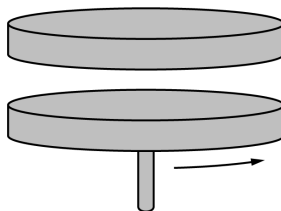


3. A horizontal circular platform with rotational inertia I_p rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the wheel until the platform reaches an angular speed ω_p after time Δt . During time Δt , the wheel stays in contact with the platform without slipping.

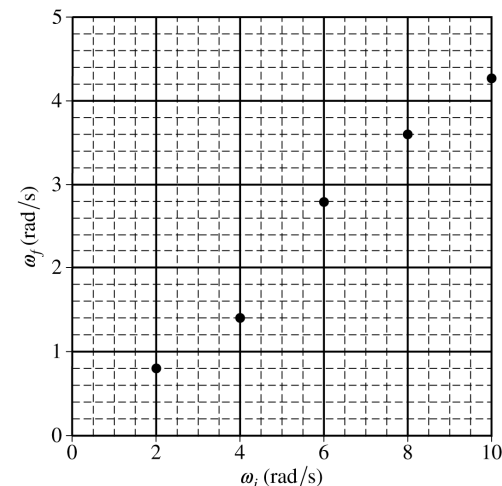
- Derive an expression for the angular speed ω_p of the platform. Express your answer in terms of I_p , r , D , F , Δt , and physical constants, as appropriate.
- Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed ω_p . Express your answer in terms of I_p , r , D , F , Δt , and physical constants, as appropriate.
- Derive an expression for the angular speed of the wheel ω_w when the platform has reached angular speed ω_p . Express your answer in terms of D , r , ω_p , and physical constants, as appropriate.



When the platform is spinning at angular speed ω_p , the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia I_p as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed ω_f .

- Derive an expression for ω_f . Express your answer in terms of ω_p , I_p , and physical constants, as appropriate.

A student now uses the rotating platform ($I_p = 3.1 \text{ kg}\cdot\text{m}^2$) to determine the rotational inertia I_U of an unknown object about a vertical axis that passes through the object's center of mass. The platform is rotating at an initial angular speed ω_i when the unknown object is dropped with its center of mass directly above the center of the platform. The platform and object are observed to be rotating together at angular speed ω_f . Trials are repeated for different values of ω_i . A graph of ω_f as a function of ω_i is shown on the axes below.



(e)

- i. On the graph on the previous page, draw a best-fit line for the data.
 - ii. Using the straight line, calculate the rotational inertia of the unknown object I_U about a vertical axis passing through its center of mass.
- (f) The kinetic energy of the spinning platform before the object is dropped on it is K_i . The total kinetic energy of the platform-object system when it reaches angular speed ω_f is K_f . Which of the following expressions is true?

_____ $K_f < K_i$ _____ $K_f = K_i$ _____ $K_f > K_i$

Justify your answer.

- (g) One of the students observes that the center of mass of the object is not actually aligned with the axis of the platform. Is the experimental value of I_U obtained in part (e) greater than, less than, or equal to the actual value of the rotational inertia of the unknown object about a vertical axis that passes through its center of mass?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

STOP

END OF EXAM