Week 4

AP Physics C: Mechanics

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

AP Physics C: Mechanics Name: ____________ Score: ______

1. A 1 kg mass is at $x = 0$ and a 3 kg mass is at $x = 4$ m. Where is the center of mass (in m)?

   ____________ m

2. Which of these is a **field** force (acts without contact)?

   ☐ friction

   ☐ gravity

   ☐ the normal force

   ☐ tension in a rope

3. A truck (3000 kg) hits a car (1000 kg). During the crash, the force the truck exerts on the car compared with the force the car exerts on the truck is...

   ☐ much larger (the truck is heavier)

   ☐ exactly equal and opposite

   ☐ much smaller

   ☐ zero

4. An object moving at constant velocity must have a nonzero net force pushing it.

   ☐ True ☐ False

5. A net force of 20 N acts on a 4 kg cart. What is its acceleration (in $\frac{\mathrm{m}}{\mathrm{s}^2}$)?

   ____________ m/s^2

6. An astronaut has a mass of 70 kg. What is her weight on Earth (in N, use $g = 9.8$)?

   ____________ N

7. A box has normal force $N = 50$ N and $\mu_s = 0.6$. What is the maximum static friction (in N)?

   ____________ N

8. A 6 N force stretches a spring by 0.30 m. What is its spring constant (in N/m)?

   ____________ N/m

9. A hidden explosion inside a floating system (no outside forces) can shift its center of mass.

   ☐ True ☐ False

10. The two forces of an action-reaction pair cancel each other out.

    ☐ True ☐ False

AP Physics C: Mechanics

1. **3 m**

   $x_{cm} = \dfrac{1(0) + 3(4)}{1 + 3} = \dfrac{12}{4} = 3$ m – close to the heavier mass.

2. **gravity**

   Gravity is a field force acting at a distance; friction, normal, and tension all require contact.

3. **exactly equal and opposite**

   Third-law forces are always equal and opposite, regardless of mass. (The car accelerates more because it has less mass, not more force.)

4. **False**

   Constant velocity means zero net force. A force is needed only to *change* velocity, not to maintain it.

5. **5 m/s^2**

   $a = \dfrac{F_{net}}{m} = \dfrac{20}{4} = 5\ \frac{\mathrm{m}}{\mathrm{s}^2}$.

6. **686 N**

   $F_g = mg = 70 \times 9.8 = 686$ N.

7. **30 N**

   $f_s^{max} = \mu_s N = 0.6 \times 50 = 30$ N.

8. **20 N/m**

   $k = F/x = 6/0.30 = 20\ \frac{\mathrm{N}}{\mathrm{m}}$.

9. **False**

   Explosion forces are *internal*, so they cannot move the center of mass. Only a net external force can.

10. **False**

    They act on *different* objects, so they never cancel. Cancellation needs both forces on the same object.

2.1 Systems and Center of Mass

Name: ______ Class: ______ Date: ______ **Total: 13 marks**

KEY WORDS center of mass 质心 • system 系统 • force 力 • kinetic 动摩擦

Objective

Build the skills to answer exam questions on **systems and the center of mass**.

You must be able to:

- treat a group of objects as a single **system** 系统
- find the **center of mass** 质心 of point masses: $x_{cm} = \dfrac{\sum m_i x_i}{\sum m_i}$
- explain that the center of mass moves as if all mass and the net external force acted there
- use symmetry to locate the center of mass of a uniform object

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Center of mass of point masses

Weight each position by its mass and divide by the total mass:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}.$$

For 2 kg at $x = 0$ and 3 kg at $x = 5$ m:

$$x_{cm} = \frac{2(0) + 3(5)}{2 + 3} = \frac{15}{5} = 3 \text{ m}.$$

It sits closer to the heavier mass.

The center of mass obeys Newton's second law

The center of mass of a system accelerates as if the total mass were there and the **net external force** acted on it. Internal forces cancel and never move it.

Symmetry

A uniform rod's center of mass is at its midpoint; a uniform disc's is at its centre.

2 Practice

Now apply the methods above.

2.1 Two masses, 1 kg at $x = 0$ and 1 kg at $x = 4$ m. Find the center of mass. [2]

2.2 Masses 2 kg at $x = 0$ and 6 kg at $x = 8$ m. Find the center of mass. [2]

2.3 State where the center of mass of a uniform metre ruler is located. [1]

2.4 Can internal forces alone move a system's center of mass? Answer yes or no. [1]

3 Exam-style questions

3.1 The center of mass of a system accelerates in response to the [1]

- **A** internal forces
- **B** net external force
- **C** largest single mass
- **D** total kinetic energy

3.2 Two equal masses sit at $x = 2$ m and $x = 6$ m. Their center of mass is at [1]

- **A** 2 m
- **B** 4 m
- **C** 6 m
- **D** 8 m

3.3 Point masses: 4 kg at $x = 1$ m and 2 kg at $x = 7$ m.

(a) Find the center of mass. [2]

(b) State whether it is closer to the 4 kg or the 2 kg mass, and why. [1]

3.4 An astronaut floating in space throws a tool.

(a) State what happens to the center of mass of (astronaut + tool). [1]

(b) Explain your answer using external forces. [1]

Solutions

2.1 $x_{cm} = \dfrac{1(0)+1(4)}{2} = 2$ m.

2.2 $x_{cm} = \dfrac{2(0)+6(8)}{8} = \dfrac{48}{8} = 6$ m.

2.3 At the 50 cm mark (its midpoint).

2.4 No — only a net external force can move the center of mass.

3.1 B — the net external force drives the center of mass.

3.2 B — equal masses give the midpoint, 4 m.

3.3 (a) $x_{cm} = \dfrac{4(1)+2(7)}{6} = \dfrac{18}{6} = 3$ m. (b) Closer to the 4 kg mass, because it is weighted more heavily.

3.4 (a) It keeps moving as before (stays at rest if it was at rest). (b) Throwing is an internal force, and internal forces cannot move the center of mass.

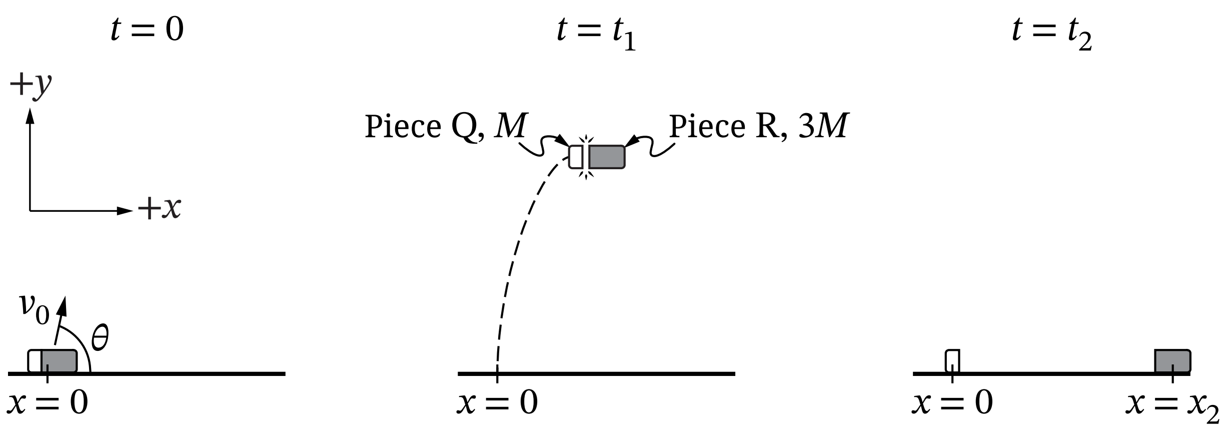
Question 2: Version J

2. A projectile of total mass $4M$ is launched from the ground at position $x = 0$ and time $t = 0$. The projectile is launched with an initial speed v_0 at an angle θ above the horizontal. When the projectile is at the highest point in its trajectory, it breaks into Pieces Q and R of masses M and $3M$, respectively.

The motion of the projectile is described for the following times.

- At $t = t_1$, immediately after the projectile breaks apart, the two pieces are moving away from each other horizontally.
- At $t = t_2$, Piece Q reaches the ground at $x = 0$ and Piece R reaches the ground at $x = x_2$, as shown in Figure 1.

Figure 1



Note: Figure not drawn to scale.

Part A

The horizontal and vertical components of a momentum vector are represented by p_x and p_y, respectively. The shaded bars in Figure 2 represent p_x and p_y of the projectile immediately after $t = 0$.

Figure 2

$t = 0$

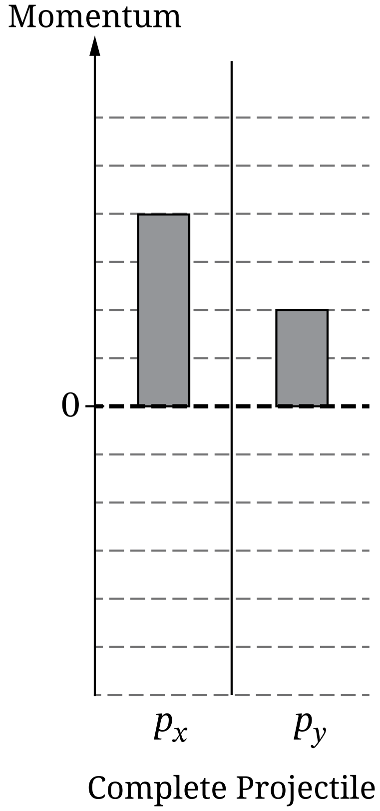
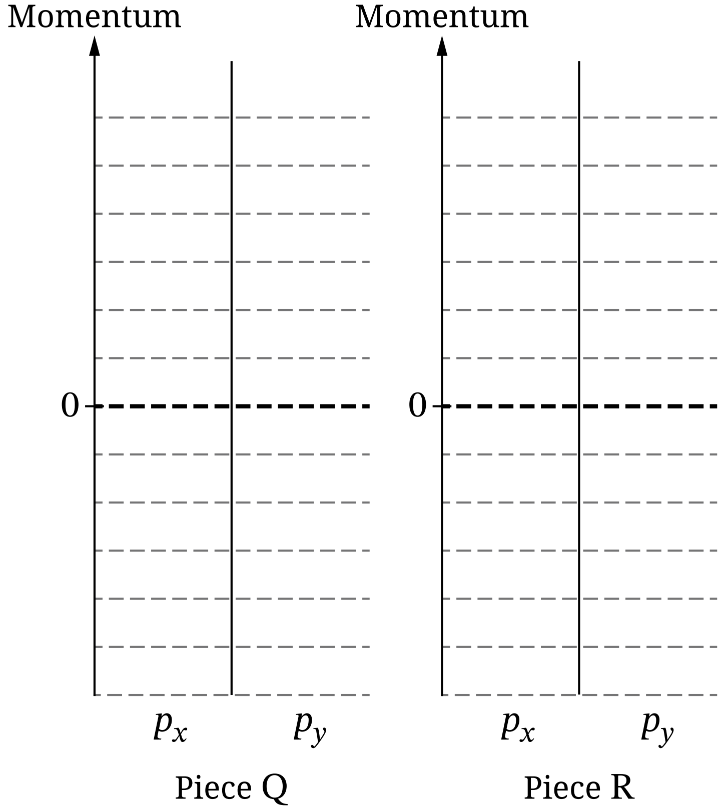


Figure 3

$t = t_1$



On Figure 3, **draw** shaded bars to represent p_x and p_y of Pieces Q and R at $t = t_1$.

- Start the shaded bars at the dashed line that represents zero momentum.
- Represent the magnitudes of the components of the momentum by using the heights of the shaded bars, consistent with the scale used in Figure 2.
- Represent any momentum component that is equal to zero by drawing a distinct line on the zero-momentum line.

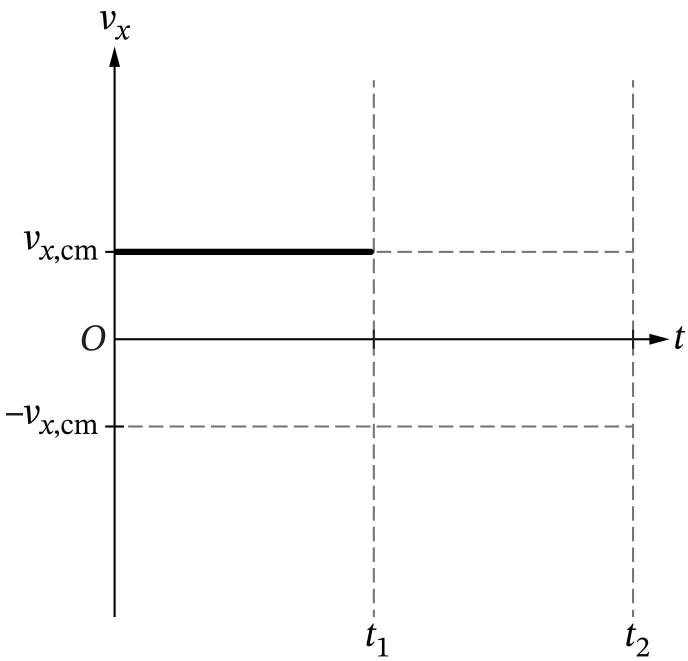
Part B

Derive an expression for x_2 in terms of v_0, θ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

The horizontal component of a velocity vector is represented by v_x. Figure 4 shows the horizontal component $v_{x,\mathrm{cm}}$ of the velocity of the center of mass of the projectile as a function of t during the time interval $0 < t < t_1$.

Figure 4



On Figure 4, **sketch** a line or curve to represent v_x as a function of t for the time interval $t_1 < t < t_2$ for each of the following.

- Piece Q
- Piece R
- The center of mass of the two-piece system

Clearly **label** all lines or curves.

Part D

Consider a case in which the projectile is launched at the same angle and initial speed as initially described. When the projectile breaks into Pieces Q and R, Piece Q falls straight down. In this case, Piece R reaches the ground at $x = x_{\mathrm{new}}$.

Indicate whether x_{new} is greater than, less than, or equal to x_2 by writing one of the following.

- $x_{\mathrm{new}} > x_2$
- $x_{\mathrm{new}} < x_2$
- $x_{\mathrm{new}} = x_2$

Briefly **justify** your answer either by referencing a feature of the representations you drew in part A or C or by using conceptual reasoning beyond algebraic solutions.

Begin your response to **QUESTION 3** on this page.

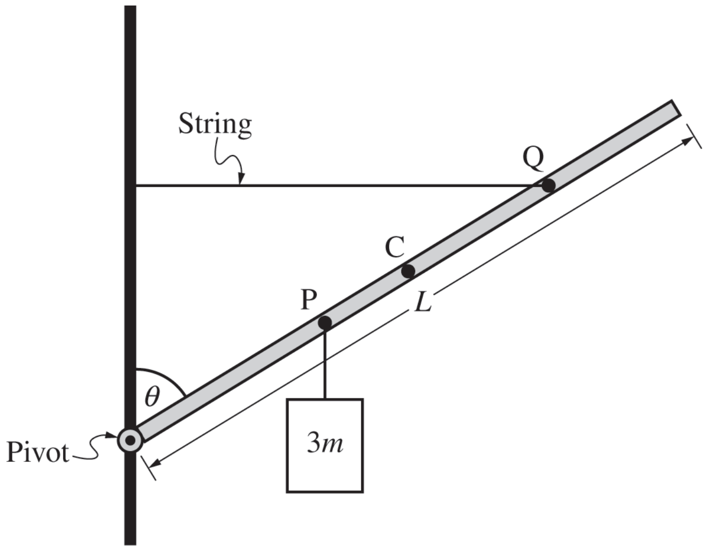
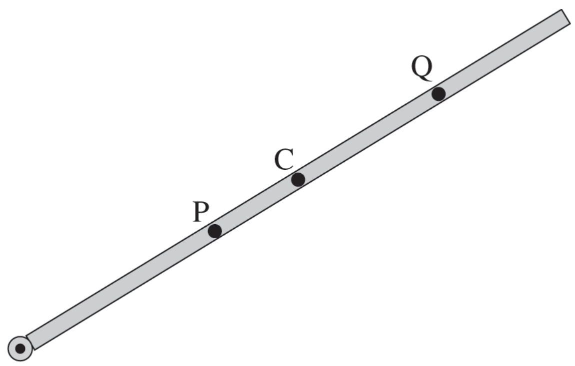


Figure 1

<u>Note</u>: Figure not drawn to scale.

3. A uniform rod of length L and mass m is attached to a pivot on a vertical pole, as shown in Figure 1. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

(a) On the following representation of the rod, **draw** and **label** the forces (not components) that are exerted on the rod. Each force must be represented by a distinct arrow that starts on and points away from the point at which the force is exerted on the rod.



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(b) In Figure 1, Point P is located $\frac{3}{8}L$ from the pivot and Point Q is located $\frac{6}{8}L$ from the pivot. **Derive** an equation for the tension F_T in the horizontal string in terms of L, m, θ, and physical constants, as appropriate.

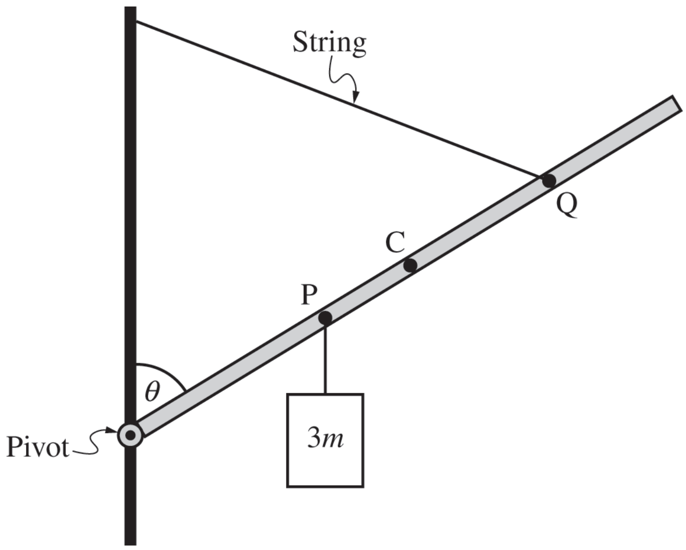

Figure 2

<u>Note:</u> Figure not drawn to scale.

(c) The original string is replaced with a longer string that connects Point Q to a higher location on the vertical pole, as shown in Figure 2. The angle θ remains the same. How does the new tension $F_{T,\text{ new}}$ compare with the original tension F_T from part (b) ? **Justify** your reasoning.

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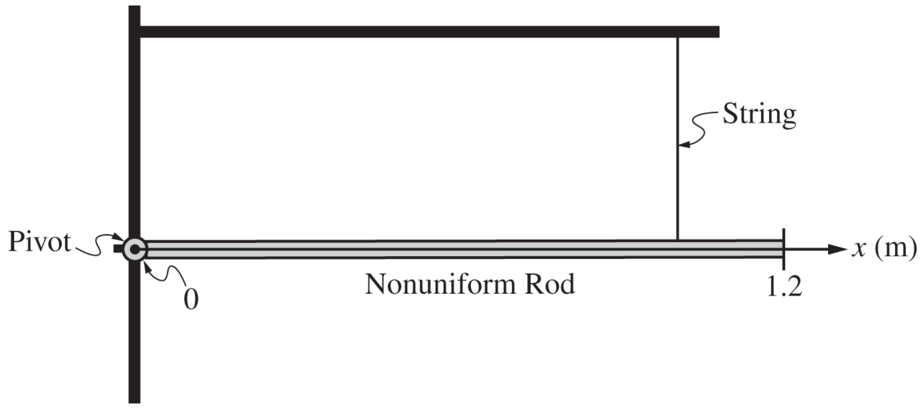

Figure 3

Note: Figure not drawn to scale.

(d) A nonuniform rod is now attached to the pivot, as shown in Figure 3. There is negligible friction between the nonuniform rod and the pivot. The rod has a length of 1.2 m and a linear mass density $\lambda(x) = A + Bx$, where x is the distance from the pivot, $A = 6.0$ kg/m, and $B = 10.0$ kg/m^2.

i. **Calculate** the mass of the rod.

ii. **Calculate** the rotational inertia of the rod about the pivot.

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END OF EXAM

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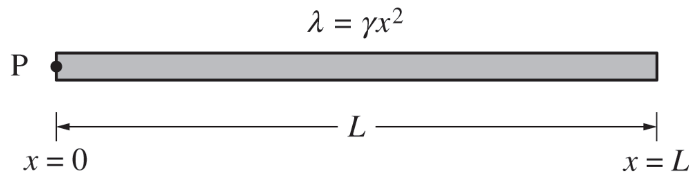

3. A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(a) Using integral calculus, show that the rotational inertia I of the rod about an axis perpendicular to the page and through point P is $\dfrac{3}{5}ML^2$.

(b) Determine the horizontal location of the center of mass of the rod relative to point P. Express your answer in terms of L.

(c) For an axis perpendicular to the page, is the value of the rotational inertia of the rod around point P greater than, less than, or equal to the value of the rotational inertia of the rod around the rod's center of mass?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

The rod is released from rest in the position shown, and the rod begins to rotate about a horizontal axis perpendicular to the page and through point P.

(d) On the axes below, sketch graphs of the magnitude of the net torque τ on the rod and the angular speed ω of the rod as functions of time t from the time the rod is released until the time its center of mass reaches its lowest point.

(e) As the rod rotates from the horizontal position down through vertical, is the magnitude of the angular acceleration on the rod increasing, decreasing, or not changing?

_____ Increasing _____ Decreasing _____ Not changing

Justify your answer.

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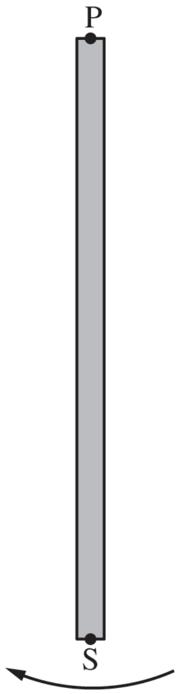

(f) The mass of the rod is 3.0 kg, and the length of the rod is 1.0 m. Calculate the linear speed v of point S as the rod swings through the vertical position shown.

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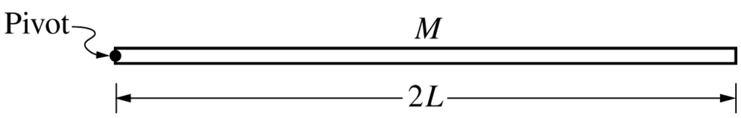
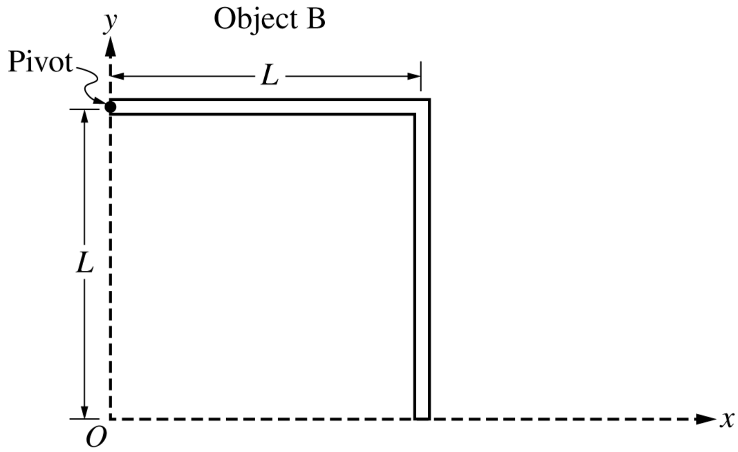
END OF EXAM

Object A



2. Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(a) Using integral calculus, derive an expression to show that the rotational inertia I_A of object A about the pivot is given by $\frac{4}{3}ML^2$.



Object B of total mass M is formed by attaching two thin, uniform, identical rods of length L at a right angle to each other. Object B is held in place, as shown above. Express your answers in part (b) in terms of L .

(b) Determine the following for the given coordinate system shown in the figure.

i. The x-coordinate of the center of mass of object B

ii. The y-coordinate of the center of mass of object B

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Object B has a rotational inertia of I_B about its pivot.

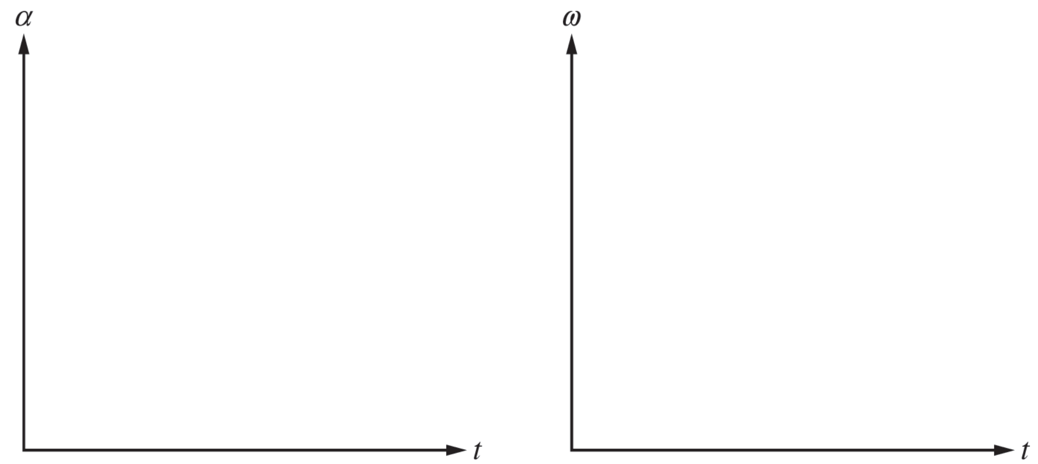
(c) Is the value of I_B greater than, less than, or equal to I_A ?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Object B is released from rest and begins to rotate about its pivot.

(d) On the axes below, sketch graphs of the magnitude of the angular acceleration α and the angular speed ω of object B as functions of time t from the time it is released to the time its center of mass reaches its lowest point.



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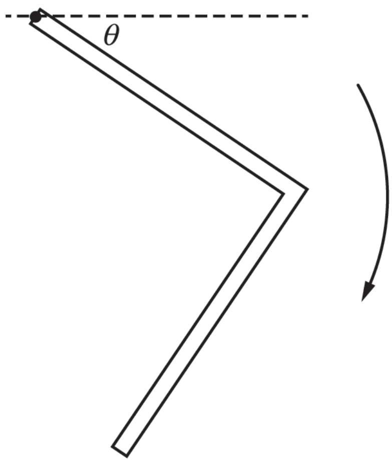
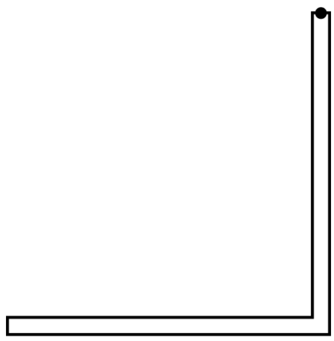
Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.



(e) While object B rotates from the horizontal position down through the angle θ shown above, is the magnitude of its angular acceleration increasing, decreasing, or not changing?

_____ Increasing _____ Decreasing _____ Not changing

Justify your answer.



Object B rotates through the position shown above.

(f) Derive an expression for the angular speed of object B when it is in the position shown above. Express your answer in terms of M, L, I_B, and physical constants, as appropriate.

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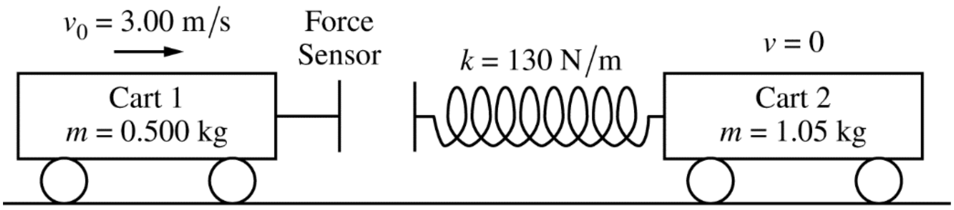
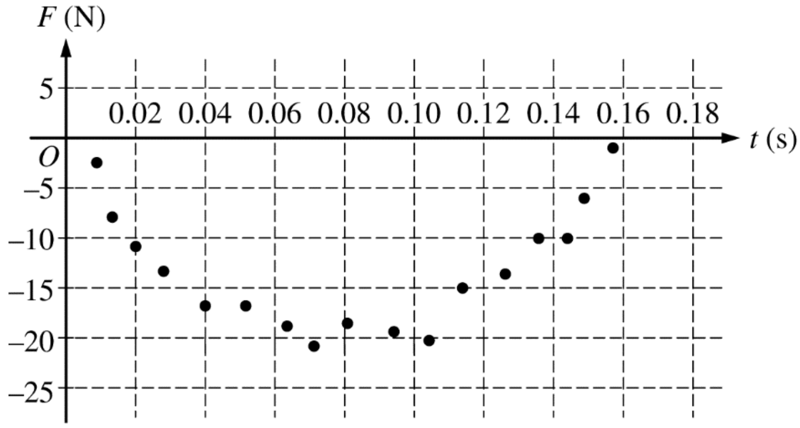


2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00\ \text{m/s}$ toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130\ \text{N/m}$. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200\ \text{N/s}^2\right)t^2 - (500\ \text{N/s})t$.



(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)

i. Calculate the speed of cart 1 after the collision.

ii. In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)

i. Calculate the speed of cart 2 after the collision.

ii. Show that the collision between the two carts is elastic.

(d)

i. Calculate the speed of the center of mass of the two-cart–spring system.

ii. Calculate the maximum elastic potential energy stored in the spring.

2.2 Forces and Free-Body Diagrams

Name: ______________ Class: __________ Date: ____________ **Total: 15 marks**

KEY WORDS friction 摩擦力 • tension 张力 • free-body diagram 受力图 • net force 净力 • force 力

Objective

Build the skills to answer exam questions on **forces and free-body diagrams**.

You must be able to:

- identify the forces acting on an object (weight, normal, tension, friction, applied)
- draw a **free-body diagram 受力图** showing each force as an arrow from the object
- find the **net force 净力** by adding forces as vectors
- resolve forces into components on an inclined surface

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Identify every force

A book resting on a table feels two forces: its **weight** mg down and the **normal force** N up from the table. They balance, so the net force is zero.

■ Draw the free-body diagram

Represent the object as a dot and draw one arrow per force, pointing the way the force acts, longer for bigger. Only forces *on* the object appear — never forces it exerts on others.

■ Net force by components

Add forces as vectors. A box pulled right with 10 N while friction pulls left with 4 N:

$$F_{\text{net}} = 10 - 4 = 6 \text{ N to the right.}$$

■ Forces on an incline

On a slope of angle θ, gravity splits into $mg\sin\theta$ (down the slope) and $mg\cos\theta$ (into the slope, balanced by the normal force).

2 Practice

Now apply the methods above.

2.1 A book rests on a table. Name the two forces acting on it. [2]

2.2 A box is pushed right with 20 N; friction opposes with 8 N. Find the net force. [2]

2.3 On a free-body diagram, whose forces are shown — those on the object, or those the object exerts? [1]

2.4 A 2 kg block sits on a 30° incline. Find the component of gravity along the slope (use $g = 10$ m s^{-2}). [2]

3 Exam-style questions

3.1 The net force on an object is the [1]

- **A** largest single force
- **B** vector sum of all forces on it
- **C** sum of its weight and mass
- **D** force it exerts on the ground

3.2 A free-body diagram should show [1]

- **A** only the weight
- **B** all forces acting on the object
- **C** the object's velocity
- **D** forces the object exerts on others

3.3 A 5 kg crate is pulled right by 30 N against 12 N of friction.

(a) Find the net force. [2]

(b) Find the acceleration. [1]

3.4 A 4 kg block rests on a 37° frictionless incline ($g = 10$ m s^{-2}).

(a) Find the component of gravity along the slope. [2]

(b) Find the block's acceleration down the slope. [1]

Solutions

2.1 Weight (mg, down) and the normal force (N, up).

2.2 $20 - 8 = 12$ N to the right.

2.3 The forces acting *on* the object.

2.4 $mg \sin\theta = 2(10) \sin 30° = 10$ N.

3.1 B — the net force is the vector sum of all forces on the object.

3.2 B — all forces acting on the object.

3.3 (a) $30 - 12 = 18$ N. (b) $a = F/m = 18/5 = 3.6$ m s^{-2}.

3.4 (a) $mg \sin\theta = 4(10) \sin 37° = 24$ N. (b) $a = g \sin\theta = 6$ m s^{-2}.

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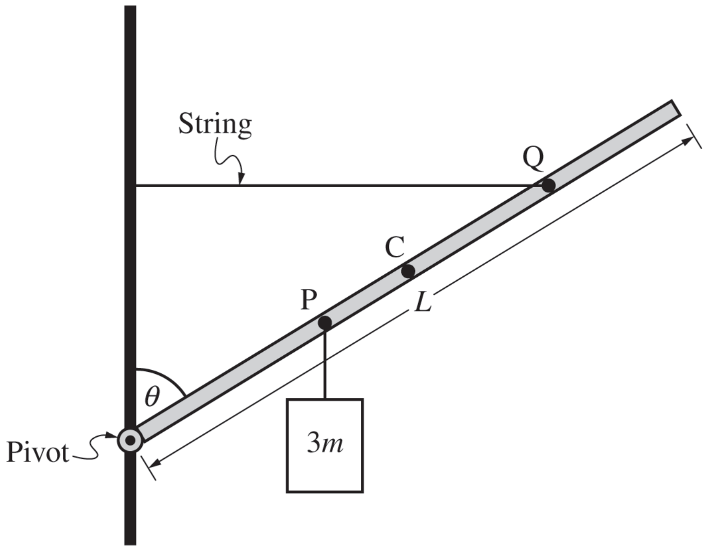
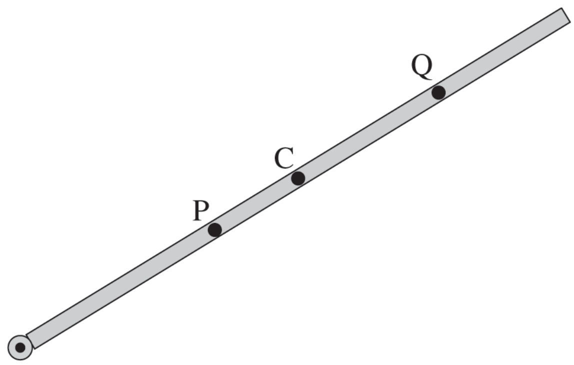


Figure 1

<u>Note</u>: Figure not drawn to scale.

3. A uniform rod of length L and mass m is attached to a pivot on a vertical pole, as shown in Figure 1. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

(a) On the following representation of the rod, **draw** and **label** the forces (not components) that are exerted on the rod. Each force must be represented by a distinct arrow that starts on and points away from the point at which the force is exerted on the rod.



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(b) In Figure 1, Point P is located $\frac{3}{8}L$ from the pivot and Point Q is located $\frac{6}{8}L$ from the pivot. **Derive** an equation for the tension F_T in the horizontal string in terms of L, m, θ, and physical constants, as appropriate.

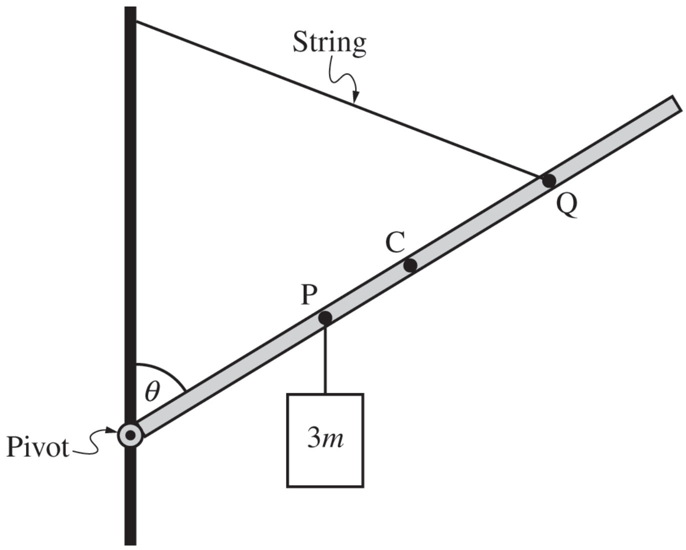

Figure 2

<u>Note:</u> Figure not drawn to scale.

(c) The original string is replaced with a longer string that connects Point Q to a higher location on the vertical pole, as shown in Figure 2. The angle θ remains the same. How does the new tension $F_{T,\text{ new}}$ compare with the original tension F_T from part (b) ? **Justify** your reasoning.

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Continue your response to **QUESTION 3** on this page.

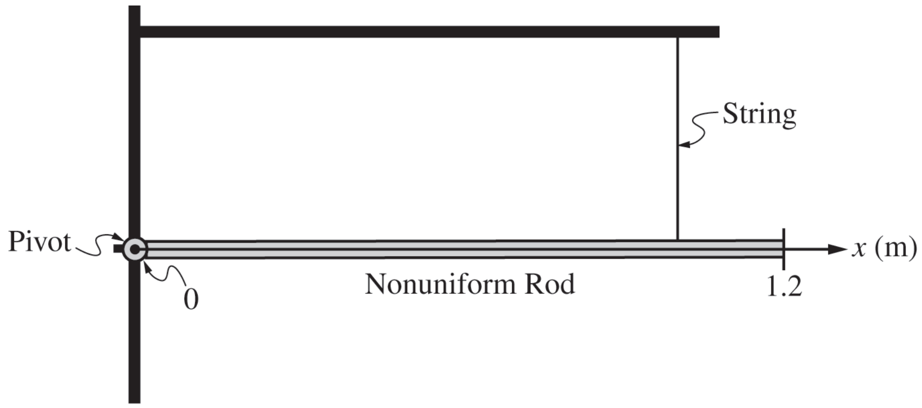

Figure 3

Note: Figure not drawn to scale.

(d) A nonuniform rod is now attached to the pivot, as shown in Figure 3. There is negligible friction between the nonuniform rod and the pivot. The rod has a length of 1.2 m and a linear mass density $\lambda(x) = A + Bx$, where x is the distance from the pivot, $A = 6.0$ kg/m, and $B = 10.0$ kg/m^2.

i. **Calculate** the mass of the rod.

ii. **Calculate** the rotational inertia of the rod about the pivot.

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END OF EXAM

Begin your response to **QUESTION 3** on this page.

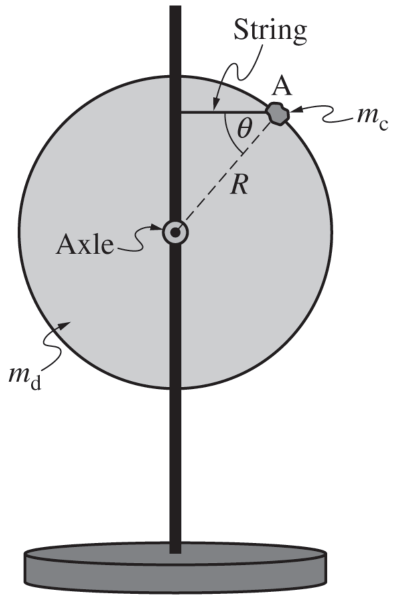
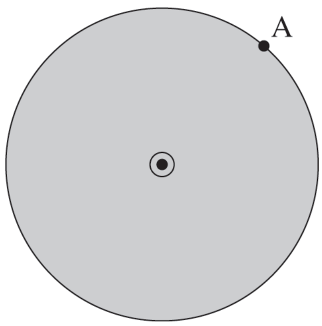


Figure 1

<u>Note:</u> Figure not drawn to scale.

3. A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(a) On the following representation of the clay-disk system, **draw** and **label** the external forces (not components) exerted on the system. Each force must be represented by a distinct arrow that starts on, and points away from, the point at which the force is exerted on the system.



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Continue your response to **QUESTION 3** on this page.

(b) **Derive** an expression for the tension F_T in the string when the clay is at Point A, as shown in Figure 1, in terms of R, m_d, m_c, θ, and physical constants, as appropriate.

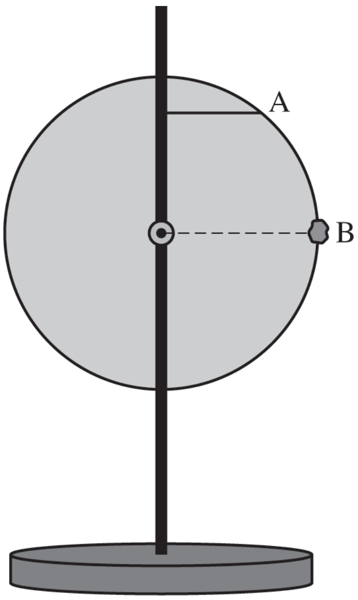

Figure 2

Note: Figure not drawn to scale.

(c) The string remains connected to the edge of the disk at Point A. The clay is moved to Point B, which is horizontally in line with the axle, as shown in Figure 2. How does the new tension $F_{T,\,new}$ compare with tension F_T from part (b)? **Justify** your reasoning.

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Continue your response to **QUESTION 3** on this page.

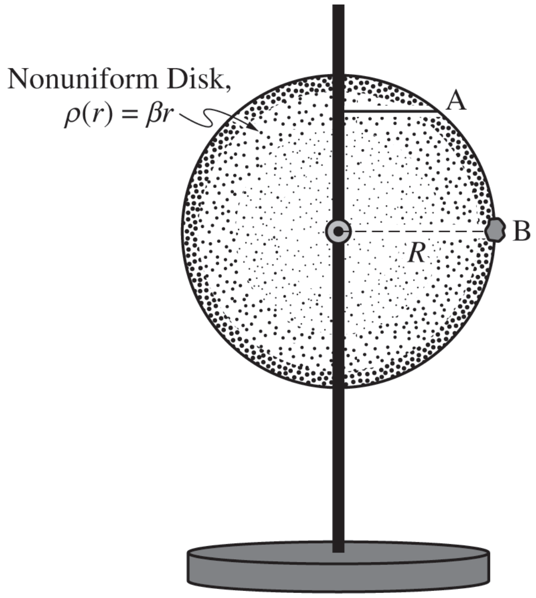

Figure 3

<u>Note:</u> Figure not drawn to scale.

(d) A nonuniform disk is now attached to the axle. The lump of clay is attached to the disk at Point B, as shown in Figure 3. The clay has mass $m_c = 0.60\text{ kg}$ and the disk has a radius $R = 0.30\text{ m}$. The mass density of the disk varies radially and can be modeled by $\rho(r) = \beta r$, where r is the radial distance from the axle and $\beta = 4.0\text{ kg/m}^3$.

i. **Calculate** the rotational inertia of the disk about the axle.

ii. The string connecting the disk to the pole is cut. **Calculate** the magnitude of the initial angular acceleration of the clay-disk system.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

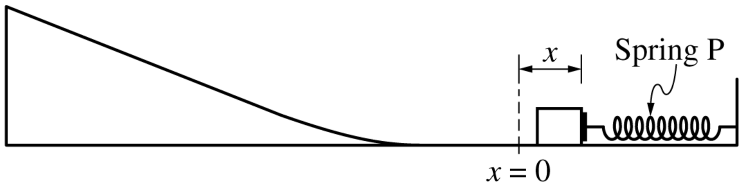

Figure 1

A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{max,P}$. Frictional forces between the block and all surfaces are negligible.

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A student compares $h_{\text{max,P}}$ for Spring P with the maximum height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x that the spring is compressed, as shown in Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0\ \text{N/m}$, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0\ \text{N/m}^{1/2}$.

