find the speed after (momentum conservation). [2]

(b) Show that kinetic energy is not conserved. [2]

Solutions

2.1 Momentum.

2.2 Kinetic energy.

2.3 They stick together and move as one.

2.4 Into heat, sound, and deformation.

3.1 B — momentum only (kinetic energy is lost).

3.2 B — sticking together is perfectly inelastic.

3.3 (a) Inelastic (KE decreased). (b) $50-32 = 18$ J lost, converted to heat/sound/deformation.

3.4 (a) $2(6) = 12 = 4v \Rightarrow v = 3$ m s^{-1}. (b) Before: $\frac{1}{2}(2)(6)^2 = 36$ J; after: $\frac{1}{2}(4)(3)^2 = 18$ J — not conserved.

Question 1: Version J

1. Two blocks, 1 and 2, slide toward each other on a horizontal surface. Block 1 has mass m and slides in the $+x$-direction with constant speed $2v_0$. Block 2 has mass $6m$ and slides in the $-x$-direction with constant speed v_0, as shown in Figure 1. The blocks then collide and stick together. The collision occurs from time $t = 0$ to $t = t_c$. After the collision, where $t > t_c$, the blocks move together with the same constant speed.

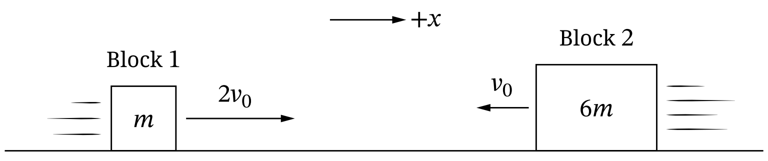


Figure 1

A. The diagrams in Figure 2 can be used to represent the momentum of blocks 1 and 2 before and after the collision. The momentum vector diagram for Block 1 before the collision is shown.

i. **Draw** arrows on the grids to represent the momentum vectors of Block 2 before the collision and the two-block system before and after the collision.

- Arrows should start at the zero-momentum line.
- The length of the arrows should be proportional to the relative magnitudes of the vectors.
- Represent an arrow of zero length by drawing a dot at zero.

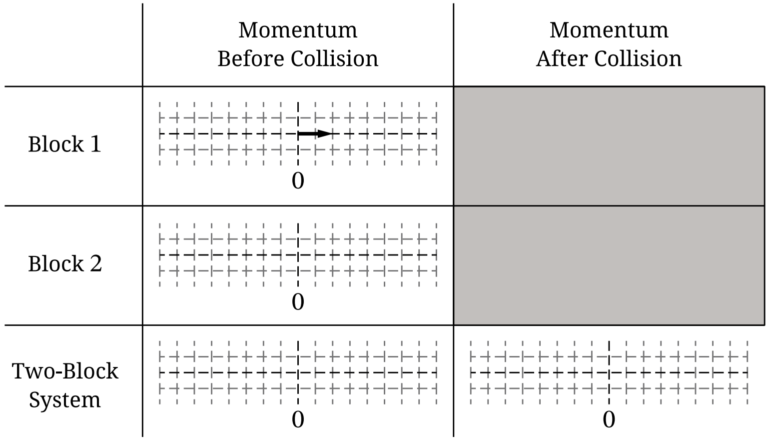


Figure 2

ii. During the time interval $0 \le t \le t_c$, a force F is exerted on Block 2 by Block 1 along the x-direction as a function of t that is modeled by $F(t) = F_{max} \sin(At)$, where A is a positive constant and F_{max} is the magnitude of the maximum force exerted on Block 2 by Block 1 during the collision.

Derive an expression for F_{max}. Express your answer in terms of m, v_0, A, t_c, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

B. Consider a new scenario where Block 1 initially slides in the $+x$-direction with a new constant speed v_1 and Block 2 again initially slides in the $-x$-direction with constant speed v_0. The blocks collide and stick together. In this new scenario, the two-block system has constant speed v_0 after the collision.

Derive an expression for v_1 in terms of v_0. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Begin your response to **QUESTION 1** on this page.

