

Question 2

15 points

A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0 \text{ m/s}^2$, $L = 0.25 \text{ m/s}^4$, and t is the time in seconds. At $t = 6.0 \text{ s}$, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

- (a) LO INT-5.E, SP 6.B, 6.C
2 points

Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0 \text{ s}$.

For an expression for calculating impulse and correct substitution of $a(t)$ and m into the correct expression.	1 point
$J = \int F(t) dt$	
$J = \int ma(t) dt = (0.50) \int (9.0 - 0.25t^2) dt$	
For integrating with correct limits or including a constant of integration	1 point
$J = (0.50) \int_{t=0}^{t=6} (9.0 - 0.25t^2) dt = (0.50) \left[9.0t - \frac{1}{12}t^3 \right]_{t=0}^{t=6}$	
$J = (0.50) \left[(9.0)(6) - \left(\frac{1}{12} \right)(6)^3 \right] - 0 = 18 \text{ N}\cdot\text{s}$	
Alternate Solution (using an alternate solution from part (b))	Alternate Points
For correctly relating impulse to the speed of the rocket	1 point
$J = \Delta p = m(v_2 - v_1)$ OR $J = m\Delta v = mv_f$	
For correctly substituting answer from part (b) into equation above	1 point
$J = (0.50 \text{ kg})(36 - 0) \therefore J = 18 \text{ N}\cdot\text{s}$	

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Question 2 (continued)

- (b) LO INT-5.A.a, SP 6.A, 6.C
2 points

Calculate the speed of the rocket at $t = 6.0 \text{ s}$.

For correctly relating impulse to the speed of the rocket	1 point
$J = \Delta p = m(v_2 - v_1)$ OR $J = m\Delta v = mv_f$	
For correctly substituting answer from part (a) into equation above	1 point
$(18 \text{ N}\cdot\text{s}) = (0.50 \text{ kg})(v_2 - 0) \therefore v_2 = 36 \text{ m/s}$	
Alternate Solution	Alternate Points
Integrate expression for $a(t)$ (This may have already been done in solving part (a).)	1 point
$v = \int a dt = \int (9.0 - 0.25t^2) dt$	
For integrating with correct limits or including a constant of integration	1 point
$v = \int_{t=0}^{t=6} (9.0 - 0.25t^2) dt = \left[9.0t - \frac{1}{12}t^3 \right]_{t=0}^{t=6} = 36 \text{ m/s}$	

- (c) i. LO INT-4.C.c, SP 6.B, 6.C
2 points

Calculate the kinetic energy of the rocket at $t = 6.0 \text{ s}$.

For substituting the mass of the rocket into the equation for kinetic energy	1 point
For substituting the answer from part (b) into the equation for kinetic energy	1 point
$K = \frac{1}{2}mv^2 = \left(\frac{1}{2} \right)(0.50 \text{ kg})(36 \text{ m/s})^2 = 324 \text{ J}$	

- ii. LO CHA-1.B, CON-1.E, SP 6.A, 6.C
3 points

Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0 \text{ s}$.

For integrating the acceleration twice to derive an expression for position	1 point
For integrating with correct limits or including a constant of integration	1 point
$v(t) = \int a(t) dt = \int (9.0 - 0.25t^2) dt = 9.0t - \frac{1}{12}t^3 + v_0 = 9.0t - \frac{1}{12}t^3$	
$\Delta y = \int v(t) dt = \int_{t=0}^{t=6} \left(9.0t - \frac{1}{12}t^3 \right) dt = \left[\frac{9}{2}t^2 - \frac{1}{48}t^4 \right]_{t=0}^{t=6}$	
$\Delta y = \left[\left(\frac{9}{2} \right)(6)^2 - \left(\frac{1}{48} \right)(6)^4 \right] - 0 = 135 \text{ m}$	
For substituting into equation for potential energy	1 point
$\Delta U_g = mg\Delta y = (0.50 \text{ kg})(9.8 \text{ m/s}^2)(135 \text{ m}) = 660 \text{ J}$	

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Question 2 (continued)

- (d) LO CHA-1.B, CON-2.B, SP 6.B, 6.C
3 points

Calculate the maximum height reached by the rocket relative to its launching point.