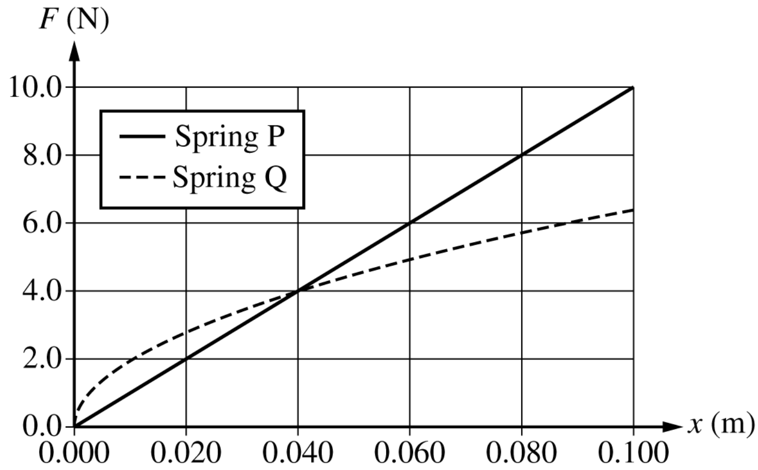
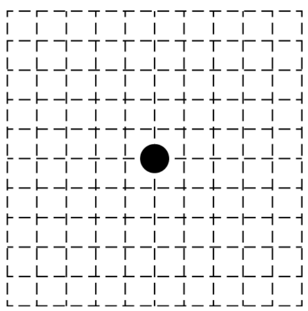


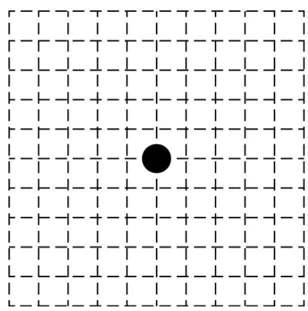
Figure 2

(a) For the experiment, the block is pushed against one of the springs, compressing the spring a distance $x = 0.010\ \text{m}$. The block is then released from rest. In Trial 1, Spring P is used, and in Trial 2, Spring Q is used. On the following dots, representing the block in Trials 1 and 2, draw and label the forces (not components) that are exerted on the block at the instant the block is released. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. The lengths of the horizontal vectors should represent the relative magnitude of the horizontal forces and the lengths of the vertical vectors should represent the relative magnitude of the vertical forces.

Trial 1
(Spring P)



Trial 2
(Spring Q)



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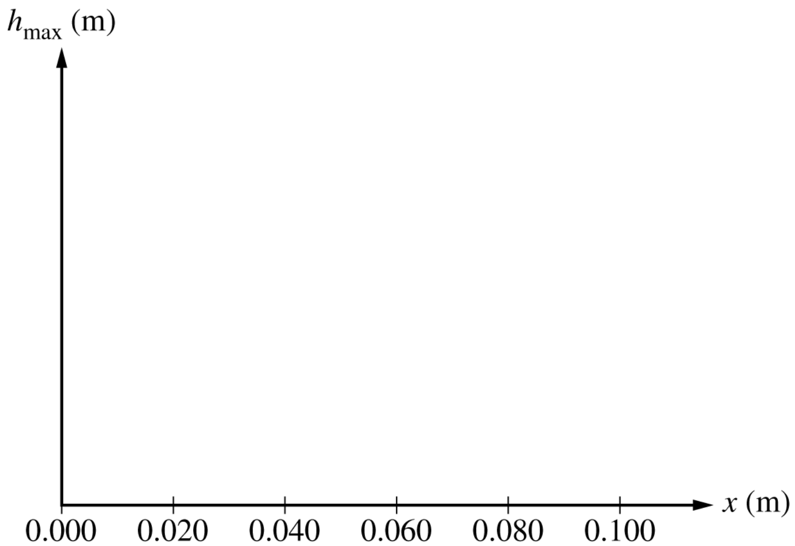
(b)

i. What feature(s) of the graph in Figure 2 could be used to estimate the work done on the block by each spring as each spring is compressed?

ii. There is one compression distance x_0 for which the maximum height h_{max} reached by the block is the same regardless of which spring, Spring P or Spring Q, is used. Predict whether the value of x_0 is greater than, less than, or equal to 0.040 m. Use the graph in Figure 2 to justify your answer.

iii. On the axes provided, for both Spring P and Spring Q, sketch a graph of the maximum height h_{max} reached by the block as a function of the distance each spring is compressed for values of x ranging from 0 to 0.100 m. Clearly label the curve for Spring P and Spring Q.



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Continue your response to **QUESTION 1** on this page.

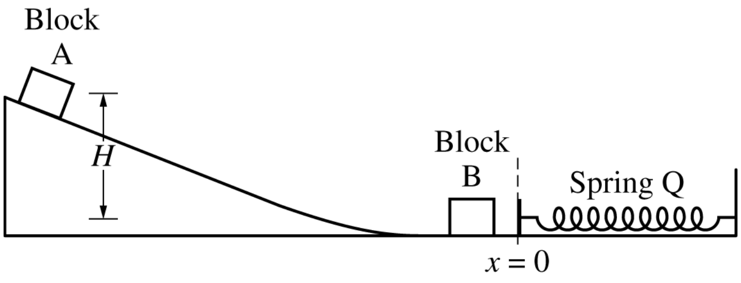

Figure 3

(c) Spring Q is attached to the wall and is at equilibrium, as shown in Figure 3. Block A has a mass of 0.120 kg and Block B has a mass of 0.070 kg. Block A is held at rest at the top of the ramp and Block B is at rest on the horizontal surface. The student releases Block A, and it moves down the ramp and collides with Block B. The change in vertical height of Block A is $H = 0.75$ m. After the collision, the blocks stick together and move to the right, compressing the spring. Frictional forces between the blocks and all surfaces are negligible.

i. Calculate the velocity of the two-block system immediately after the collision between Blocks A and B.

ii. Calculate the maximum compression of the spring.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(d) Spring Q where $F_Q(x) = Cx^{1/2}$ is replaced by a different nonlinear spring, Spring R, and the procedure described in part (c) is repeated. For Spring R, $F_R(x) = Dx^{1/2}$. The maximum compression of Spring R is greater than the maximum compression of Spring Q. Which of the following correctly compares the constants C and D ?

__________ $C < D$ __________ $C > D$ __________ $C = D$

Briefly justify your answer.

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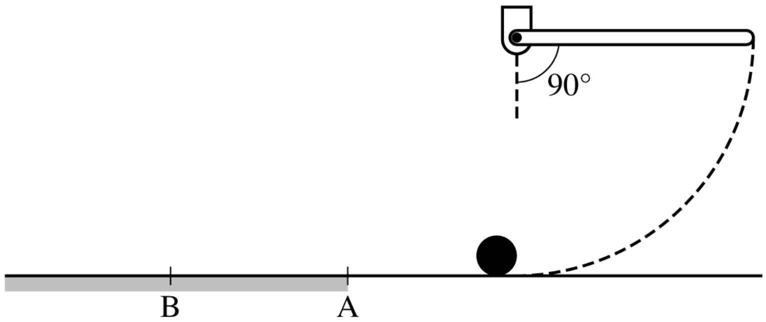
Begin your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

Figure 1

3. A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(a) Derive an expression for the angular speed of the rod just before striking the sphere in terms of the length ℓ and physical constants as appropriate.

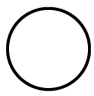
GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(b) Derive an expression for the linear speed v_0 of the sphere immediately after colliding with the rod in terms of the length ℓ and physical constants as appropriate.

After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ, beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

(c) In the following diagram, which represents the sphere while the sphere is traveling between Points A and B, draw and label the forces (not components) that act on the sphere. Each force must be represented by a distinct arrow starting on, and pointing away from, the point of application on the sphere.



(d) Derive an expression for each of the following as the sphere is rotating and sliding between points A and B in terms of v_0, μ, R, t, and physical constants as appropriate.

i. The linear velocity v of the center of mass of the sphere as a function of time t

ii. The angular velocity ω of the sphere as a function of time t

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(e)

i. Derive an expression for the time it takes the sphere to travel from Point A to Point B in terms of v_0, μ, and physical constants as appropriate.

ii. Derive an expression for the linear velocity of the sphere upon reaching Point B in terms of v_0.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

Begin your response to **QUESTION 1** on this page.

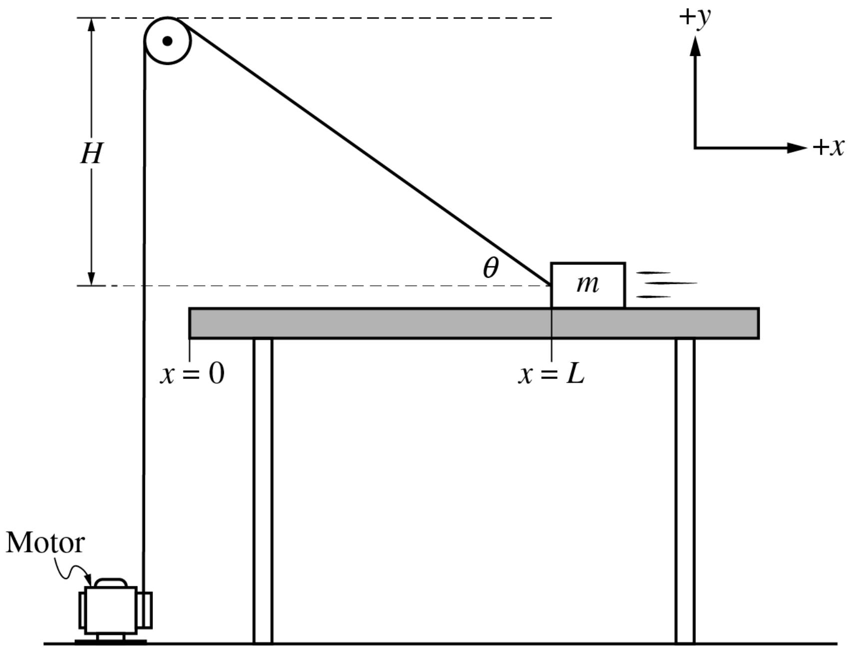
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

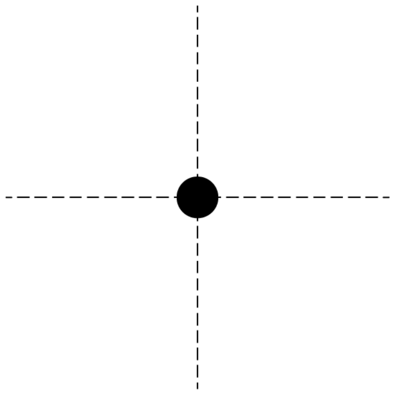


1. A block of mass m is pulled across a rough horizontal table by a string connected to a motor that is attached to the floor. The string passes over a pulley with negligible friction that is vertically aligned with the left edge of the table as shown. The string and pulley both have negligible mass. The pulley is at height H above the table. The motor exerts a constant force of tension F_T on the string, and the block remains in contact with the table at all times as the block slides across the table from $x = L$ to $x = 0$. The coefficient of kinetic friction between the table and the block is μ_k. Express all algebraic answers in terms of m, H, F_T, x, μ_k, L, and physical constants as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(a) On the dot below that represents the block, draw and label the forces (not components) that act on the block when the block is at $x = L$. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



(b) Derive an expression for the angle θ that the string makes with the horizontal as a function of x.

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Continue your response to **QUESTION 1** on this page.

(c)

i. Derive an expression for the normal force F_N exerted on the block by the table as a function of the block's position x.

ii. Derive an expression for the magnitude of the net horizontal force F_{net} exerted on the block as a function of the position x.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(d) Write, but do not solve, an integral expression that could be used to solve for the work W done by the string on the block as the block moves from $x = L$ to $x = 0$.

(e) Does the string do more, less, or the same amount of work on the block as the block moves from $x = L$ to $x = \frac{L}{2}$ compared to when the block moves from $x = \frac{L}{2}$ to $x = 0$?

_______ More work when the block moves from $x = L$ to $x = \frac{L}{2}$

_______ Less work when the block moves from $x = L$ to $x = \frac{L}{2}$

_______ The same amount of work when the block moves from $x = L$ to $x = \frac{L}{2}$

Justify your answer.

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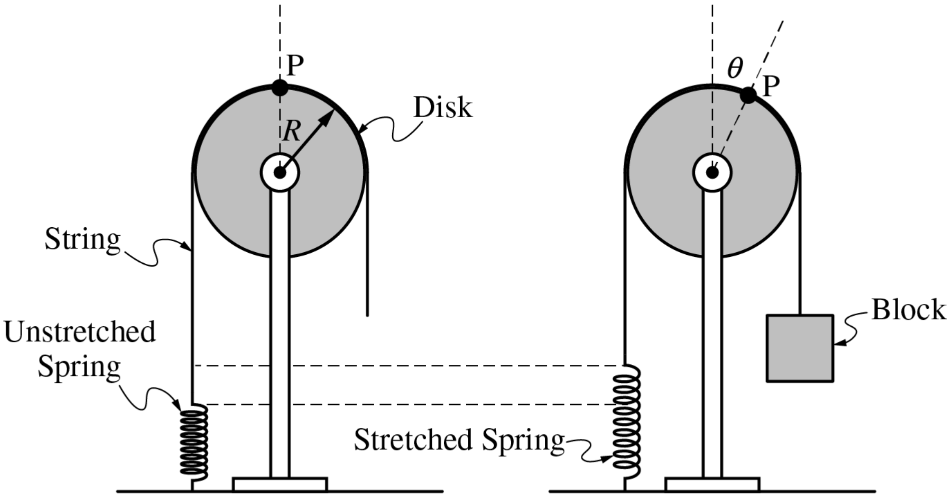


Figure 1 Figure 2

<u>Note:</u> Figures not drawn to scale.

. A solid uniform disk is supported by a vertical stand. The disk is able to rotate with negligible friction about an axle that passes through the center of the disk. The mass and radius of the disk are given by M_d and R, respectively. The rotational inertia of the disk is $I_d = \frac{1}{2} M_d R^2$. A string of negligible mass is draped over the disk and attached to the top of the disk at point P. One end of the string is connected to an unstretched ideal spring of spring constant k, which is fixed to the ground as shown in Figure 1.

A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

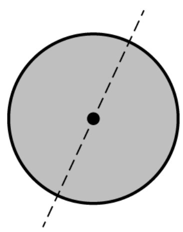
Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(a) Derive an expression for the mass m_B of the block.

(b) At time $t = 0$, the string on the right side of the disk is cut and the block falls to the ground. On the circle below, which represents the disk, draw and label the forces (not components) that act on the disk immediately after the string is cut and the block is falling to the ground. Each force should be represented by an arrow that starts on and is directed away from the point of application.

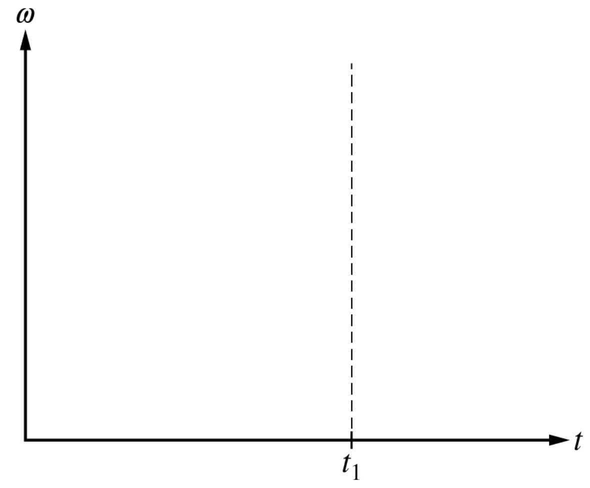


(c) Derive an expression for the angular acceleration α of the disk immediately after the string is cut.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(d) At $t = t_1$, the disk has rotated and point P is again directly above the axle. Sketch a graph of the magnitude of the angular velocity ω of the disk as a function of time t from $t = 0$ to $t = t_1$.



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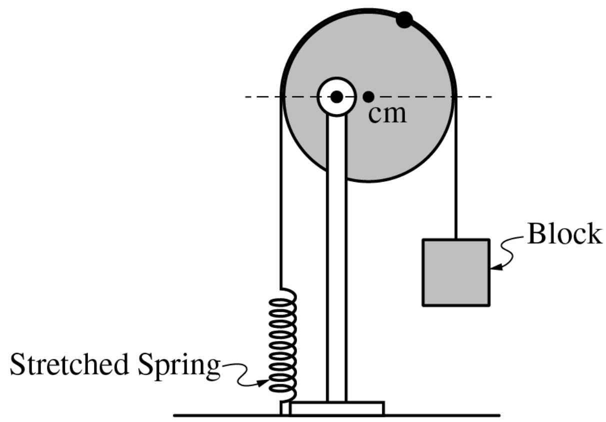


Figure 3

Note: Figure not drawn to scale.

(e) The disk is adjusted on the support so that the axle does not pass through the center of mass of the disk. The block is again hung on the right side of the disk and the spring-disk-block system comes to equilibrium, as shown in Figure 3. The axle does not exert a torque on the disk. For each force on the disk, indicate whether the magnitude of the torque about the axle caused by that force increases, decreases, or stays the same relative to part (b).

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

Begin your response to **QUESTION 1** on this page.

PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

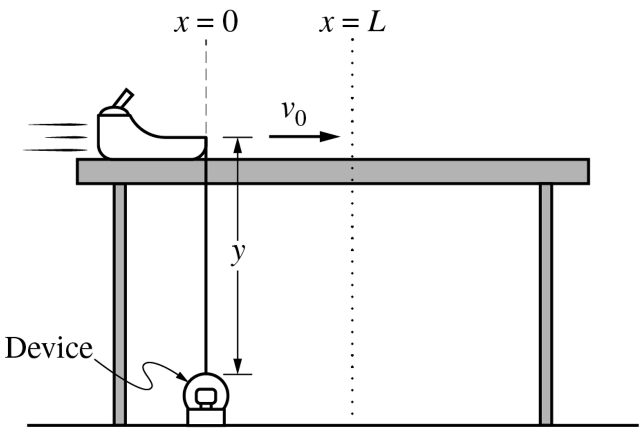


Figure 1

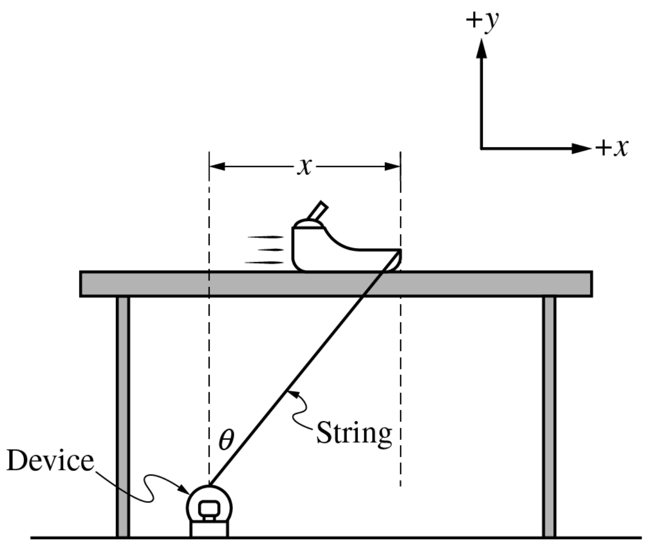


Figure 2

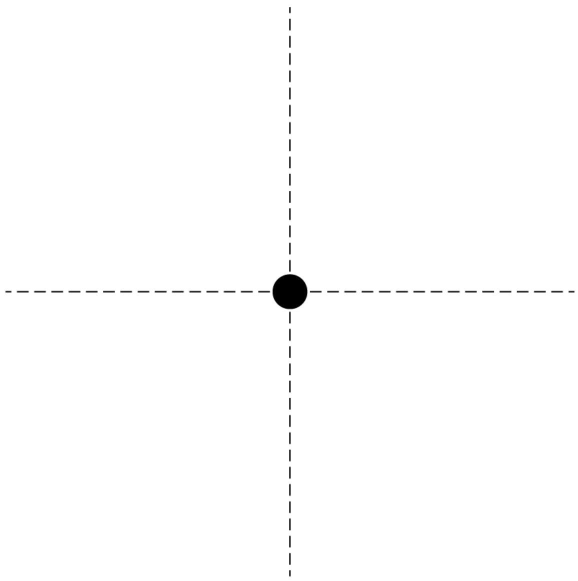
<u>Note:</u> Figures not drawn to scale.

1. A small sled slides across a rough horizontal table with an initial velocity v_0. The coefficient of kinetic friction between the sled and the table is μ_k. A string connects the sled to a device on the ground. The device maintains constant tension F_T in the string by unwinding the string as the sled slides to the right. The total mass of the sled is m. The string is attached to the device at $x = 0$ and at a height of y, as shown in Figure 1. The horizontal position of the sled is represented by x, as shown in Figure 2. Express all algebraic answers in terms of m, μ_k, F_T, x, y, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(a) On the dot below that represents the sled, draw and label the forces (not components) that are exerted on the sled a short time after $t = 0$ but before the sled has come to rest. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



(b) Determine an expression for the angle θ that the string makes with the vertical when the sled has traveled a horizontal distance x.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(c)

i. Derive an expression for the normal force F_N exerted on the sled by the table as a function of the position x.

ii. Derive an expression for the magnitude of the net horizontal force F_{net} exerted on the sled as a function of the position x.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(d) Derive an expression for the work W done by the string on the sled as the sled moves from $x = 0$ to $x = L$.

(e) The sled comes to rest after traveling a horizontal distance $x = 2L$. As the system slides from $x = 0$ to $x = L$, the energy dissipated by friction is E_1. As the sled slides from $x = L$ to $x = 2L$, the energy dissipated by friction is E_2. Is E_1 greater than, less than, or equal to E_2 ?

___ $E_1 > E_2$ ___ $E_1 < E_2$ ___ $E_1 = E_2$

Justify your answer.

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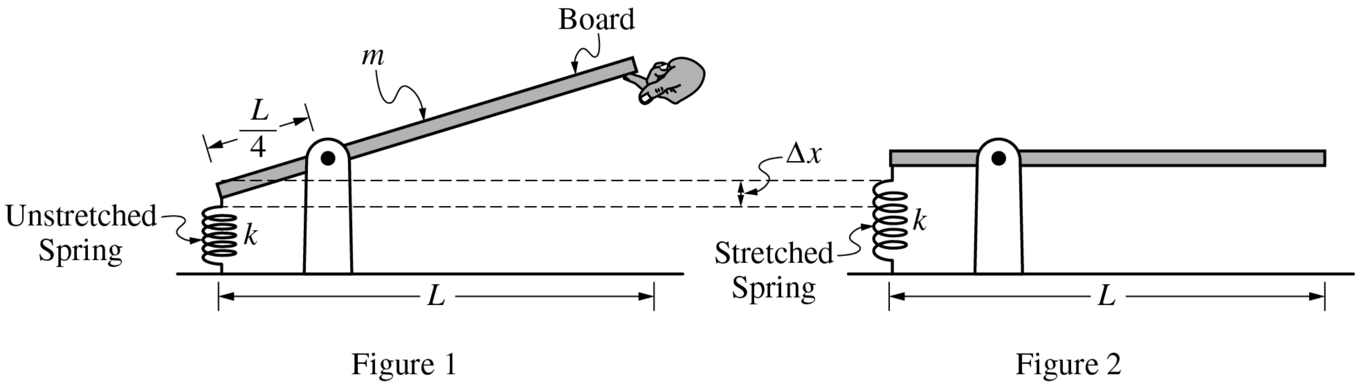
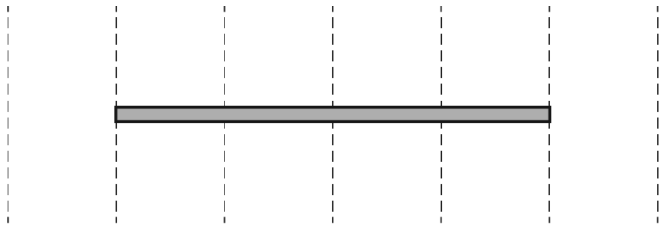


Note: Figures not drawn to scale.

. A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The right end of the board is initially held by a student so that the spring is unstretched, as shown in Figure 1. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring stretched, as shown in Figure 2. The rotational inertia of the board about the pivot is I.

(a) On the rectangle below, which represents the board, draw and label the forces (not components) that act on the board while the board-spring system is in equilibrium. Each force should be represented by an arrow that starts on, and points away from, the board, and should represent the location at which that force acts.



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Continue your response to **QUESTION 3** on this page.

(b) Derive an expression for the distance the spring stretches, Δx, when the board is in equilibrium. Express your answer in terms of k, L, m, and physical constants, as appropriate.

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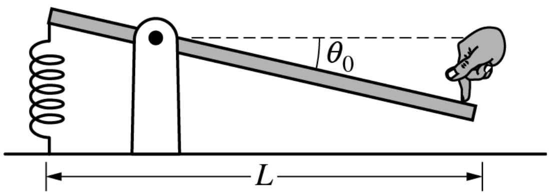
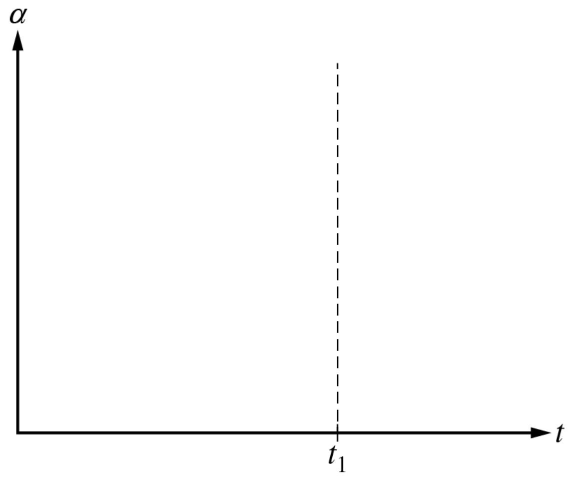


Note: Figure not drawn to scale.

(c) A student pushes the board down on the right side, stretching the spring a new distance Δx_2 from the unstretched position. The board is held at a small angle θ_0 with the horizontal, as shown. The student then releases the board from rest.

i. At time $t = 0$, the board is released. At $t = t_1$, the board first crosses the horizontal. Sketch a graph of the magnitude of the angular acceleration α of the board as a function of time t from $t = 0$ to $t = t_1$.



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Continue your response to **QUESTION 3** on this page.

ii. Derive an expression for the angular acceleration α_0 of the board immediately after the board is released. Express your answer in terms of k, L, m, I, Δx_2, θ_0, and physical constants, as appropriate.

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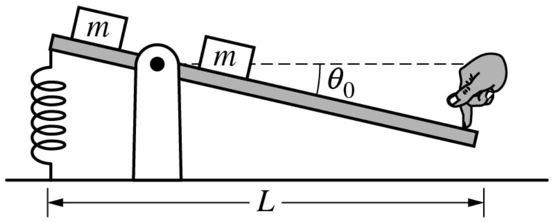
Continue your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

(d) Two blocks of equal mass m are attached to the board equal distances from the pivot point, as shown. The board is again pushed down on the right side so that the spring stretches the same distance Δx_2 as in part (c). The board is then released. How does the new angular acceleration α' when the blocks are attached compare to the angular acceleration α_0 from part (c) ?

___ $\alpha' > \alpha_0$ ___ $\alpha' < \alpha_0$ ___ $\alpha' = \alpha_0$

Justify your answer.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

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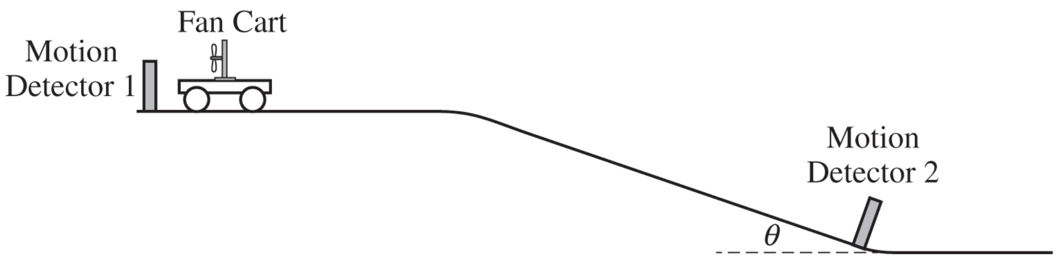
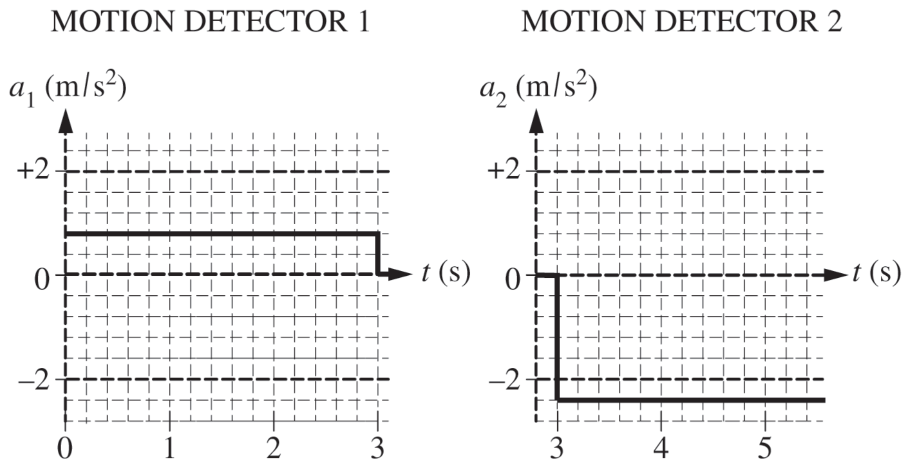


Note: Figure not drawn to scale.

1. A 0.50 kg fan cart is placed on a level, horizontal track of negligible friction, as shown. The fan is turned on, and the fan cart is released from rest and moves to the right. The cart travels along the horizontal track and then down an incline. Motion detector 1 measures the acceleration a of the cart from time $t = 0$ to $t = 3$ s. At $t = 3$ s, the cart makes a smooth transition to the incline, and motion detector 2 measures the acceleration of the cart after $t = 3$ s. The fan exerts the same magnitude of force on the cart during the entire motion. The graphs below show a as functions of t. For each motion detector, the positive direction is away from the detector.



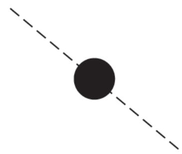
GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(a) On the dots below that represent the cart at two different locations, draw and label the forces (not components) that act on the cart at each location. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

Cart on Horizontal Track

Cart on Incline



(b) Calculate the magnitude of the net force exerted on the fan cart when it is on the horizontal track.

(c) Calculate the angle θ of the incline.

(d) Suppose careful measurement determines the angle of the incline to be $3°$ larger than that calculated in part (c). Consider the following explanation.

“The scale used to measure the mass of the fan cart was not calibrated properly before the measurement, and this could account for the observed difference in the angle.”

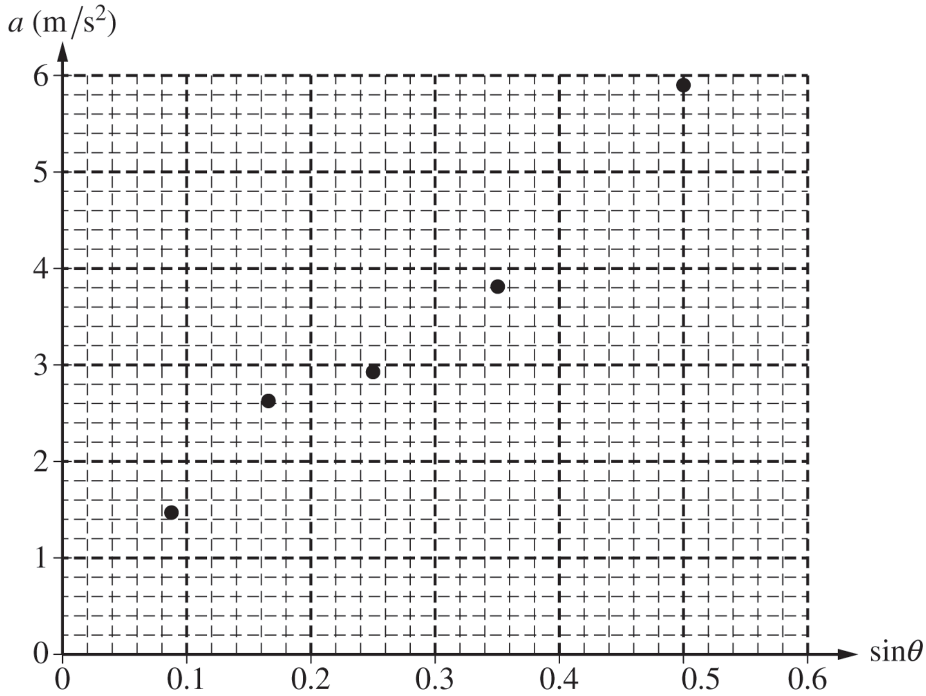
Does the explanation sufficiently account for the observed discrepancy?

______ Yes   ______ No

Justify your answer.

GO ON TO THE NEXT PAGE.

The experiment is repeated for several trials, each with a different angle for the incline. The acceleration of the cart down the incline is measured for each angle. The graph below shows the plot of the acceleration a of the cart as a function of the sine of the angle $\sin\theta$.



(e)

i. Draw a best-fit line for the data.

ii. Using the straight line, calculate an experimental value for the acceleration due to gravity g.

(f) If the cart were replaced with a second cart of mass 1.0 kg that has a fan that exerts the same magnitude of force as the original fan, explain how the graph given in part (e) would change.

GO ON TO THE NEXT PAGE.

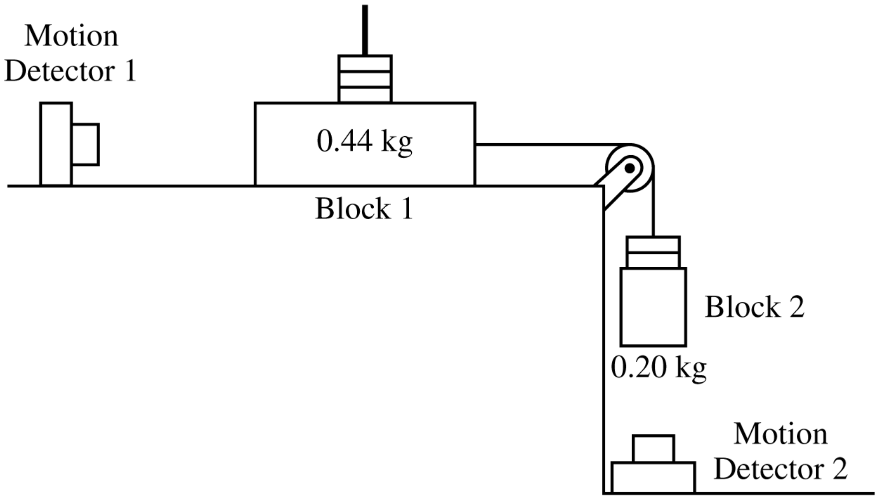
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

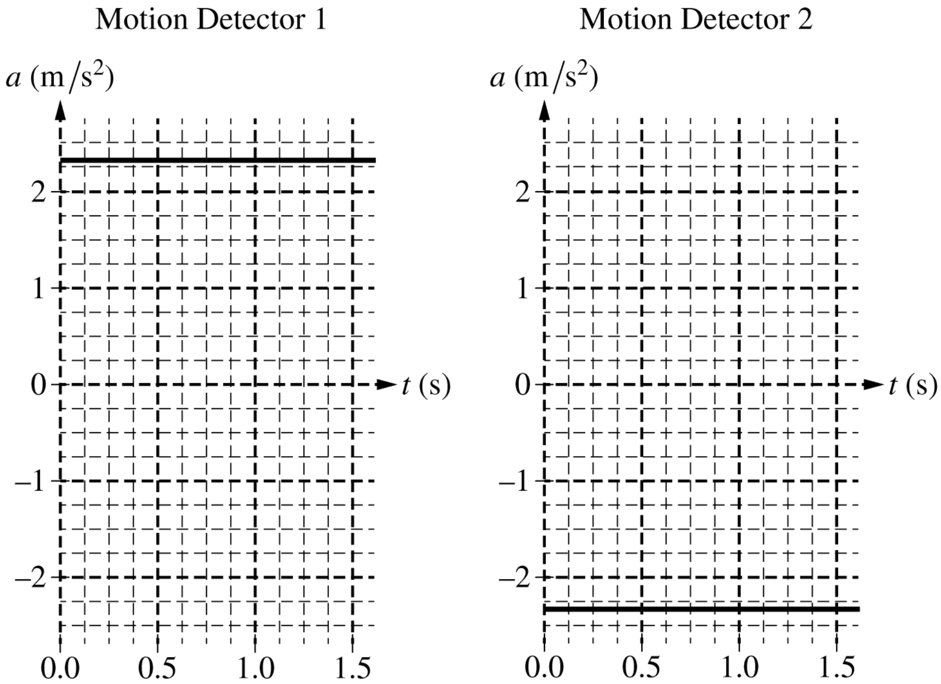


Students design an experiment using blocks of adjustable mass to investigate friction using the setup shown. Block 1 of initial mass 0.44 kg is placed on a rough horizontal surface and connected by a string to block 2 of initial mass 0.20 kg. The string extends over a pulley that has negligible mass and friction.

(a) Calculate the minimum value of the coefficient of static friction μ_s that would keep the two-block system at rest.

GO ON TO THE NEXT PAGE.

The coefficient of friction is such that when block 2 is released from rest, block 1 travels across the surface. The acceleration a of each block is recorded with motion detectors 1 and 2, as shown in the figure. The data for the motion detectors as functions of time t are shown on the graphs. For each motion detector, the positive direction is away from the detector.



(b) On the dots below, which represent the blocks, draw and label the forces (not components) that act on each block. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

Block 1 Block 2

(c) Calculate the coefficient of kinetic friction μ_k between block 1 and the table.

GO ON TO THE NEXT PAGE.

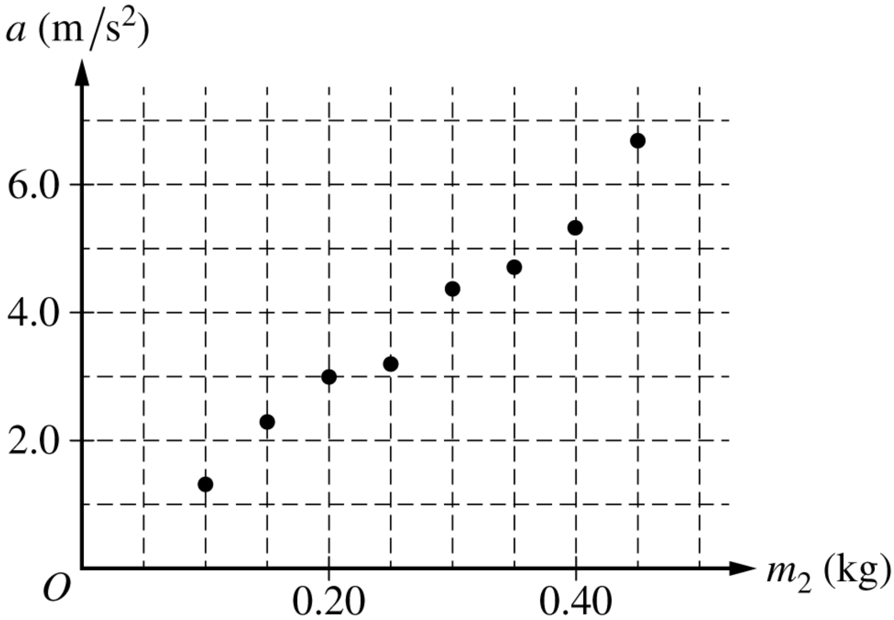
(d) Careful measurements determine that the coefficient of kinetic friction is larger than the value calculated in part (c). Does the following explanation sufficiently account for the observed discrepancy?

"The horizontal table was not perfectly level before the experiment was conducted. The observed difference in the angle accounts for the difference in the expected and calculated values of μ_k."

_____Yes _____ No

Justify your answer.

The experiment is moved to a surface with negligible friction and run for eight trials. In each trial, the students vary the masses m_1 and m_2 of blocks 1 and 2, respectively, while keeping the total mass $(m_1 + m_2) = 0.64\,\text{kg}$ constant. The data for the acceleration a of block 1 as a function of m_2 are shown on the graph below.



(e)

i. Draw a best-fit line for the data points.

ii. Using the straight line, calculate an experimental value for the acceleration due to gravity g.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(f) The students lift the left end of the surface so that the surface is inclined at an angle to the horizontal, and the experiment for $m_2 = 0.20\,\text{kg}$ is repeated. Would the acceleration of the system be greater than, less than, or equal to the acceleration of the system in the original experiment?

_____ Greater than _____ Less than _____ Equal to

Justify your claim.

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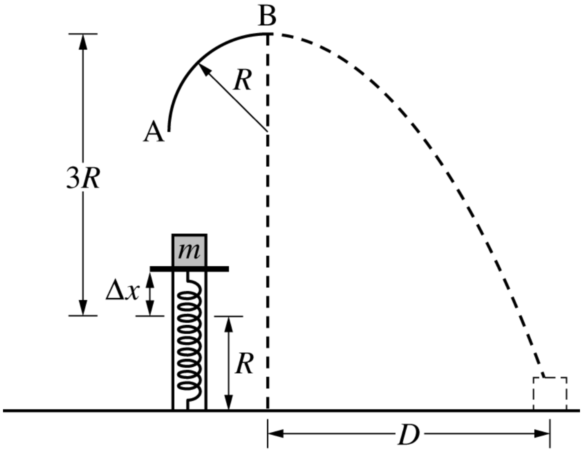
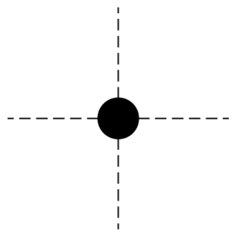


Note: Figure not drawn to scale.

3. A block of mass m is placed on top of an ideal spring of spring constant k. The block is pushed against the spring, compressing the spring a distance Δx. The block is released from rest, leaves the spring at the position shown in the figure, travels upward, and enters a track with a constant radius of curvature R that has negligible friction. The block enters the track at point A, maintains contact with the track, and exits horizontally at point B, a distance $3R$ above the point the block was released. The block then falls to the ground and lands a horizontal distance D from the end of the track. Express all algebraic answers in terms of m, k, Δx, R, and physical constants, as appropriate. The size of the block is much smaller than the radius of curvature of the track.

(a) On the dot below, which represents the block, draw and label the forces (not components) that act on the block while still in contact with the track at point B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



Justify your choice of vectors.

GO ON TO THE NEXT PAGE.

(b)

i. Derive an expression for the speed v of the block at point B.

ii. Derive an expression for the magnitude of the net force F on the block at point B.

(c) Derive an expression for the minimum value of Δx_{min} required in order for the block to maintain contact with the track through point B.

The procedure is repeated several times with the distance $\Delta x > \Delta x_{min}$.

(d) Calculate the distance D that the block travels.

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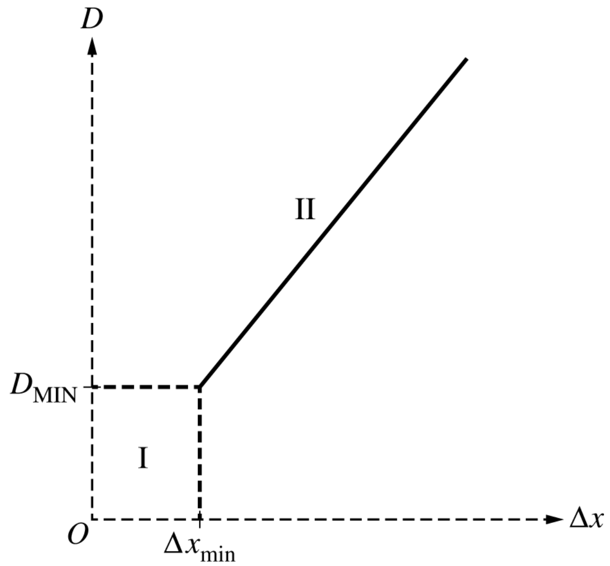
(e) The graph below shows the best-fit line drawn by the students through their data of D as a function of Δx.



i. Explain why there are no data for section I of the graph.

ii. Explain the reason for the shape and minimum value of section II on the graph.

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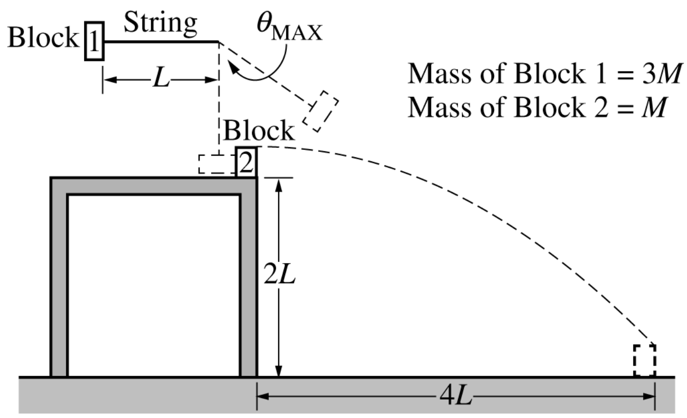
STOP

END OF EXAM

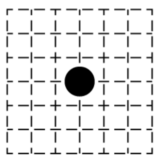


Note: Figure not drawn to scale.