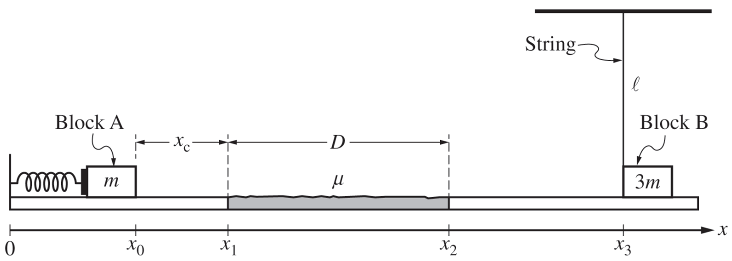
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



<u>Note:</u> Figure not drawn to scale.

Figure 1

1. Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position.
- At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction.
- At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1.

ii. **Derive** an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3.

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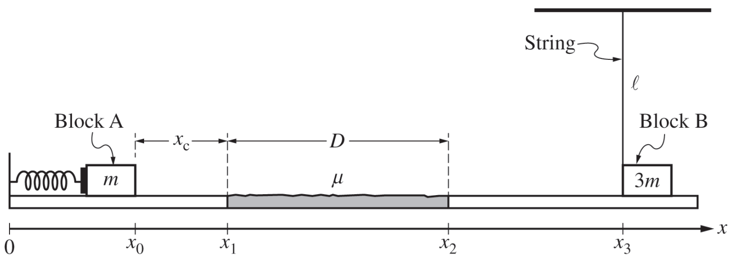
Continue your response to **QUESTION 1** on this page.

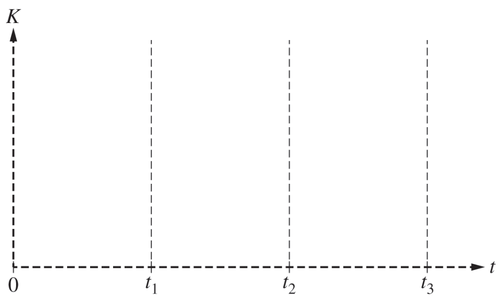


Note: Figure not drawn to scale.

Figure 1

(b)

i. On the following axes, **sketch** a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to time t_3.



ii. Use principles of work and energy to **justify** the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

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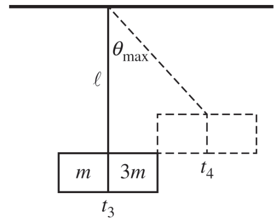
Continue your response to **QUESTION 1** on this page.



Note: Figure not drawn to scale.

Figure 2

After the collision, the two-block system instantaneously comes to rest at time t_4, which occurs when the string makes a small angle θ_{max} with the vertical, as shown in Figure 2. For times $t > t_4$, the system oscillates with frequency f_ℓ. The support holding the string is raised, and the procedure is then repeated using a new string of length 2ℓ.

(c) **Indicate** how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length 2ℓ will compare to the frequency of oscillation f_ℓ from the original procedure.

____ $f_{2\ell} > f_\ell$ ____ $f_{2\ell} < f_\ell$ ____ $f_{2\ell} = f_\ell$

Briefly **justify** your answer.

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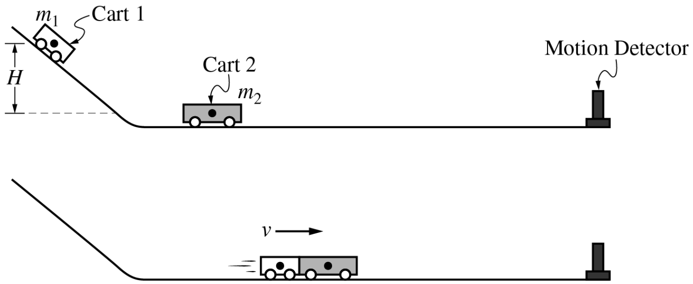
Begin your response to **QUESTION 2** on this page.



. Cart 1 of mass m_1 is held at rest above the bottom of an incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and smoothly transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance of H, as shown. At time t_C, Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(a) During the collision, is the impulse on Cart 1 from Cart 2 greater than, less than, or equal to the magnitude of the impulse on Cart 2 from Cart 1 ?

