AP PHYSICS C: MECHANICS • EXERCISE SHEET

2.1 Systems and Center of Mass

Name: _____ Class: _____ Date: _____ **Total: 15 marks**

Objective

Build the skills to answer exam questions on **systems and the center of mass**.

You must be able to:

- treat a group of objects as a single **system** 系统
- find the **center of mass** 质心 of point masses: $x_{cm} = \dfrac{\sum m_i x_i}{\sum m_i}$
- explain that the center of mass moves as if all mass and the net external force acted there
- use symmetry to locate the center of mass of a uniform object

1 Worked examples

Study these first. Each one shows the method for a question type used later —follow the steps and you can do the Practice and Exam-style questions yourself.

■ Center of mass of point masses

Weight each position by its mass and divide by the total mass:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}.$$

For 2 kg at $x = 0$ and 3 kg at $x = 5$ m:

$$x_{cm} = \frac{2(0) + 3(5)}{2 + 3} = \frac{15}{5} = 3 \text{ m}.$$

It sits closer to the heavier mass.

■ The center of mass obeys Newton's second law

The center of mass of a system accelerates as if the total mass were there and the **net external force** acted on it. Internal forces cancel and never move it.

■ Symmetry

A uniform rod's center of mass is at its midpoint; a uniform disc's is at its centre.

2 Practice

Now apply the methods above.

2.1 Two masses, 1 kg at $x = 0$ and 1 kg at $x = 4$ m. Find the center of mass. [2]

2.2 Masses 2 kg at $x = 0$ and 6 kg at $x = 8$ m. Find the center of mass. [2]

2.3 State where the center of mass of a uniform metre ruler is located. [1]

2.4 Can internal forces alone move a system's center of mass? Answer yes or no. [1]

3 Exam-style questions

3.1 The center of mass of a system accelerates in response to the [1]

- **A** internal forces
- **B** net external force
- **C** largest single mass
- **D** total kinetic energy

3.2 Two equal masses sit at $x = 2$ m and $x = 6$ m. Their center of mass is at [1]

- **A** 2 m
- **B** 4 m
- **C** 6 m
- **D** 8 m

3.3 Point masses: 4 kg at $x = 1$ m and 2 kg at $x = 7$ m.

(a) Find the center of mass. [2]

(b) State whether it is closer to the 4 kg or the 2 kg mass, and why. [1]

3.4 An astronaut floating in space throws a tool.

(a) State what happens to the center of mass of (astronaut + tool). [1]

(b) Explain your answer using external forces. [1]

4 Go further

You are ready for more on this subtopic:

- work through the interactive **2.1 Systems and Center of Mass** lesson on the **Learn** page;
- read the **Force and Translational Dynamics** section of the AP Physics C: Mechanics handout on the **Know** page for the full explanation and worked diagrams.

Solutions

2.1 $x_{cm} = \dfrac{1(0) + 1(4)}{2} = 2$ m.

2.2 $x_{cm} = \dfrac{2(0) + 6(8)}{8} = \dfrac{48}{8} = 6$ m.

2.3 At the 50 cm mark (its midpoint).

2.4 No —only a net external force can move the center of mass.

3.1 B —the net external force drives the center of mass.

3.2 B —equal masses give the midpoint, 4 m.

3.3 (a) $x_{cm} = \dfrac{4(1) + 2(7)}{6} = \dfrac{18}{6} = 3$ m. (b) Closer to the 4 kg mass, because it is weighted more heavily.

3.4 (a) It keeps moving as before (stays at rest if it was at rest). (b) Throwing is an internal force, and internal forces cannot move the center of mass.