2. A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(a) Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

(b) On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



(c) Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

(d) Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

(e) Calculate the speed of block 2 as it leaves the table.

(f) Calculate the speed of block 1 just after it collides with block 2.

(g) Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.

2.3 Newton's Third Law

Name: ______________ Class: __________ Date: ____________ **Total: 12 marks**

KEY WORDS newton's third law 牛顿第三定律 • action-reaction pair 作用 • force 力 • free-body diagram 受力图 • normal force 法向力

Objective

Build the skills to answer exam questions on **Newton's third law**.

You must be able to:

- state **Newton's third law 牛顿第三定律**: every force has an equal and opposite reaction
- identify an **action-reaction pair 作用-反作用对** acting on two different objects
- explain why the two forces do not cancel (they act on different bodies)
- apply it to walking, rockets, and collisions

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Equal and opposite, on different objects

If A pushes B, then B pushes A with an equal and opposite force. The two forces are the same size, opposite in direction, and act on **different** objects.

Why they don't cancel

Action and reaction act on *different* bodies, so they never appear in the same free-body diagram and never cancel. Only forces on the *same* object can cancel.

Everyday examples

- Walking: your foot pushes back on the ground; the ground pushes you forward.
- Rocket: the engine pushes gas down; the gas pushes the rocket up.

2 Practice

Now apply the methods above.

2.1 A hammer hits a nail with 50 N. State the force the nail exerts on the hammer. [1]

2.2 State the two properties an action-reaction pair always shares. [2]

2.3 Why do action and reaction forces not cancel each other out? [1]

2.4 When you walk forward, what pushes you forward? [1]

3 Exam-style questions

3.1 Newton's third law says forces come in pairs that are [1]

- **A** equal and in the same direction
- **B** equal and opposite, on the same object
- **C** equal and opposite, on different objects
- **D** unequal and opposite

3.2 A car tyre pushes back on the road with 2000 N. The road pushes the tyre forward with [1]

- **A** 0 N
- **B** 1000 N
- **C** 2000 N
- **D** 4000 N

3.3 A book of weight 8 N rests on a table.

(a) State the reaction to the Earth's gravitational pull on the book. [1]

(b) Explain why this is *not* the normal force from the table. [1]

3.4 A swimmer pushes water backward to move forward.

(a) Identify the action-reaction pair. [2]

(b) State which force of the pair moves the swimmer. [1]

Solutions

2.1 50 N in the opposite direction (back on the hammer).

2.2 Equal in size and opposite in direction (and they act on different objects).

2.3 They act on different objects, so they never appear together on one free-body diagram.

2.4 The ground pushes you forward (the reaction to your foot pushing back on it).

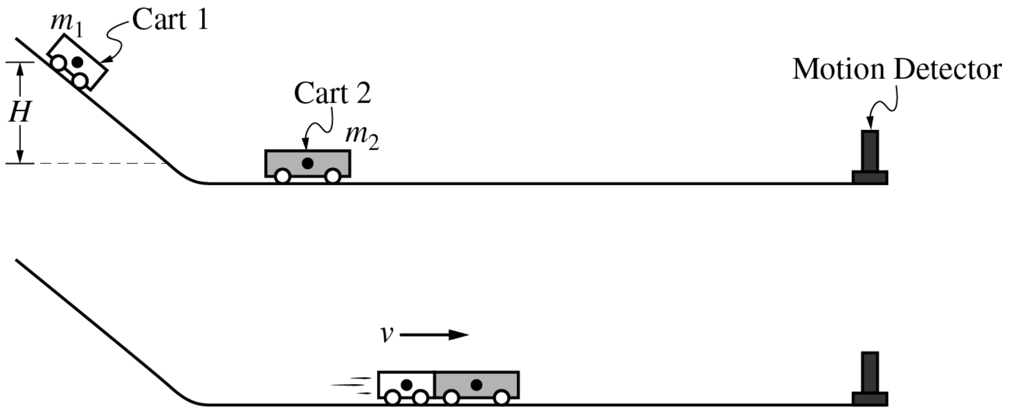
3.1 C — equal and opposite, on different objects.

3.2 C — the reaction is equal, 2000 N.

3.3 (a) The book's gravitational pull on the Earth, equal and upward. (b) The normal force acts on the book from the table, a different pair; the reaction to gravity acts on the Earth.

3.4 (a) The swimmer pushes water back; the water pushes the swimmer forward. (b) The forward push of the water on the swimmer.

Begin your response to **QUESTION 2** on this page.



. Cart 1 of mass m_1 is held at rest above the bottom of an incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and smoothly transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance of H, as shown. At time t_C, Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(a) During the collision, is the impulse on Cart 1 from Cart 2 greater than, less than, or equal to the magnitude of the impulse on Cart 2 from Cart 1 ?

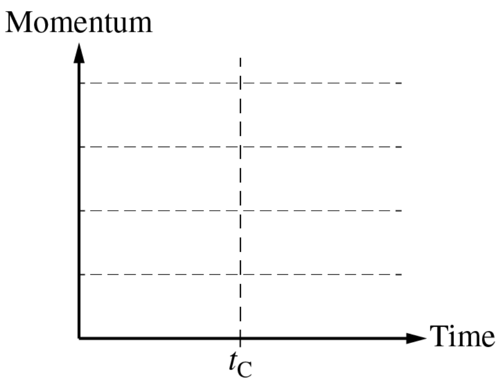
___ Greater than ___ Less than ___ Equal to

Justify your answer.

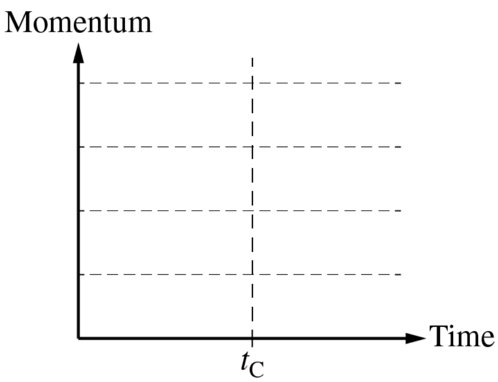
GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b) On the following axes, draw graphs of the magnitude of the momentum of each cart as a function of time t, before and after t_C. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.



Cart 1



Cart 2

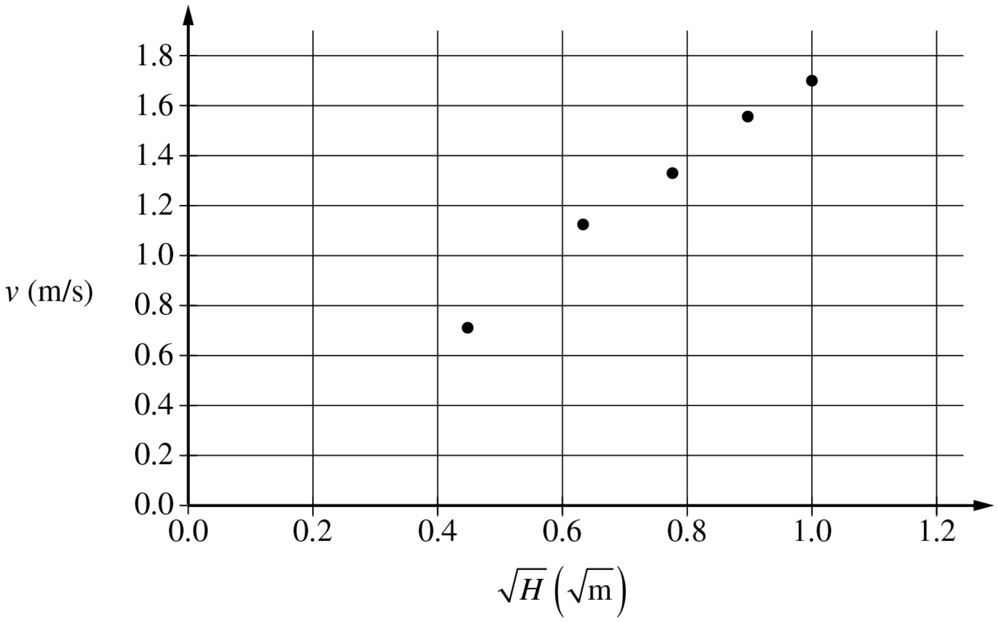
(c) Show that the velocity v of the two-cart system after the collision is given by the equation

$$v = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}.$$

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) A group of students use the setup to perform an experiment. They measure the mass of Cart 1 to be $m_1 = 0.250\ \text{kg}$. The mass of Cart 2 is unknown. The students perform several trials and in each trial, Cart 1 is released from a different height H and the final velocity of the two-cart system is measured. The students graph v as a function of $\sqrt{H}$, as shown below.



i. Draw a line that represents the best fit to the data points shown.

ii. Use the best-fit line to calculate the mass of Cart 2.

GO ON TO THE NEXT PAGE.

(e) After the experiment, the students use a balance to measure the mass of Cart 2 and find it to be less than what was determined in part (d). To explain this discrepancy, one of the students proposes that the mass of Cart 1 was incorrectly measured at the beginning of the experiment. The students measure the mass of Cart 1 again and record a new value, m_1'.

Should the students expect that m_1' will be greater than 0.250 kg, less than 0.250 kg, or equal to 0.250 kg ?

_____ $m_1' > 0.250\text{ kg}$ _____ $m_1' < 0.250\text{ kg}$ _____ $m_1' = 0.250\text{ kg}$

Justify your answer.

GO ON TO THE NEXT PAGE.

Mech.2.

A block of mass 2*M* rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where *B* is a positive constant and *x* is the displacement from equilibrium for the spring. A block of mass 3*M* and initial speed v_0 is moving to the left as shown.

(a) On the dots below, which represent the blocks of mass 2*M* and 3*M*, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance *D*, as shown above. Express your answers to parts (b), (c), and (d) in terms of *M*, *B*, v_0, and physical constants, as appropriate.

(b) Derive an expression for the speed of the blocks immediately after the collision.

(c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.

(d) Derive an expression for the maximum distance *D* that the spring is compressed.

(e)

i. In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

____ Left ____ Right ____ The net force on the block of mass $2M$ is zero.

Justify your answer.

ii. Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

____ The magnitude of the net force is greater on the block of mass $2M$.

____ The magnitude of the net force is greater on the block of mass $3M$.

____ The magnitude of the net force on each block has the same nonzero value.

____ The magnitude of the net force on each block is zero.

Justify your answer.

(f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

____ Yes ____ No

Justify your answer.

2.4 Newton's First Law

Name: ______________ Class: __________ Date: ____________ **Total: 12 marks**

KEY WORDS equilibrium 平衡 • inertia 惯性 • net force 合力 • newton's first law 牛顿第一定律 • force 力

Objective

Build the skills to answer exam questions on **Newton's first law**.

You must be able to:

- state **Newton's first law 牛顿第一定律**: an object keeps constant velocity unless a net force acts
- describe **inertia 惯性** as the resistance to a change in motion
- apply the **equilibrium 平衡** condition: net force zero means constant velocity (or rest)
- relate mass to inertia

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ The law of inertia

With no net force, an object at rest stays at rest and a moving object keeps a constant velocity in a straight line. Motion needs no force to continue – only a *change* in motion needs one.

■ Equilibrium

If the net force is zero, the object is in equilibrium: either at rest or moving at constant velocity. On a free-body diagram, the forces balance.

■ Mass is inertia

More mass means more inertia – harder to start, stop, or turn. A loaded truck resists changes in motion far more than a bicycle.

2 Practice

Now apply the methods above.

2.1 State what happens to a moving object when no net force acts on it. [1]

2.2 Define inertia in one sentence. [1]

2.3 A crate moves at constant velocity. State the net force on it. [1]

2.4 Which has more inertia: a 1 kg ball or a 10 kg ball? [1]

3 Exam-style questions

3.1 An object moves at constant velocity. The net force on it is [1]

- **A** in the direction of motion
- **B** opposite to the motion
- **C** zero
- **D** equal to its weight

3.2 Inertia depends on an object's [1]

- **A** velocity
- **B** mass
- **C** colour
- **D** position

3.3 A hovering drone stays perfectly still in mid-air.

(a) State the net force on it. [1]

(b) Name the two forces that must balance. [2]

3.4 A passenger lurches forward when a bus suddenly brakes.

(a) Explain this using inertia. [2]

(b) State whether a force pushes the passenger forward. [1]

Solutions

2.1 It keeps moving at constant velocity in a straight line.

2.2 Inertia is an object's resistance to a change in its motion.

2.3 Zero (it is in equilibrium).

2.4 The 10 kg ball (more mass, more inertia).

3.1 C — constant velocity means zero net force.

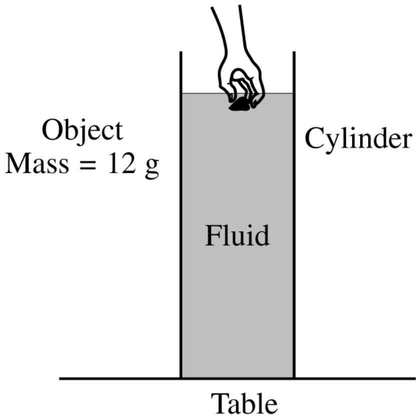
3.2 B — inertia depends on mass.

3.3 (a) Zero. (b) Weight (down) and the upward lift/thrust (up).

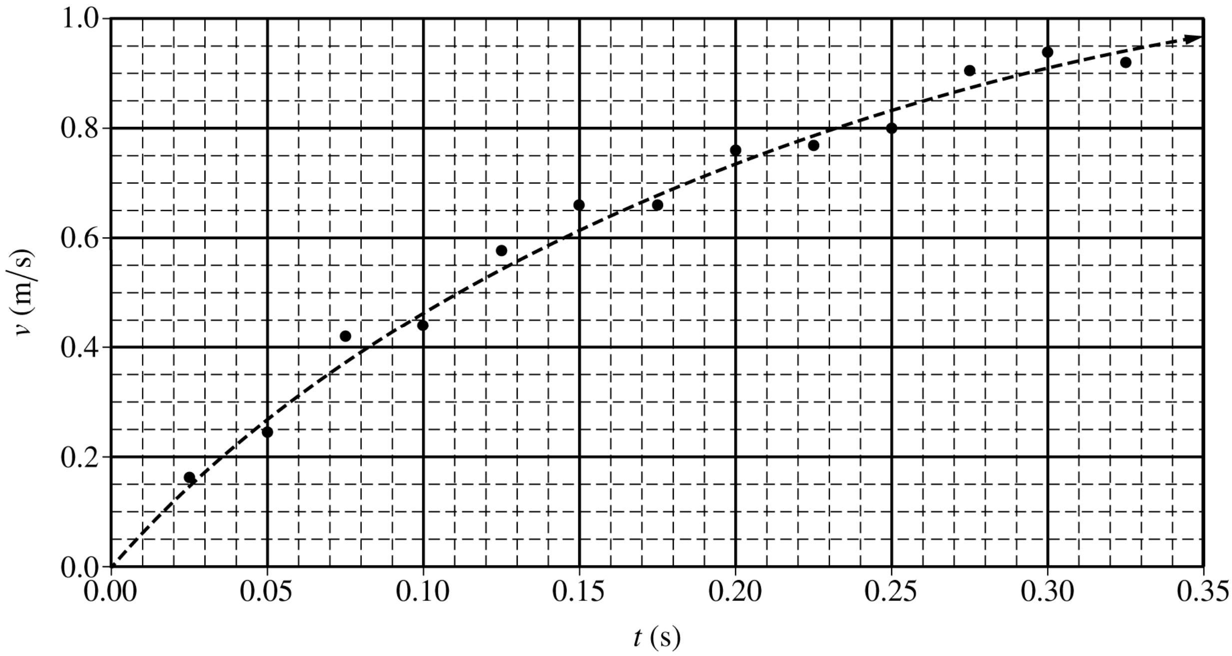
3.4 (a) The passenger's inertia keeps them moving forward while the bus decelerates beneath them. (b) No — no forward force acts; it is the absence of one.

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of m = 12 g is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.



(a)

i. Does the speed of the object increase, decrease, or remain the same?

____ Increase ____ Decrease ____ Remain the same

ii. In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.

iii. Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20$ s.

The students use the equation $v = A\left(1 - e^{-Bt}\right)$ to model the speed of the falling object and find the best fit coefficients to be $A = 1.18\ \text{m/s}$ and $B = 5\ \text{s}^{-1}$.

(b) Use the above equation to:

i. Derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time t.

ii. Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time t.

The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed.

(c)

i. Determine the constant speed of the object.

Justify your answer.

ii. Determine the force exerted by the fluid on the object at this time.

Justify your answer.

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

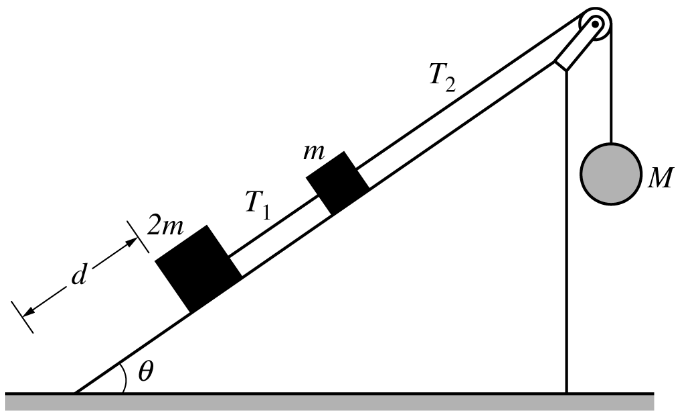


Figure 1

1. Blocks of mass m and $2m$ are connected by a light string and placed on a frictionless inclined plane that makes an angle θ with the horizontal, as shown in Figure 1 above. Another light string connecting the block of mass m to a hanging sphere of mass M passes over a pulley of negligible mass and negligible friction. The entire system is initially at rest and in equilibrium.

   (a) On the dots below that represent the block of mass m and the sphere of mass M, draw and label the forces (not components) that act on each of the objects shown. Each force must be represented by a distinct arrow starting on and pointing away from the dot.

(b) Derive expressions for the magnitude of each of the following. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figures in part (a).

i. The force T_2 exerted on the block of mass m by the string. Express your answers in terms of m, θ, and physical constants, as appropriate.

ii. The mass M for which the system can remain in equilibrium. Express your answers in terms of m, θ, and physical constants, as appropriate.

(c) Now suppose that mass M is large enough to descend and that the sphere reaches the floor before the blocks reach the pulley. Answer the following for the moment immediately after the sphere reaches the floor.

i. Does the tension T_1 increase, decrease to a nonzero value, decrease to zero, or stay the same?

____ Increase ____ Decrease to a nonzero value

____ Decrease to zero ____ Stay the same

ii. Is the velocity of the block of mass m up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

iii. Is the acceleration of the block of mass m up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

(d) Consider the initial setup in Figure 1. Now suppose the surface of the incline is rough and the coefficient of static friction between the blocks and the inclined plane is μ_s. Derive an expression for the minimum possible value of M that will keep the blocks from moving down the incline. Express your answer in terms of m, μ_s, θ, and fundamental constants, as appropriate.

(e) The string connecting block m and the sphere of mass M then breaks, and the blocks begin to move from rest down the incline. The lower block starts a distance d from the bottom of the incline, as shown in Figure 1. The coefficient of kinetic friction between the blocks and the inclined plane is μ_k. Derive an expression for the speed of the blocks when the lower block reaches the bottom of the incline. Express your answer in terms of m, d, μ_k, θ, and fundamental constants, as appropriate.

AP PHYSICS C: MECHANICS • EXERCISE SHEET

2.5 Newton's Second Law

Name: ______________ Class: __________ Date: ____________ **Total: 16 marks**

KEY WORDS newton's second law 牛顿第二定律 • friction 摩擦力 • force 力 • net force 合力 • free-body diagram 受力图

Objective

Build the skills to answer exam questions on **Newton's second law**.

You must be able to:

- apply **Newton's second law 牛顿第二定律**: $\vec{F}_{net} = m\vec{a}$
- find acceleration from a net force, or net force from an acceleration
- solve for one unknown among force, mass, and acceleration
- combine with free-body diagrams to find the net force first

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ The core equation

$$\vec{F}_{net} = m\vec{a}.$$

A 4 kg block with a net force of 12 N accelerates at

$$a = \frac{F_{net}}{m} = \frac{12}{4} = 3 \text{ m s}^{-2}.$$

■ Find the force

Rearrange: $F_{net} = ma$. A 2 kg ball accelerating at 5 m s^{-2} needs $F_{net} = 2 \times 5 = 10$ N.

■ Use the net force

First combine the forces, then apply $F_{net} = ma$. A 3 kg box pulled by 20 N against 8 N of friction: $F_{net} = 12$ N, so $a = 12/3 = 4$ m s^{-2}.

2 Practice

Now apply the methods above.

2.1 A net force of 18 N acts on a 6 kg mass. Find the acceleration. [2]

2.2 A 3 kg object accelerates at 4 m s^{-2}. Find the net force. [2]

2.3 A 10 N net force gives an acceleration of 2 m s^{-2}. Find the mass. [2]

2.4 A 5 kg crate is pushed by 25 N against 10 N friction. Find the acceleration. [2]

3 Exam-style questions

3.1 For a fixed net force, doubling the mass [1]

- **A** doubles the acceleration
- **B** halves the acceleration
- **C** leaves the acceleration unchanged
- **D** quadruples the acceleration

3.2 A 2 kg ball has a net force of 6 N. Its acceleration is [1]

- **A** 12 m s^{-2}
- **B** 3 m s^{-2}
- **C** 8 m s^{-2}
- **D** 0.33 m s^{-2}

3.3 A 1200 kg car speeds up at 2.5 m s^{-2}.

(a) Find the net driving force. [2]

(b) State what happens to the acceleration if the same force acts on a heavier car. [1]

3.4 A 2 kg block on a frictionless surface is pulled by two horizontal forces: 12 N right and 4 N left.

(a) Find the net force. [1]

(b) Find the acceleration. [2]

Solutions

2.1 $a = F/m = 18/6 = 3$ m s^{-2}.

2.2 $F = ma = 3 \times 4 = 12$ N.

2.3 $m = F/a = 10/2 = 5$ kg.

2.4 $F_{net} = 25 - 10 = 15$ N; $a = 15/5 = 3$ m s^{-2}.

3.1 B — $a = F/m$, so doubling m halves a.

3.2 B — $a = 6/2 = 3$ m s^{-2}.

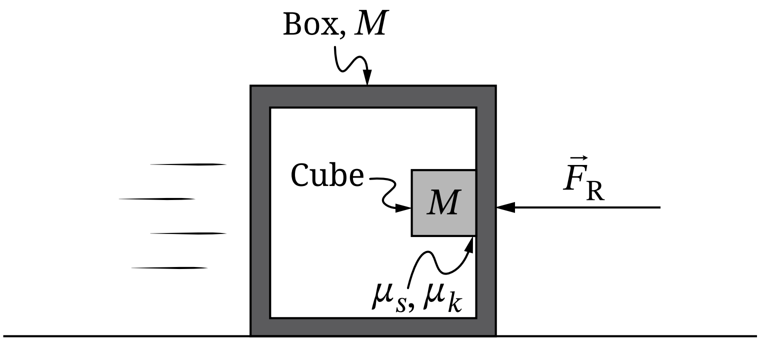
3.3 (a) $F = ma = 1200 \times 2.5 = 3000$ N. (b) It decreases (more mass, less acceleration for the same force).

3.4 (a) $12 - 4 = 8$ N. (b) $a = 8/2 = 4$ m s^{-2}.

Question 1: Version J

1. A box of mass M slides to the right on a horizontal surface. A small cube, also of mass M, is inside the box. The cube is against the right wall of the box and above the bottom of the box, as shown in Figure 1. The coefficients of static and kinetic friction between the cube and the wall of the box are μ_s and μ_k, respectively, where $\mu_s > \mu_k$.

Figure 1



The speed of the box is decreasing as a result of a resistive force that is exerted on the box. The resistive force $\vec{F}_R$ is modeled by $\vec{F}_R = -b\vec{v}$, where b is a positive constant and $\vec{v}$ is the velocity of the box. Friction between the box and horizontal surface is negligible.

Consider the motion of the box and cube for the following times.

- At time $t = 0$, the box-cube system slides to the right with speed v_0.
- For $0 \le t < t_{crit}$, the cube remains in contact with the wall of the box at a constant height above the bottom of the box.
- At $t = t_{crit}$, the cube begins to slide downward along the wall of the box.

Part A

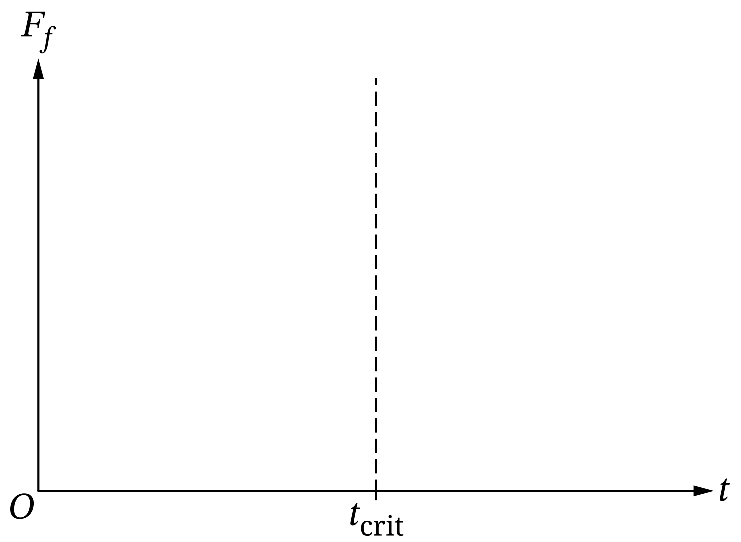
The magnitude of the normal force exerted on the cube by the wall of the box is F_N.

i. **Determine** an expression for the magnitude of the acceleration of the cube. Express your answer in terms of M, F_N, and physical constants, as appropriate.

ii. **Derive** an expression for F_N as a function of t from $t = 0$ until immediately before the cube reaches the bottom of the box. Express your answer in terms of M, b, v_0, t, and physical constants, as appropriate. Begin your derivation with a fundamental physics principle or an equation from the reference information.

iii. On the axes shown in Figure 2, **sketch** a graph of the magnitude F_f of the frictional force exerted on the cube by the wall of the box as a function of t from $t = 0$ until immediately before the cube reaches the bottom of the box.

Figure 2



Part B

Derive an expression for t_{crit} in terms of M, b, μ_s, v_0, and physical constants, as appropriate. Begin your derivation with a fundamental physics principle or an equation from the reference information.

Begin your response to **QUESTION 2** on this page.

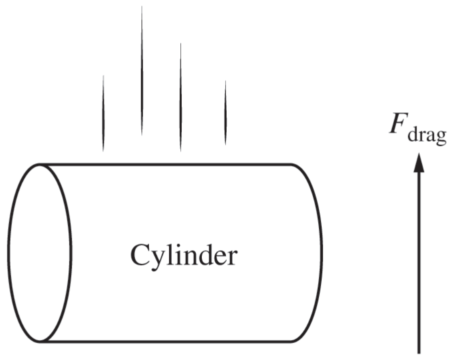


Figure 1

2. A student drops a cylinder of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the cylinder, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv^2$, where v is the speed of the cylinder and b is a positive constant with appropriate units.

   (a) **Derive**, but do NOT solve, a differential equation that could be used to determine the speed v of the cylinder as a function of time t. Express your answer in terms of given quantities and physical constants, as appropriate.

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Continue your response to **QUESTION 2** on this page.

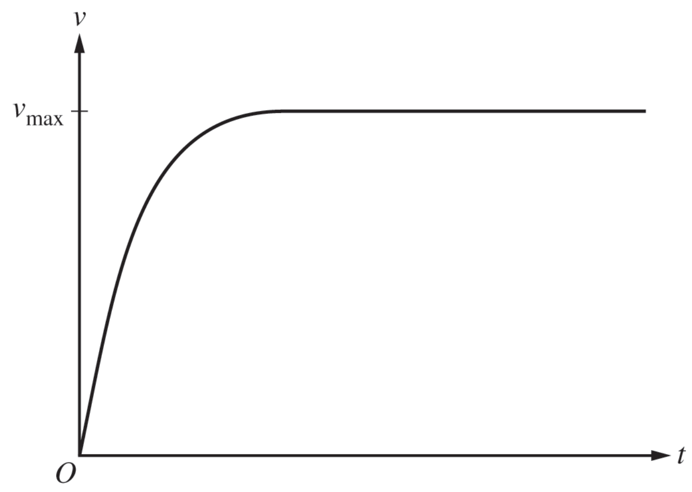


Figure 2

(b) The student correctly sketches the speed v of the cylinder as a function of time t, as shown in Figure 2.

i. **Draw** a vertical line on the sketch in Figure 2 to indicate the earliest time at which F_{drag} on the cylinder is equal to the magnitude of the weight of the cylinder. Label this time as t_1 on the time axis.

ii. **Justify** the location of t_1. Explicitly reference appropriate features of the sketch in Figure 2.

(c) Rather than dropping the cylinder from rest, the student throws the cylinder upward with a nonzero initial speed. The cylinder is in the same orientation as when the cylinder was previously dropped. The student allows the cylinder to fall toward the ground.

Indicate whether the magnitude of the cylinder's maximum downward speed after being thrown upward would be greater than, less than, or equal to the maximum speed v_{max} in Figure 2.

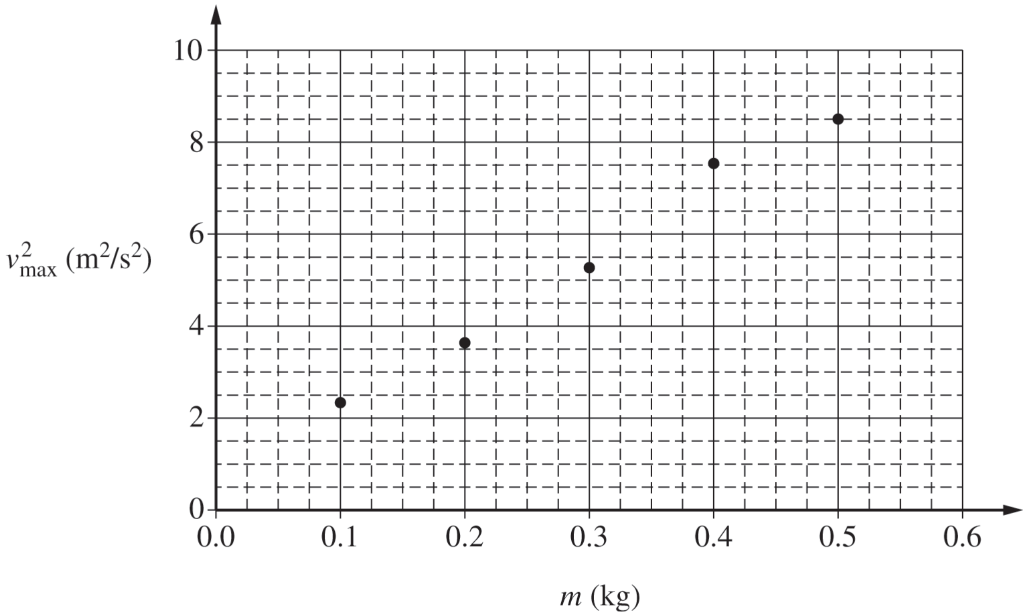
______ Greater than ______ Less than ______ Equal to

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) The student conducts an experiment to better understand the relationship between maximum speed v_{max} and mass. The student collects data to determine the maximum speed for cylinders dropped from rest, each with the same physical size and shape but a different mass m. The student then graphs v_{max}^2 as a function of mass.



i. **Draw** the best-fit line for the data.

ii. Use the best-fit line to **calculate** an experimental value for b.

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Continue your response to **QUESTION 2** on this page.

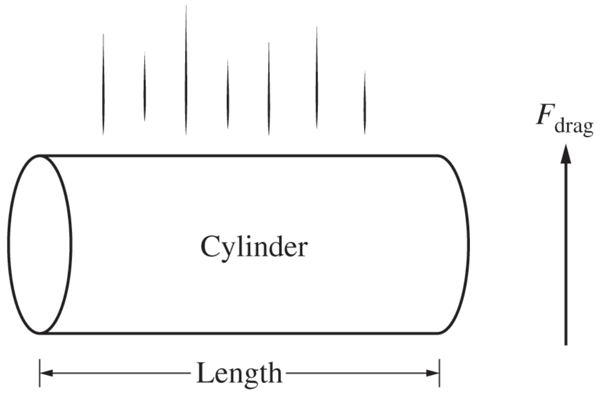

Figure 3

A student claims that the magnitude of the maximum speed of a cylinder dropped from rest depends on the length of the cylinder. The student designs an experiment to collect data that can be used to provide evidence to support the claim. The student drops cylinders with the orientation shown in Figure 3.

(e) The student has access to but does not have to use all of the following equipment.

- Cylinder Set 1: cylinders of the same known length with different known masses
- Cylinder Set 2: cylinders of the same known mass with different known lengths
- A motion detector that can measure velocity as a function of time

i. **Indicate** two quantities that when graphed could be used to determine whether the length of the cylinder affects the maximum speed.

Vertical axis: ______________ Horizontal axis: ______________

ii. Briefly **describe** how the quantities graphed could be used to determine the relationship between cylinder length and maximum speed.

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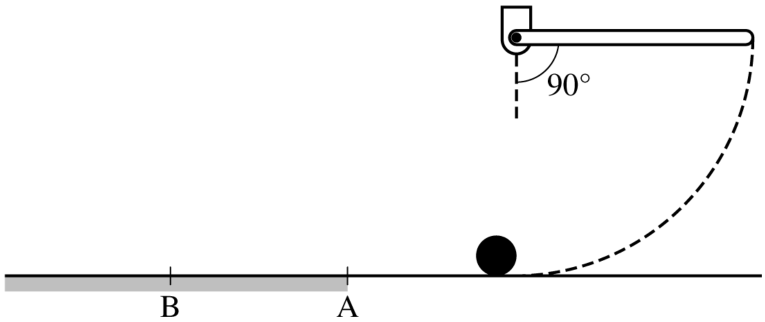


Note: Figure not drawn to scale.

Figure 1

3. A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

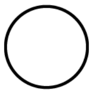
(a) Derive an expression for the angular speed of the rod just before striking the sphere in terms of the length ℓ and physical constants as appropriate.

GO ON TO THE NEXT PAGE.

(b) Derive an expression for the linear speed v_0 of the sphere immediately after colliding with the rod in terms of the length ℓ and physical constants as appropriate.

After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ, beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

(c) In the following diagram, which represents the sphere while the sphere is traveling between Points A and B, draw and label the forces (not components) that act on the sphere. Each force must be represented by a distinct arrow starting on, and pointing away from, the point of application on the sphere.



(d) Derive an expression for each of the following as the sphere is rotating and sliding between points A and B in terms of v_0, μ, R, t, and physical constants as appropriate.

i. The linear velocity v of the center of mass of the sphere as a function of time t

ii. The angular velocity ω of the sphere as a function of time t

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(e)

i. Derive an expression for the time it takes the sphere to travel from Point A to Point B in terms of v_0, μ, and physical constants as appropriate.

ii. Derive an expression for the linear velocity of the sphere upon reaching Point B in terms of v_0.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

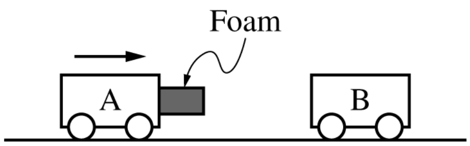
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

irections: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, hich are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this ooklet in the spaces provided after each part.



. Scientists have created a new type of lightweight foam and are performing experiments to investigate the properties of the foam. The mass of Cart A is 1000 kg and the mass of Cart B is 2000 kg. A piece of foam with negligible mass is attached to the front of Cart A, as shown. Cart A moves with a constant speed toward Cart B, which is initially at rest. At time $t = 0$ s, the foam connected to Cart A makes contact with Cart B. The foam remains in contact with Cart B for 0.5 s, after which the carts separate and both carts move with constant velocities.

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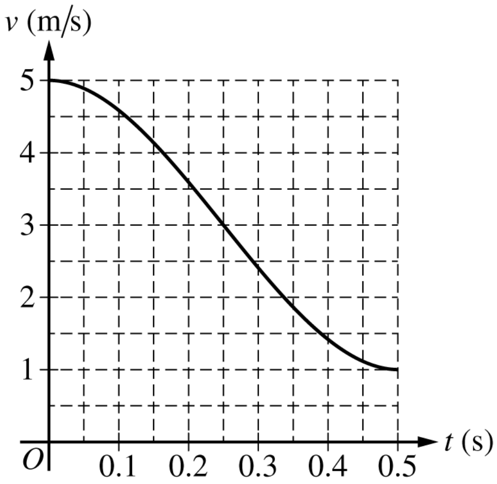
Continue your response to **QUESTION 1** on this page.

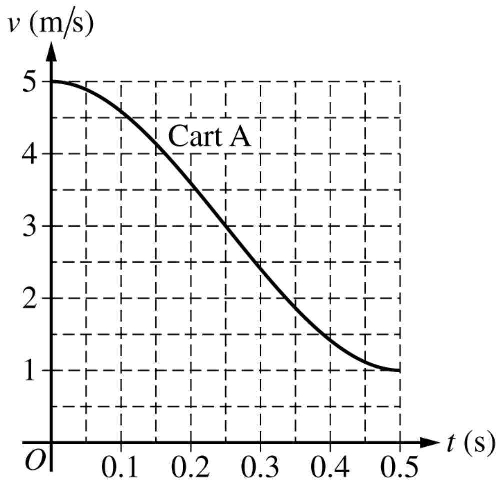


(a) The graph shows the velocity v of Cart A as a function of time t for the time interval when the foam and Cart B are in contact.

i. What feature(s) of the graph could be used to estimate the displacement of Cart A during the collision?

ii. Using the information shown in the graph, determine the speed of Cart B at $t = 0.5$ s.

iii. On the following grid, draw a smooth curve of the velocity of Cart B as a function of time.



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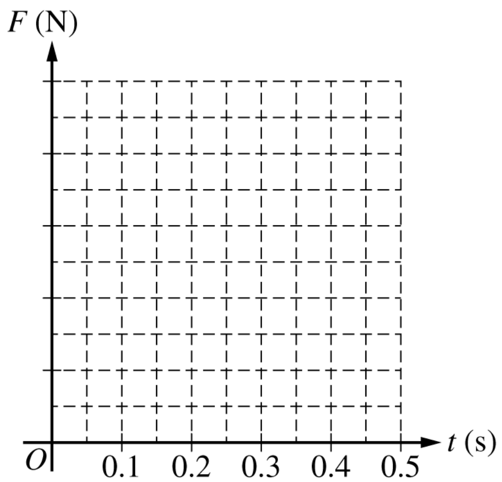
(b) For $0 \leq t \leq 0.50$ s, the velocity v of Cart A can be described by the function $v(t) = 64t^3 - 48t^2 + 5$.

i. Calculate the magnitude of the maximum net force acting on Cart A during this interval.

ii. On the following grid, draw a smooth curve of the magnitude of the force acting on Cart A as a function of time. Clearly indicate the value of the maximum force on the vertical axis.



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The foam is removed from the front of Cart A and the experiment is repeated. The carts collide, with both Cart A and Cart B having the same initial and final velocities as in the original collision. The time intervals during which the carts are in contact are different in the collision with the foam and the collision without the foam. In the collision without the foam, Cart A is in contact with Cart B for a shorter duration than in the original collision, when the foam was present.

For the original collision when the foam is present, the magnitude of the average net force exerted on Cart B is F_1. For the collision without the foam, the magnitude of the average net force exerted on Cart B is F_2.

(c) What is the relationship between the magnitude of the average net force F_1 exerted on Cart B for the collision with the foam and the magnitude of the average net force F_2 exerted on Cart B for the collision without the foam?

____ $F_1 > F_2$ ____ $F_1 < F_2$ ____ $F_1 = F_2$

Justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 1** on this page.

PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

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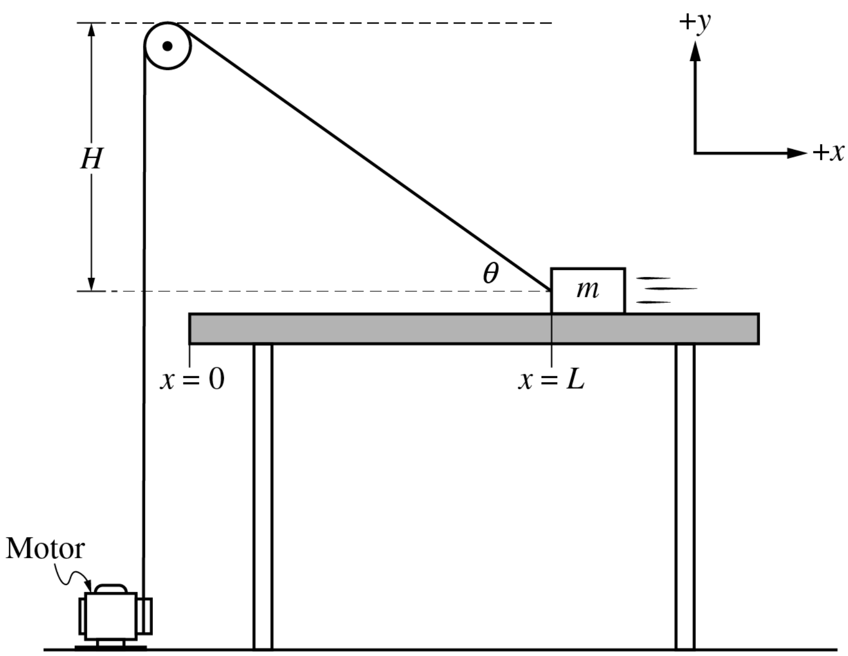
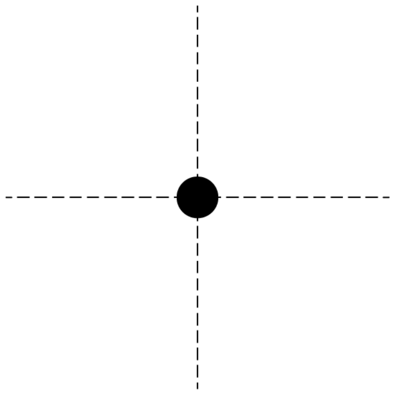


1. A block of mass m is pulled across a rough horizontal table by a string connected to a motor that is attached to the floor. The string passes over a pulley with negligible friction that is vertically aligned with the left edge of the table as shown. The string and pulley both have negligible mass. The pulley is at height H above the table. The motor exerts a constant force of tension F_T on the string, and the block remains in contact with the table at all times as the block slides across the table from $x = L$ to $x = 0$. The coefficient of kinetic friction between the table and the block is μ_k. Express all algebraic answers in terms of m, H, F_T, x, μ_k, L, and physical constants as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(a) On the dot below that represents the block, draw and label the forces (not components) that act on the block when the block is at $x = L$. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



(b) Derive an expression for the angle θ that the string makes with the horizontal as a function of x.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(c)

i. Derive an expression for the normal force F_N exerted on the block by the table as a function of the block's position x.

ii. Derive an expression for the magnitude of the net horizontal force F_{net} exerted on the block as a function of the position x.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(d) Write, but do not solve, an integral expression that could be used to solve for the work W done by the string on the block as the block moves from $x = L$ to $x = 0$.

(e) Does the string do more, less, or the same amount of work on the block as the block moves from $x = L$ to $x = \frac{L}{2}$ compared to when the block moves from $x = \frac{L}{2}$ to $x = 0$?

_______ More work when the block moves from $x = L$ to $x = \frac{L}{2}$

_______ Less work when the block moves from $x = L$ to $x = \frac{L}{2}$

_______ The same amount of work when the block moves from $x = L$ to $x = \frac{L}{2}$

Justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 1** on this page.

PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

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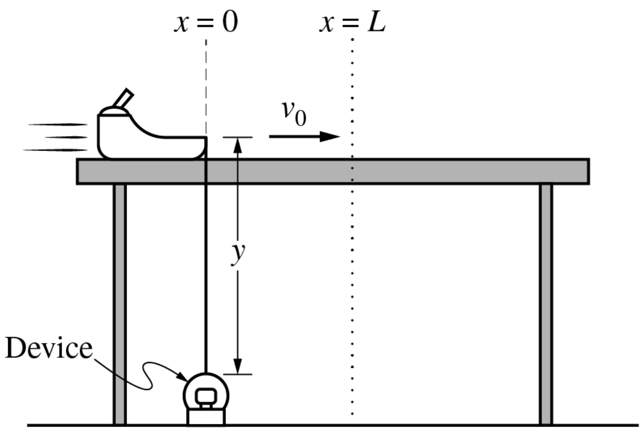


Figure 1

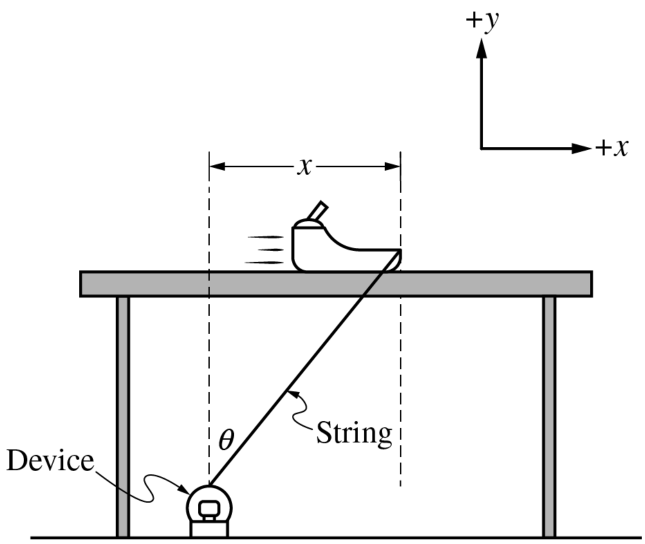


Figure 2

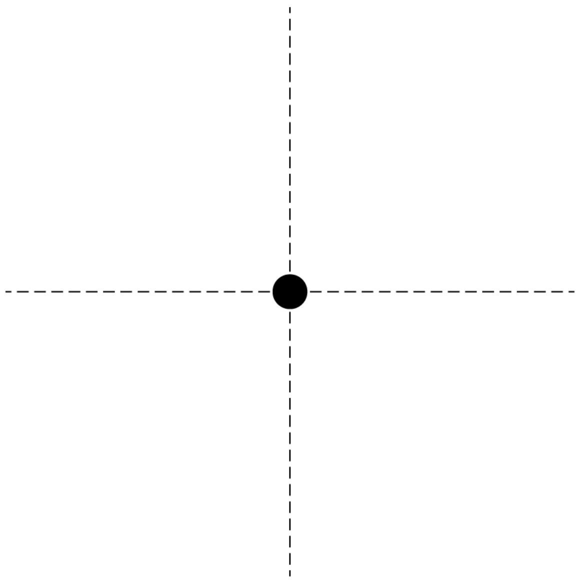
Note: Figures not drawn to scale.

1. A small sled slides across a rough horizontal table with an initial velocity v_0. The coefficient of kinetic friction between the sled and the table is μ_k. A string connects the sled to a device on the ground. The device maintains constant tension F_T in the string by unwinding the string as the sled slides to the right. The total mass of the sled is m. The string is attached to the device at $x = 0$ and at a height of y, as shown in Figure 1. The horizontal position of the sled is represented by x, as shown in Figure 2. Express all algebraic answers in terms of m, μ_k, F_T, x, y, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(a) On the dot below that represents the sled, draw and label the forces (not components) that are exerted on the sled a short time after $t = 0$ but before the sled has come to rest. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



(b) Determine an expression for the angle θ that the string makes with the vertical when the sled has traveled a horizontal distance x.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(c)

i. Derive an expression for the normal force F_N exerted on the sled by the table as a function of the position x.

ii. Derive an expression for the magnitude of the net horizontal force F_{net} exerted on the sled as a function of the position x.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(d) Derive an expression for the work W done by the string on the sled as the sled moves from $x = 0$ to $x = L$.

(e) The sled comes to rest after traveling a horizontal distance $x = 2L$. As the system slides from $x = 0$ to $x = L$, the energy dissipated by friction is E_1. As the sled slides from $x = L$ to $x = 2L$, the energy dissipated by friction is E_2. Is E_1 greater than, less than, or equal to E_2 ?

___ $E_1 > E_2$ ___ $E_1 < E_2$ ___ $E_1 = E_2$

Justify your answer.

GO ON TO THE NEXT PAGE.

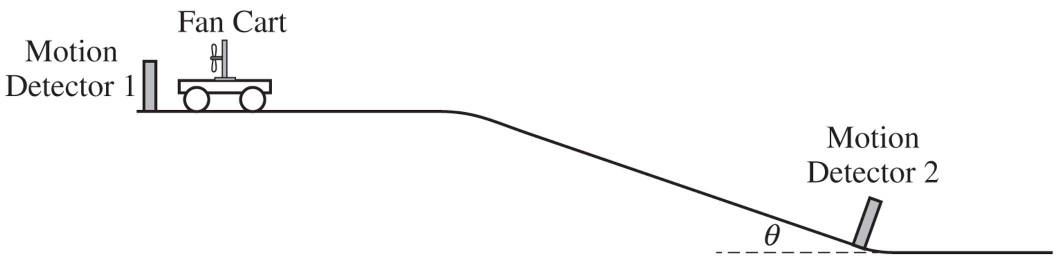
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

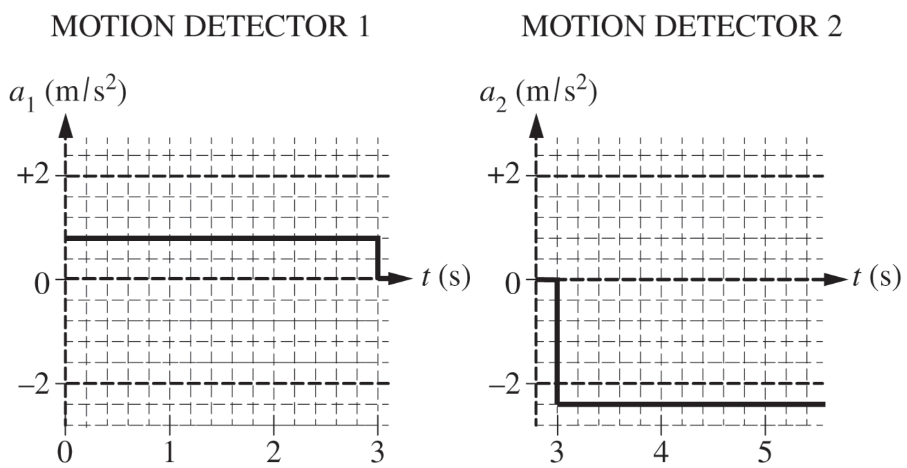
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

1. A 0.50 kg fan cart is placed on a level, horizontal track of negligible friction, as shown. The fan is turned on, and the fan cart is released from rest and moves to the right. The cart travels along the horizontal track and then down an incline. Motion detector 1 measures the acceleration a of the cart from time $t = 0$ to $t = 3$ s. At $t = 3$ s, the cart makes a smooth transition to the incline, and motion detector 2 measures the acceleration of the cart after $t = 3$ s. The fan exerts the same magnitude of force on the cart during the entire motion. The graphs below show a as functions of t. For each motion detector, the positive direction is away from the detector.



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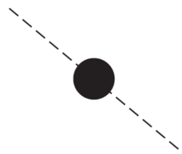
Continue your response to QUESTION 1 on this page.

(a) On the dots below that represent the cart at two different locations, draw and label the forces (not components) that act on the cart at each location. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

Cart on Horizontal Track

Cart on Incline



(b) Calculate the magnitude of the net force exerted on the fan cart when it is on the horizontal track.

(c) Calculate the angle θ of the incline.

(d) Suppose careful measurement determines the angle of the incline to be $3°$ larger than that calculated in part (c). Consider the following explanation.

"The scale used to measure the mass of the fan cart was not calibrated properly before the measurement, and this could account for the observed difference in the angle."

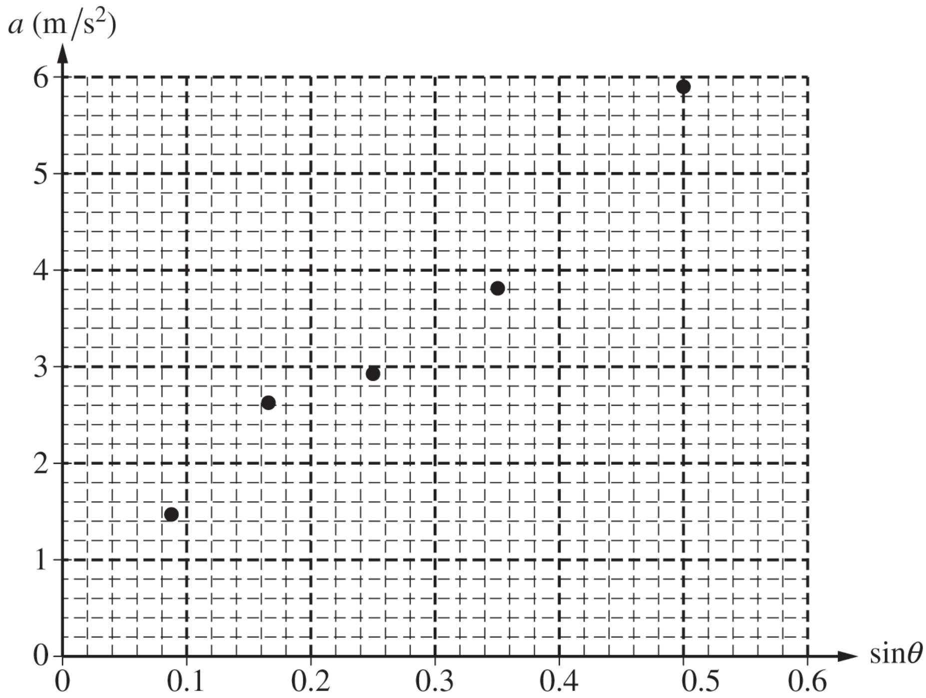
Does the explanation sufficiently account for the observed discrepancy?

_____ Yes　　　_____ No

Justify your answer.

GO ON TO THE NEXT PAGE.

The experiment is repeated for several trials, each with a different angle for the incline. The acceleration of the cart down the incline is measured for each angle. The graph below shows the plot of the acceleration a of the cart as a function of the sine of the angle $\sin\theta$.



(e)

i. Draw a best-fit line for the data.

ii. Using the straight line, calculate an experimental value for the acceleration due to gravity g.

(f) If the cart were replaced with a second cart of mass 1.0 kg that has a fan that exerts the same magnitude of force as the original fan, explain how the graph given in part (e) would change.

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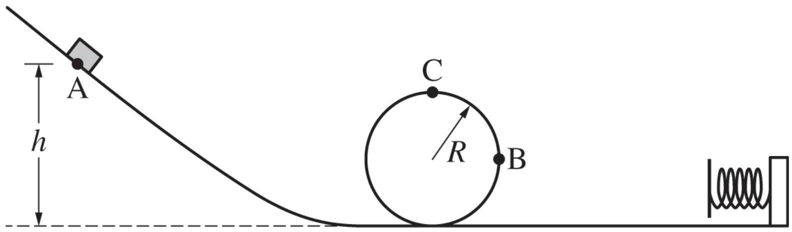
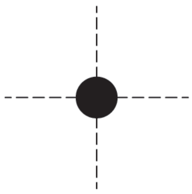


Note: Figure not drawn to scale.

2. A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \dfrac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

(a) On the dot below, which represents the block, draw an arrow that represents the direction of the acceleration of the block at point B in the figure above. The arrow must start on and point away from the dot.



Justify your answer.

(b)

i. Derive an expression for the speed v of the block at point B.

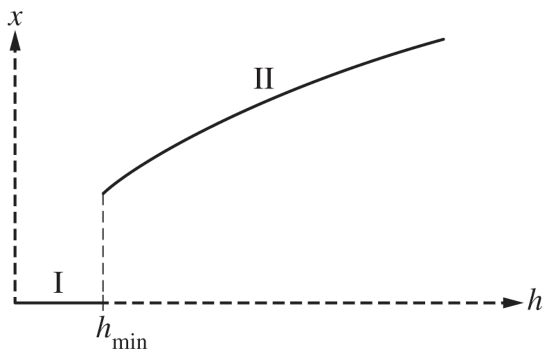
ii. Derive an expression for the magnitude of the net force F on the block at point B.

GO ON TO THE NEXT PAGE.

(c) In terms of R, derive an expression for the minimum height h_{min} necessary for the block to maintain contact with the track through point C.

(d) It is determined that $h = 0.30\,\text{m}$ and $R = 0.10\,\text{m}$. If the block is released from a height greater than that found in part (c), what would be the maximum compression x_{MAX} of the spring?

(e) A graph of the maximum compression of the spring as a function of height is shown below. The height h_{min} is the height calculated in part (c).



i. Explain why section I appears as a horizontal line segment on the horizontal axis.

ii. Explain the reason for the shape of section II on the graph.

GO ON TO THE NEXT PAGE.

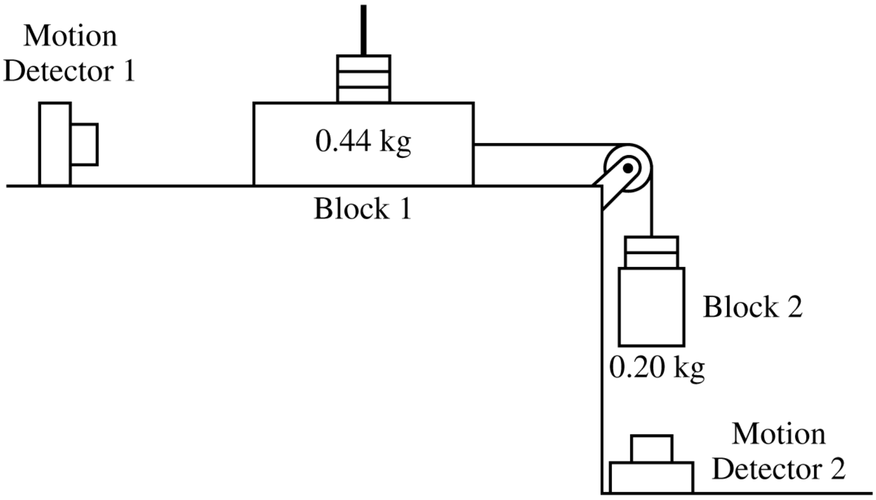
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



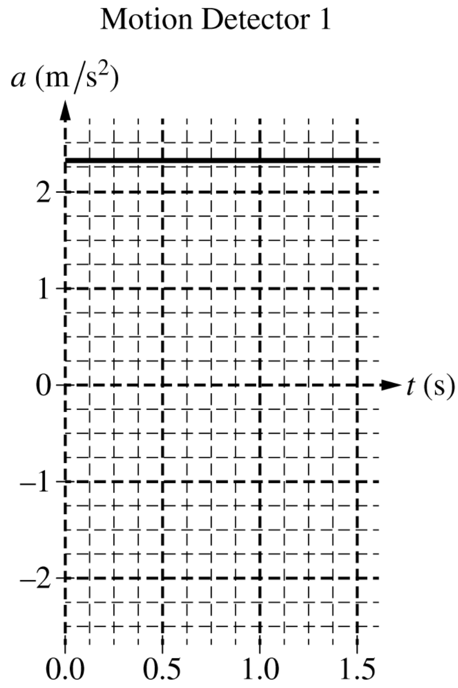
Students design an experiment using blocks of adjustable mass to investigate friction using the setup shown. Block 1 of initial mass 0.44 kg is placed on a rough horizontal surface and connected by a string to block 2 of initial mass 0.20 kg. The string extends over a pulley that has negligible mass and friction.

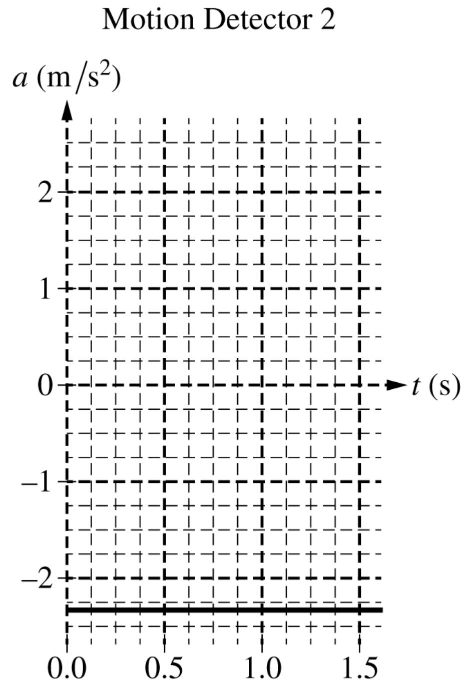
(a) Calculate the minimum value of the coefficient of static friction μ_s that would keep the two-block system at rest.

GO ON TO THE NEXT PAGE.

Continue your response to QUESTION 1 on this page.

The coefficient of friction is such that when block 2 is released from rest, block 1 travels across the surface. The acceleration a of each block is recorded with motion detectors 1 and 2, as shown in the figure. The data for the motion detectors as functions of time t are shown on the graphs. For each motion detector, the positive direction is away from the detector.





(b) On the dots below, which represent the blocks, draw and label the forces (not components) that act on each block. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

Block 1

Block 2

(c) Calculate the coefficient of kinetic friction μ_k between block 1 and the table.

GO ON TO THE NEXT PAGE.

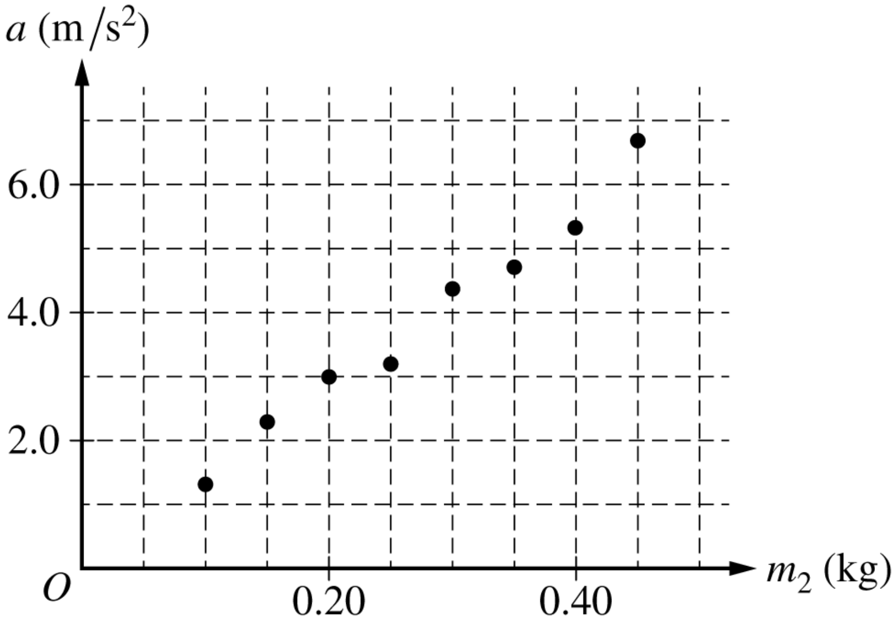
(d) Careful measurements determine that the coefficient of kinetic friction is larger than the value calculated in part (c). Does the following explanation sufficiently account for the observed discrepancy?

"The horizontal table was not perfectly level before the experiment was conducted. The observed difference in the angle accounts for the difference in the expected and calculated values of μ_k."

_____Yes   _____ No

Justify your answer.

The experiment is moved to a surface with negligible friction and run for eight trials. In each trial, the students vary the masses m_1 and m_2 of blocks 1 and 2, respectively, while keeping the total mass $(m_1 + m_2) = 0.64\,\text{kg}$ constant. The data for the acceleration a of block 1 as a function of m_2 are shown on the graph below.



(e)

i. Draw a best-fit line for the data points.

ii. Using the straight line, calculate an experimental value for the acceleration due to gravity g.

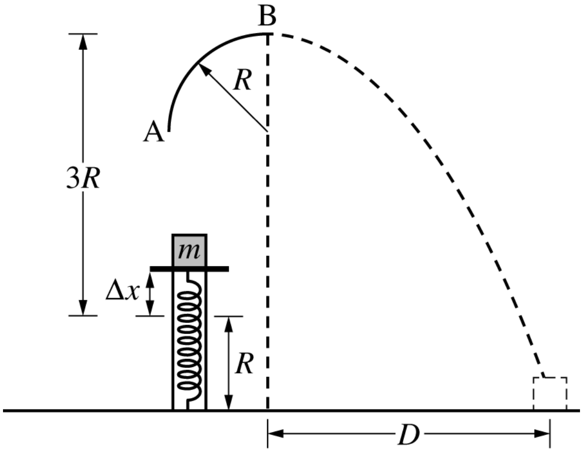
GO ON TO THE NEXT PAGE.

(f) The students lift the left end of the surface so that the surface is inclined at an angle to the horizontal, and the experiment for $m_2 = 0.20\,\text{kg}$ is repeated. Would the acceleration of the system be greater than, less than, or equal to the acceleration of the system in the original experiment?

_____ Greater than _____ Less than _____ Equal to

Justify your claim.

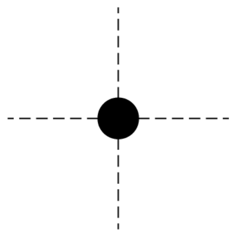
Begin your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

3. A block of mass m is placed on top of an ideal spring of spring constant k. The block is pushed against the spring, compressing the spring a distance Δx. The block is released from rest, leaves the spring at the position shown in the figure, travels upward, and enters a track with a constant radius of curvature R that has negligible friction. The block enters the track at point A, maintains contact with the track, and exits horizontally at point B, a distance $3R$ above the point the block was released. The block then falls to the ground and lands a horizontal distance D from the end of the track. Express all algebraic answers in terms of m, k, Δx, R, and physical constants, as appropriate. The size of the block is much smaller than the radius of curvature of the track.

(a) On the dot below, which represents the block, draw and label the forces (not components) that act on the block while still in contact with the track at point B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



Justify your choice of vectors.

GO ON TO THE NEXT PAGE.

(b)

i. Derive an expression for the speed v of the block at point B.

ii. Derive an expression for the magnitude of the net force F on the block at point B.

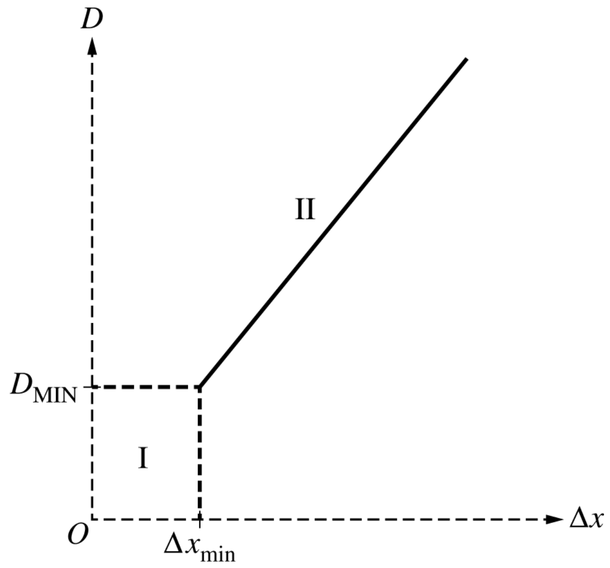
(c) Derive an expression for the minimum value of Δx_{min} required in order for the block to maintain contact with the track through point B.

The procedure is repeated several times with the distance $\Delta x > \Delta x_{\text{min}}$.

(d) Calculate the distance D that the block travels.

GO ON TO THE NEXT PAGE.

(e) The graph below shows the best-fit line drawn by the students through their data of D as a function of Δx.



i. Explain why there are no data for section I of the graph.

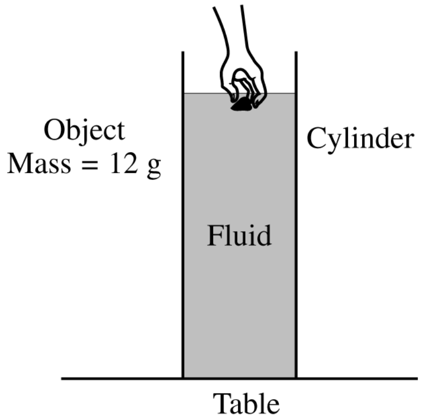
ii. Explain the reason for the shape and minimum value of section II on the graph.

STOP

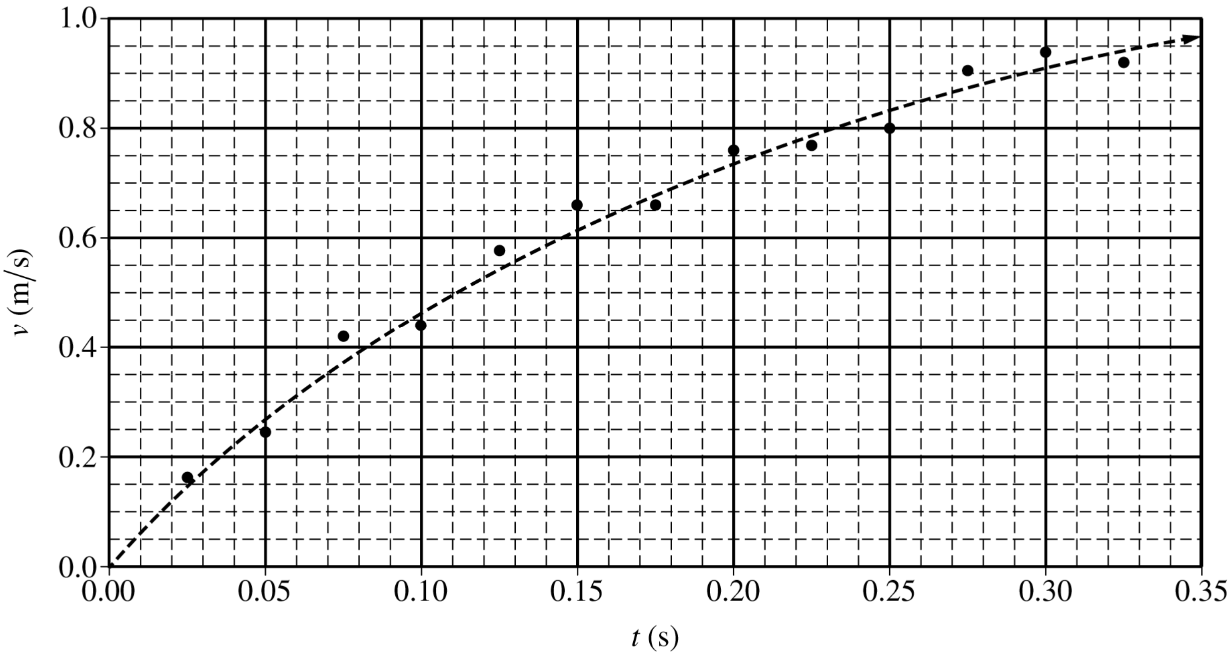
END OF EXAM

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of m = 12 g is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.



(a)

i. Does the speed of the object increase, decrease, or remain the same?

____ Increase ____ Decrease ____ Remain the same

ii. In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.

iii. Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20$ s.

The students use the equation $v = A\left(1 - e^{-Bt}\right)$ to model the speed of the falling object and find the best fit coefficients to be $A = 1.18\ \text{m/s}$ and $B = 5\ \text{s}^{-1}$.

(b) Use the above equation to:

i. Derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time t.

ii. Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time t.

The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed.

(c)

i. Determine the constant speed of the object.

Justify your answer.

ii. Determine the force exerted by the fluid on the object at this time.

Justify your answer.

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

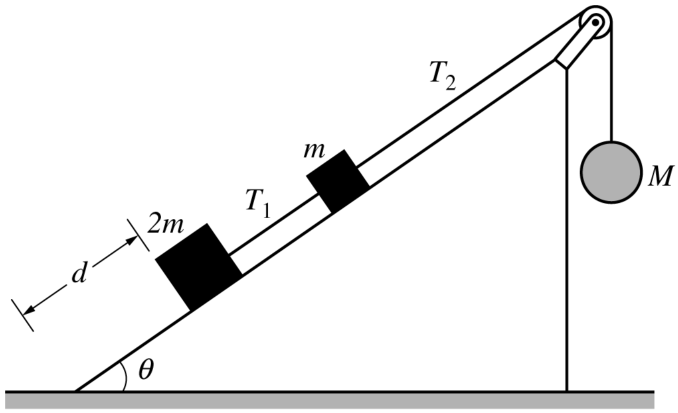


Figure 1

1. Blocks of mass m and $2m$ are connected by a light string and placed on a frictionless inclined plane that makes an angle θ with the horizontal, as shown in Figure 1 above. Another light string connecting the block of mass m to a hanging sphere of mass M passes over a pulley of negligible mass and negligible friction. The entire system is initially at rest and in equilibrium.

   (a) On the dots below that represent the block of mass m and the sphere of mass M, draw and label the forces (not components) that act on each of the objects shown. Each force must be represented by a distinct arrow starting on and pointing away from the dot.

(b) Derive expressions for the magnitude of each of the following. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figures in part (a).

i. The force T_2 exerted on the block of mass m by the string. Express your answers in terms of m, θ, and physical constants, as appropriate.

ii. The mass M for which the system can remain in equilibrium. Express your answers in terms of m, θ, and physical constants, as appropriate.

(c) Now suppose that mass M is large enough to descend and that the sphere reaches the floor before the blocks reach the pulley. Answer the following for the moment immediately after the sphere reaches the floor.

i. Does the tension T_1 increase, decrease to a nonzero value, decrease to zero, or stay the same?

____ Increase ____ Decrease to a nonzero value

____ Decrease to zero ____ Stay the same

ii. Is the velocity of the block of mass m up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

iii. Is the acceleration of the block of mass m up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

(d) Consider the initial setup in Figure 1. Now suppose the surface of the incline is rough and the coefficient of static friction between the blocks and the inclined plane is μ_s. Derive an expression for the minimum possible value of M that will keep the blocks from moving down the incline. Express your answer in terms of m, μ_s, θ, and fundamental constants, as appropriate.

(e) The string connecting block m and the sphere of mass M then breaks, and the blocks begin to move from rest down the incline. The lower block starts a distance d from the bottom of the incline, as shown in Figure 1. The coefficient of kinetic friction between the blocks and the inclined plane is μ_k. Derive an expression for the speed of the blocks when the lower block reaches the bottom of the incline. Express your answer in terms of m, d, μ_k, θ, and fundamental constants, as appropriate.

2.6 Gravitational Force

Name: ______________ Class: __________ Date: ____________ **Total: 15 marks**

KEY WORDS weight 重量 • force 力

Objective

Build the skills to answer exam questions on **gravitational force**.

You must be able to:

- find **weight 重量** near Earth's surface: $W = mg$
- use **Newton's law of universal gravitation 万有引力定律**: $F = \dfrac{GMm}{r^2}$
- explain that gravity is an inverse-square force
- distinguish mass (constant) from weight (depends on g)

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Weight near a surface

$$W = mg.$$

A 5 kg mass on Earth ($g = 9.8$ m s^{-2}) weighs $W = 5 \times 9.8 = 49$ N.

■ Universal gravitation

Between any two masses,

$$F = \frac{GMm}{r^2},$$

where r is the centre-to-centre distance. Doubling r quarters the force (inverse square).

■ Mass versus weight

Mass is the same everywhere; weight changes with g. On the Moon ($g \approx 1.6$ m s^{-2}) the 5 kg mass weighs only 8 N, but its mass is still 5 kg.

2 Practice

Now apply the methods above.

2.1 Find the weight of a 12 kg object on Earth ($g = 9.8$ m s^{-2}). [2]

2.2 In $F = GMm/r^2$, what happens to the force if r doubles? [1]

2.3 State one quantity that stays the same and one that changes when you travel from Earth to the Moon. [2]

2.4 A 2 kg mass weighs 3.2 N on the Moon. Find the Moon's g. [2]

3 Exam-style questions

3.1 The gravitational force between two masses is proportional to [1]

- **A** r
- **B** $1/r$
- **C** $1/r^2$
- **D** r^2

3.2 An astronaut's mass on the Moon compared with on Earth is [1]

- **A** smaller
- **B** larger
- **C** the same
- **D** zero

3.3 Two satellites orbit a planet. Satellite B is twice as far from the centre as satellite A.

(a) State how the gravitational force on B compares with that on A (same mass). [1]

(b) Explain using the inverse-square law. [2]

3.4 A rock has a mass of 3 kg.

(a) Find its weight on Earth ($g = 9.8$ m s^{-2}). [1]

(b) State its weight and mass on a planet where $g = 20$ m s^{-2}. [2]

Solutions

2.1 $W = mg = 12 \times 9.8 = 118$ N.

2.2 It falls to one quarter (inverse square).

2.3 Mass stays the same; weight changes (because g changes).

2.4 $g = W/m = 3.2/2 = 1.6$ m s^{-2}.

3.1 C — gravity follows an inverse-square law.

3.2 C — mass is the same everywhere.

3.3 (a) The force on B is one quarter of that on A. (b) Doubling r makes $1/r^2$ one quarter, so the force is quartered.

3.4 (a) $W = 3 \times 9.8 = 29.4$ N. (b) Mass still 3 kg; weight $= 3 \times 20 = 60$ N.

Question 3

3. A box is connected to one end of a rigid rod. Both the box and the rod have negligible mass. The other end of the rod is connected to a pivot. The box is open on one side, and a block is placed inside the box.

The center of mass of the block is displaced a vertical distance h, as shown in Figure 1. The block-box system is then released from rest and swings downward. There is negligible friction about the pivot. When the system is at the lowest point of its swing, the rod collides with a rigid stopper, as shown in Figure 2. The box comes to rest, and the block is launched horizontally out of the box. The block moves across a horizontal surface toward a motion sensor that measures the speed of the block. All frictional forces are negligible.

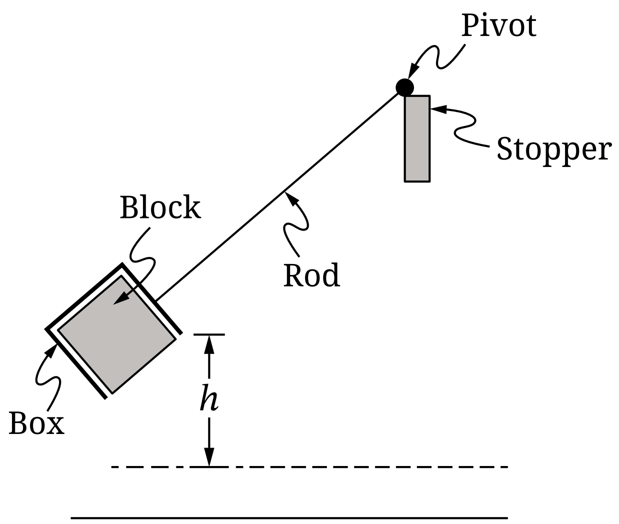


Figure 1

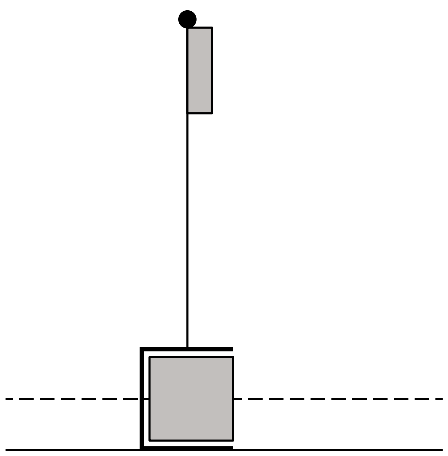


Figure 2

A. Students are asked to experimentally determine the acceleration due to gravity g using a linear graph. To determine g, the students are permitted to use measurements from only a meterstick and the motion sensor.

Describe an experimental procedure using the described setup to collect data that would allow the students to determine an experimental value of g using a linear graph. Include any steps necessary to reduce experimental uncertainty.

B. **Describe** how the data collected in part A could be graphed and how that graph would be analyzed to determine the value of g.

C. The experiment is repeated, but the horizontal surface on which the block slides is replaced with a new rough surface, as shown in Figure 3. The coefficient of kinetic friction between the block and the new surface is μ.

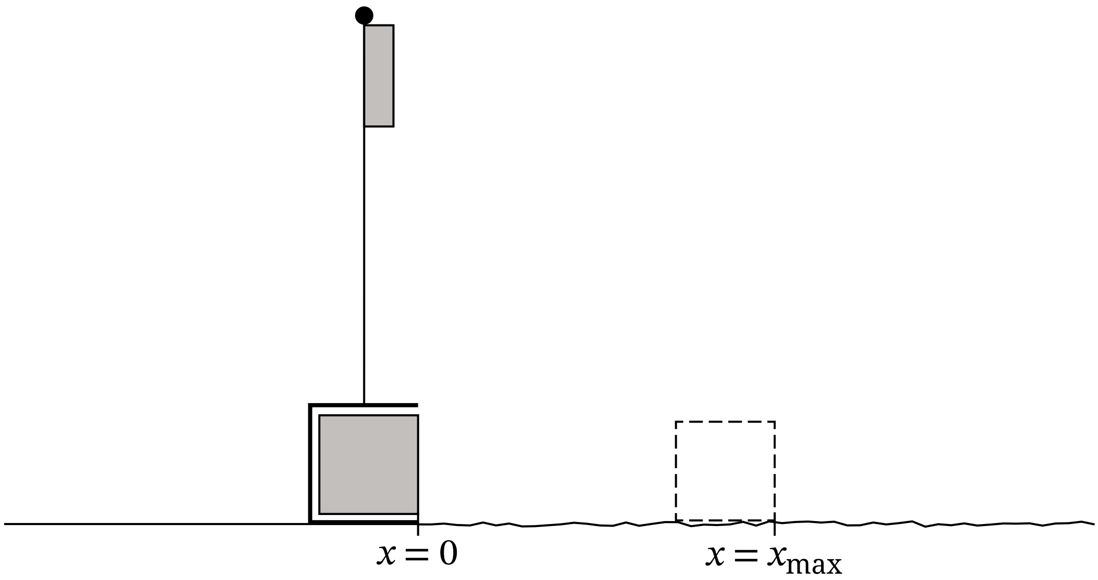


Figure 3

The block-box system is pulled aside so that the center of mass of the block is displaced various vertical distances h and then released from rest. For each vertical distance, students measure the position $x = x_{max}$ at which the block comes to rest.

The students' measurements of h and x_{max} are shown in Table 1.

Table 1

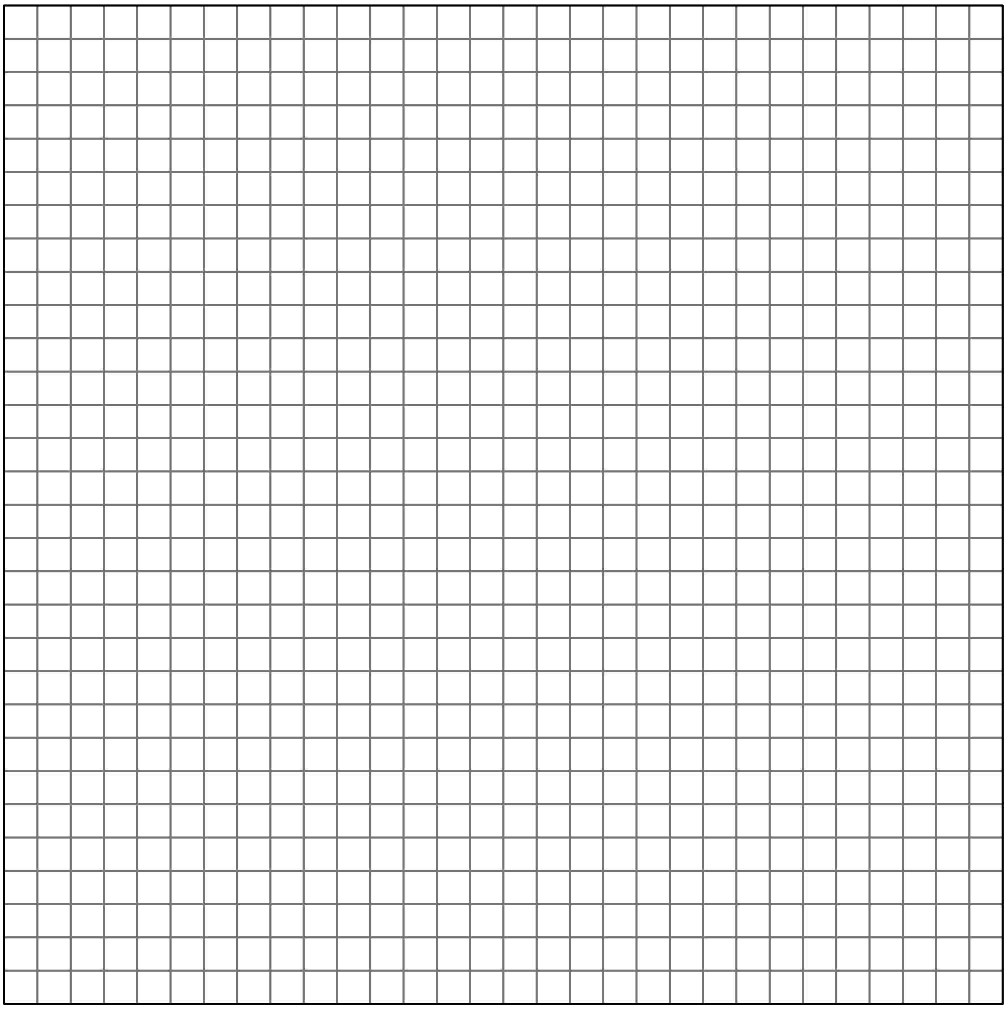
h (m)	x_{max} (m)
0.30	0.76
0.45	1.10
0.60	1.40
0.75	1.90
0.90	2.30

i. **Indicate** two quantities, either measured quantities from Table 1 or additional calculated quantities, that could be graphed to produce a straight line that could be used to determine μ.

Vertical axis: ______ Horizontal axis: ______

ii. On the grid provided, create a graph of the quantities indicated in part C (i).

- Use Table 2 to record the measured or calculated quantities that you will plot.
- Clearly **label** the axes, including units as appropriate.
- **Plot** the points you recorded in Table 2.



iii. **Draw** a best-fit line to the data graphed in part C (ii).

D. Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for μ.

PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

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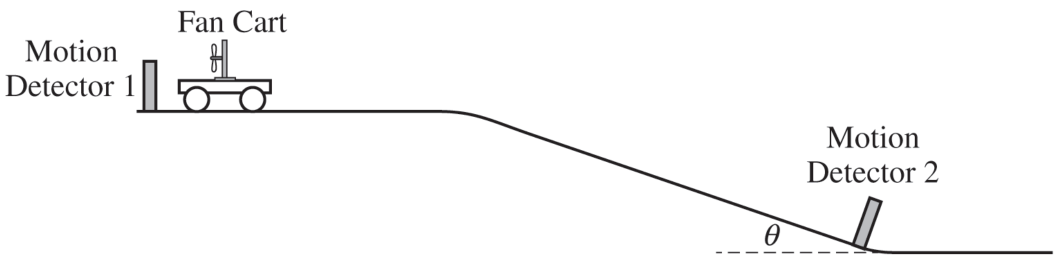
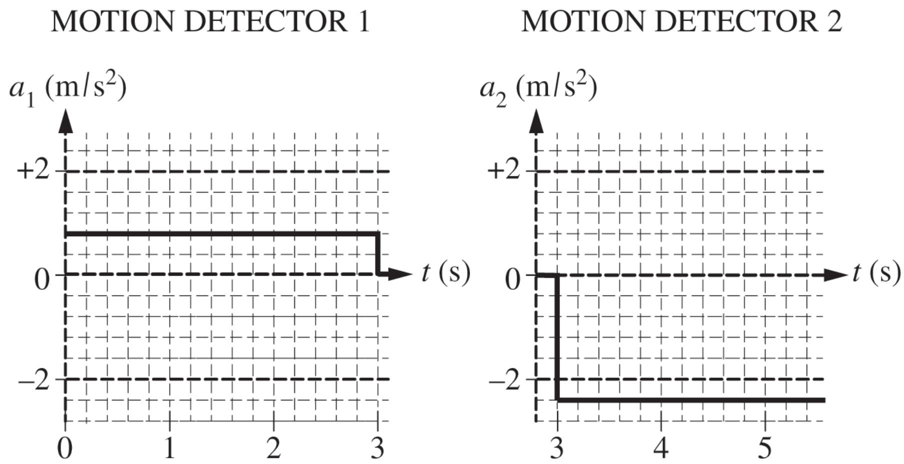


Note: Figure not drawn to scale.