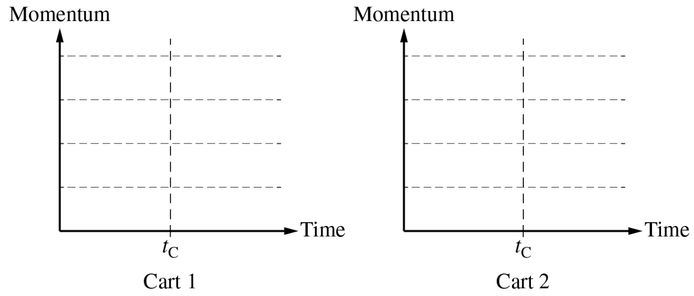
___ Greater than ___ Less than ___ Equal to

Justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b) On the following axes, draw graphs of the magnitude of the momentum of each cart as a function of time t, before and after t_C. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.



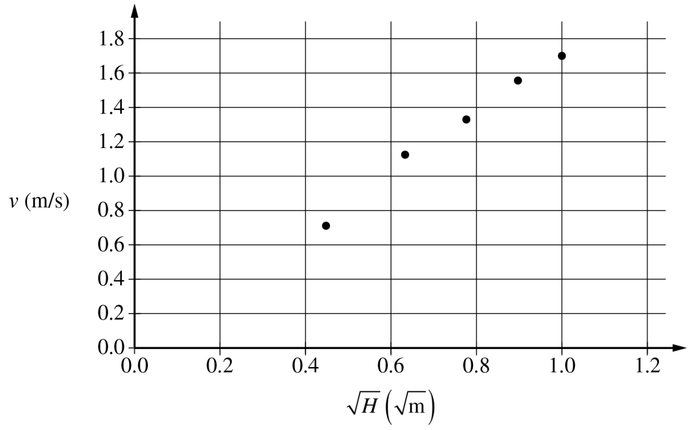
(c) Show that the velocity v of the two-cart system after the collision is given by the equation

$$v = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}.$$

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) A group of students use the setup to perform an experiment. They measure the mass of Cart 1 to be $m_1 = 0.250$ kg. The mass of Cart 2 is unknown. The students perform several trials and in each trial, Cart 1 is released from a different height H and the final velocity of the two-cart system is measured. The students graph v as a function of $\sqrt{H}$, as shown below.



i. Draw a line that represents the best fit to the data points shown.

ii. Use the best-fit line to calculate the mass of Cart 2.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(e) After the experiment, the students use a balance to measure the mass of Cart 2 and find it to be less than what was determined in part (d). To explain this discrepancy, one of the students proposes that the mass of Cart 1 was incorrectly measured at the beginning of the experiment. The students measure the mass of Cart 1 again and record a new value, m_1'.

Should the students expect that m_1' will be greater than 0.250 kg, less than 0.250 kg, or equal to 0.250 kg ?

_____ $m_1' > 0.250$ kg   _____ $m_1' < 0.250$ kg   _____ $m_1' = 0.250$ kg

Justify your answer.

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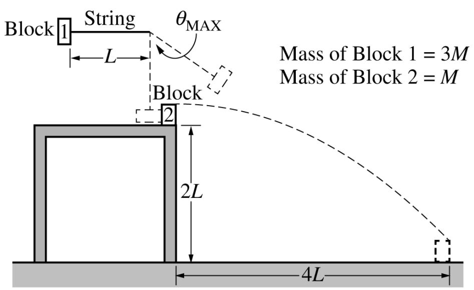


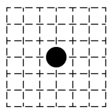
Mass of Block 1 = $3M$
Mass of Block 2 = M

Note: Figure not drawn to scale.

