AP PHYSICS C: MECHANICS • EXERCISE SHEET

6.5 Rolling

Name: ____________ Class: ________ Date: __________ **Total: 15 marks**

KEY WORDS hoop 圆环 • rolling without slipping 无滑滚动 • rotational inertia 转动惯量

Objective

Build the skills to answer exam questions on **rolling**.

You must be able to:

- use the **rolling without slipping** 无滑滚动 condition: $v = \omega r$ (and $a = \alpha r$)
- state that a rolling object's total kinetic energy is $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
- explain that the contact point is instantaneously at rest
- reason about which shape wins a downhill race

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ The no-slip condition

When a wheel rolls without slipping, its speed and spin are locked:

$$v = \omega r.$$

A wheel of radius 0.3 m spinning at 10 rad s^{-1} rolls forward at $v = 3$ m s^{-1}.

■ Total kinetic energy

A rolling object carries both translational and rotational KE:

$$K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2.$$

■ The downhill race

Sharing energy between motion and spin, a shape with a **smaller** rotational inertia (per mr^2) keeps more for speed and reaches the bottom first: solid sphere before cylinder before ring.

2 Practice

Now apply the methods above.

2.1 A wheel of radius 0.4 m rolls without slipping at $\omega = 5$ rad s^{-1}. Find its forward speed. [2]

2.2 State the speed of the point where a rolling wheel touches the ground. [1]

2.3 Write the two terms in the total kinetic energy of a rolling object. [2]

2.4 A rolling ball of radius 0.1 m moves at 2 m s^{-1}. Find its angular velocity. [2]

3 Exam-style questions

3.1 For rolling without slipping, the link between speed and spin is [1]

- **A** $v = \omega/r$
- **B** $v = \omega r$
- **C** $v = r/\omega$
- **D** $v = \omega^2 r$

3.2 Released together down the same ramp, which reaches the bottom first? [1]

- **A** a hollow ring
- **B** a solid cylinder
- **C** a solid sphere
- **D** they tie

3.3 A hoop and a solid disc (same mass and radius) roll from rest down the same ramp.

(a) State which arrives first. [1]

(b) Explain why, in terms of how the energy is shared. [2]

3.4 A ball of radius 0.2 m rolls without slipping at 6 m s^{-1}.

(a) Find its angular velocity. [2]

(b) State what happens to v if it begins to skid (slip). [1]

Solutions

2.1 $v = \omega r = 5 \times 0.4 = 2$ m s^{-1}.

2.2 Zero (instantaneously at rest).

2.3 $\frac{1}{2}mv^2$ (translational) and $\frac{1}{2}I\omega^2$ (rotational).

2.4 $\omega = v/r = 2/0.1 = 20$ rad s^{-1}.

3.1 **B** — $v = \omega r$.

3.2 **C** — the solid sphere (smallest rotational inertia per mr^2) wins.

3.3 (a) The solid disc. (b) The hoop puts more energy into spin (larger I), leaving less for speed, so it is slower.

3.4 (a) $\omega = v/r = 6/0.2 = 30$ rad s^{-1}. (b) $v = \omega r$ no longer holds — the speeds decouple.