1. A 0.50 kg fan cart is placed on a level, horizontal track of negligible friction, as shown. The fan is turned on, and the fan cart is released from rest and moves to the right. The cart travels along the horizontal track and then down an incline. Motion detector 1 measures the acceleration a of the cart from time $t = 0$ to $t = 3$ s. At $t = 3$ s, the cart makes a smooth transition to the incline, and motion detector 2 measures the acceleration of the cart after $t = 3$ s. The fan exerts the same magnitude of force on the cart during the entire motion. The graphs below show a as functions of t. For each motion detector, the positive direction is away from the detector.



Continue your response to **QUESTION 1** on this page.

(a) On the dots below that represent the cart at two different locations, draw and label the forces (not components) that act on the cart at each location. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

Cart on Horizontal Track　　　　Cart on Incline

(b) Calculate the magnitude of the net force exerted on the fan cart when it is on the horizontal track.

(c) Calculate the angle θ of the incline.

(d) Suppose careful measurement determines the angle of the incline to be $3°$ larger than that calculated in part (c). Consider the following explanation.

“The scale used to measure the mass of the fan cart was not calibrated properly before the measurement, and this could account for the observed difference in the angle.”

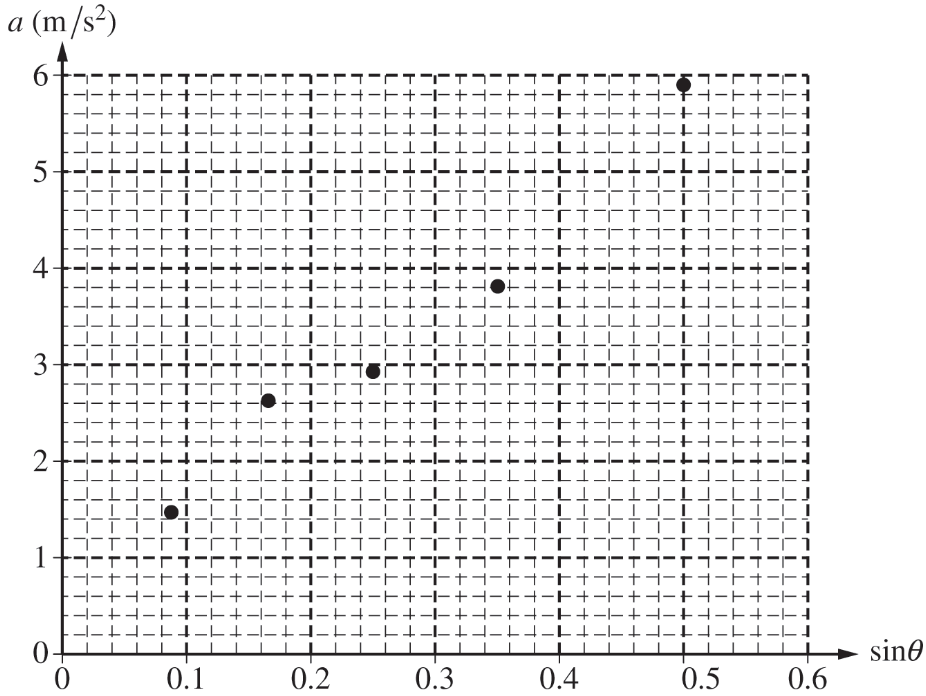
Does the explanation sufficiently account for the observed discrepancy?

______ Yes　　　______ No

Justify your answer.

GO ON TO THE NEXT PAGE.

The experiment is repeated for several trials, each with a different angle for the incline. The acceleration of the cart down the incline is measured for each angle. The graph below shows the plot of the acceleration a of the cart as a function of the sine of the angle $\sin\theta$.



(e)

i. Draw a best-fit line for the data.

ii. Using the straight line, calculate an experimental value for the acceleration due to gravity g.

(f) If the cart were replaced with a second cart of mass 1.0 kg that has a fan that exerts the same magnitude of force as the original fan, explain how the graph given in part (e) would change.

GO ON TO THE NEXT PAGE.

PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

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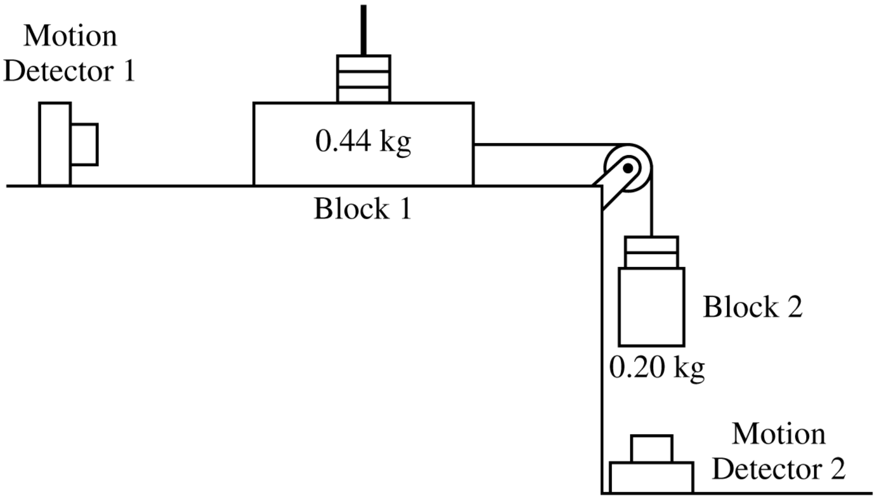
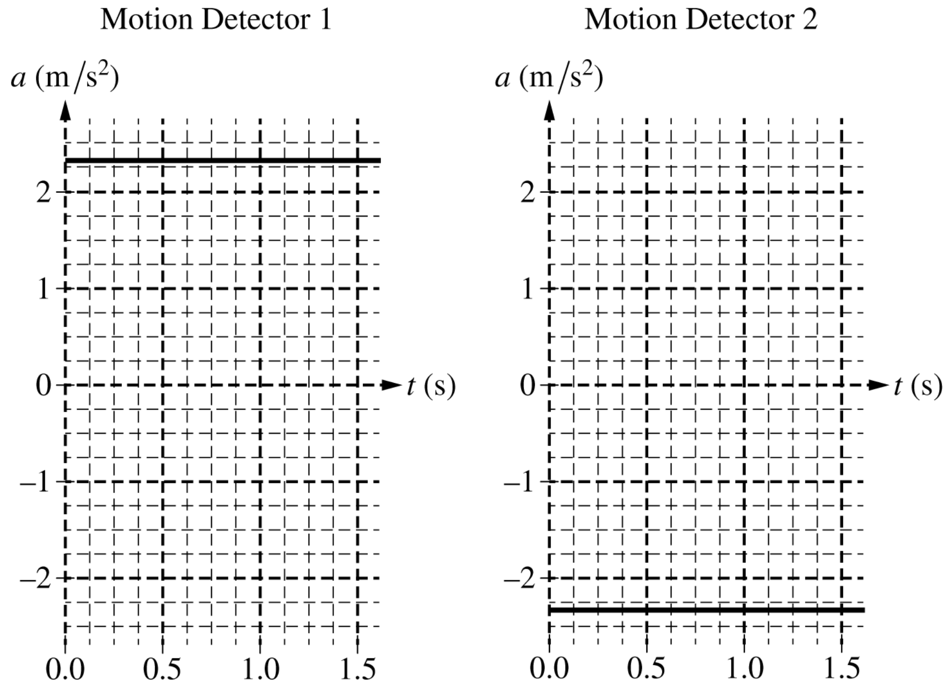


Students design an experiment using blocks of adjustable mass to investigate friction using the setup shown. Block 1 of initial mass 0.44 kg is placed on a rough horizontal surface and connected by a string to block 2 of initial mass 0.20 kg. The string extends over a pulley that has negligible mass and friction.

(a) Calculate the minimum value of the coefficient of static friction μ_s that would keep the two-block system at rest.

GO ON TO THE NEXT PAGE.

The coefficient of friction is such that when block 2 is released from rest, block 1 travels across the surface. The acceleration a of each block is recorded with motion detectors 1 and 2, as shown in the figure. The data for the motion detectors as functions of time t are shown on the graphs. For each motion detector, the positive direction is away from the detector.



(b) On the dots below, which represent the blocks, draw and label the forces (not components) that act on each block. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

(c) Calculate the coefficient of kinetic friction μ_k between block 1 and the table.

GO ON TO THE NEXT PAGE.

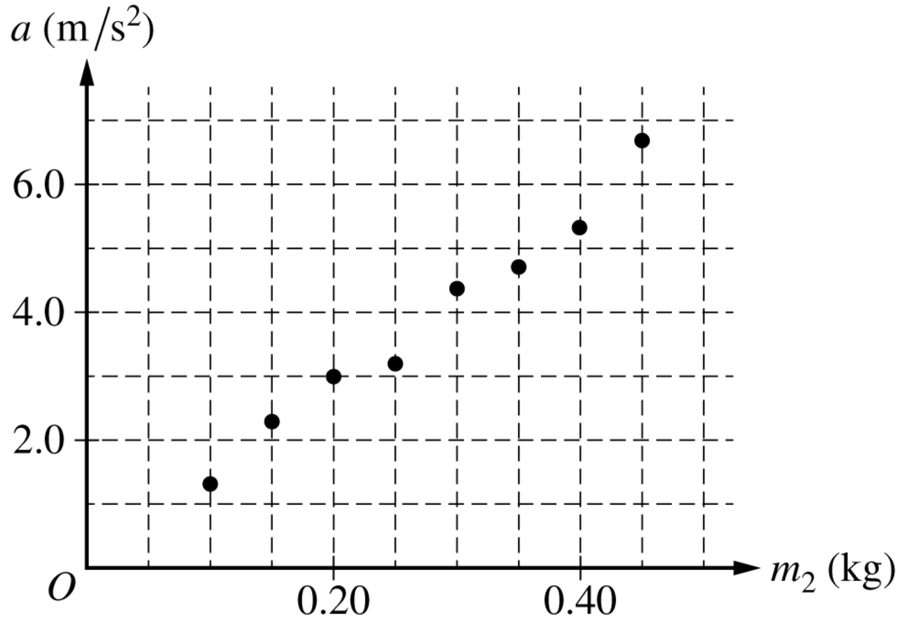
(d) Careful measurements determine that the coefficient of kinetic friction is larger than the value calculated in part (c). Does the following explanation sufficiently account for the observed discrepancy?

"The horizontal table was not perfectly level before the experiment was conducted. The observed difference in the angle accounts for the difference in the expected and calculated values of μ_k."

_____Yes _____ No

Justify your answer.

The experiment is moved to a surface with negligible friction and run for eight trials. In each trial, the students vary the masses m_1 and m_2 of blocks 1 and 2, respectively, while keeping the total mass $(m_1 + m_2) = 0.64\ \text{kg}$ constant. The data for the acceleration a of block 1 as a function of m_2 are shown on the graph below.



(e)

i. Draw a best-fit line for the data points.

ii. Using the straight line, calculate an experimental value for the acceleration due to gravity g.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(f) The students lift the left end of the surface so that the surface is inclined at an angle to the horizontal, and the experiment for $m_2 = 0.20\,\text{kg}$ is repeated. Would the acceleration of the system be greater than, less than, or equal to the acceleration of the system in the original experiment?

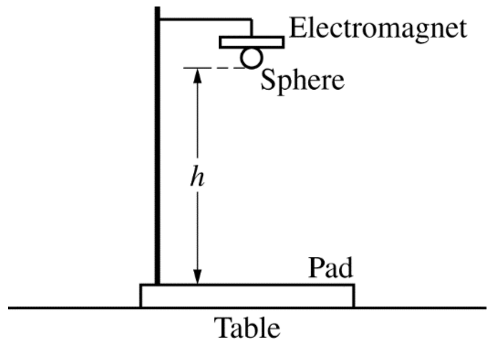
_____ Greater than   _____ Less than   _____ Equal to

Justify your claim.

GO ON TO THE NEXT PAGE.

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

(a) While taking the first data point, the student notices that the electromagnet actually releases the sphere after the timer begins. Would the value of g calculated from this one measurement be greater than, less than, or equal to the actual value of g at the student's location?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

The electromagnet is replaced so that the timer begins when the sphere is released. The student varies the distance h. The student measures and records the time Δt of the fall for each particular height, resulting in the following data table.

h (m)	0.10	0.20	0.60	0.80	1.00
Δt (s)	0.105	0.213	0.342	0.401	0.451

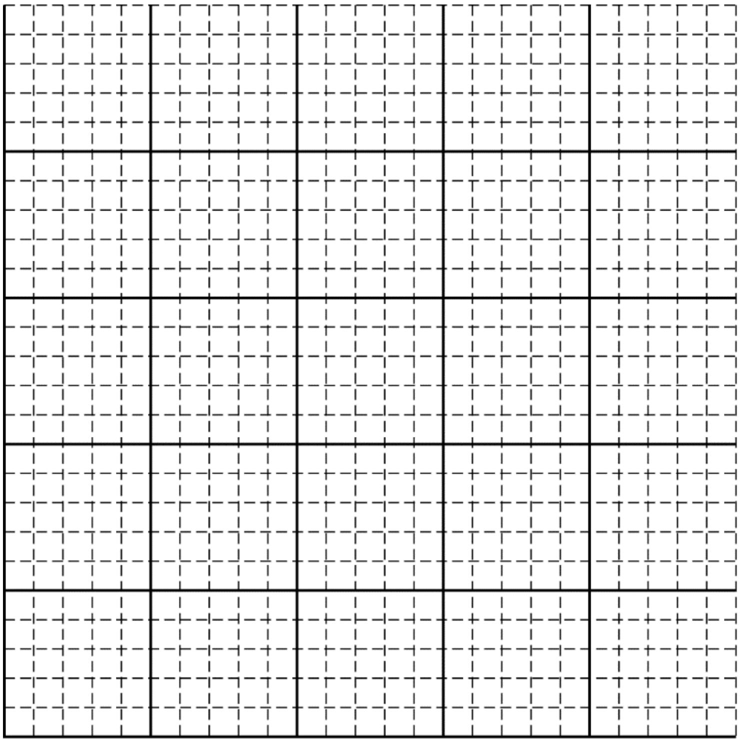
(b) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for g.

Vertical axis: ____________

Horizontal axis: ____________

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given in the table. Label each row you use and include units.

(c) Plot the data points for the quantities indicated in part (b) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(d) Using the straight line, calculate an experimental value for g.

Another student fits the data in the table to a quadratic equation. The student's equation for the distance fallen y as a function of time t is $y = At^2 + Bt + C$, where $A = 5.75\ \text{m/s}^2$, $B = -0.524\ \text{m/s}$, and $C = +0.080\ \text{m}$. Vertically down is the positive direction.

(e) Using the student's equation above, do the following.

i. Derive an expression for the velocity of the sphere as a function of time.

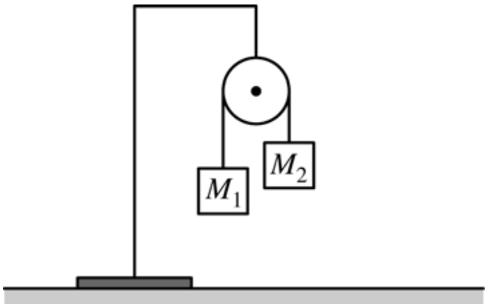
ii. Calculate the new experimental value for g.

iii. Using $9.81\ \text{m/s}^2$ as the accepted value for g at this location, calculate the percent error for the value found in part (e)ii.

iv. Assuming the sphere is at a height of 1.40 m at $t = 0$, calculate the velocity of the sphere just before it strikes the pad.

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

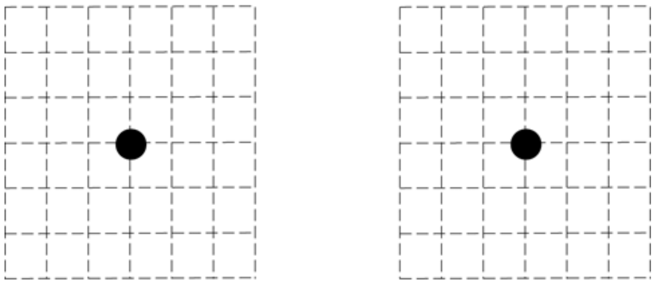
Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. An Atwood's machine consists of two blocks connected by a light string that passes over a frictionless pulley of negligible mass, as shown in the figure above. The masses of the two blocks, M_1 and M_2, can be varied. M_2 is always greater than M_1.

   (a) On the dots below, which represent the blocks, draw and label the forces (not components) that act on the blocks. Each force must be represented by a distinct arrow starting on and pointing away from the appropriate dot. The relative lengths of the arrows should show the relative magnitudes of the forces.

Block of Mass M_1 | Block of Mass M_2



   (b) Using the forces in your diagrams above, write an equation applying Newton's second law to each block and use these two equations to derive the magnitude of the acceleration of the blocks and show that it is given by the equation: $a = \dfrac{(M_2 - M_1)}{(M_1 + M_2)}g$

The magnitude of the acceleration a was measured for different values of M_1 and M_2, and the data are shown below.

M_1 (kg)	1.0	2.0	5.0	6.0	10.0
M_2 (kg)	2.0	3.0	12.0	8.0	14.0
a (m/s^2)	3.02	1.82	4.21	1.15	1.71

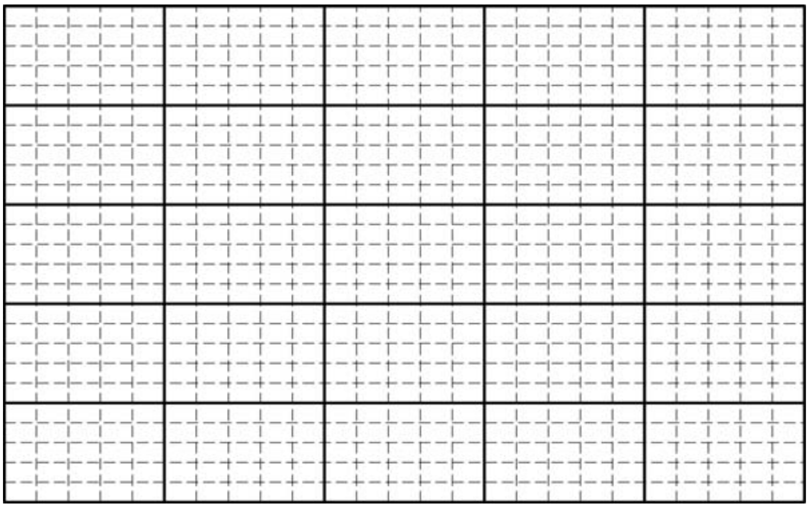
(c) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the acceleration due to gravity g.

Vertical axis: ____________________

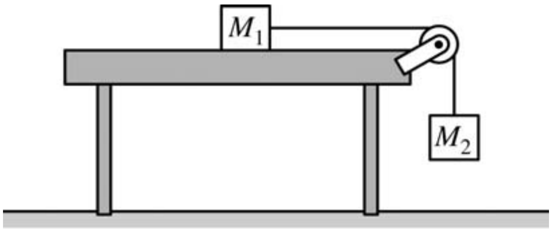
Horizontal axis: ____________________

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given.

(d) Plot the data points for the quantities indicated in part (c) on the graph below. Clearly scale and label all axes including units, if appropriate. Draw a straight line that best represents the data.



(e) Using your straight line, determine an experimental value for g.



The experiment is now repeated with a modification. The Atwood's machine is now set up so that the block of mass M_1 is on a smooth, horizontal table and the block of mass M_2 is hanging over the side of the table, as shown in the figure above.

(f) For the same values of M_1 and M_2, is the magnitude of the tension in the string when the blocks are moving higher, lower, or equal to the magnitude of the tension in the string when the blocks are moving in the first experiment?

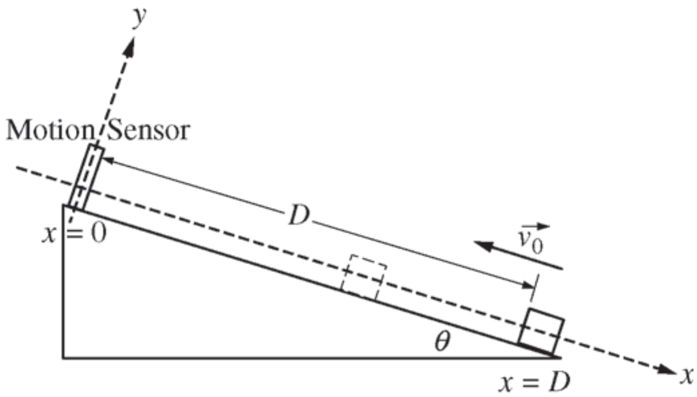
____ Higher ____ Lower ____ Equal to

Justify your answer.

(g) The value determined for the acceleration due to gravity g is lower than in the first experiment. Give one physical factor that could account for this lower value and explain how this factor affected the experiment.

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Mech.1.

A block of mass m is projected up from the bottom of an inclined ramp with an initial velocity of magnitude v_0. The ramp has negligible friction and makes an angle θ with the horizontal. A motion sensor aimed down the ramp is mounted at the top of the incline so that the positive direction is down the ramp. The block starts a distance D from the motion sensor, as shown above. The block slides partway up the ramp, stops before reaching the sensor, and then slides back down.

(a) Consider the motion of the block at some time t after it has been projected up the ramp. Express your answers in terms of m, D, v_0, t, θ and physical constants, as appropriate.

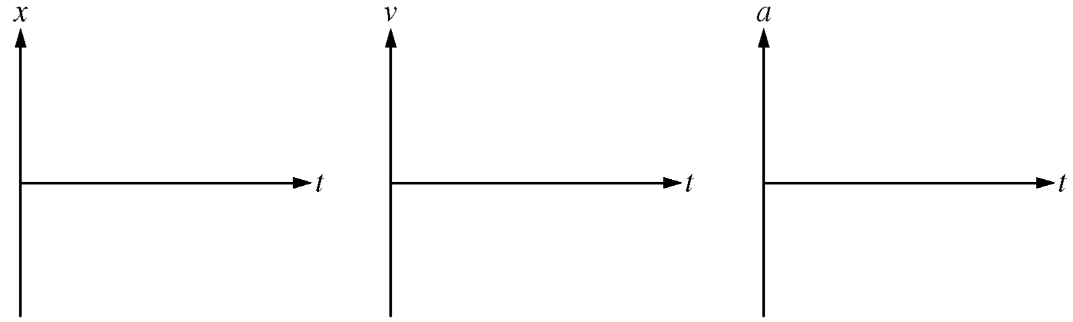
i. Determine the acceleration a of the block.

ii. Determine an expression for the velocity v of the block.

iii. Determine an expression for the position x of the block.

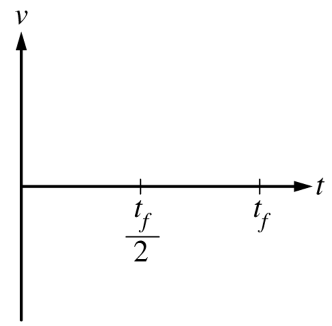
(b) Derive an expression for the position x_{min} of the block when it is closest to the motion sensor. Express your answer in terms of m, D, v_0, θ, and physical constants, as appropriate.

(c) On the axes provided below, sketch graphs of position x, velocity v, and acceleration a as functions of time t for the motion of the block while it goes up and back down the ramp. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



(d) After the block slides back down and leaves the bottom of the ramp, it slides on a horizontal surface with a coefficient of friction given by μ_k. Derive an expression for the distance the block slides before stopping. Express your answer in terms of m, D, v_0, θ, μ_k, and physical constants, as appropriate.

(e) Suppose the ramp now has friction. The same block is projected up with the same initial speed v_0 and comes back down the ramp. On the axes provided below, sketch a graph of the velocity v as a function of time t for the motion of the block while it goes up and back down the ramp, arriving at the bottom of the ramp at time t_f. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



2.7 Kinetic and Static Friction

Name: ______________ Class: __________ Date: ____________ **Total: 15 marks**

KEY WORDS kinetic 动摩擦 • static 静摩擦 • normal force 法向力 • friction 摩擦力 • force 力

Objective

Build the skills to answer exam questions on **kinetic and static friction**.

You must be able to:

- use **kinetic friction 动摩擦**: $f_k = \mu_k N$
- use the **static friction 静摩擦** limit: $f_s \leq \mu_s N$
- explain that static friction adjusts up to a maximum before slipping
- find the normal force N first, then the friction

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Kinetic friction

Once sliding, $f_k = \mu_k N$. A 10 kg block on the flat ($N = mg = 100$ N) with $\mu_k = 0.3$ feels

$$f_k = 0.3 \times 100 = 30 \text{ N},$$

opposing its motion.

■ Static friction adjusts

Before sliding, static friction matches the applied force up to a maximum $f_{s,\max} = \mu_s N$. Push with 20 N on a block whose $f_{s,\max} = 40$ N and it does not move – static friction is exactly 20 N.

■ Find the normal force first

On a flat surface $N = mg$; on an incline $N = mg\cos\theta$. Always find N, then multiply by μ.

2 Practice

Now apply the methods above.

2.1 A block with $N = 50$ N slides with $\mu_k = 0.2$. Find the kinetic friction. [2]

2.2 State the direction of the friction force on a sliding block. [1]

2.3 A block has $f_{s,\max} = 30$ N. You push with 25 N and it stays still. State the static friction force. [1]

2.4 A 4 kg block on the flat has $\mu_k = 0.25$ ($g = 10$ m s^{-2}). Find the kinetic friction while it slides. [2]

3 Exam-style questions

3.1 Kinetic friction on a sliding object is given by [1]

- **A** $\mu_k N$
- **B** μ_k / N
- **C** N / μ_k
- **D** $\mu_k m$

3.2 Static friction [1]

- **A** is always at its maximum
- **B** adjusts up to a maximum, then the object slips
- **C** is always larger than kinetic friction while sliding
- **D** points in the direction of motion

3.3 A 6 kg box on the flat has $\mu_s = 0.5$ and $\mu_k = 0.4$ ($g = 10$ m s^{-2}).

(a) Find the maximum static friction. [2]

(b) Find the kinetic friction once it slides. [1]

3.4 A 5 kg block is pushed along the flat by 30 N with $\mu_k = 0.2$ ($g = 10$ m s^{-2}).

(a) Find the kinetic friction. [2]

(b) Find the net force and the acceleration. [2]

Solutions

2.1 $f_k = \mu_k N = 0.2 \times 50 = 10$ N.

2.2 Opposite to the motion.

2.3 25 N (static friction matches the applied force).

2.4 $N = mg = 40$ N; $f_k = 0.25 \times 40 = 10$ N.

3.1 **A** — $f_k = \mu_k N$.

3.2 **B** — static friction adjusts up to a maximum, then slipping begins.

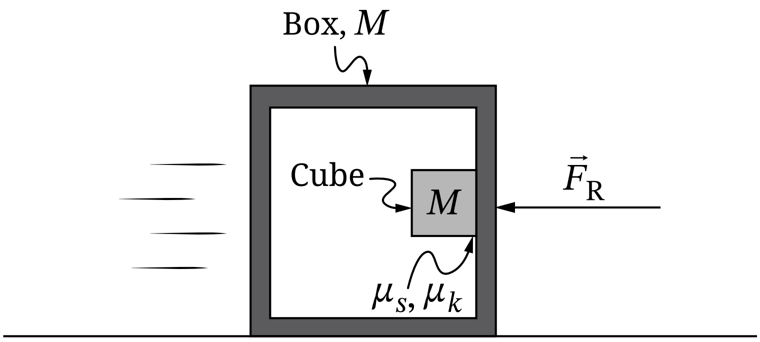
3.3 (a) $N = 60$ N, $f_{s,\max} = 0.5 \times 60 = 30$ N. (b) $f_k = 0.4 \times 60 = 24$ N.

3.4 (a) $N = 50$ N, $f_k = 0.2 \times 50 = 10$ N. (b) $F_{net} = 30 - 10 = 20$ N; $a = 20/5 = 4$ m s^{-2}.

Question 1: Version J

1. A box of mass M slides to the right on a horizontal surface. A small cube, also of mass M, is inside the box. The cube is against the right wall of the box and above the bottom of the box, as shown in Figure 1. The coefficients of static and kinetic friction between the cube and the wall of the box are μ_s and μ_k, respectively, where $\mu_s > \mu_k$.

Figure 1



The speed of the box is decreasing as a result of a resistive force that is exerted on the box. The resistive force $\vec{F}_R$ is modeled by $\vec{F}_R = -b\vec{v}$, where b is a positive constant and $\vec{v}$ is the velocity of the box. Friction between the box and horizontal surface is negligible.

Consider the motion of the box and cube for the following times.

- At time $t = 0$, the box-cube system slides to the right with speed v_0.
- For $0 \le t < t_{crit}$, the cube remains in contact with the wall of the box at a constant height above the bottom of the box.
- At $t = t_{crit}$, the cube begins to slide downward along the wall of the box.

Part A

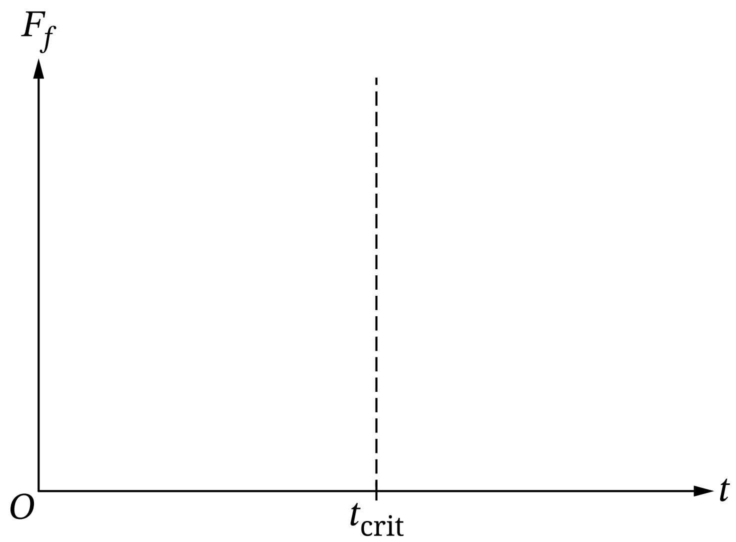
The magnitude of the normal force exerted on the cube by the wall of the box is F_N.

i. **Determine** an expression for the magnitude of the acceleration of the cube. Express your answer in terms of M, F_N, and physical constants, as appropriate.

ii. **Derive** an expression for F_N as a function of t from $t = 0$ until immediately before the cube reaches the bottom of the box. Express your answer in terms of M, b, v_0, t, and physical constants, as appropriate. Begin your derivation with a fundamental physics principle or an equation from the reference information.

iii. On the axes shown in Figure 2, **sketch** a graph of the magnitude F_f of the frictional force exerted on the cube by the wall of the box as a function of t from $t = 0$ until immediately before the cube reaches the bottom of the box.

Figure 2



Part B

Derive an expression for t_{crit} in terms of M, b, μ_s, v_0, and physical constants, as appropriate. Begin your derivation with a fundamental physics principle or an equation from the reference information.

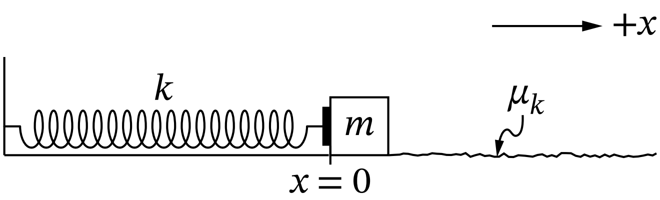
Question 3

The following information applies to parts A and B.

---

3. A horizontal spring of known spring constant k is attached to a wall. A block of known mass m is placed on a horizontal surface next to but not attached to the spring. The spring is at its relaxed length when the block is at position $x = 0$, as shown in Figure 1. There is friction between the block and the surface only for positions $x > 0$.

Figure 1



A student is asked to experimentally determine the coefficient of kinetic friction μ_k between the block and the surface using a graph. The student is permitted to use only measurements from a meterstick.

Part A

i. **Indicate** quantities that could be measured by the student that would allow them to determine μ_k using a graph.

ii. Briefly **describe** a method to reduce experimental uncertainty for the measured quantities.

Part B

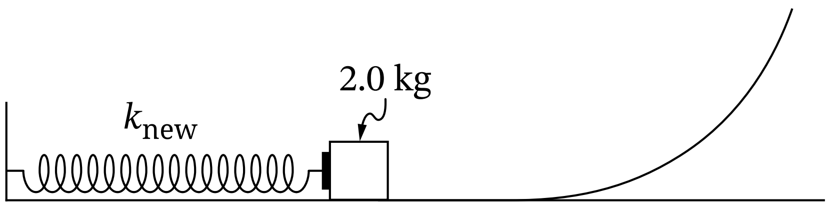
i. **Indicate** what quantities the student could graph on the horizontal and vertical axes to create a graph that can be used to determine μ_k.

ii. Briefly **describe** the relationship between μ_k and a feature of the graph from part B (i). Your answer may include an equation that relates μ_k and the chosen feature of the graph.

The following information applies to parts C and D.

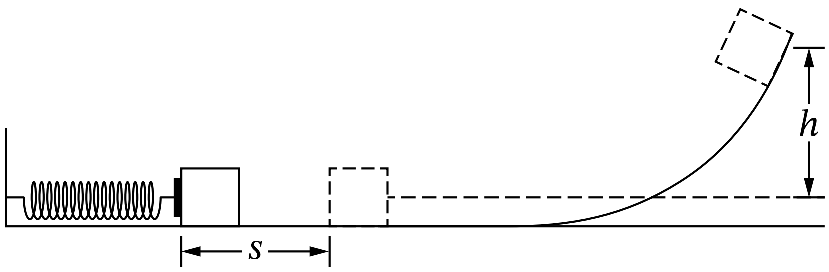
In a different experiment, the student is asked to determine the spring constant k_{new} of a new spring that is attached to a wall. A block of mass 2.0 kg is placed next to the spring on a surface where frictional forces are negligible. The surface is horizontal near the spring and slopes upward into a ramp to the right of the initial position of the block, as shown in Figure 2.

Figure 2



The student pushes the block, compressing the spring a distance s, as shown in Figure 3. The block is released from rest.

Figure 3



The student uses a meterstick to measure the maximum height h of the block on the ramp. The experiment is repeated several times with different compression distances s. The student's measurements of s and h are shown in Table 1.

Table 1

s (m)	h (m)
0.020	0.02
0.040	0.04
0.060	0.13
0.080	0.18
0.100	0.33

Part C

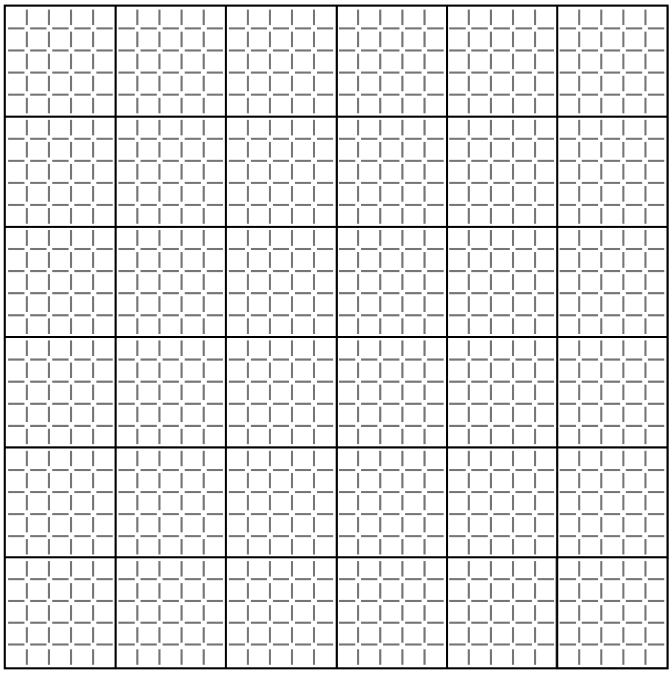
i. **Label** the axes of the grid provided with measured or calculated quantities. Include units, as appropriate. The graphed quantities should yield a linear graph that can be used to determine k_{new}.

ii. On the grid provided, create a graph of the quantities indicated in part C (i).

- Clearly **label** the vertical and horizontal axes with numerical scales.
- **Plot** the corresponding data points on the grid.
- Table 2 is provided in your booklet for scratch work and will not be scored.

Quantity (units, if appropriate)

____________ ____________



Quantity (units, if appropriate)

____________ ____________

iii. **Draw** a best-fit line for the data graphed in part C (ii).

Part D

Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for k_{new}.

Question 2: Version J

2. In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

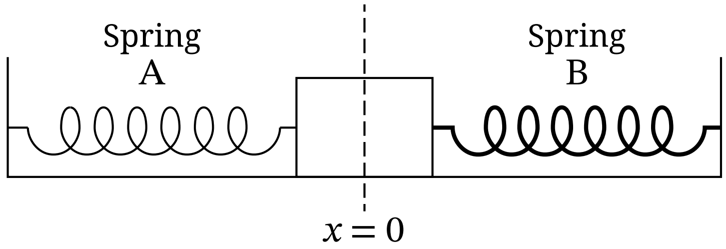


Figure 1

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

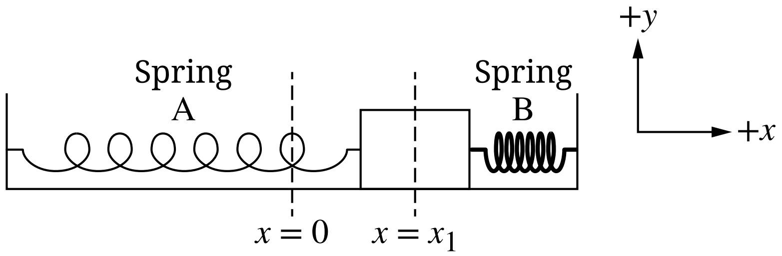


Figure 2

A. An energy bar chart can be used to represent the elastic potential energy U_A of Spring A, the elastic potential energy U_B of Spring B, and the kinetic energy K_{block} of the block. On the energy bar chart in Figure 3, **draw** shaded bars to represent the energy of the system for when the block is at $x = x_1$.

- The height of the shaded bars should be proportional to the relative values of U_A, U_B, and K_{block}.
- Any energy that is equal to zero should be represented by a distinct line on the zero-energy line.

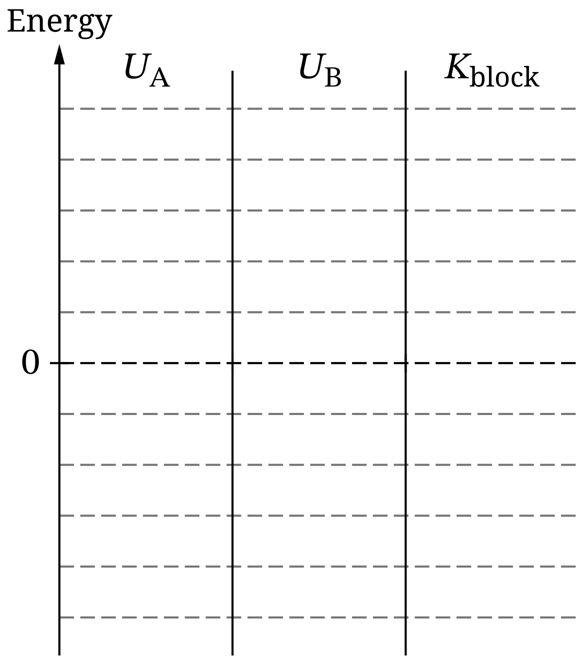
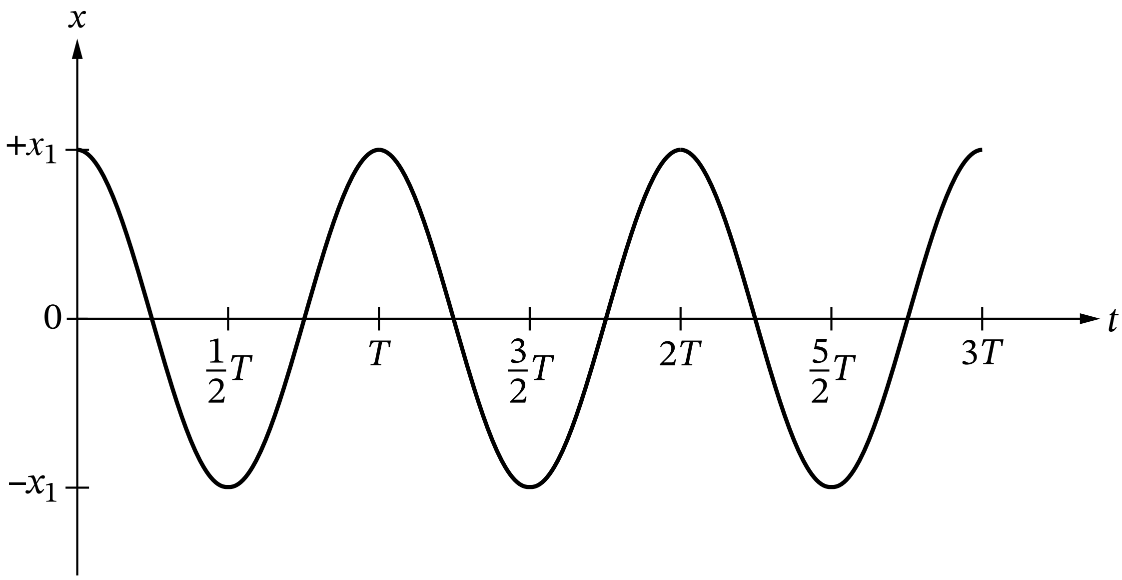


Figure 3

B. The block is released from rest at $x = x_1$ and begins to oscillate. **Derive** an expression for the speed v of the block as the block passes through $x = \frac{1}{2}x_1$. Express your answer in terms of m, k, x_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

C. In Scenario 1, the block oscillates with period T. The position x of the block in Scenario 1 as a function of time t is shown in Figure 4.

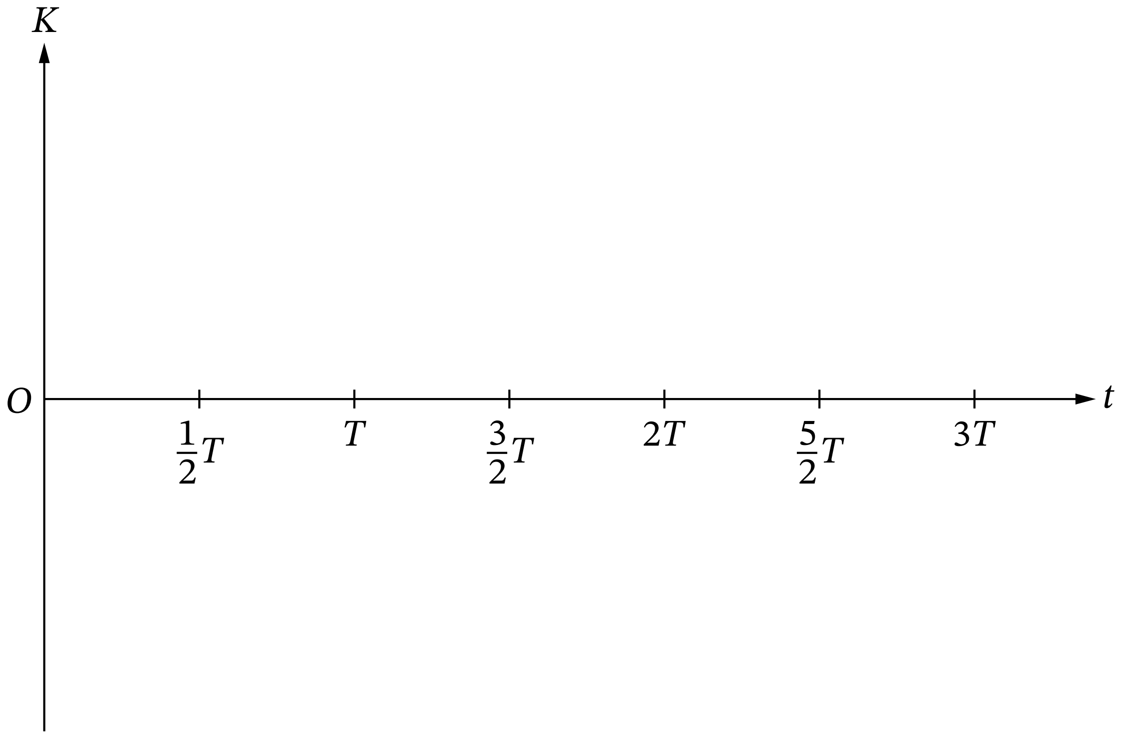


Scenario 1

Figure 4

In Scenario 2, the block-springs system is placed on a new surface. There is friction between the block and the new surface. The block is again moved to the same position $x = x_1$ and released from rest. The block completes multiple oscillations with the same period as in Scenario 1 before coming to rest.

