Question 2: Free-Response Question — 15 points

(a) For integrating using the correct limits or constant of integration — **1 point**

$$I = \int_{r=0}^{r=2L} \lambda r^2 dr = \lambda\left[\frac{r^3}{3}\right]_{r=0}^{r=2L} = \frac{\lambda}{3}\left((2L)^3 - 0\right)$$

For correctly relating λ to M and L — **1 point**

$$\lambda = \frac{m}{\ell} = \frac{M}{2L} \therefore I = \left(\frac{1}{3}\right)\left(\frac{M}{2L}\right)\left(8L^3\right) = \frac{4}{3}ML^2$$

Total for part (a) 2 points

(b) i. For correctly substituting into an equation for the center of mass of an object in the horizontal direction — **1 point**

$$X_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\left[\left(\frac{M}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{M}{2}\right)(L)\right]}{\left(\frac{M}{2} + \frac{M}{2}\right)} = \frac{\left(\frac{ML}{4} + \frac{ML}{2}\right)}{M} = \frac{3}{4}L$$

ii. For correctly substituting into an equation for the center of mass of an object in the vertical direction — **1 point**

$$Y_{CM} = \frac{\sum m_i y_i}{\sum m_i} = \frac{\left[\left(\frac{M}{2}\right)(L) + \left(\frac{M}{2}\right)\left(\frac{L}{2}\right)\right]}{\left(\frac{M}{2} + \frac{M}{2}\right)} = \frac{\left(\frac{ML}{2} + \frac{ML}{4}\right)}{M} = \frac{3}{4}L$$

Total for part (b) 2 points

(c) For selecting "Less than" and attempting a relevant justification — **1 point**

For a correct justification — **1 point**

Example response for part (c)

Because object B has more of its mass closer to the pivot than object A, the rotational inertia of object B must be less than that of object A.

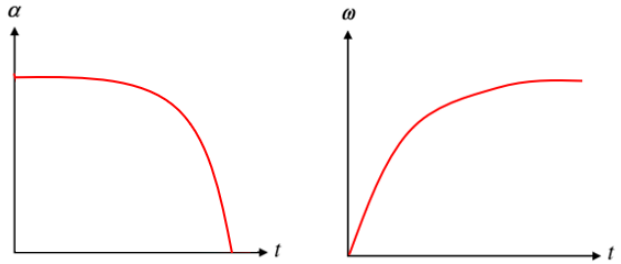
Total for part (c) 2 points

(d) For an acceleration graph that is concave down and begins horizontally — **1 point**

For an angular speed graph that is concave down and ends horizontally — **1 point**

For consistency between the angular acceleration and angular speed graphs — **1 point**

Example responses for part (d)



Total for part (d) 3 points

(e) For selecting "Decreasing" and attempting a relevant justification — **1 point**

For a justification that indicates the lever arm for the torque is decreasing — **1 point**

Example response for part (e)

Because the horizontal position of the center of mass for the object is moving closer to the pivot, the lever arm for the force of gravity is decreasing so the angular acceleration decreases.

Total for part (e) 2 points

(f) For using conservation of energy — **1 point**

$$U_{g1} = K_2$$

For correctly relating the change in rotational kinetic energy to the change in gravitational potential energy — **1 point**

$$mgh = \frac{1}{2}I\omega^2$$

For correctly substituting for h into the equation above — **1 point**

$$mg\left(\frac{L}{2}\right) = \frac{1}{2}I\omega^2$$

For an expression for ω that uses only the allowed symbols and is algebraically consistent with the previous steps — **1 point**

$$\omega = \sqrt{\frac{2mgh}{I}} = \sqrt{\frac{2Mg(L/2)}{I_B}}$$

$$\omega = \sqrt{\frac{MgL}{I_B}}$$

Total for part (f) 4 points

Total for question 2 15 points