For using $a = g$ in a correct kinematics equation to solve for height	1 point
$v_2^2 = v_1^2 + 2a(y_2 - y_1) \therefore 0 = v_1^2 + 2a(y_2 - y_1)$	
$y_2 - y_1 = -\frac{v_1^2}{2a} \therefore y_2 = -\frac{v_1^2}{2a} + y_1$	
For substituting the speed from part (b)	1 point
$y_2 = -\frac{(36 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} + y_1$	
For substituting the height from part (c)	1 point
$y_2 = 66 \text{ m} + 135 \text{ m} = 201 \text{ m}$ (199.8 m if $g = 10 \text{ m/s}^2$)	
<i>Alternate Solution 1</i>	<i>Alternate Points</i>
For using energy conservation to find maximum height, consistent with the speed found in part (b).	1 point
$mg\Delta y = \frac{1}{2}mv^2 \therefore \Delta y = \frac{v^2}{2g}$	
For substituting the speed from part (b)	1 point
$\Delta y = \frac{(36 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 66 \text{ m}$	
For substituting the height from part (c)	1 point
$y_2 = 66 \text{ m} + 135 \text{ m} = 201 \text{ m}$ (199.8 m if $g = 10 \text{ m/s}^2$)	
<i>Alternate Solution 2</i>	<i>Alternate Points</i>
For using energy conservation to find maximum height, from kinetic and potential energies found in part (c)	1 point
$K_1 + U_1 = 0 + U_{top}$	
For substituting the kinetic energy from part (c)(i) into the equation above	1 point
For substituting the potential energy from part (c)(ii) into the equation above	1 point
$324 + 660 = (0.5)(9.8)\Delta y$	
$\Delta y = 201 \text{ m}$ (199.8 m if $g = 10 \text{ m/s}^2$)	

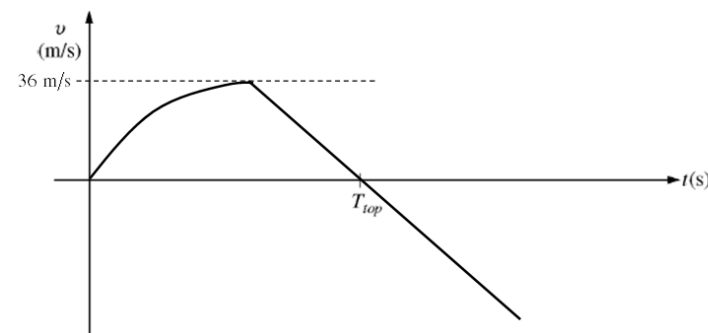
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Question 2 (continued)

- (e) LO CHA-1.C, SP 3.C
3 points

On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground.

T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



For an initial concave down curve that starts at the origin.	1 point
For a transition that occurs before T_{top} into a straight line with a negative slope.	1 point
For labeling the maximum value of the velocity, consistent with part (b), and a line that crosses the x-axis at T_{top}	1 point

2019 SCORING GUIDELINES**Question 2 (continued)****Learning Objectives**

CHA-1.B: Determine functions of position, velocity, and acceleration that are consistent with each other, for the motion of an object with a nonuniform acceleration.

CHA-1.C: Describe the motion of an object in terms of the consistency that exists between position and time, velocity and time, and acceleration and time.

CON-1.E: Calculate the potential energy of a system consisting of an object in a uniform gravitational field.

CON-2.B: Describe kinetic energy, potential energy, and total energy in relation to time (or position) for a “conservative” mechanical system.

INT-4.C.c: Calculate changes in an object’s kinetic energy or changes in speed that result from the application of specified forces.

INT-5.A.a: Calculate the total momentum of an object or system of objects.

INT-5.E: Calculate the change in momentum of an object given a nonlinear function, $F(t)$, for a net force acting on the object.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.

6.A: Extract quantities from narratives or mathematical relationships to solve problems.

6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.

6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.