On the axes shown in Figure 5, **sketch** a graph of the kinetic energy K of the block as a function of t for Scenario 2.



Scenario 2

Figure 5

D. In Scenario 3, the block is replaced with a new block of larger mass. The coefficient of kinetic friction between the new block and the surface in Scenario 3 is the same as the coefficient of kinetic friction between the original block and the surface in Scenario 2.

The new block is moved to position $x = x_1$ and released from rest. The kinetic energy of the new block is plotted as a function of time.

Describe how one feature of the graph of K as a function of t in Scenario 3 would differ from the graph you drew in Figure 5 for Scenario 2.

Briefly **justify** your answer.

Question 3

3. A box is connected to one end of a rigid rod. Both the box and the rod have negligible mass. The other end of the rod is connected to a pivot. The box is open on one side, and a block is placed inside the box.

The center of mass of the block is displaced a vertical distance *h*, as shown in Figure 1. The block-box system is then released from rest and swings downward. There is negligible friction about the pivot. When the system is at the lowest point of its swing, the rod collides with a rigid stopper, as shown in Figure 2. The box comes to rest, and the block is launched horizontally out of the box. The block moves across a horizontal surface toward a motion sensor that measures the speed of the block. All frictional forces are negligible.

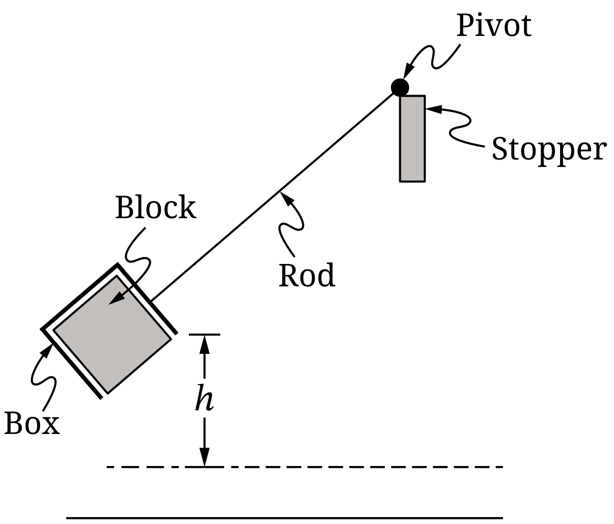


Figure 1

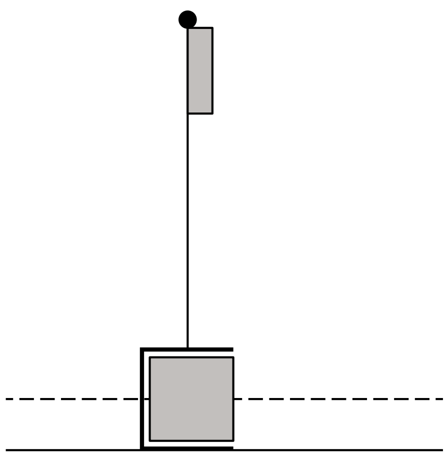


Figure 2

A. Students are asked to experimentally determine the acceleration due to gravity *g* using a linear graph. To determine *g*, the students are permitted to use measurements from only a meterstick and the motion sensor.

Describe an experimental procedure using the described setup to collect data that would allow the students to determine an experimental value of *g* using a linear graph. Include any steps necessary to reduce experimental uncertainty.

B. **Describe** how the data collected in part A could be graphed and how that graph would be analyzed to determine the value of *g*.

C. The experiment is repeated, but the horizontal surface on which the block slides is replaced with a new rough surface, as shown in Figure 3. The coefficient of kinetic friction between the block and the new surface is μ.

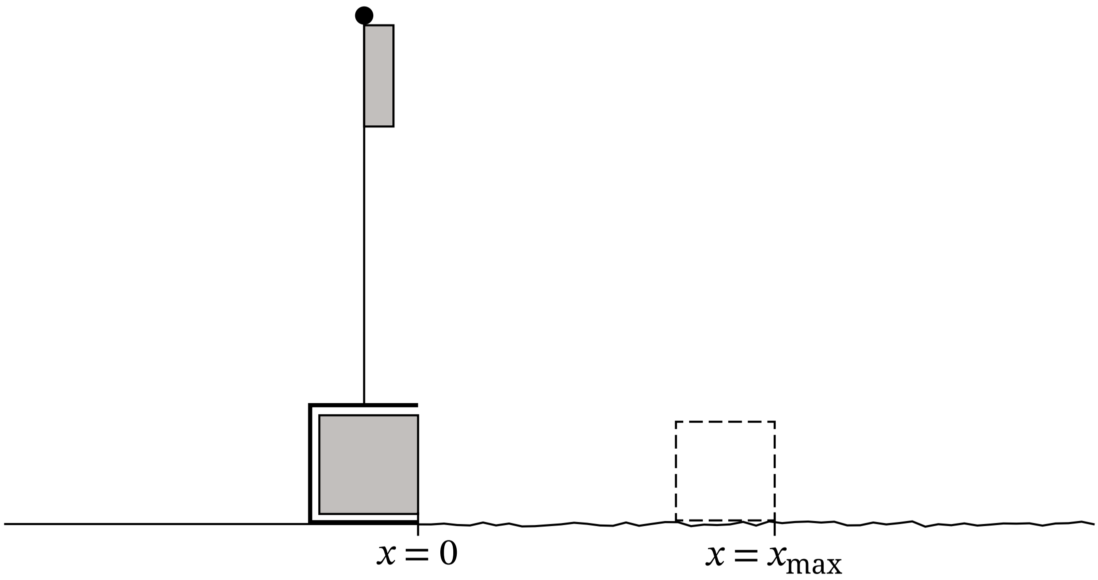


Figure 3

The block-box system is pulled aside so that the center of mass of the block is displaced various vertical distances h and then released from rest. For each vertical distance, students measure the position $x = x_{max}$ at which the block comes to rest.

The students' measurements of h and x_{max} are shown in Table 1.

Table 1

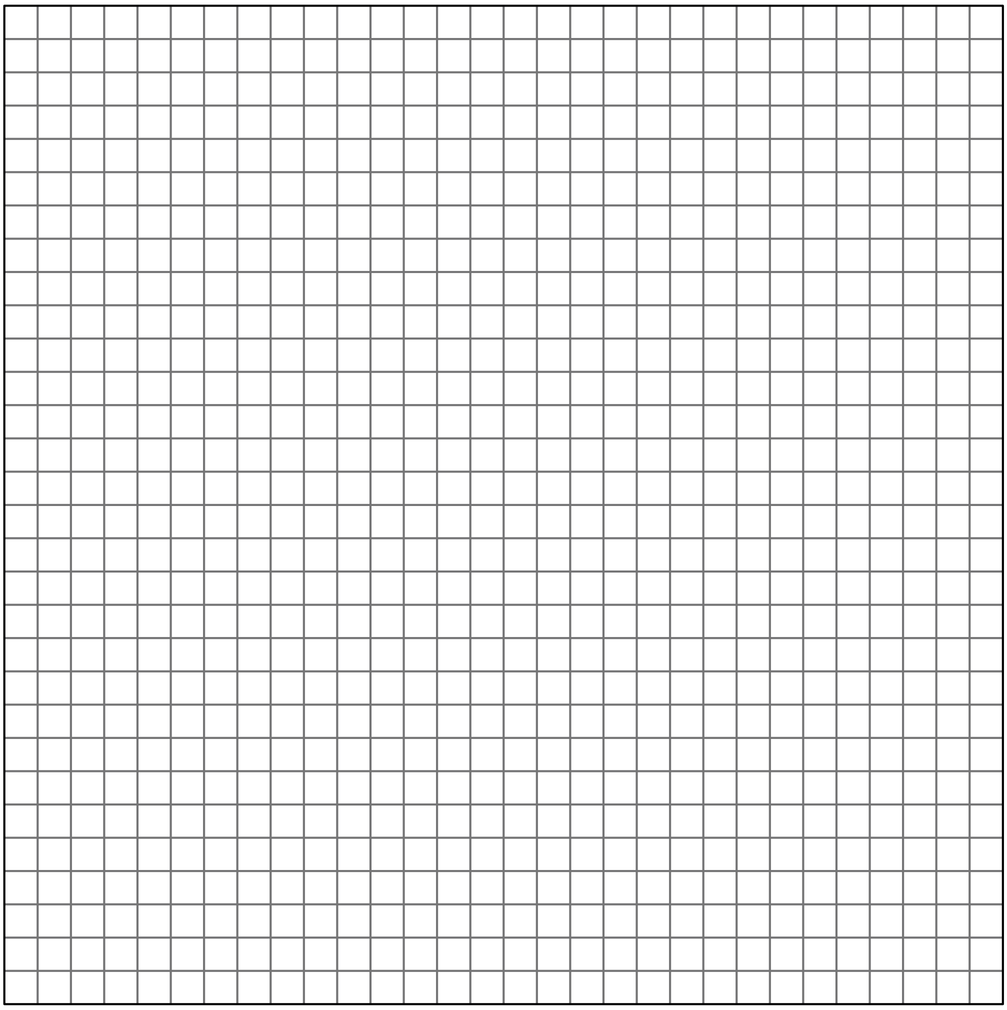
h (m)	x_{max} (m)
0.30	0.76
0.45	1.10
0.60	1.40
0.75	1.90
0.90	2.30

i. **Indicate** two quantities, either measured quantities from Table 1 or additional calculated quantities, that could be graphed to produce a straight line that could be used to determine μ.

Vertical axis: ______ Horizontal axis: ______

ii. On the grid provided, create a graph of the quantities indicated in part C (i).

- Use Table 2 to record the measured or calculated quantities that you will plot.
- Clearly **label** the axes, including units as appropriate.
- **Plot** the points you recorded in Table 2.



iii. **Draw** a best-fit line to the data graphed in part C (ii).

D. Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for μ.

Question 4

4. A uniform disk and ring, each of mass M and radius R, roll without slipping along a horizontal surface, as shown in Figure 1. The outer edges of the disk and ring are made of the same material. The center of mass of the disk and the center of mass of the ring each initially move with the same constant speed v.

The disk and the ring then smoothly transition to a ramp that is inclined at an angle θ above the horizontal. Both the disk and the ring continue to roll without slipping as they move up the ramp, as shown in Figure 2.

The ring travels a greater distance along the ramp than the disk travels before each momentarily comes to rest.

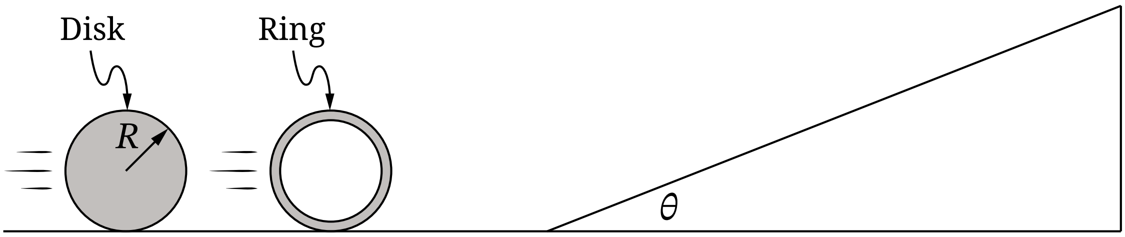


Figure 1

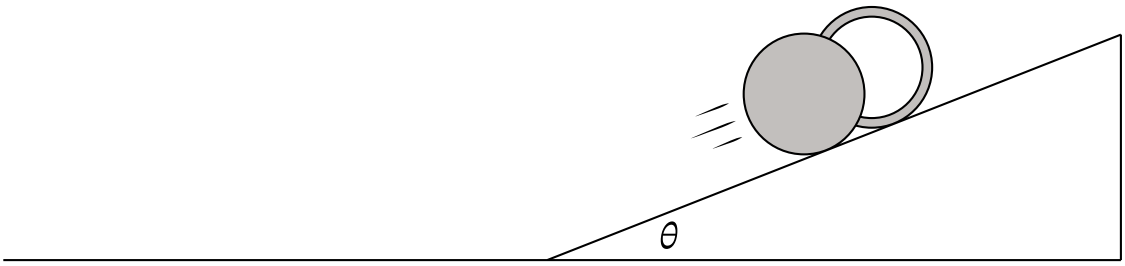


Figure 2

A. While the disk and the ring are rolling on the ramp without slipping, the magnitudes of the static frictional force exerted on the disk and on the ring by the ramp are f_D and f_R, respectively.

Indicate whether f_D is greater than, less than, or equal to f_R by writing one of the following.

- $f_D > f_R$
- $f_D < f_R$
- $f_D = f_R$

Justify your answer using qualitative reasoning beyond referencing equations.

B. A cylinder has mass M, radius R, and rotational inertia I about its central axis. The cylinder rolls without slipping up a ramp that is inclined at an angle θ above the horizontal.

Derive an expression for the magnitude of the static frictional force f exerted on the cylinder by the ramp. Express your answer in terms of M, R, I, θ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

C. In a different scenario, the centers of mass of the original disk and ring each have the same initial speed v as they did in the original scenario. The ramp is replaced by a new ramp on which the disk and the ring initially slip as they roll up the new ramp.

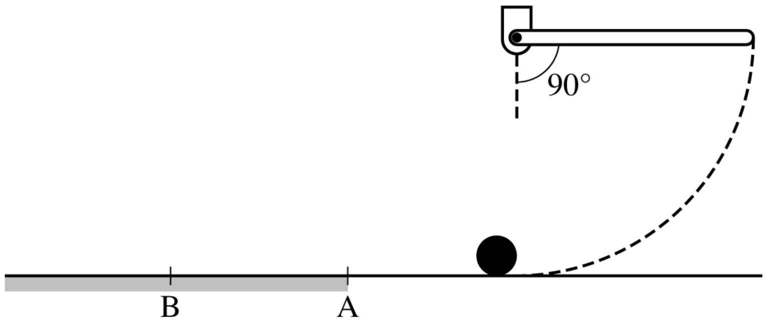
Indicate whether the magnitude of the kinetic frictional force exerted on the disk by the new ramp is greater than, less than, or equal to the magnitude of the kinetic frictional force exerted on the ring by the new ramp while both are slipping.

Briefly **justify** your answer.

STOP

END OF EXAM

Begin your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

Figure 1

3. A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

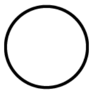
(a) Derive an expression for the angular speed of the rod just before striking the sphere in terms of the length ℓ and physical constants as appropriate.

GO ON TO THE NEXT PAGE.

(b) Derive an expression for the linear speed v_0 of the sphere immediately after colliding with the rod in terms of the length ℓ and physical constants as appropriate.

After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ, beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

(c) In the following diagram, which represents the sphere while the sphere is traveling between Points A and B, draw and label the forces (not components) that act on the sphere. Each force must be represented by a distinct arrow starting on, and pointing away from, the point of application on the sphere.



(d) Derive an expression for each of the following as the sphere is rotating and sliding between points A and B in terms of v_0, μ, R, t, and physical constants as appropriate.

i. The linear velocity v of the center of mass of the sphere as a function of time t

ii. The angular velocity ω of the sphere as a function of time t

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(e)

i. Derive an expression for the time it takes the sphere to travel from Point A to Point B in terms of v_0, μ, and physical constants as appropriate.

ii. Derive an expression for the linear velocity of the sphere upon reaching Point B in terms of v_0.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

Begin your response to **QUESTION 1** on this page.

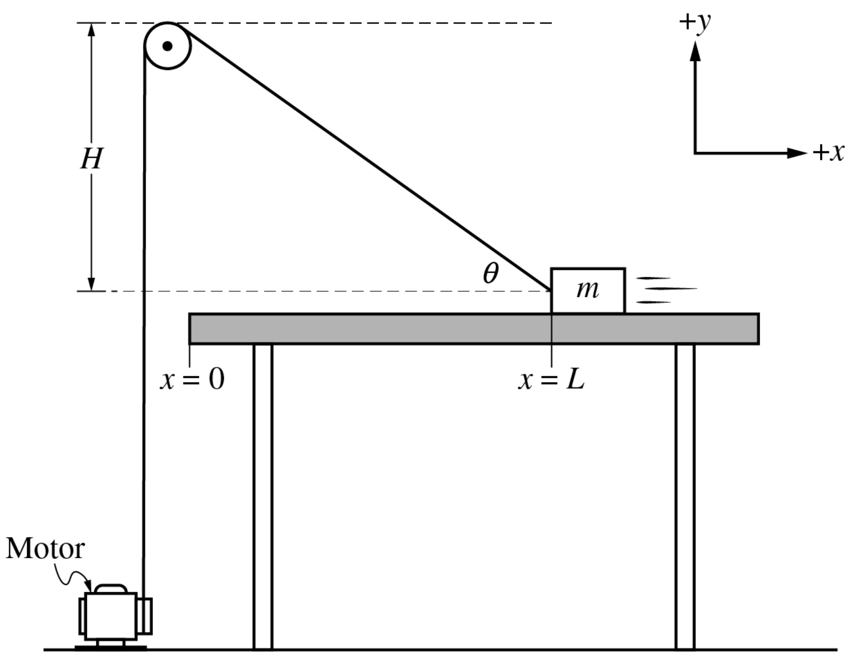
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

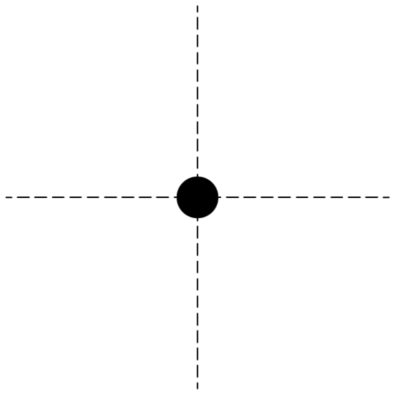


1. A block of mass m is pulled across a rough horizontal table by a string connected to a motor that is attached to the floor. The string passes over a pulley with negligible friction that is vertically aligned with the left edge of the table as shown. The string and pulley both have negligible mass. The pulley is at height H above the table. The motor exerts a constant force of tension F_T on the string, and the block remains in contact with the table at all times as the block slides across the table from $x = L$ to $x = 0$. The coefficient of kinetic friction between the table and the block is μ_k. Express all algebraic answers in terms of m, H, F_T, x, μ_k, L, and physical constants as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(a) On the dot below that represents the block, draw and label the forces (not components) that act on the block when the block is at $x = L$. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



(b) Derive an expression for the angle θ that the string makes with the horizontal as a function of x.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(c)

i. Derive an expression for the normal force F_N exerted on the block by the table as a function of the block's position x.

ii. Derive an expression for the magnitude of the net horizontal force F_{net} exerted on the block as a function of the position x.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(d) Write, but do not solve, an integral expression that could be used to solve for the work W done by the string on the block as the block moves from $x = L$ to $x = 0$.

(e) Does the string do more, less, or the same amount of work on the block as the block moves from $x = L$ to $x = \frac{L}{2}$ compared to when the block moves from $x = \frac{L}{2}$ to $x = 0$?

_______ More work when the block moves from $x = L$ to $x = \frac{L}{2}$

_______ Less work when the block moves from $x = L$ to $x = \frac{L}{2}$

_______ The same amount of work when the block moves from $x = L$ to $x = \frac{L}{2}$

Justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 1** on this page.

PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

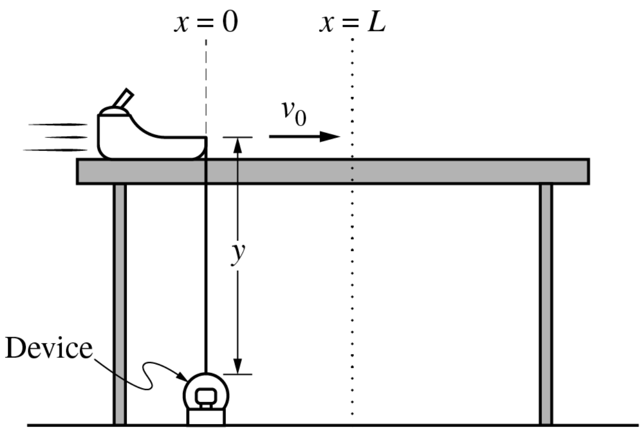


Figure 1

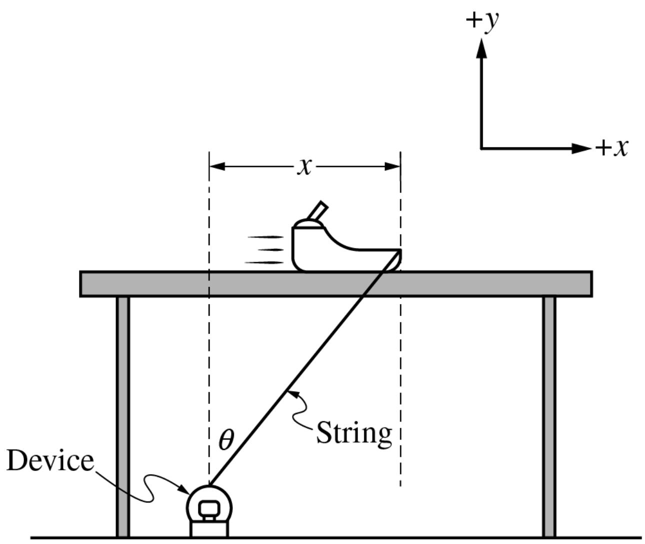


Figure 2

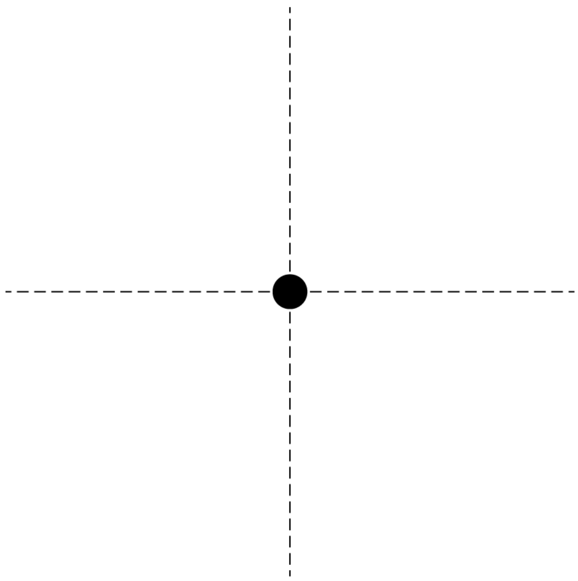
Note: Figures not drawn to scale.

1. A small sled slides across a rough horizontal table with an initial velocity v_0. The coefficient of kinetic friction between the sled and the table is μ_k. A string connects the sled to a device on the ground. The device maintains constant tension F_T in the string by unwinding the string as the sled slides to the right. The total mass of the sled is m. The string is attached to the device at $x = 0$ and at a height of y, as shown in Figure 1. The horizontal position of the sled is represented by x, as shown in Figure 2. Express all algebraic answers in terms of m, μ_k, F_T, x, y, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(a) On the dot below that represents the sled, draw and label the forces (not components) that are exerted on the sled a short time after $t = 0$ but before the sled has come to rest. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



(b) Determine an expression for the angle θ that the string makes with the vertical when the sled has traveled a horizontal distance x.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(c)

i. Derive an expression for the normal force F_N exerted on the sled by the table as a function of the position x.

ii. Derive an expression for the magnitude of the net horizontal force F_{net} exerted on the sled as a function of the position x.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(d) Derive an expression for the work W done by the string on the sled as the sled moves from $x = 0$ to $x = L$.

(e) The sled comes to rest after traveling a horizontal distance $x = 2L$. As the system slides from $x = 0$ to $x = L$, the energy dissipated by friction is E_1. As the sled slides from $x = L$ to $x = 2L$, the energy dissipated by friction is E_2. Is E_1 greater than, less than, or equal to E_2 ?

____ $E_1 > E_2$ ____ $E_1 < E_2$ ____ $E_1 = E_2$

Justify your answer.

GO ON TO THE NEXT PAGE.

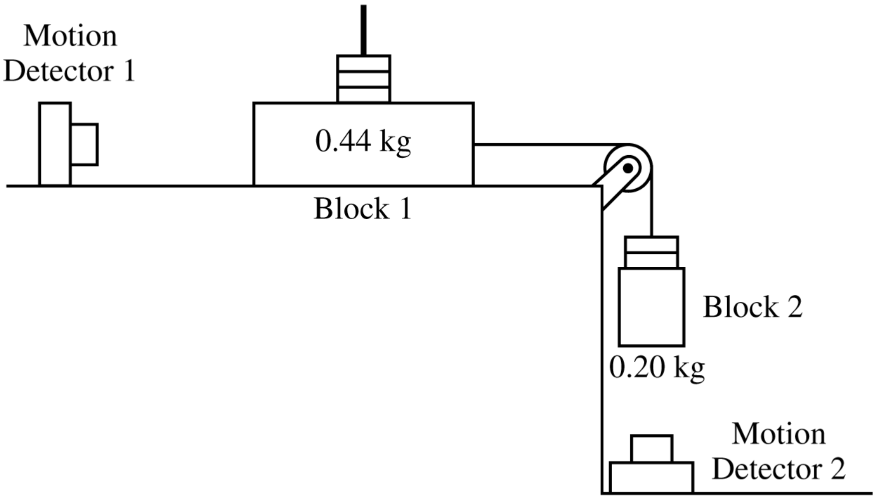
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



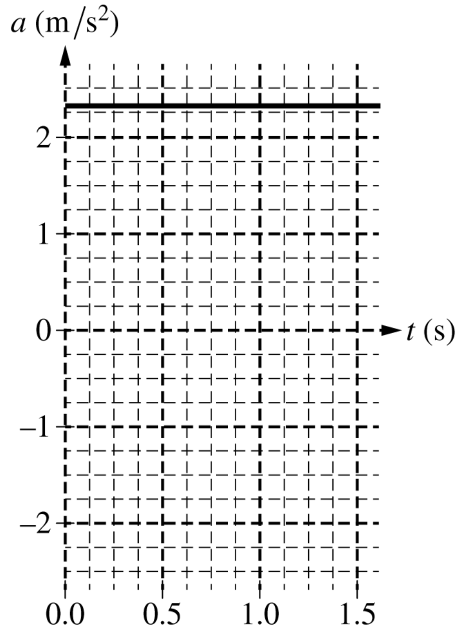
Students design an experiment using blocks of adjustable mass to investigate friction using the setup shown. Block 1 of initial mass 0.44 kg is placed on a rough horizontal surface and connected by a string to block 2 of initial mass 0.20 kg. The string extends over a pulley that has negligible mass and friction.

(a) Calculate the minimum value of the coefficient of static friction μ_s that would keep the two-block system at rest.

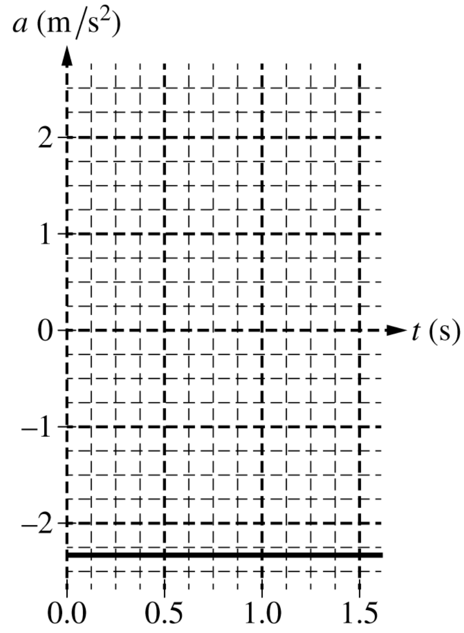
GO ON TO THE NEXT PAGE.

The coefficient of friction is such that when block 2 is released from rest, block 1 travels across the surface. The acceleration a of each block is recorded with motion detectors 1 and 2, as shown in the figure. The data for the motion detectors as functions of time t are shown on the graphs. For each motion detector, the positive direction is away from the detector.

Motion Detector 1



Motion Detector 2



(b) On the dots below, which represent the blocks, draw and label the forces (not components) that act on each block. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

Block 1 Block 2

(c) Calculate the coefficient of kinetic friction μ_k between block 1 and the table.

GO ON TO THE NEXT PAGE.

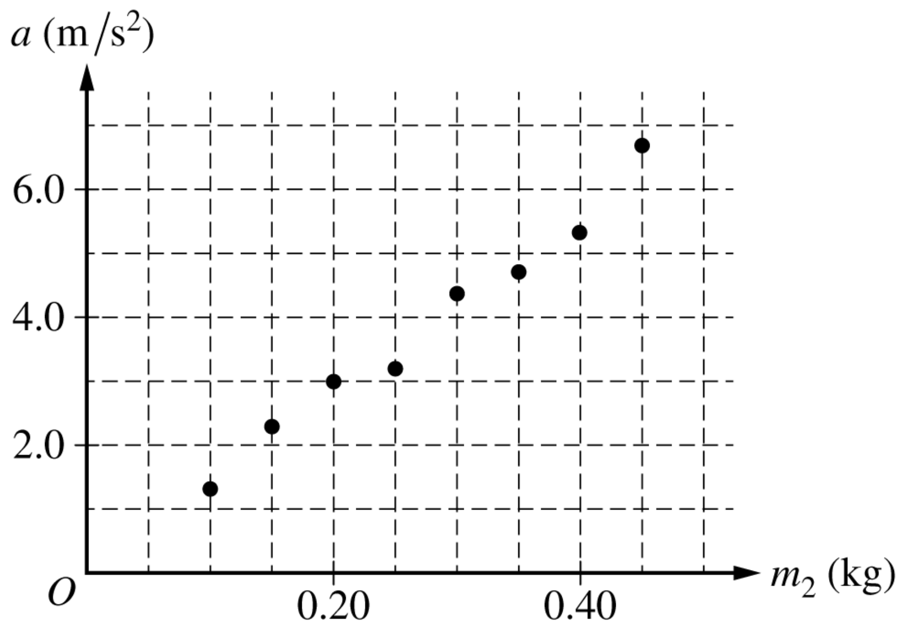
(d) Careful measurements determine that the coefficient of kinetic friction is larger than the value calculated in part (c). Does the following explanation sufficiently account for the observed discrepancy?

"The horizontal table was not perfectly level before the experiment was conducted. The observed difference in the angle accounts for the difference in the expected and calculated values of μ_k."

_____Yes _____ No

Justify your answer.

The experiment is moved to a surface with negligible friction and run for eight trials. In each trial, the students vary the masses m_1 and m_2 of blocks 1 and 2, respectively, while keeping the total mass $(m_1 + m_2) = 0.64\ \text{kg}$ constant. The data for the acceleration a of block 1 as a function of m_2 are shown on the graph below.



(e)

i. Draw a best-fit line for the data points.

ii. Using the straight line, calculate an experimental value for the acceleration due to gravity g.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(f) The students lift the left end of the surface so that the surface is inclined at an angle to the horizontal, and the experiment for $m_2 = 0.20\,\text{kg}$ is repeated. Would the acceleration of the system be greater than, less than, or equal to the acceleration of the system in the original experiment?

_____ Greater than   _____ Less than   _____ Equal to

Justify your claim.

GO ON TO THE NEXT PAGE.

PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

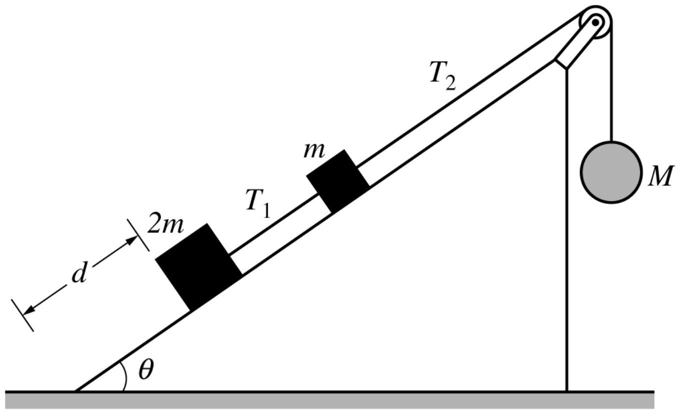


Figure 1

1. Blocks of mass m and $2m$ are connected by a light string and placed on a frictionless inclined plane that makes an angle θ with the horizontal, as shown in Figure 1 above. Another light string connecting the block of mass m to a hanging sphere of mass M passes over a pulley of negligible mass and negligible friction. The entire system is initially at rest and in equilibrium.

   (a) On the dots below that represent the block of mass m and the sphere of mass M, draw and label the forces (not components) that act on each of the objects shown. Each force must be represented by a distinct arrow starting on and pointing away from the dot.

(b) Derive expressions for the magnitude of each of the following. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figures in part (a).

i. The force T_2 exerted on the block of mass *m* by the string. Express your answers in terms of *m*, θ, and physical constants, as appropriate.

ii. The mass *M* for which the system can remain in equilibrium. Express your answers in terms of *m*, θ, and physical constants, as appropriate.

(c) Now suppose that mass *M* is large enough to descend and that the sphere reaches the floor before the blocks reach the pulley. Answer the following for the moment immediately after the sphere reaches the floor.

i. Does the tension T_1 increase, decrease to a nonzero value, decrease to zero, or stay the same?

____ Increase ____ Decrease to a nonzero value

____ Decrease to zero ____ Stay the same

ii. Is the velocity of the block of mass *m* up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

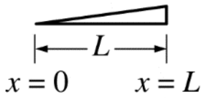
iii. Is the acceleration of the block of mass *m* up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

(d) Consider the initial setup in Figure 1. Now suppose the surface of the incline is rough and the coefficient of static friction between the blocks and the inclined plane is μ_s. Derive an expression for the minimum possible value of *M* that will keep the blocks from moving down the incline. Express your answer in terms of *m*, μ_s, θ, and fundamental constants, as appropriate.

(e) The string connecting block *m* and the sphere of mass *M* then breaks, and the blocks begin to move from rest down the incline. The lower block starts a distance *d* from the bottom of the incline, as shown in Figure 1. The coefficient of kinetic friction between the blocks and the inclined plane is μ_k. Derive an expression for the speed of the blocks when the lower block reaches the bottom of the incline. Express your answer in terms of *m*, *d*, μ_k, θ, and fundamental constants, as appropriate.

$$\lambda = \left(\frac{2M}{L^2}\right)x$$



3. A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(a) Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

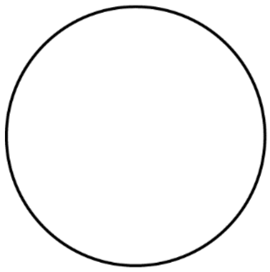


Figure 1

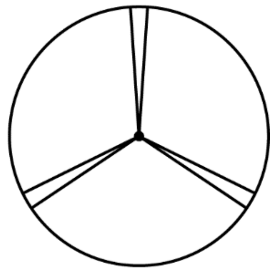


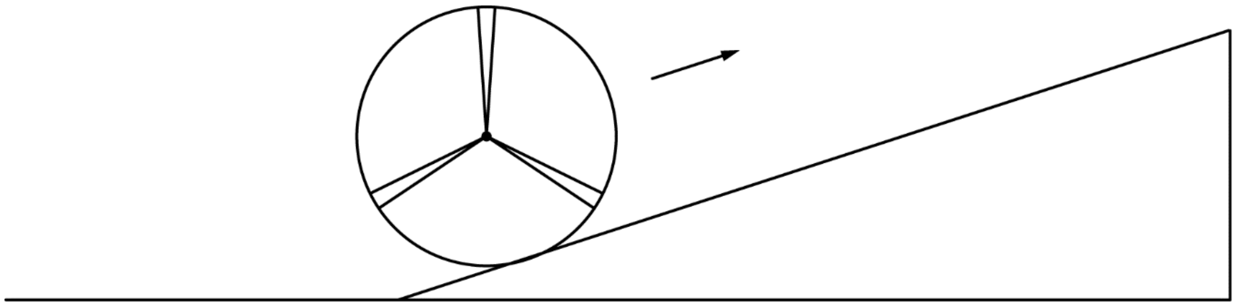
Figure 2

The thin hoop shown above in Figure 1 has a mass M, radius L, and a rotational inertia around its center of ML^2. Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

(b) Derive an expression for the rotational inertia I_{tot} of the hoop-rods system about the center of the hoop. Express your answer in terms of M, L, and physical constants, as appropriate.

The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt.

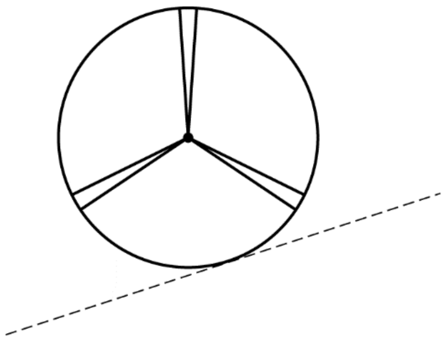
(c) Derive an expression for the final angular speed ω of the hoop-rods system. Express your answer in terms of M, L, F, Δt, and physical constants, as appropriate.



The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed ω found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

(d)

i. On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.



ii. Justify your choice for the direction of each of the forces drawn in part (d)i.

(e) Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of M, L, I_{tot}, ω, and physical constants, as appropriate.

STOP

END OF EXAM

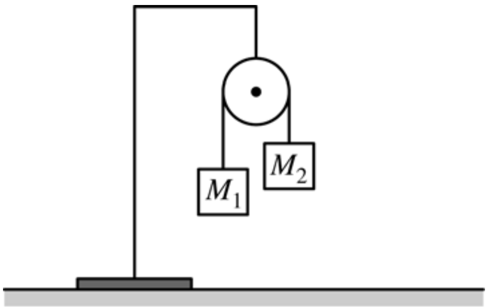
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

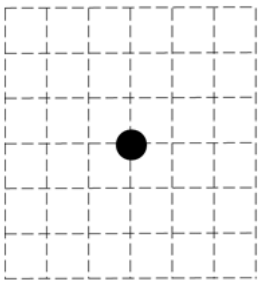
Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



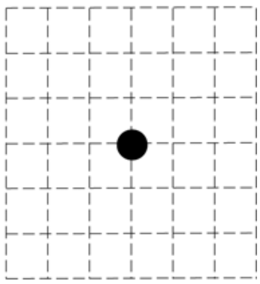
1. An Atwood's machine consists of two blocks connected by a light string that passes over a frictionless pulley of negligible mass, as shown in the figure above. The masses of the two blocks, M_1 and M_2, can be varied. M_2 is always greater than M_1.

   (a) On the dots below, which represent the blocks, draw and label the forces (not components) that act on the blocks. Each force must be represented by a distinct arrow starting on and pointing away from the appropriate dot. The relative lengths of the arrows should show the relative magnitudes of the forces.

   Block of Mass M_1

   Block of Mass M_2

   (b) Using the forces in your diagrams above, write an equation applying Newton's second law to <u>each</u> block and use these two equations to derive the magnitude of the acceleration of the blocks and show that it is given by the equation: $a = \dfrac{(M_2 - M_1)}{(M_1 + M_2)} g$

The magnitude of the acceleration a was measured for different values of M_1 and M_2, and the data are shown below.

M_1 (kg)	1.0	2.0	5.0	6.0	10.0
M_2 (kg)	2.0	3.0	12.0	8.0	14.0
a (m/s^2)	3.02	1.82	4.21	1.15	1.71

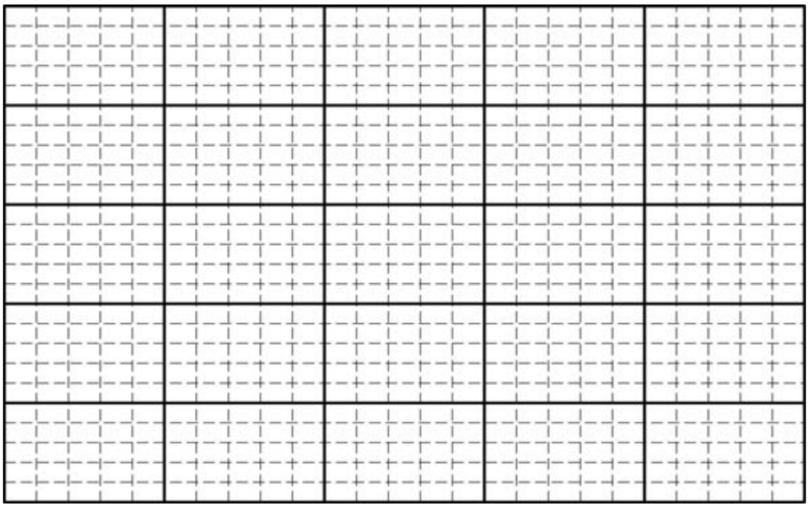
(c) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the acceleration due to gravity g.

Vertical axis: ____________________

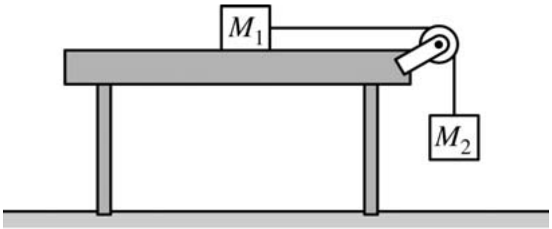
Horizontal axis: ____________________

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given.

(d) Plot the data points for the quantities indicated in part (c) on the graph below. Clearly scale and label all axes including units, if appropriate. Draw a straight line that best represents the data.



(e) Using your straight line, determine an experimental value for g.



The experiment is now repeated with a modification. The Atwood's machine is now set up so that the block of mass M_1 is on a smooth, horizontal table and the block of mass M_2 is hanging over the side of the table, as shown in the figure above.

(f) For the same values of M_1 and M_2, is the magnitude of the tension in the string when the blocks are moving higher, lower, or equal to the magnitude of the tension in the string when the blocks are moving in the first experiment?

____ Higher ____ Lower ____ Equal to

Justify your answer.

(g) The value determined for the acceleration due to gravity g is lower than in the first experiment. Give one physical factor that could account for this lower value and explain how this factor affected the experiment.

2.8 Spring Forces

Name: ______________ Class: __________ Date: ____________ **Total: 14 marks**

KEY WORDS spring constant 劲度系数 • hooke's law 胡克定律 • force 力

Objective

Build the skills to answer exam questions on **spring forces**.

You must be able to:

- use **Hooke's law 胡克定律**: the restoring force has magnitude $F = kx$
- explain the minus sign in $F = -kx$ (the force opposes the displacement)
- use the **spring constant 劲度系数** k as force per unit stretch
- relate stiffness to k (larger k = stiffer)

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Hooke's law

The size of the spring force grows with the stretch or compression:

$$F = kx.$$

A spring with $k = 200$ N m^{-1} stretched 0.1 m pulls back with $F = 200 \times 0.1 = 20$ N.

■ The restoring direction

Written with direction, $F = -kx$: the minus sign means the force always points back toward the natural length, opposing the displacement.

■ The spring constant

k is the force needed per metre of stretch. A stiffer spring has a larger k. From data, $k = F/x$: if 15 N stretches a spring 0.05 m, then $k = 15/0.05 = 300$ N m^{-1}.

2 Practice

Now apply the methods above.

2.1 A spring with $k = 100$ N m^{-1} is stretched 0.2 m. Find the force. [2]

2.2 State the meaning of the minus sign in $F = -kx$. [1]

2.3 A force of 12 N stretches a spring 0.03 m. Find k. [2]

2.4 Two springs stretch the same distance; spring A needs more force. Which has the larger k? [1]

3 Exam-style questions

3.1 The magnitude of a spring's force is proportional to [1]

- **A** the mass on it
- **B** its displacement from the natural length
- **C** the square of the displacement
- **D** time

3.2 A spring force always points [1]

- **A** in the direction of the stretch
- **B** back toward the natural length
- **C** downward
- **D** perpendicular to the spring

3.3 A spring with $k = 500$ N m^{-1} is compressed 0.04 m.

(a) Find the size of the spring force. [2]

(b) State the direction of the force relative to the compression. [1]

3.4 A 2 kg mass hangs at rest from a spring, stretching it 0.1 m ($g = 10$ m s^{-2}).

(a) State the spring force (it balances the weight). [1]

(b) Find the spring constant k. [2]

Solutions

2.1 $F = kx = 100 \times 0.2 = 20$ N.