2. A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(a) Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

(b) On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



(c) Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

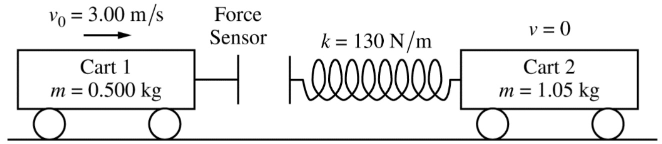
For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

(d) Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

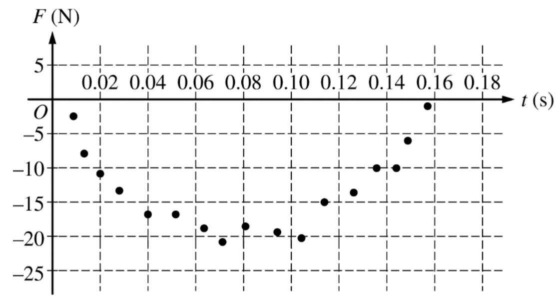
(e) Calculate the speed of block 2 as it leaves the table.

(f) Calculate the speed of block 1 just after it collides with block 2.

(g) Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.



2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00$ m/s toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130$ N/m. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200\ \text{N/s}^2\right)t^2 - (500\ \text{N/s})t$.



(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)

i. Calculate the speed of cart 1 after the collision.

ii. In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)

i. Calculate the speed of cart 2 after the collision.

ii. Show that the collision between the two carts is elastic.

(d)

i. Calculate the speed of the center of mass of the two-cart–spring system.

ii. Calculate the maximum elastic potential energy stored in the spring.

Mech.2.

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(a) On the dots below, which represent the blocks of mass $2M$ and $3M$, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

Block of Mass $2M$ Block of Mass $3M$

The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance D, as shown above. Express your answers to parts (b), (c), and (d) in terms of M, B, v_0, and physical constants, as appropriate.

(b) Derive an expression for the speed of the blocks immediately after the collision.

(c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.

(d) Derive an expression for the maximum distance D that the spring is compressed.

(e)

i. In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

____ Left ____ Right ____ The net force on the block of mass $2M$ is zero.

Justify your answer.

ii. Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

____ The magnitude of the net force is greater on the block of mass $2M$.

____ The magnitude of the net force is greater on the block of mass $3M$.

____ The magnitude of the net force on each block has the same nonzero value.

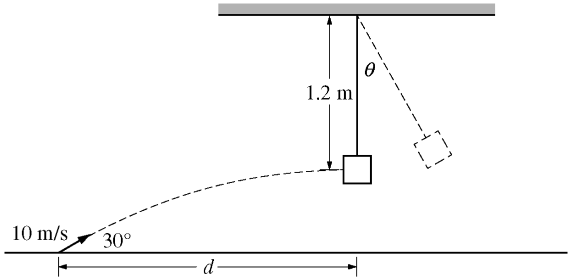
____ The magnitude of the net force on each block is zero.

Justify your answer.

(f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

____ Yes ____ No

Justify your answer.



Mech.2.

A small dart of mass 0.020 kg is launched at an angle of 30° above the horizontal with an initial speed of 10 m/s. At the moment it reaches the highest point in its path and is moving horizontally, it collides with and sticks to a wooden block of mass 0.10 kg that is suspended at the end of a massless string. The center of mass of the block is 1.2 m below the pivot point of the string. The block and dart then swing up until the string makes an angle θ with the vertical, as shown above. Air resistance is negligible.

(a) Determine the speed of the dart just before it strikes the block.

(b) Calculate the horizontal distance d between the launching point of the dart and a point on the floor directly below the block.

(c) Calculate the speed of the block just after the dart strikes.

(d) Calculate the angle θ through which the dart and block on the string will rise before coming momentarily to rest.

(e) The block then continues to swing as a simple pendulum. Calculate the time between when the dart collides with the block and when the block first returns to its original position.

(f) In a second experiment, a dart with more mass is launched at the same speed and angle. The dart collides with and sticks to the same wooden block.

i. Would the angle θ that the dart and block swing to increase, decrease, or stay the same?

_____ Increase _____ Decrease _____ Stay the same

Justify your answer.

ii. Would the period of oscillation after the collision increase, decrease, or stay the same?

_____ Increase _____ Decrease _____ Stay the same

Justify your answer.