Question 1: Free-Response Question — 15 points

(a)(i) For a multi-step derivation with an application of the conservation of mechanical energy that indicates that all of the energy of the system is initially U_s — **1 point**

Example Response

$$E_{\text{initial}} = E_{\text{final}}$$
$$\frac{1}{2}kx_{\text{c}}^2 = \frac{1}{2}mv^2$$

For a correct solution for v — **1 point**

Example Response

$$v = x_{\text{c}}\sqrt{\frac{k}{m}}$$

Example Solution

$$E_{\text{initial}} = E_{\text{final}}$$
$$U_s = K$$
$$\frac{1}{2}kx_{\text{c}}^2 = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{kx_{\text{c}}^2}{m}}$$
$$v = x_{\text{c}}\sqrt{\frac{k}{m}}$$

(a)(ii) For a derivation to solve for the speed at x_2 that includes **one** of the following: **1 point**

- An appropriate application of the conservation of energy
- An appropriate kinematics equation

Example Responses

$K_{\text{initial}} - \Delta E_{\text{friction}} = K_{\text{final}}$ **OR** $v^2 = v_0^2 + 2a\Delta x$

For **one** of the following that is consistent with the previous point in the response for part (a)(ii): **1 point**

- A correct expression for the energy dissipated by friction
- A correct expression for the acceleration of the block in the region with nonnegligible friction

Example Responses

$\Delta E_{\text{friction}} = \mu mgD$ **OR** $a = -\mu g$

For attempting to derive an expression for $v_{\text{A,B}}$ by using the conservation of momentum **1 point**

Example Response

$m_{\text{initial}}v_{\text{initial}} = m_{\text{A,B}}v_{\text{A,B}}$

For substituting the expression for the speed at x_2 that is consistent with the first point of the response in part (a)(ii) and substituting the correct masses into an expression for conservation of momentum **1 point**

Example Response

$$v_{\text{A,B}} = \frac{m\sqrt{\dfrac{k{x_c}^2}{m} - 2\mu gD}}{4m}$$

Example Solutions

$$E_{x_1} = E_{\text{before collision}}$$

$$K_{x_1} - \Delta E_{\text{friction}} = K_{\text{before collision}}$$

$$\frac{1}{2}m\left(\sqrt{\frac{kx_c^{\,2}}{m}}\right)^2 - \mu mgD = \frac{1}{2}mv_2^{\,2}$$

$$v_2 = \sqrt{\frac{kx_c^{\,2}}{m} - 2\mu gD}$$

$$\Sigma p_{\text{before collision}} = \Sigma p_{\text{after collision}}$$

$$m_A v_2 = m_{A,B} v_{A,B}$$

$$m_A v_2 = (m + 3m) v_{A,B}$$

$$v_{A,B} = \frac{m\sqrt{\frac{kx_c^{\,2}}{m} - 2\mu gD}}{4m}$$

$$v_{A,B} = \frac{1}{4}\sqrt{\frac{kx_c^{\,2}}{m} - 2\mu gD}$$

OR

$$v_2^{\,2} = v^2 + 2a\Delta x$$

$$v_2^{\,2} = \left(x_c\sqrt{\frac{k}{m}}\right)^2 + 2aD$$

$$v_2 = \sqrt{\frac{kx_c^{\,2}}{m} + 2aD}$$

$$\Sigma F_x = -F_f = ma$$

$$-\mu mg = ma$$

$$a = -\mu g$$

$$v_2 = \sqrt{\frac{kx_c^{\,2}}{m} - 2\mu gD}$$

$$\Sigma p_{\text{before collision}} = \Sigma p_{\text{after collision}}$$

$$m_A v_2 = m_{A,B} v_{A,B}$$

$$m_A v_2 = (m + 3m) v_{A,B}$$

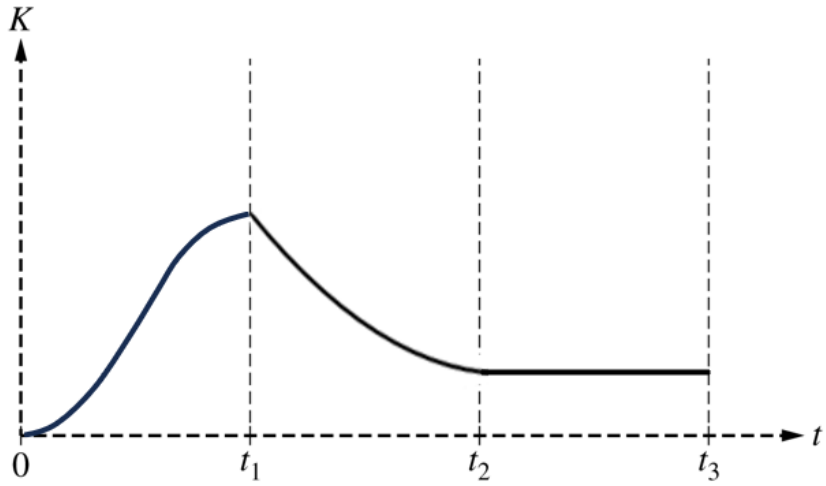
$$v_{A,B} = \frac{m\sqrt{\frac{kx_c^{\,2}}{m} - 2\mu gD}}{4m}$$

$$v_{A,B} = \frac{1}{4}\sqrt{\frac{kx_c^{\,2}}{m} - 2\mu gD}$$

Total for part (a) 6 points

(b)(i)	For a nonlinear sketch that begins at zero and increases for the entire time interval $0 \le t \le t_1$	**1 point**
	For a sketch that decreases for the entire time interval $t_1 \le t \le t_2$ but does not go to zero	**1 point**
	For a sketch that is concave up for the time interval $t_1 \le t \le t_2$	**1 point**
	For a continuous function for the time interval $t_1 \le t \le t_3$ that has a horizontal line that is greater than zero for the time interval $t_2 \le t \le t_3$	**1 point**

Example Response



(b)(ii) For a statement about the change in kinetic energy that is consistent with the graph drawn in the response for part (b)(i) — **1 point**

For a correct explanation for why the kinetic energy is increasing, such as **one** of the following: — **1 point**

- An increasing graph means positive work is being done on the block.
- An external force is exerted on Block A, causing the velocity of the block to increase and the kinetic energy of the block to increase.
- Mechanical energy is conserved and/or there is no work done for the block-spring system, and the potential energy decreases.

For a correct explanation for why the graph is nonlinear, such as **one** of the following: — **1 point**

- The rate at which the slope of the graph changes is related to the rate at which work is being done on the block.
- The external force exerted on Block A is changing, which causes a nonuniform change in the velocity of Block A, which results in a nonuniform change in kinetic energy.

Example Response

From $0 < t < t_1$, the kinetic energy of Block A increases. The force exerted on the block by the compressed spring transfers the elastic potential energy in the block-spring system to the kinetic energy of the block. Because the force exerted by the spring is not applied at a constant rate, the kinetic energy of the block does not increase at a constant rate.

Total for part (b) — 7 points

(c)	For selecting $f_{2\ell} < f_{\ell}$ with an attempt at a relevant justification	**1 point**
	For correctly applying an equation that relates the length of a pendulum to the period or frequency of the pendulum	**1 point**
	Example Response *The period of a pendulum is calculated by using* $T = 2\pi\sqrt{\frac{l}{g}}$ *. Therefore, as the length is increased, the period will also increase. Because frequency and period are inversely related, an increase in period will result in a decrease in frequency.*	
	Total for part (c)	**2 points**
	Total for question 1	**15 points**