2.2 The force opposes the displacement (points back toward the natural length).

2.3 $k = F/x = 12/0.03 = 400$ N m^{-1}.

2.4 Spring A (more force for the same stretch means a larger k).

3.1 B — $F = kx$, proportional to displacement.

3.2 B — the restoring force points back toward the natural length.

3.3 (a) $F = kx = 500 \times 0.04 = 20$ N. (b) Opposite to the compression (pushing outward).

3.4 (a) $F = mg = 20$ N. (b) $k = F/x = 20/0.1 = 200$ N m^{-1}.

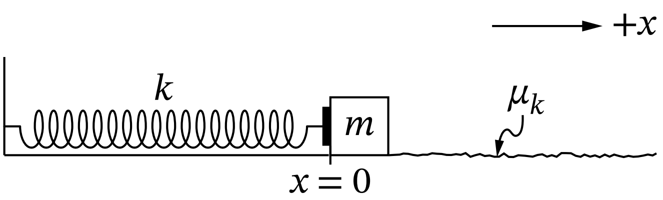
Question 3

The following information applies to parts A and B.

---

3. A horizontal spring of known spring constant k is attached to a wall. A block of known mass m is placed on a horizontal surface next to but not attached to the spring. The spring is at its relaxed length when the block is at position $x = 0$, as shown in Figure 1. There is friction between the block and the surface only for positions $x > 0$.

Figure 1



A student is asked to experimentally determine the coefficient of kinetic friction μ_k between the block and the surface using a graph. The student is permitted to use only measurements from a meterstick.

Part A

i. **Indicate** quantities that could be measured by the student that would allow them to determine μ_k using a graph.

ii. Briefly **describe** a method to reduce experimental uncertainty for the measured quantities.

Part B

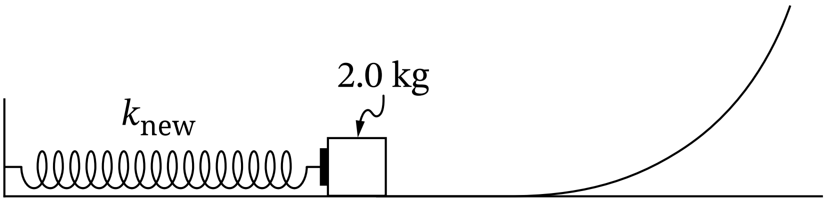
i. **Indicate** what quantities the student could graph on the horizontal and vertical axes to create a graph that can be used to determine μ_k.

ii. Briefly **describe** the relationship between μ_k and a feature of the graph from part B (i). Your answer may include an equation that relates μ_k and the chosen feature of the graph.

The following information applies to parts C and D.

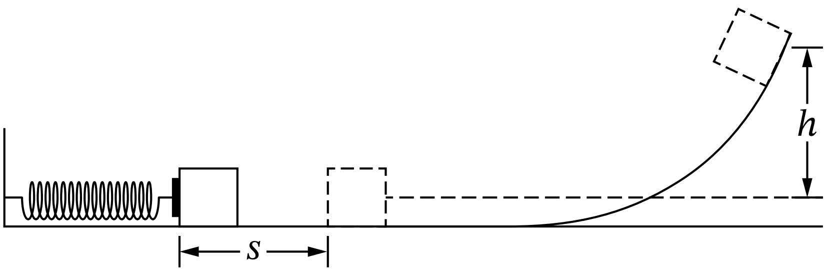
In a different experiment, the student is asked to determine the spring constant k_{new} of a new spring that is attached to a wall. A block of mass 2.0 kg is placed next to the spring on a surface where frictional forces are negligible. The surface is horizontal near the spring and slopes upward into a ramp to the right of the initial position of the block, as shown in Figure 2.

Figure 2



The student pushes the block, compressing the spring a distance s, as shown in Figure 3. The block is released from rest.

Figure 3



The student uses a meterstick to measure the maximum height h of the block on the ramp. The experiment is repeated several times with different compression distances s. The student's measurements of s and h are shown in Table 1.

Table 1

s (m)	h (m)
0.020	0.02
0.040	0.04
0.060	0.13
0.080	0.18
0.100	0.33

Part C

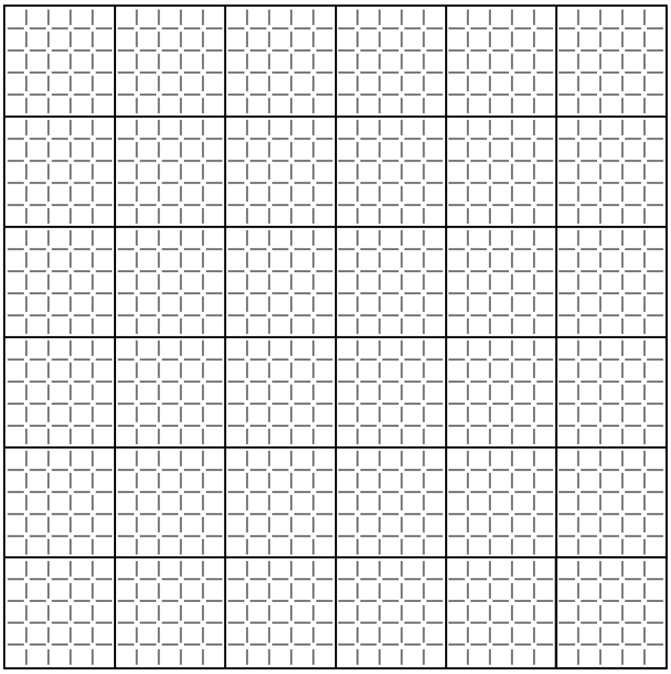
i. **Label** the axes of the grid provided with measured or calculated quantities. Include units, as appropriate. The graphed quantities should yield a linear graph that can be used to determine k_{new}.

ii. On the grid provided, create a graph of the quantities indicated in part C (i).

- Clearly **label** the vertical and horizontal axes with numerical scales.
- **Plot** the corresponding data points on the grid.
- Table 2 is provided in your booklet for scratch work and will not be scored.

Quantity (units, if appropriate)

____________ ____________



Quantity (units, if appropriate)

____________ ____________

iii. **Draw** a best-fit line for the data graphed in part C (ii).

Part D

Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for k_{new}.

Question 2: Version J

2. In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

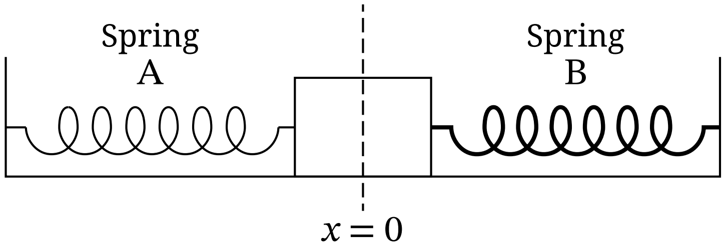


Figure 1

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

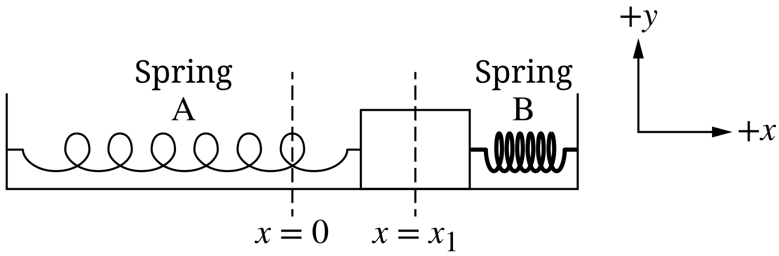


Figure 2

A. An energy bar chart can be used to represent the elastic potential energy U_A of Spring A, the elastic potential energy U_B of Spring B, and the kinetic energy K_{block} of the block. On the energy bar chart in Figure 3, **draw** shaded bars to represent the energy of the system for when the block is at $x = x_1$.

- The height of the shaded bars should be proportional to the relative values of U_A, U_B, and K_{block}.
- Any energy that is equal to zero should be represented by a distinct line on the zero-energy line.

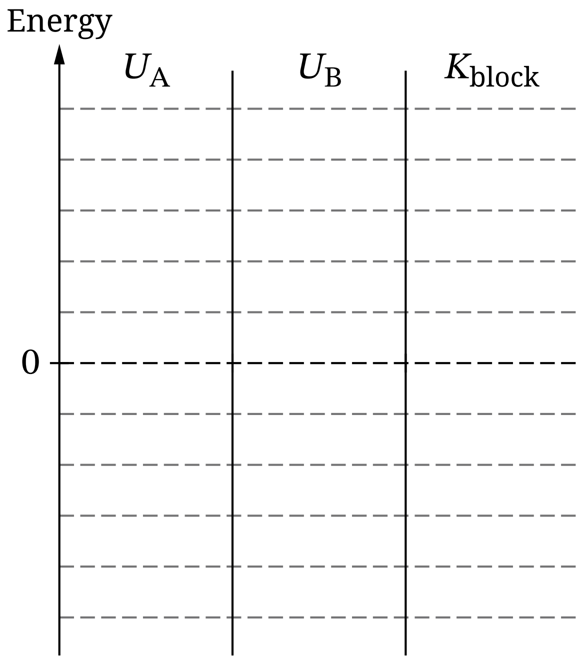
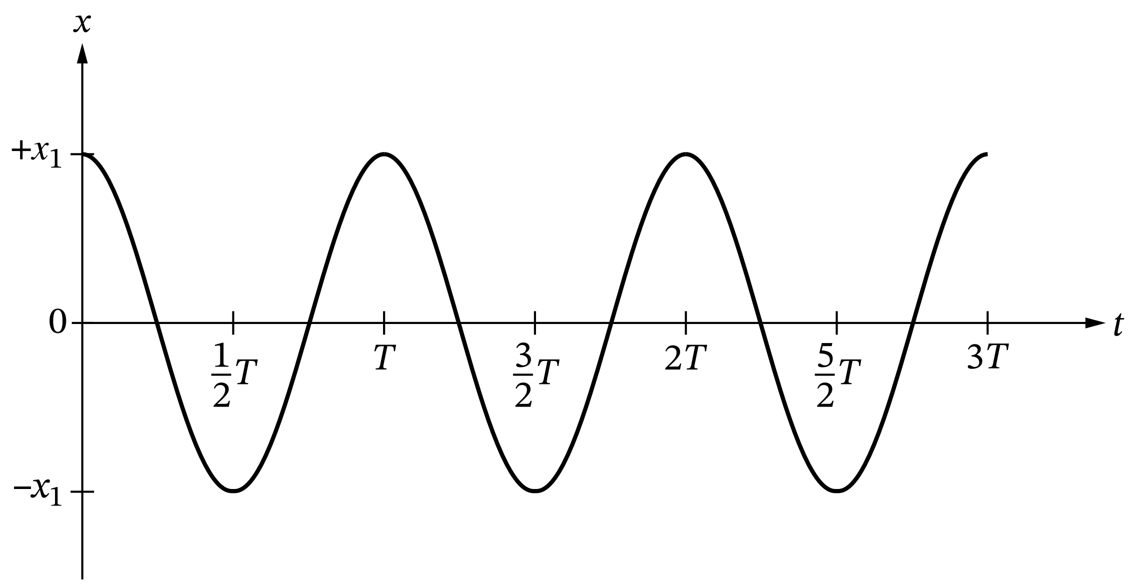


Figure 3

B. The block is released from rest at $x = x_1$ and begins to oscillate. **Derive** an expression for the speed v of the block as the block passes through $x = \frac{1}{2}x_1$. Express your answer in terms of m, k, x_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

C. In Scenario 1, the block oscillates with period T. The position x of the block in Scenario 1 as a function of time t is shown in Figure 4.

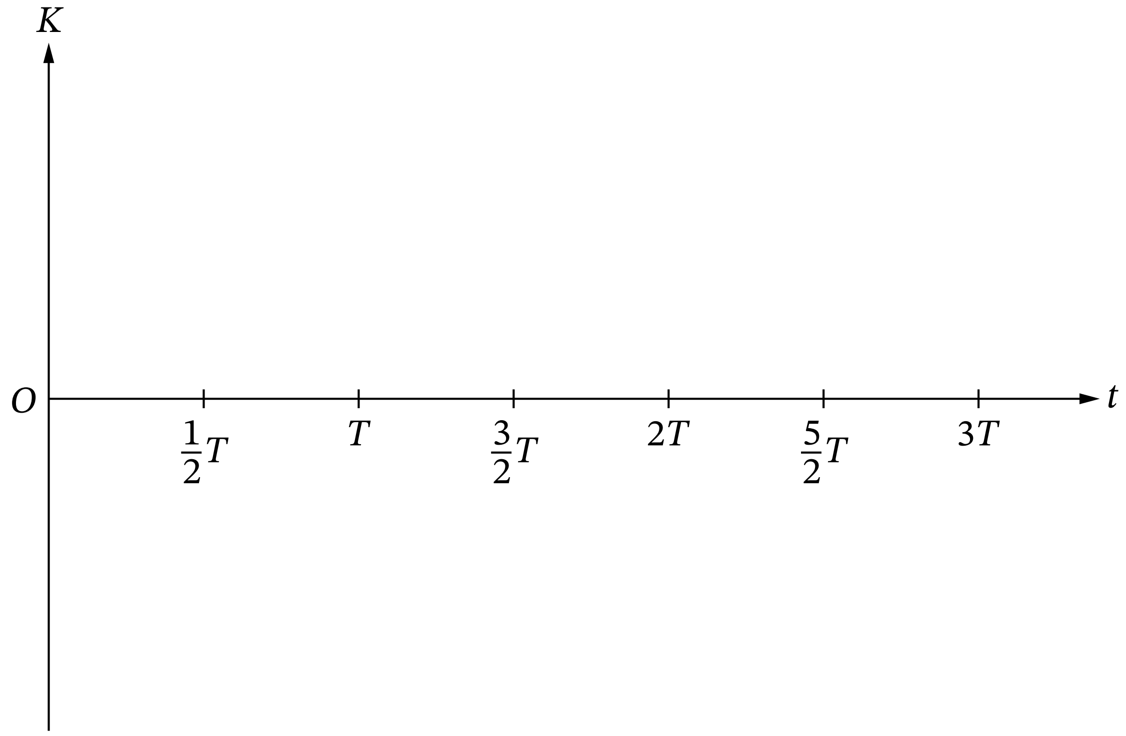


Scenario 1

Figure 4

In Scenario 2, the block-springs system is placed on a new surface. There is friction between the block and the new surface. The block is again moved to the same position $x = x_1$ and released from rest. The block completes multiple oscillations with the same period as in Scenario 1 before coming to rest.

On the axes shown in Figure 5, **sketch** a graph of the kinetic energy K of the block as a function of t for Scenario 2.



Scenario 2

Figure 5

D. In Scenario 3, the block is replaced with a new block of larger mass. The coefficient of kinetic friction between the new block and the surface in Scenario 3 is the same as the coefficient of kinetic friction between the original block and the surface in Scenario 2.

The new block is moved to position $x = x_1$ and released from rest. The kinetic energy of the new block is plotted as a function of time.

Describe how one feature of the graph of K as a function of t in Scenario 3 would differ from the graph you drew in Figure 5 for Scenario 2.

Briefly **justify** your answer.

Begin your response to **QUESTION 1** on this page.

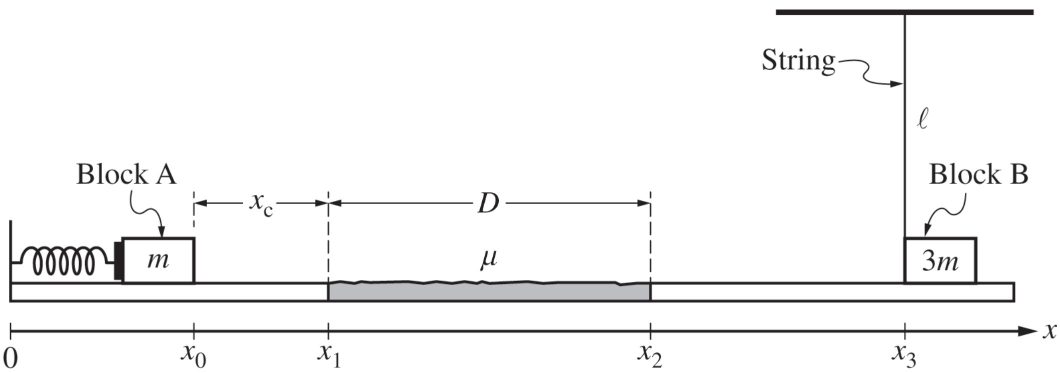
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



<u>Note:</u> Figure not drawn to scale.

Figure 1

1. Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position.
- At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction.
- At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1.

ii. **Derive** an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3.

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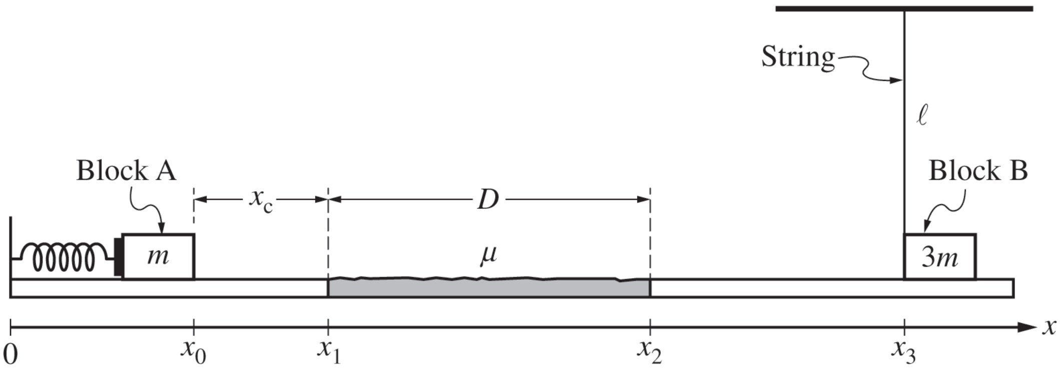
Continue your response to **QUESTION 1** on this page.

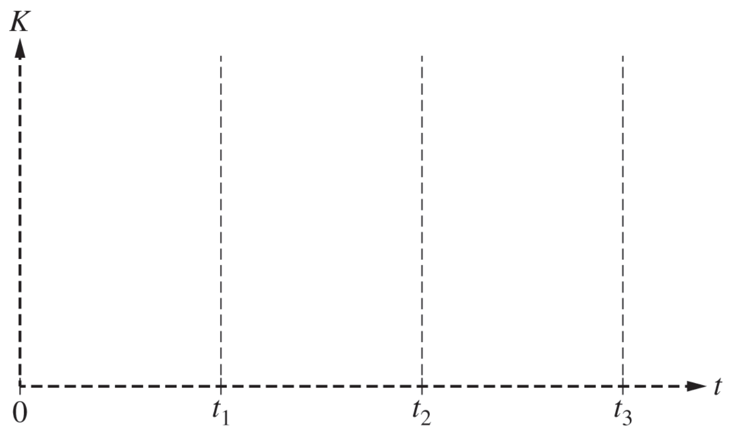


Note: Figure not drawn to scale.

Figure 1

(b)

i. On the following axes, **sketch** a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to time t_3.



ii. Use principles of work and energy to **justify** the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

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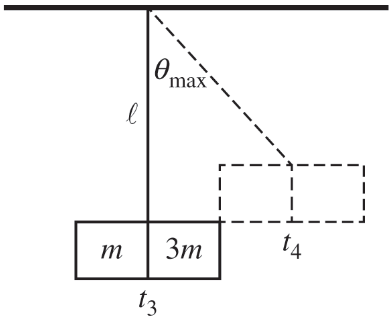
Continue your response to **QUESTION 1** on this page.



Note: Figure not drawn to scale.

Figure 2

After the collision, the two-block system instantaneously comes to rest at time t_4, which occurs when the string makes a small angle θ_{max} with the vertical, as shown in Figure 2. For times $t > t_4$, the system oscillates with frequency f_ℓ. The support holding the string is raised, and the procedure is then repeated using a new string of length 2ℓ.

(c) **Indicate** how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length 2ℓ will compare to the frequency of oscillation f_ℓ from the original procedure.

____ $f_{2\ell} > f_\ell$ ____ $f_{2\ell} < f_\ell$ ____ $f_{2\ell} = f_\ell$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 1** on this page.

PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

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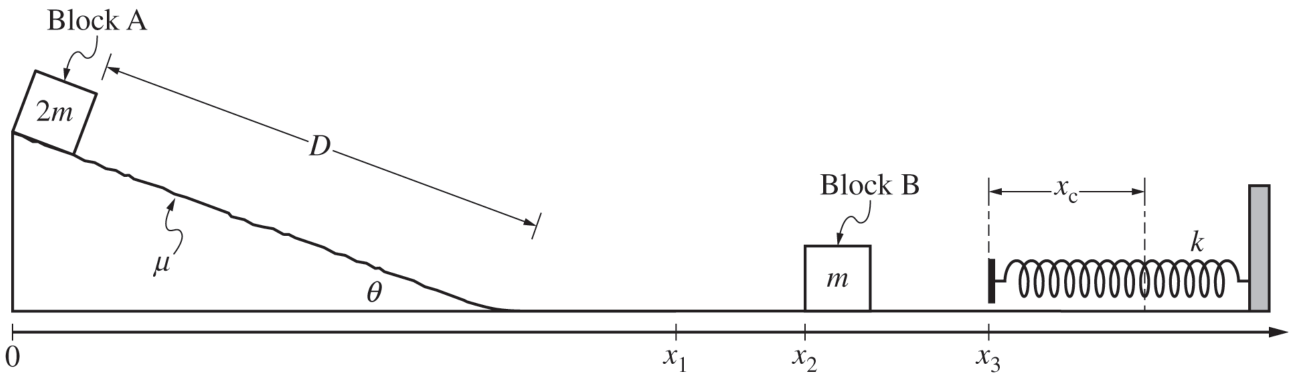

Note: Figure not drawn to scale.

1. Blocks A and B of masses $2m$ and m, respectively, are arranged in a setup consisting of a ramp that makes an angle θ with a smooth horizontal table and an ideal spring of spring constant k fixed to a wall, as shown. Block A is held at rest a distance D up the ramp, and Block B is at rest on the horizontal table. The coefficient of kinetic friction between Block A and the rough ramp is μ in the region of length D, and there is negligible friction between the blocks and the smooth table.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$.
- At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B.
- At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position.
- At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, θ, D, μ, x_c, and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1.

ii. **Derive** an expression for the spring constant k of the spring.

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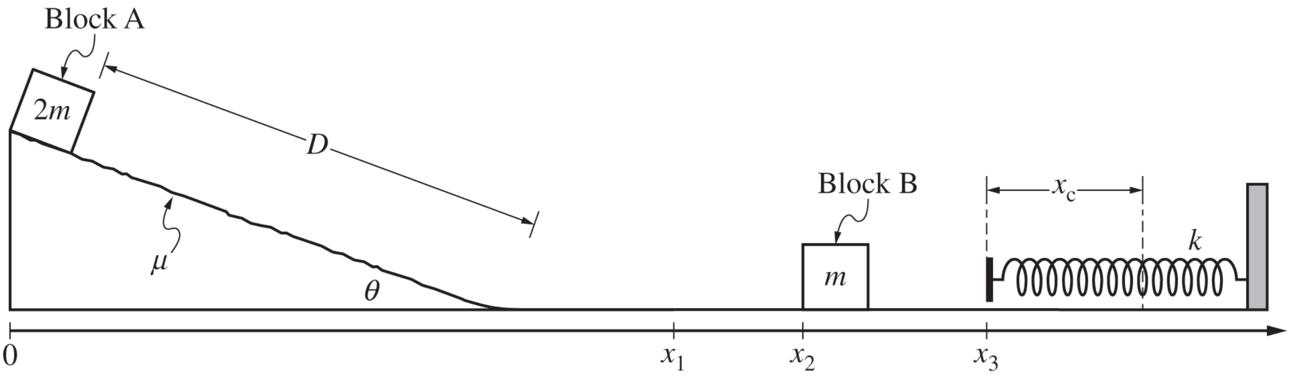
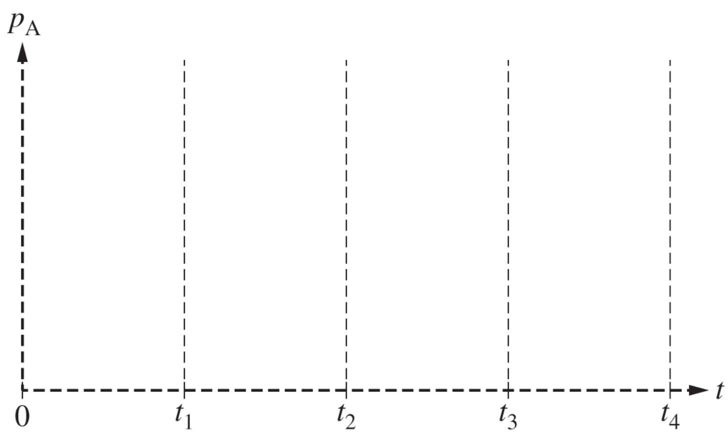
Continue your response to **QUESTION 1** on this page.



Note: Figure not drawn to scale.

(b)

i. On the following axes, **sketch** a graph of the magnitude of the momentum p_A of Block A as a function of time t from $t = 0$ to t_4.



ii. Use principles of forces to **justify** the graph drawn in part (b)(i) for the time interval $t = t_3$ to $t = t_4$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

For times $t > t_4$, the two-block-spring system oscillates with period T_O. The procedure is then repeated using a new ramp, where there is negligible friction between Block A and the ramp.

(c) **Indicate** how the new period of oscillation T_N in the procedure that uses the new ramp compares with the period of oscillation T_O from the original procedure.

_____ $T_N > T_O$ _____ $T_N < T_O$ _____ $T_N = T_O$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

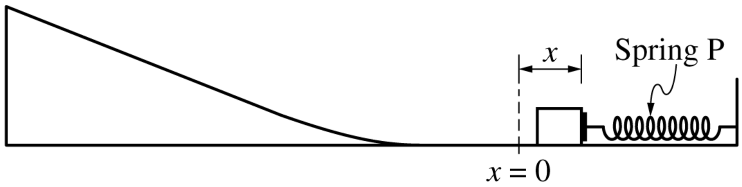


Figure 1

. A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{max,P}$. Frictional forces between the block and all surfaces are negligible.

GO ON TO THE NEXT PAGE.

A student compares $h_{max,P}$ for Spring P with the maximum height $h_{max,Q}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x that the spring is compressed, as shown in Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0$ N/m, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0\ \text{N/m}^{1/2}$.

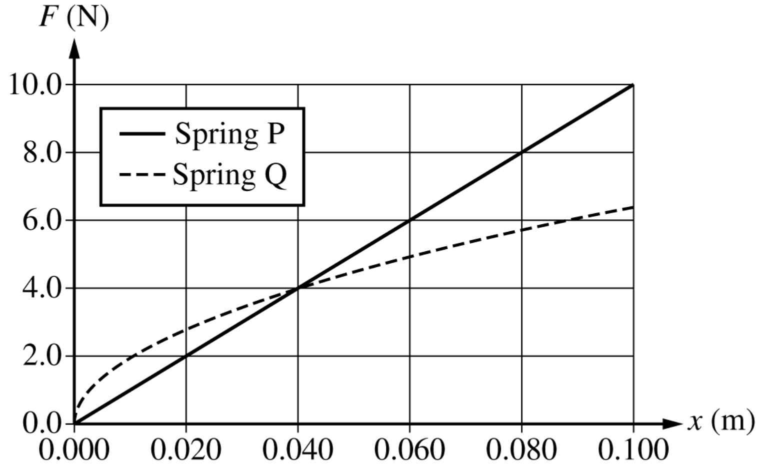
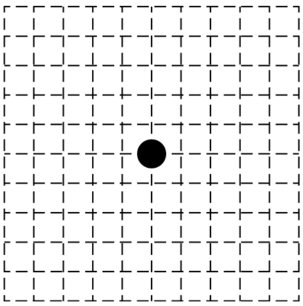


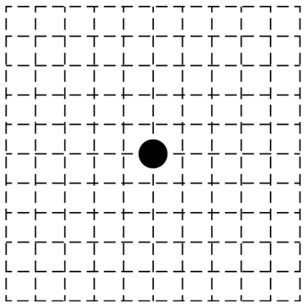
Figure 2

(a) For the experiment, the block is pushed against one of the springs, compressing the spring a distance $x = 0.010$ m. The block is then released from rest. In Trial 1, Spring P is used, and in Trial 2, Spring Q is used. On the following dots, representing the block in Trials 1 and 2, draw and label the forces (not components) that are exerted on the block at the instant the block is released. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. The lengths of the horizontal vectors should represent the relative magnitude of the horizontal forces and the lengths of the vertical vectors should represent the relative magnitude of the vertical forces.

Trial 1
(Spring P)



Trial 2
(Spring Q)



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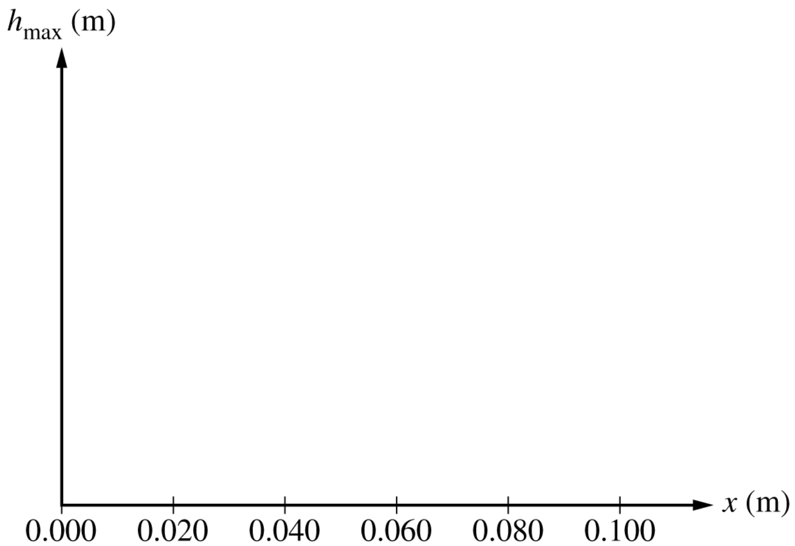
Continue your response to **QUESTION 1** on this page.

(b)

i. What feature(s) of the graph in Figure 2 could be used to estimate the work done on the block by each spring as each spring is compressed?

ii. There is one compression distance x_0 for which the maximum height h_{max} reached by the block is the same regardless of which spring, Spring P or Spring Q, is used. Predict whether the value of x_0 is greater than, less than, or equal to 0.040 m. Use the graph in Figure 2 to justify your answer.

iii. On the axes provided, for both Spring P and Spring Q, sketch a graph of the maximum height h_{max} reached by the block as a function of the distance each spring is compressed for values of x ranging from 0 to 0.100 m. Clearly label the curve for Spring P and Spring Q.



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Continue your response to **QUESTION 1** on this page.

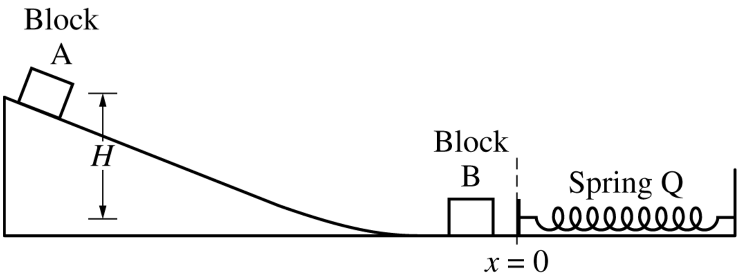


Figure 3

(c) Spring Q is attached to the wall and is at equilibrium, as shown in Figure 3. Block A has a mass of 0.120 kg and Block B has a mass of 0.070 kg. Block A is held at rest at the top of the ramp and Block B is at rest on the horizontal surface. The student releases Block A, and it moves down the ramp and collides with Block B. The change in vertical height of Block A is $H = 0.75$ m. After the collision, the blocks stick together and move to the right, compressing the spring. Frictional forces between the blocks and all surfaces are negligible.

i. Calculate the velocity of the two-block system immediately after the collision between Blocks A and B.

ii. Calculate the maximum compression of the spring.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

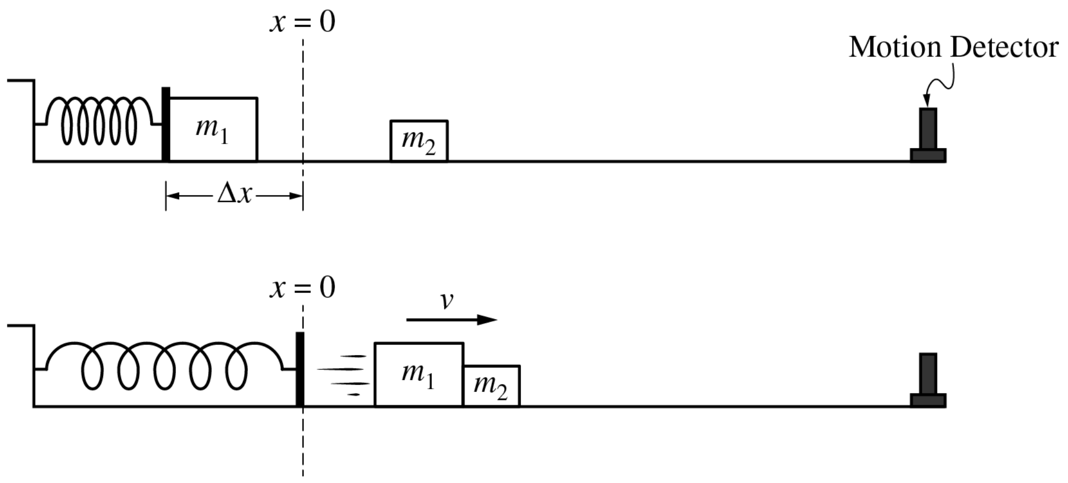
(d) Spring Q where $F_Q(x) = Cx^{1/2}$ is replaced by a different nonlinear spring, Spring R, and the procedure described in part (c) is repeated. For Spring R, $F_R(x) = Dx^{1/2}$. The maximum compression of Spring R is greater than the maximum compression of Spring Q. Which of the following correctly compares the constants C and D ?

__________ $C < D$ __________ $C > D$ __________ $C = D$

Briefly justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 2** on this page.



. Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v, as shown. Frictional effects are negligible.

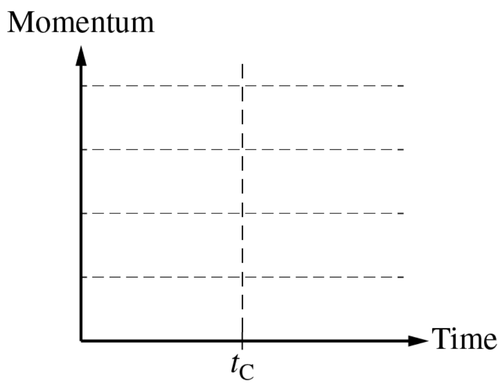
(a) The impulse on Block 1 from the spring during the time interval $0 < t < t_C$ is J_S. The impulse on Block 1 from Block 2 during the collision is J_2. Which of the following expressions correctly compares the magnitudes of J_S and J_2 ?

___ $J_S > J_2$ ___ $J_S < J_2$ ___ $J_S = J_2$

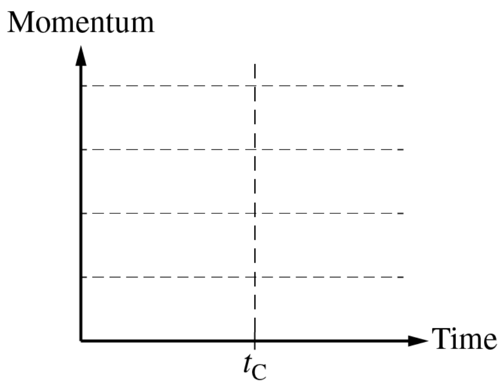
Justify your answer.

Continue your response to **QUESTION 2** on this page.

(b) On the following axes, draw graphs of the magnitude of the momentum of each block as a function of time, before and after t_C. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.



Block 1



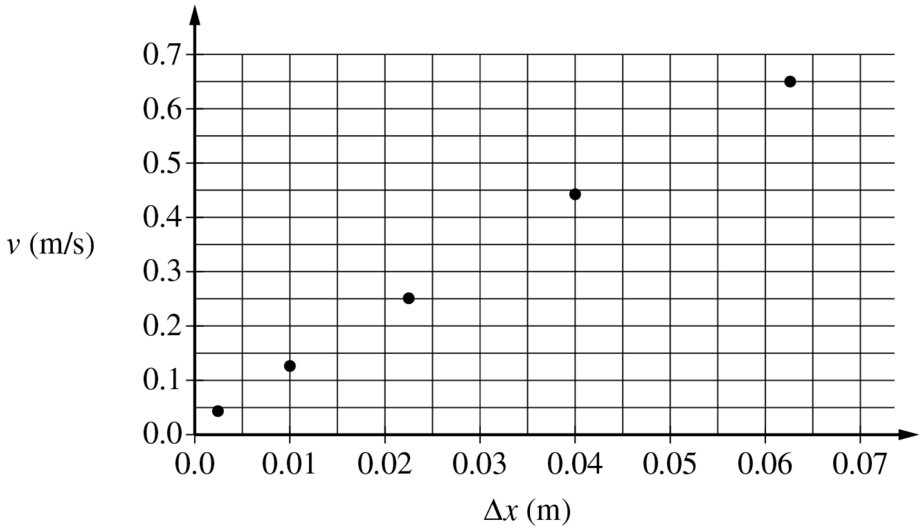
Block 2

(c) Show that the velocity v of the two-block system after the collision is given by the equation $v = \dfrac{\sqrt{km_1}}{m_1 + m_2}\Delta x$.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) A group of students use the setup to perform an experiment. They measure the mass of Block 1 to be $m_1 = 0.500$ kg, and the spring constant k of the spring to be 150 N/m. The mass of Block 2 is unknown. They perform several trials and in each trial the spring is compressed a different distance Δx and the final velocity v of the two-block system is measured. They graph v as a function of Δx, as shown below.



i. Draw a line that represents the best fit to the data points shown.

ii. Use the best-fit line to calculate the mass of Block 2.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(e) After the experiment, the students use a balance to measure the mass of Block 2 and find it to be greater than what was determined in part (d). To explain this discrepancy, one of the students proposes that the spring constant was incorrectly measured at the beginning of the experiment. The students measure the spring constant again and record a new value, k'.

Should the students expect that k' be greater than 150 N / m, less than 150 N / m, or equal to 150 N / m ?

_____ $k' > 150$ N / m   _____ $k' < 150$ N / m   _____ $k' = 150$ N / m

Justify your answer.

GO ON TO THE NEXT PAGE.

Mech.2.

A block of mass 2*M* rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where *B* is a positive constant and *x* is the displacement from equilibrium for the spring. A block of mass 3*M* and initial speed v_0 is moving to the left as shown.

(a) On the dots below, which represent the blocks of mass 2*M* and 3*M*, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

Block of Mass 2*M* Block of Mass 3*M*

The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance *D*, as shown above. Express your answers to parts (b), (c), and (d) in terms of *M*, *B*, v_0, and physical constants, as appropriate.

(b) Derive an expression for the speed of the blocks immediately after the collision.

(c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.

(d) Derive an expression for the maximum distance *D* that the spring is compressed.

(e)

i. In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

____ Left ____ Right ____ The net force on the block of mass $2M$ is zero.

Justify your answer.

ii. Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

____ The magnitude of the net force is greater on the block of mass $2M$.

____ The magnitude of the net force is greater on the block of mass $3M$.

____ The magnitude of the net force on each block has the same nonzero value.

____ The magnitude of the net force on each block is zero.

Justify your answer.

(f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

____ Yes ____ No

Justify your answer.

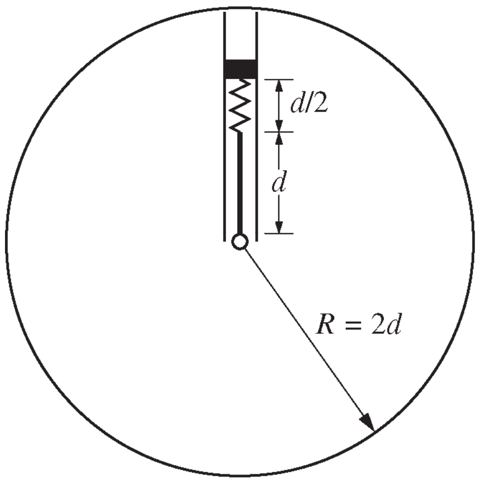


Figure 1

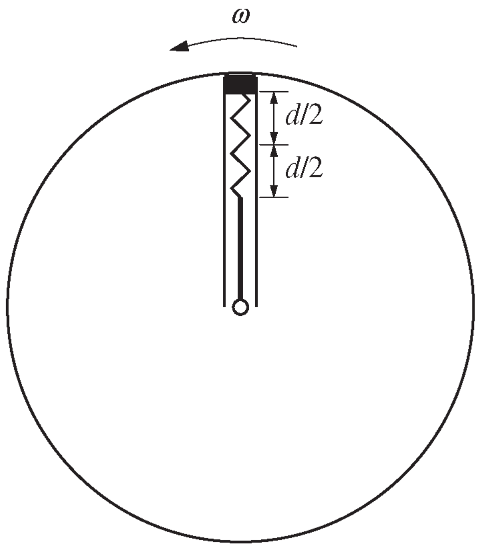


Figure 2

Mech.3.

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

	Mass	Rotational Inertia
Block	m	
Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)
Platform	$m_P = 5m$	$\frac{m_P R^2}{2}$ (about the central axis)

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(a) Derive an expression for the spring constant of the spring.

(b)

i. Determine an expression for the rotational inertia of the block around the axis of the platform.

ii. Derive an expression for the rotational inertia of the entire system about the axis of the platform.

(c) Determine an expression for the angular momentum of the entire system about the axis of the platform.

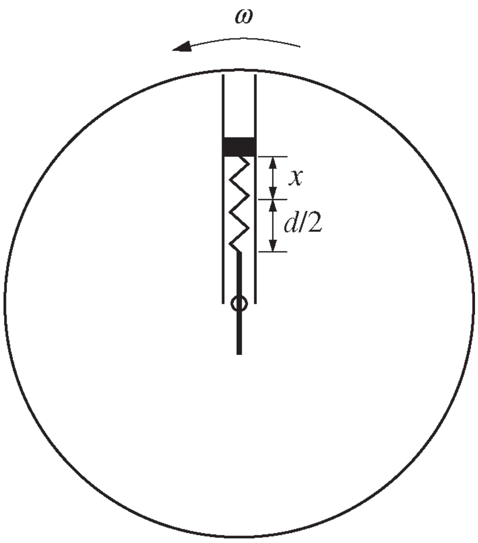


Figure 3

While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed ω is maintained by the motor driving the platform.

(d) Derive an expression for the distance x that the spring is stretched when the rod reaches the position shown in Figure 3 above.

For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

(e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

_____ Increasing _____ Decreasing _____ Staying the same

Justify your answer.

(f) In order to keep the system rotating with constant angular speed ω, is the motor doing positive work, negative work, or no work on the rotating system?

_____ Positive _____ Negative _____ No work

Justify your answer.

(g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.

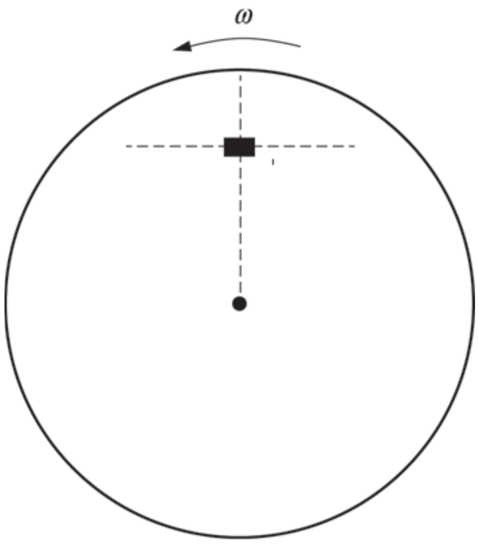


Figure 4

STOP

END OF EXAM

2.9 Resistive Forces

Name: ____________ Class: ________ Date: __________ **Total: 13 marks**

KEY WORDS drag force 阻力 • terminal velocity 终极速度 • force 力 • net force 合力

Objective

Build the skills to answer exam questions on **resistive forces**.

