

Question 3: Free-Response Question

15 points

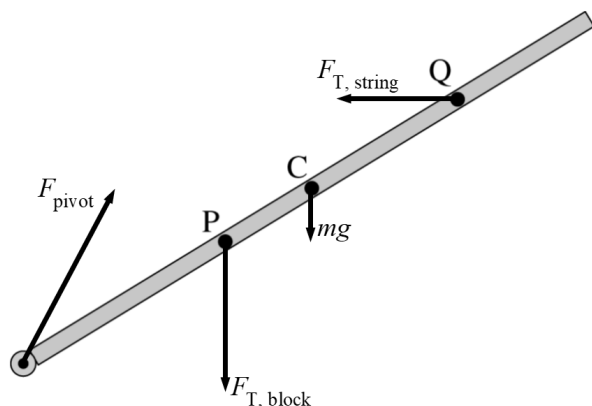
- (a) For drawing and appropriately labeling the downward forces that are exerted on the rod at points P and C **1 point**

Scoring Note: Labeling the downward force of tension as F_{block} , $3mg$, or similar, may earn this point.

For drawing and appropriately labeling a leftward force that is exerted on the rod at Point Q **1 point**

For drawing and appropriately labeling a force that is directed up and to the right that is exerted on the rod at the pivot, and no extraneous forces are present **1 point**

Example Response



Scoring Note: Examples of appropriate labels for the force due to gravity include: F_G , F_g , F_{grav} , W , mg , Mg , “grav force,” “F Earth on block,” “F on block by Earth,” $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity.

Scoring Note: Examples of appropriate labels for the force from the pivot include: F_p , F_{pivot} , F_n , F_N , N , “normal force,” “pivot force.”

Scoring Note: Examples of appropriate labels for the tension force include F_t , F_T , T ,

F_{string} , and “Force from string.”

Total for part (a) 3 points

- (b) For indicating that the net torque exerted on the rod is equal to zero **1 point**

Example Response

$$\Sigma \tau = 0$$

For a correct expression for the torque exerted on the rod by the hanging mass **1 point**

Example Response

$$3mg \sin \theta \left(\frac{3L}{8} \right)$$

For a correct expression for the torque exerted on the rod by the gravitational force **1 point**

Example Response

$$mg \sin \theta \left(\frac{4L}{8} \right)$$

For a correct expression for the torque exerted on the rod by the string **1 point**

Example Responses

$$F_T \sin(90^\circ - \theta) \left(\frac{6L}{8} \right) \quad \text{OR} \quad F_T \cos \theta \left(\frac{6L}{8} \right)$$

Example Solution

$$\Sigma \tau = 0$$

$$3mg \sin \theta \left(\frac{3L}{8} \right) + mg \sin \theta \left(\frac{4L}{8} \right) - F_T \sin(90^\circ - \theta) \left(\frac{6L}{8} \right) = 0$$

$$3mg \sin \theta \left(\frac{3L}{8} \right) + mg \sin \theta \left(\frac{4L}{8} \right) - F_T \cos \theta \left(\frac{6L}{8} \right) = 0$$

$$\left(\frac{9}{8} \right) mg \sin \theta + \left(\frac{4}{8} \right) mg \sin \theta = \left(\frac{6}{8} \right) F_T \cos \theta$$

$$13mg \sin \theta = 6F_T \cos \theta$$

$$F_T = \frac{13}{6} mg \tan \theta$$

Scoring Note: A maximum of three points may be earned if the trigonometric functions (sin and cos) are reversed for all three torque terms.

Total for part (b) 4 points

- (c) For indicating that the torque exerted on the rod by the string is always the same **1 point**
- For stating that as the angle between the string and the rod increases, the force exerted on the rod by the string decreases **1 point**

Example Response

Because the torque exerted on the rod by the string is always the same, as the angle between the string and the rod increases, the tension $F_{T, \text{new}}$ must decrease.

Total for part (c) 2 points

- (d)(i) For indicating the total mass is the sum of differentiable masses along the length of the rod **1 point**

Example Response

$$M = \int dm$$

For correctly writing dm in terms of x **1 point**

Example Response

$$M = \int_0^{1.2} (6 + 10x) dx$$

For a correct numeric answer with correct units **1 point**

Example Response

$$M = 14.4 \text{ kg}$$

Example Solution

$$M = \int dm$$

$$M = \int \lambda dx$$

$$M = \int_0^{1.2} (6 + 10x) dx$$

$$M = \left(6x + \frac{10x^2}{2} \right) \bigg|_0^{1.2 \text{ m}}$$

$$M = 6 \text{ kg/m}(1.2 \text{ m}) + \frac{10 \text{ kg/m}^2 (1.2 \text{ m})^2}{2}$$

$$M = 14.4 \text{ kg}$$

- (d)(ii) For a correct substitution of λ into an integral expression of rotational inertia **1 point**

Example Response

$$I = \int_0^{1.2 \text{ m}} (A + Bx)x^2 dx$$

For a correct integration **1 point**

Example Response

$$I = \left(\frac{Ax^3}{3} + \frac{Bx^4}{4} \right) \bigg|_0^{1.2 \text{ m}}$$

For a correct numeric answer with correct units **1 point**

Example Response

$$I = 8.64 \text{ kg} \cdot \text{m}^2$$

Example Solution

$$I = \int r^2 dm \quad dm = \lambda dr \quad \text{and} \quad r = x$$

$$I = \int_0^{1.2 \text{ m}} \lambda x^2 dx$$

$$I = \int_0^{1.2 \text{ m}} (A + Bx)x^2 dx$$

$$I = \left(\frac{Ax^3}{3} + \frac{Bx^4}{4} \right) \bigg|_0^{1.2 \text{ m}}$$

$$I = \frac{(6.0 \text{ kg/m})(1.2 \text{ m})^3}{3} + \frac{(10.0 \text{ kg/m}^2)(1.2 \text{ m})^4}{4}$$

$$I = 8.64 \text{ kg} \cdot \text{m}^2$$

Total for part (d) 6 points**Total for question 3 15 points**