You must be able to:

- describe a **drag force** 阻力 that opposes motion and grows with speed
- explain **terminal velocity** 终极速度: reached when drag balances weight
- describe the acceleration as it falls from g to zero at terminal velocity
- reason about a velocity-time graph that levels off

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Drag opposes motion and grows with speed

A resistive (drag) force points opposite to the velocity and increases as the object speeds up. At low speed it is small; at high speed it can become large.

■ Terminal velocity

A falling object speeds up, so drag grows, until drag equals weight. Then the net force is zero and the velocity stops changing:

$$\text{drag} = mg \quad \Rightarrow \quad \text{terminal velocity.}$$

■ The acceleration fades

Just after release, drag is tiny, so $a \approx g$. As speed rises, drag grows and a falls, reaching zero at terminal velocity. The v-t graph curves and levels off.

2 Practice

Now apply the methods above.

2.1 State the direction of a drag force relative to the motion. [1]

2.2 What condition is met at terminal velocity? [1]

2.3 State the acceleration of an object at its terminal velocity. [1]

2.4 Just after a skydiver jumps, what is their approximate acceleration, and why? [2]

3 Exam-style questions

3.1 As a falling object speeds up, the drag force on it [1]

- **A** decreases
- **B** increases
- **C** stays constant
- **D** becomes zero

3.2 At terminal velocity, the net force on the object is [1]

- **A** equal to its weight
- **B** zero
- **C** upward
- **D** increasing

3.3 A raindrop falls and reaches terminal velocity.

(a) Explain, in terms of forces, why the velocity stops increasing. [2]

(b) Sketch in words the shape of its velocity-time graph. [1]

3.4 A 70 kg skydiver reaches terminal velocity ($g = 10$ m s^{-2}).

(a) State the size of the drag force. [2]

(b) State the acceleration at that instant. [1]

Solutions

2.1 Opposite to the motion.

2.2 Drag equals weight (net force zero).

2.3 Zero.

2.4 About g ($\approx 9.8\ \mathrm{m\ s^{-2}}$) — the speed is still low, so drag is negligible.

3.1 B — drag increases with speed.

3.2 B — at terminal velocity the forces balance, so the net force is zero.

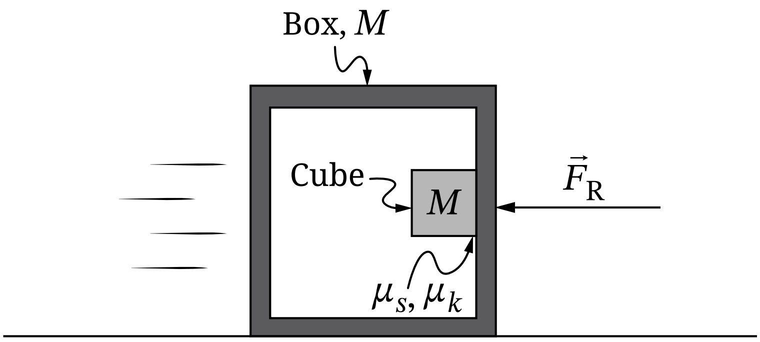
3.3 (a) Drag grows with speed until it equals the weight; the net force becomes zero, so the velocity stops changing. (b) It rises steeply, curves over, and levels off at a constant value.

3.4 (a) drag $= mg = 70 \times 10 = 700$ N. (b) Zero.

Question 1: Version J

1. A box of mass M slides to the right on a horizontal surface. A small cube, also of mass M, is inside the box. The cube is against the right wall of the box and above the bottom of the box, as shown in Figure 1. The coefficients of static and kinetic friction between the cube and the wall of the box are μ_s and μ_k, respectively, where $\mu_s > \mu_k$.

Figure 1



The speed of the box is decreasing as a result of a resistive force that is exerted on the box. The resistive force $\vec{F}_\mathrm{R}$ is modeled by $\vec{F}_\mathrm{R} = -b\vec{v}$, where b is a positive constant and $\vec{v}$ is the velocity of the box. Friction between the box and horizontal surface is negligible.

Consider the motion of the box and cube for the following times.

- At time $t = 0$, the box-cube system slides to the right with speed v_0.
- For $0 \leq t < t_\mathrm{crit}$, the cube remains in contact with the wall of the box at a constant height above the bottom of the box.
- At $t = t_\mathrm{crit}$, the cube begins to slide downward along the wall of the box.

Part A

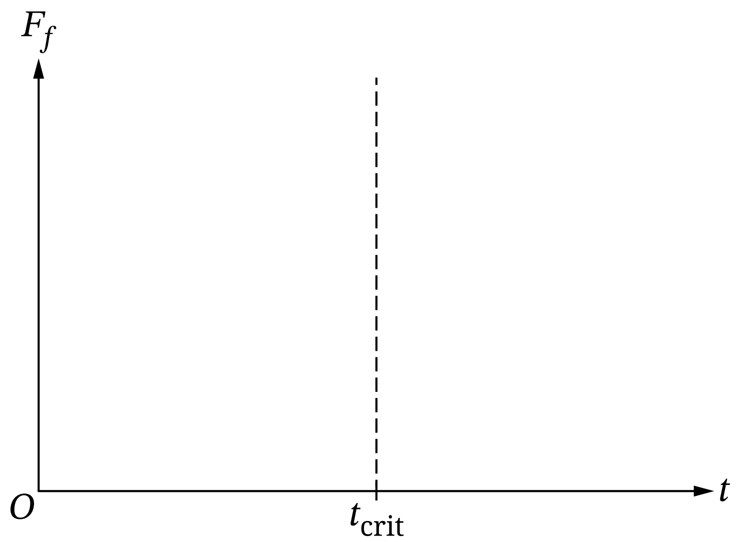
The magnitude of the normal force exerted on the cube by the wall of the box is F_N.

i. **Determine** an expression for the magnitude of the acceleration of the cube. Express your answer in terms of M, F_N, and physical constants, as appropriate.

ii. **Derive** an expression for F_N as a function of t from $t = 0$ until immediately before the cube reaches the bottom of the box. Express your answer in terms of M, b, v_0, t, and physical constants, as appropriate. Begin your derivation with a fundamental physics principle or an equation from the reference information.

iii. On the axes shown in Figure 2, **sketch** a graph of the magnitude F_f of the frictional force exerted on the cube by the wall of the box as a function of t from $t = 0$ until immediately before the cube reaches the bottom of the box.

Figure 2



Part B

Derive an expression for t_{crit} in terms of M, b, μ_s, v_0, and physical constants, as appropriate. Begin your derivation with a fundamental physics principle or an equation from the reference information.

Begin your response to **QUESTION 2** on this page.

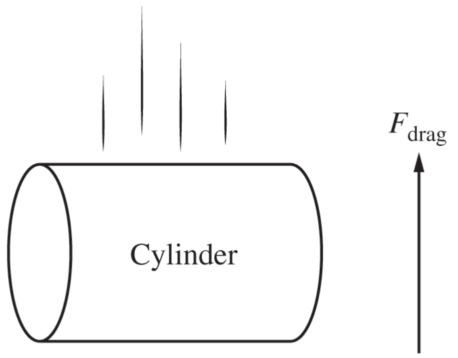


Figure 1

2. A student drops a cylinder of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the cylinder, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv^2$, where v is the speed of the cylinder and b is a positive constant with appropriate units.

(a) **Derive**, but do NOT solve, a differential equation that could be used to determine the speed v of the cylinder as a function of time t. Express your answer in terms of given quantities and physical constants, as appropriate.

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Continue your response to **QUESTION 2** on this page.

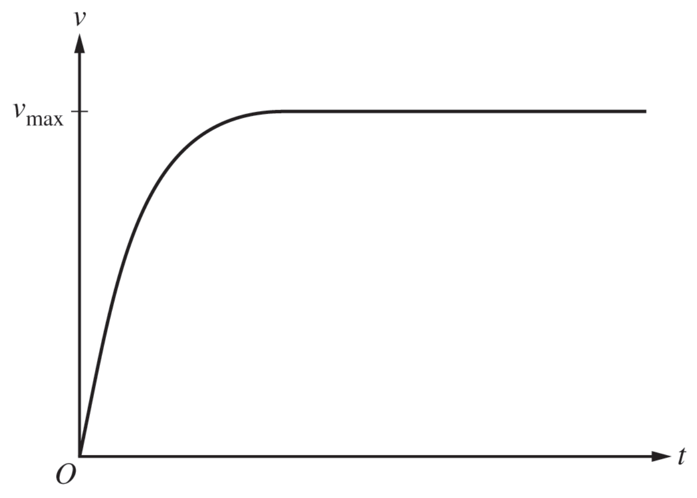

Figure 2

(b) The student correctly sketches the speed v of the cylinder as a function of time t, as shown in Figure 2.

i. **Draw** a vertical line on the sketch in Figure 2 to indicate the earliest time at which F_{drag} on the cylinder is equal to the magnitude of the weight of the cylinder. Label this time as t_1 on the time axis.

ii. **Justify** the location of t_1. Explicitly reference appropriate features of the sketch in Figure 2.

(c) Rather than dropping the cylinder from rest, the student throws the cylinder upward with a nonzero initial speed. The cylinder is in the same orientation as when the cylinder was previously dropped. The student allows the cylinder to fall toward the ground.

Indicate whether the magnitude of the cylinder's maximum downward speed after being thrown upward would be greater than, less than, or equal to the maximum speed v_{max} in Figure 2.

______ Greater than ______ Less than ______ Equal to

Briefly **justify** your answer.

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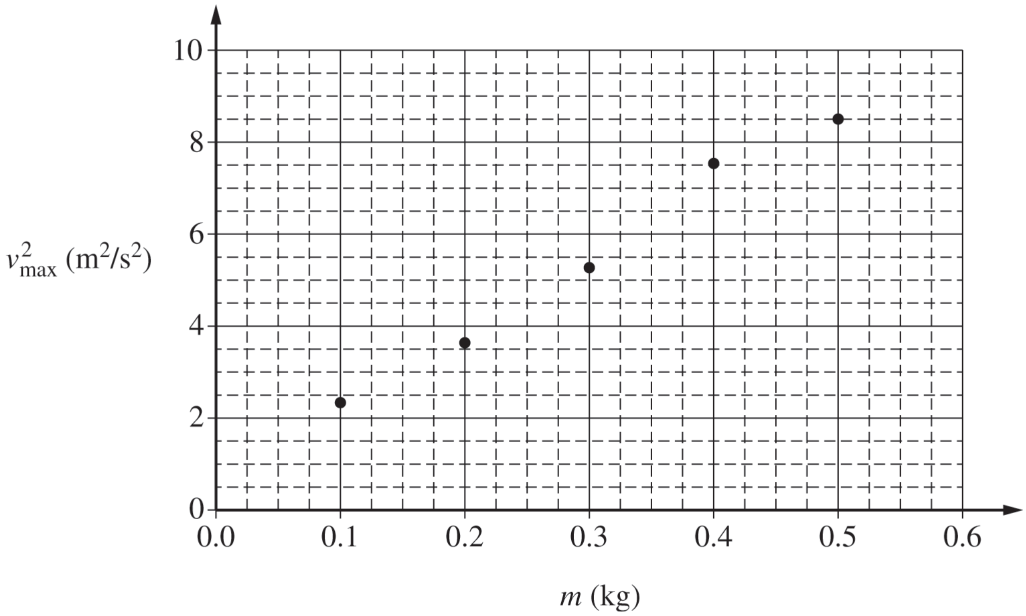
Continue your response to **QUESTION 2** on this page.

(d) The student conducts an experiment to better understand the relationship between maximum speed v_{max} and mass. The student collects data to determine the maximum speed for cylinders dropped from rest, each with the same physical size and shape but a different mass m. The student then graphs v_{max}^2 as a function of mass.



i. **Draw** the best-fit line for the data.

ii. Use the best-fit line to **calculate** an experimental value for b.

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Continue your response to **QUESTION 2** on this page.

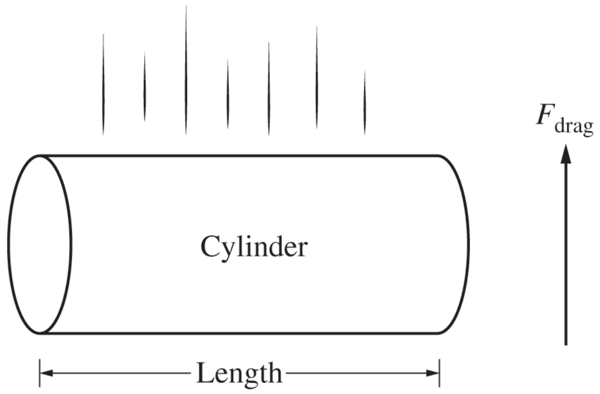

Figure 3

A student claims that the magnitude of the maximum speed of a cylinder dropped from rest depends on the length of the cylinder. The student designs an experiment to collect data that can be used to provide evidence to support the claim. The student drops cylinders with the orientation shown in Figure 3.

(e) The student has access to but does not have to use all of the following equipment.

- Cylinder Set 1: cylinders of the same known length with different known masses
- Cylinder Set 2: cylinders of the same known mass with different known lengths
- A motion detector that can measure velocity as a function of time

i. **Indicate** two quantities that when graphed could be used to determine whether the length of the cylinder affects the maximum speed.

Vertical axis: ______________ Horizontal axis: ______________

ii. Briefly **describe** how the quantities graphed could be used to determine the relationship between cylinder length and maximum speed.

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Begin your response to **QUESTION 2** on this page.

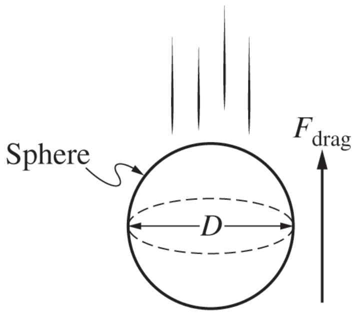

Figure 1

2. A student drops a sphere of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the sphere, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv$, where v is the speed of the sphere and b is a positive constant with appropriate units.

   (a) **Derive**, but do NOT solve, a differential equation that could be used to determine the speed v of the sphere as a function of time t. Express your answer in terms of given quantities and physical constants, as appropriate.

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Continue your response to **QUESTION 2** on this page.

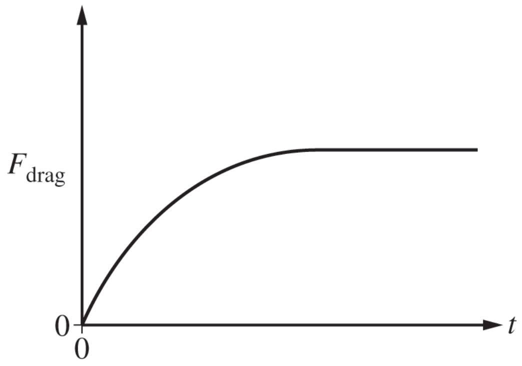


Figure 2

(b) The student sketches the drag force F_{drag} exerted on the sphere as a function of time t, as shown in Figure 2.

i. **Draw** a vertical line on the sketch in Figure 2 to indicate the earliest time at which F_{drag} is equal to the magnitude of the weight of the sphere, which occurs when the sphere reaches terminal speed. Label this time as t_T on the time axis.

ii. **Justify** the location of t_T. Explicitly reference appropriate features of the sketch in Figure 2.

(c) Suppose the student throws the same sphere downward with a nonzero initial speed. The magnitude of the new drag force at terminal speed after being thrown downward is F_{new}.

Indicate whether F_{new} would be greater than, less than, or equal to the magnitude of F_{drag} at terminal speed represented in Figure 2.

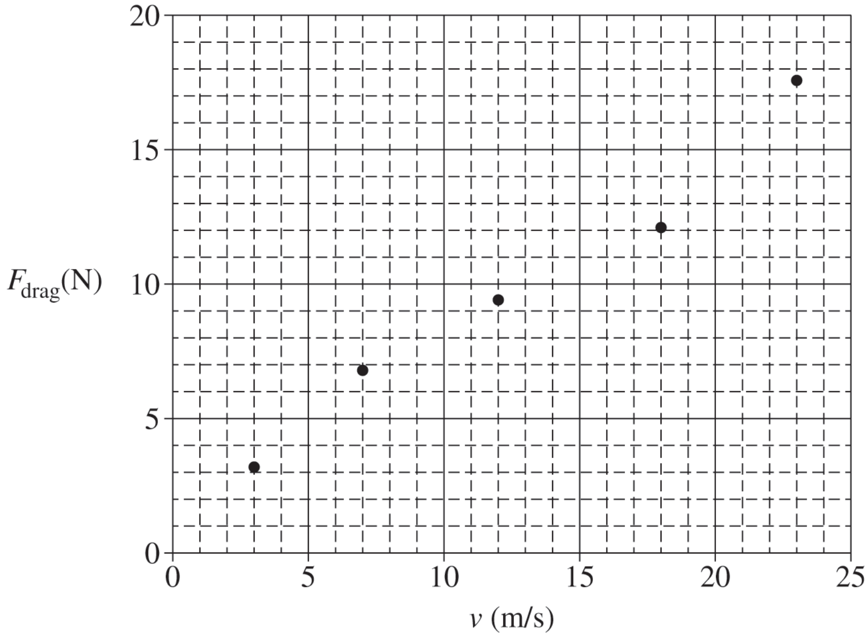
_____ Greater than _____ Less than _____ Equal to

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) The student conducts an experiment to better understand the relationship between F_{drag} and v. The student makes measurements to calculate and graph the magnitude of F_{drag} as a function of v for the falling sphere.



i. **Draw** the best-fit line for the data.

ii. Use the best-fit line to **calculate** an experimental value for b.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

A student claims that the terminal speed v_T of the sphere depends on the diameter D of the sphere. The student designs an experiment to collect data that can be used to provide evidence to support the claim.

(e) The student has access to but does not have to use all of the following equipment.

- Sphere Set 1: spheres of the same known mass with different known diameters
- Sphere Set 2: spheres of the same known diameter with different known masses
- A motion detector that can measure velocity as a function of time

i. **Indicate** two quantities that when graphed could be used to determine whether the diameter of the sphere affects the terminal speed.

Vertical axis: ________________ Horizontal axis: ________________

ii. Briefly **describe** how the quantities graphed could be used to determine the relationship between sphere diameter and terminal speed.

GO ON TO THE NEXT PAGE.

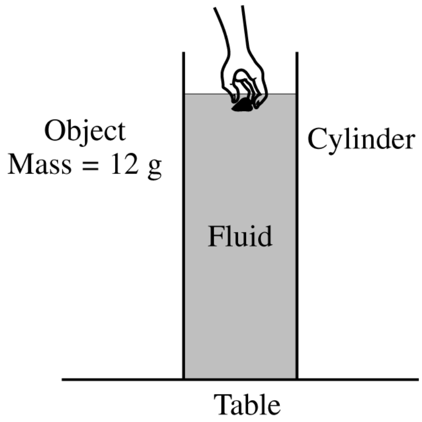
PHYSICS C: MECHANICS

SECTION II

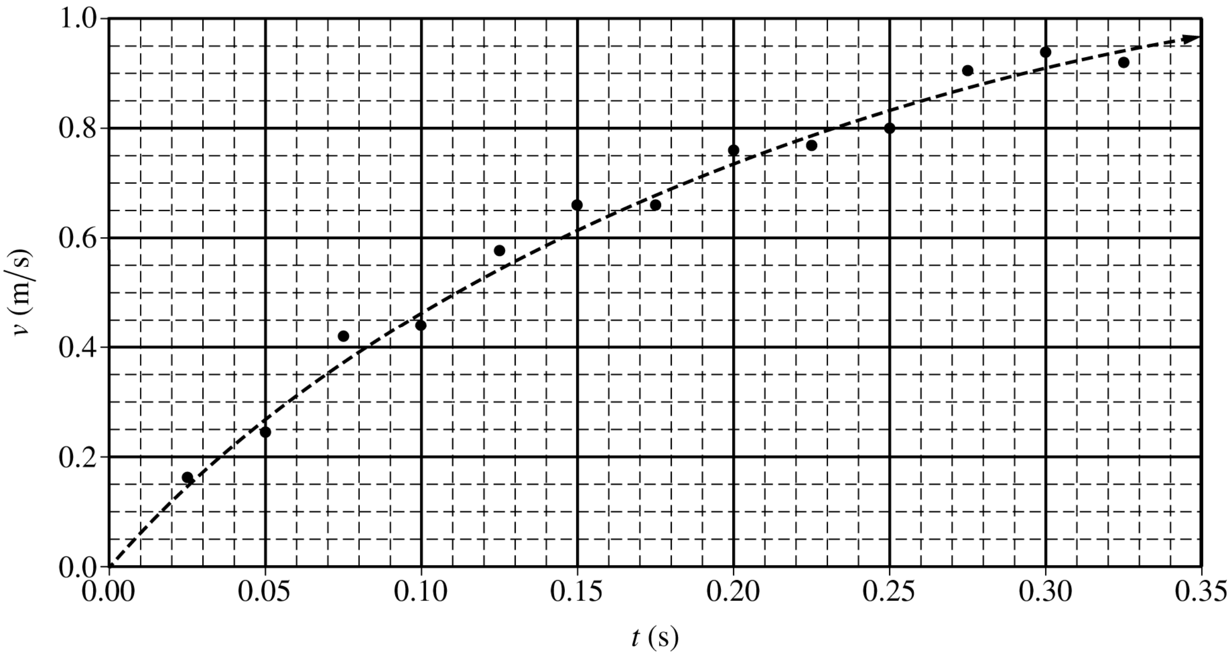
Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of m = 12 g is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.



(a)

i. Does the speed of the object increase, decrease, or remain the same?

____ Increase ____ Decrease ____ Remain the same

ii. In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.

iii. Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20$ s.

The students use the equation $v = A\left(1 - e^{-Bt}\right)$ to model the speed of the falling object and find the best fit coefficients to be $A = 1.18\ \text{m/s}$ and $B = 5\ \text{s}^{-1}$.

(b) Use the above equation to:

i. Derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time t.

ii. Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time t.

The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed.

(c)

i. Determine the constant speed of the object.

Justify your answer.

ii. Determine the force exerted by the fluid on the object at this time.

Justify your answer.

Note: Figure not drawn to scale.

2. A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

(a)

i. Derive an expression for the speed of the block just before it collides with the spring.

ii. Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

____ Greater than ______ Less than ____ Equal to

Justify your answer.

(b) Derive an expression for the maximum compression of the spring.

(c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

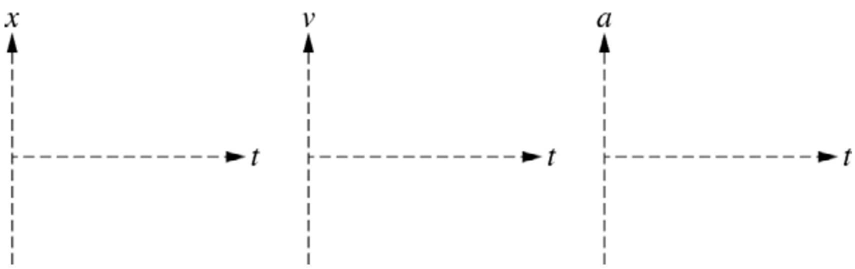
The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0. Now suppose the block experiences a resistive force as it slides on the horizontal surface. The magnitude of the resistive force F is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of $\mathrm{kg/m}$.

(d)

i. Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m, h, k, β, v, and physical constants, as appropriate.

ii. Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time t can be written in the form $\dfrac{1}{v(t)} = \dfrac{1}{v_0} + \dfrac{\beta t}{m}$, where v_0 is the speed at $t = 0$.

(e) Sketch graphs of position x as a function of time t, velocity v as a function of time t, and acceleration a as a function of time t for the block as it is moving on the horizontal surface before it reaches the spring.



2.10 Circular Motion

Name: ______________ Class: __________ Date: ____________ **Total: 14 marks**

KEY WORDS centripetal force 向心力 • centripetal acceleration 向心加速度 • friction 摩擦力 • force 力 • net force 合力

Objective

Build the skills to answer exam questions on **circular motion**.

You must be able to:

- use the **centripetal acceleration** 向心加速度: $a_c = \dfrac{v^2}{r}$
- use the **centripetal force** 向心力: $F_c = \dfrac{mv^2}{r}$, directed toward the centre
- identify which real force provides the centripetal force (gravity, tension, friction, normal)
- explain that speed can be constant while velocity changes direction

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Centripetal acceleration and force

An object moving in a circle of radius r at speed v accelerates toward the centre:

$$a_c = \frac{v^2}{r}, \qquad F_c = ma_c = \frac{mv^2}{r}.$$

For $m = 2$ kg, $v = 4$ m s^{-1}, $r = 8$ m: $F_c = \dfrac{2(4)^2}{8} = 4$ N.

■ A real force supplies it

Centripetal force is not a new force – it is provided by tension (a ball on a string), gravity (a satellite), friction (a car turning), or the normal force.

■ Constant speed, changing velocity

In uniform circular motion the speed is constant, but the direction keeps changing, so the velocity changes – there is an acceleration toward the centre.

2 Practice

Now apply the methods above.

2.1 State the direction of the centripetal acceleration. [1]

2.2 A 3 kg ball moves in a circle of radius 2 m at 4 m s^{-1}. Find the centripetal force. [2]

2.3 Name the force that keeps a car turning on a flat road. [1]

2.4 An object has $v = 6$ m s^{-1} and $r = 3$ m. Find its centripetal acceleration. [2]

3 Exam-style questions

3.1 For uniform circular motion, the net force points [1]

- **A** along the velocity
- **B** toward the centre
- **C** away from the centre
- **D** upward

3.2 Doubling the speed of an object in a fixed circle changes the centripetal force by a factor of [1]

- **A** 2
- **B** 4
- **C** 1/2
- **D** 1

3.3 A 0.5 kg ball on a string moves in a horizontal circle of radius 1 m at 3 m s^{-1}.

(a) Find the centripetal force. [2]

(b) State what provides this force. [1]

3.4 A car of mass 1000 kg rounds a curve of radius 50 m at 10 m s^{-1}.

(a) Find the centripetal acceleration. [2]

(b) Find the friction force needed to keep it on the curve. [1]

Solutions

2.1 Toward the centre of the circle.

2.2 $F_c = \dfrac{mv^2}{r} = \dfrac{3(4)^2}{2} = 24$ N.

2.3 Friction (between the tyres and the road).

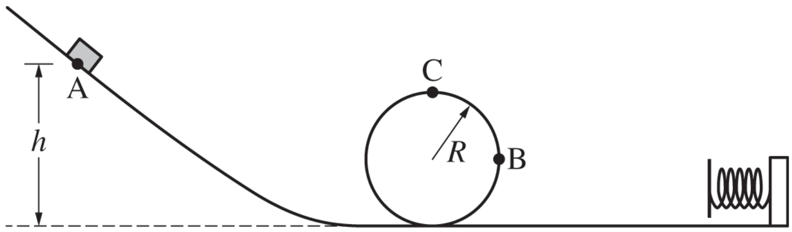
2.4 $a_c = \dfrac{v^2}{r} = \dfrac{6^2}{3} = 12$ m s^{-2}.

3.1 B — the centripetal force points toward the centre.

3.2 B — $F_c \propto v^2$, so doubling v gives $2^2 = 4$ times.

3.3 (a) $F_c = \dfrac{0.5(3)^2}{1} = 4.5$ N. (b) The tension in the string.

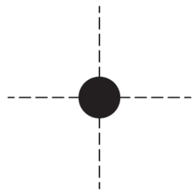
3.4 (a) $a_c = \dfrac{10^2}{50} = 2$ m s^{-2}. (b) $F = ma_c = 1000 \times 2 = 2000$ N.



Note: Figure not drawn to scale.

2. A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \dfrac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

(a) On the dot below, which represents the block, draw an arrow that represents the direction of the acceleration of the block at point B in the figure above. The arrow must start on and point away from the dot.



Justify your answer.

(b)

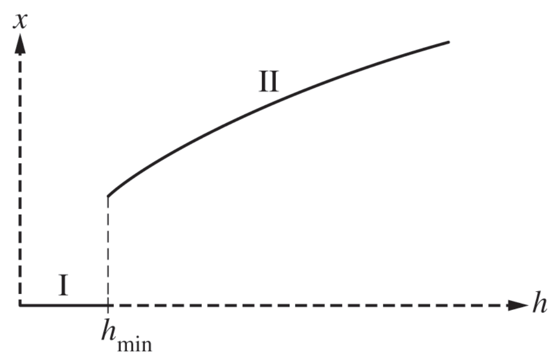
i. Derive an expression for the speed v of the block at point B.

ii. Derive an expression for the magnitude of the net force F on the block at point B.

(c) In terms of R, derive an expression for the minimum height h_{min} necessary for the block to maintain contact with the track through point C.

(d) It is determined that $h = 0.30\,\text{m}$ and $R = 0.10\,\text{m}$. If the block is released from a height greater than that found in part (c), what would be the maximum compression x_{MAX} of the spring?

(e) A graph of the maximum compression of the spring as a function of height is shown below. The height h_{min} is the height calculated in part (c).

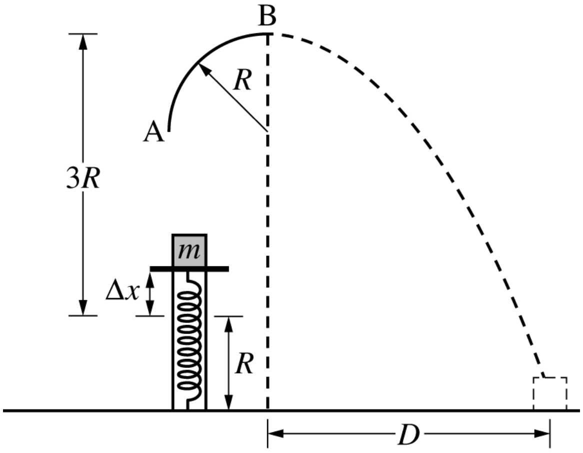


i. Explain why section I appears as a horizontal line segment on the horizontal axis.

ii. Explain the reason for the shape of section II on the graph.

GO ON TO THE NEXT PAGE.

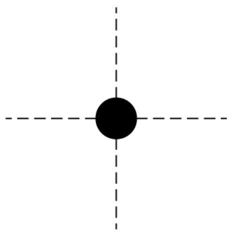
Begin your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

3. A block of mass m is placed on top of an ideal spring of spring constant k. The block is pushed against the spring, compressing the spring a distance Δx. The block is released from rest, leaves the spring at the position shown in the figure, travels upward, and enters a track with a constant radius of curvature R that has negligible friction. The block enters the track at point A, maintains contact with the track, and exits horizontally at point B, a distance $3R$ above the point the block was released. The block then falls to the ground and lands a horizontal distance D from the end of the track. Express all algebraic answers in terms of m, k, Δx, R, and physical constants, as appropriate. The size of the block is much smaller than the radius of curvature of the track.

(a) On the dot below, which represents the block, draw and label the forces (not components) that act on the block while still in contact with the track at point B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



Justify your choice of vectors.

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Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.

(b)

i. Derive an expression for the speed v of the block at point B.

ii. Derive an expression for the magnitude of the net force F on the block at point B.

(c) Derive an expression for the minimum value of Δx_{min} required in order for the block to maintain contact with the track through point B.

The procedure is repeated several times with the distance $\Delta x > \Delta x_{\text{min}}$.

(d) Calculate the distance D that the block travels.

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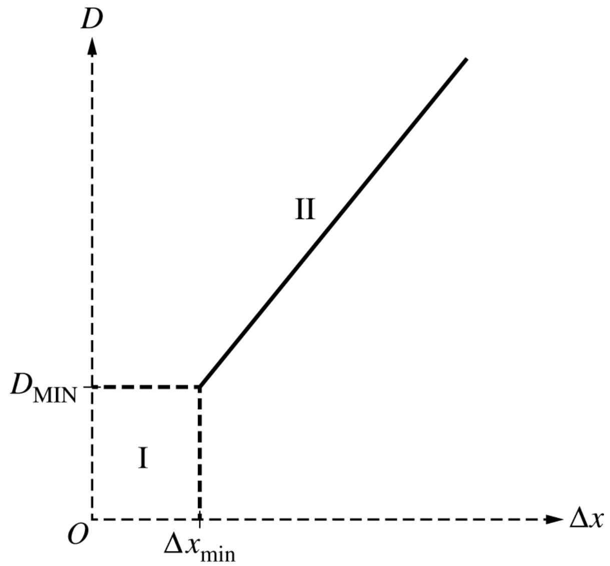
(e) The graph below shows the best-fit line drawn by the students through their data of D as a function of Δx.



i. Explain why there are no data for section I of the graph.

ii. Explain the reason for the shape and minimum value of section II on the graph.

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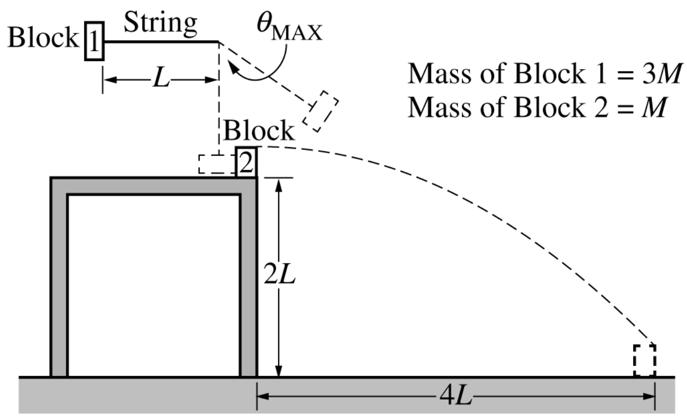
STOP

END OF EXAM

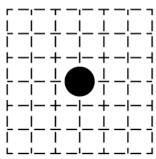


Note: Figure not drawn to scale.

2. A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(a) Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

(b) On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



(c) Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

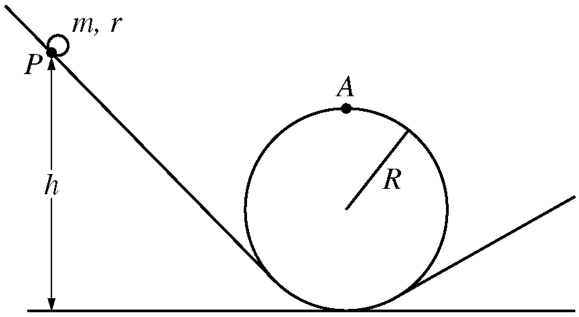
For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

(d) Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

(e) Calculate the speed of block 2 as it leaves the table.

(f) Calculate the speed of block 1 just after it collides with block 2.

(g) Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.



Note: Figure not drawn to scale.

3. The rotational inertia of a rolling object may be written in terms of its mass m and radius r as $I = bmr^2$, where b is a numerical value based on the distribution of mass within the rolling object. Students wish to conduct an experiment to determine the value of b for a partially hollowed sphere. The students use a looped track of radius $R >> r$, as shown in the figure above. The sphere is released from rest a height h above the floor and rolls around the loop.

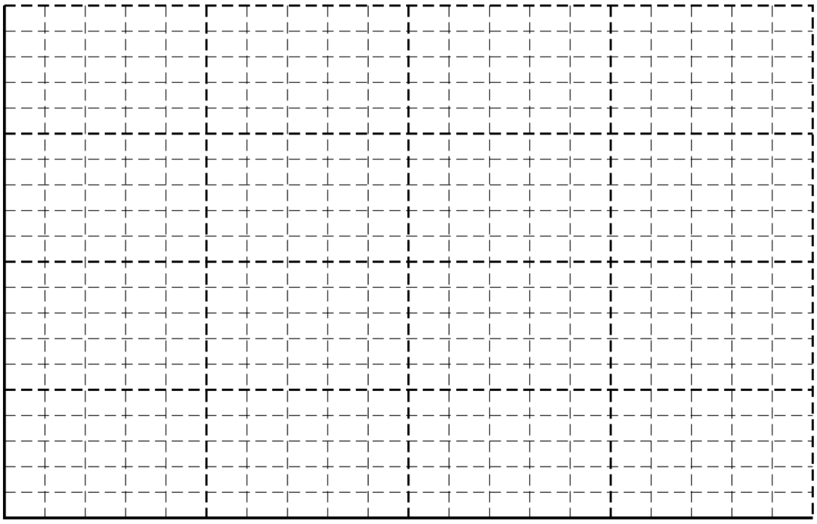
   (a) Derive an expression for the minimum speed of the sphere's center of mass that will allow the sphere to just pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

   (b) Suppose the sphere is released from rest at some point P and rolls without slipping. Derive an equation for the minimum release height h that will allow the sphere to pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

   The students perform an experiment by determining the minimum release height h for various other objects of radius r and known values of b. They collect the following data.

Object	b	h (m)
Solid sphere	0.40	1.08
Hollow sphere	0.67	1.13
Solid cylinder	0.50	1.10
Hollow cylinder	1.0	1.20

(c) On the grid below, plot the release height *h* as a function of *b*. Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



(d) The students repeat the experiment with the partially hollowed sphere and determine the minimum release height to be 1.16 m. Using the straight line from part (c), determine the value of *b* for the partially hollowed sphere.

(e) Calculate *R*, the radius of the loop.

(f) In part (b), the radius *r* of the rolling sphere was assumed to be much smaller than the radius *R* of the loop. If the radius *r* of the rolling sphere was not negligible, would the value of the minimum release height *h* be greater, less, or the same?

____ Greater ____ Less ____ The same

Justify your answer.

STOP

END OF EXAM

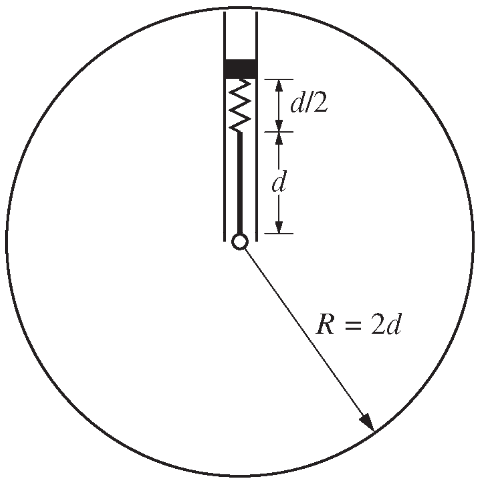


Figure 1

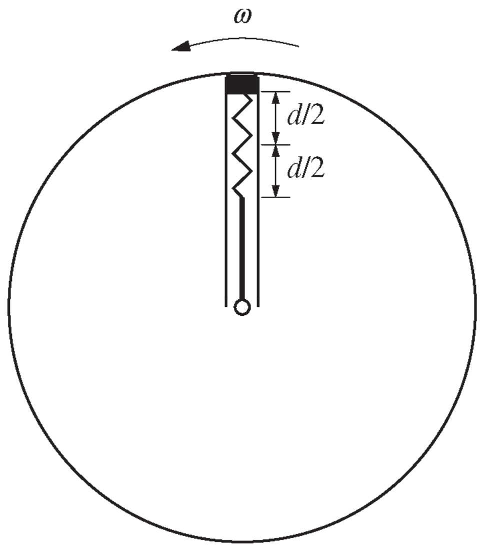


Figure 2

Mech.3.

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

	Mass	Rotational Inertia
Block	m	
Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)
Platform	$m_P = 5m$	$\frac{m_P R^2}{2}$ (about the central axis)

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(a) Derive an expression for the spring constant of the spring.

(b)

i. Determine an expression for the rotational inertia of the block around the axis of the platform.

ii. Derive an expression for the rotational inertia of the entire system about the axis of the platform.

(c) Determine an expression for the angular momentum of the entire system about the axis of the platform.

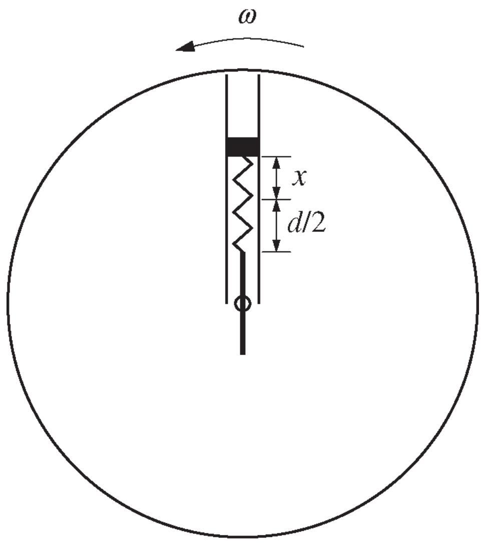


Figure 3

While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed ω is maintained by the motor driving the platform.

(d) Derive an expression for the distance x that the spring is stretched when the rod reaches the position shown in Figure 3 above.

For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

(e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

______ Increasing   ______ Decreasing   ______ Staying the same

Justify your answer.

(f) In order to keep the system rotating with constant angular speed ω, is the motor doing positive work, negative work, or no work on the rotating system?

______ Positive   ______ Negative   ______ No work

Justify your answer.

(g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.

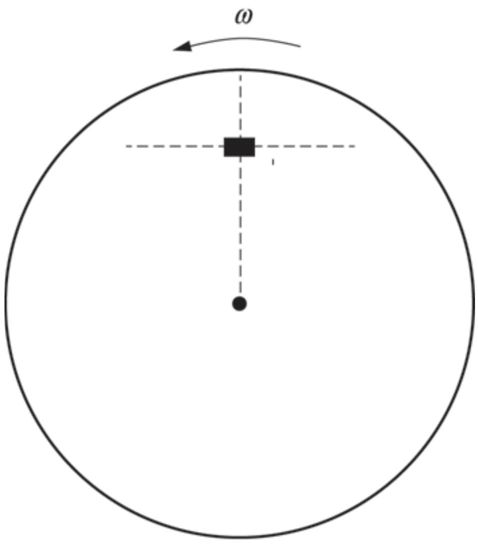
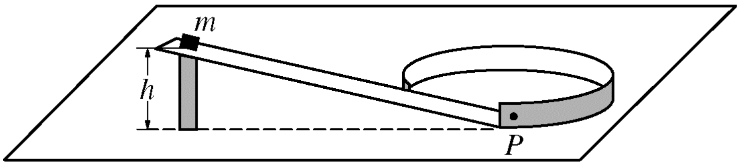


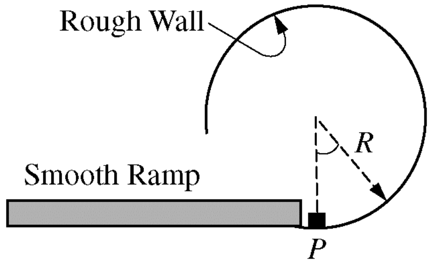
Figure 4

STOP

END OF EXAM



Side View



Top View

Mech. 2.

A small block of mass m starts from rest at the top of a frictionless ramp, which is at a height h above a horizontal tabletop, as shown in the side view above. The block slides down the smooth ramp and reaches point P with a speed v_0. After the block reaches point P at the bottom of the ramp, it slides on the tabletop guided by a circular vertical wall with radius R, as shown in the top view. The tabletop has negligible friction, and the coefficient of kinetic friction between the block and the circular wall is μ.

(a) Derive an expression for the height of the ramp h. Express your answer in terms of v_0, m, and fundamental constants, as appropriate.

A short time after passing point P, the block is in contact with the wall and moves with a speed of v.

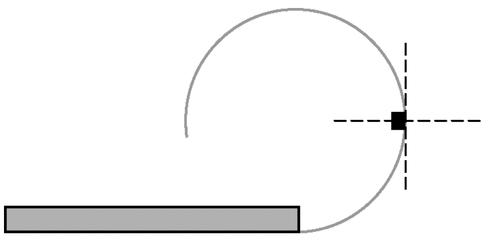
(b)

i. Is the vertical component of the net force on the block upward, downward, or zero?

____ Upward ____ Downward ____ Zero

Justify your answer.

ii. On the figure below, draw an arrow starting on the block to indicate the direction of the horizontal component of the net force on the moving block when it is at the position shown.



Top View

Justify your answer.

Express your answers to the following in terms of v_0, v, m, R, μ, and fundamental constants, as appropriate.

(c) Determine an expression for the magnitude of the normal force N exerted on the block by the circular wall as a function of v.

(d) Derive an expression for the magnitude of the tangential acceleration of the block at the instant the block has attained a speed of v.

(e) Derive an expression for $v(t)$, the speed of the block as a function of time t after passing point P on the track.