AP PHYSICS C: MECHANICS • EXERCISE SHEET

1.2 Displacement, Velocity, and Acceleration

Name: _____ Class: _____ Date: _____ **Total: 17 marks**

Objective

Build the skills to answer exam questions on **displacement, velocity, and acceleration** —the calculus links between them.

You must be able to:

- use **velocity** 速度 as the derivative of position, $v = \dfrac{dx}{dt}$
- use **acceleration** 加速度 as the derivative of velocity, $a = \dfrac{dv}{dt}$
- go the other way by **integration** 积分: $x = \int v\,dt$ and $v = \int a\,dt$
- read velocity as the slope of an x-t graph and displacement as the area under a v-t graph

1 Worked examples

Study these first. Each one shows the method for a question type used later —follow the steps and you can do the Practice and Exam-style questions yourself.

■ Derivatives: position to velocity to acceleration

Differentiate to move down the chain. If $x(t) = 3t^2$, then

$$v = \frac{dx}{dt} = 6t, \qquad a = \frac{dv}{dt} = 6.$$

At $t = 2$ s: $v = 12$ m s^{-1}, $a = 6$ m s^{-2}.

■ Integration: acceleration back to velocity

Integrate to move up the chain, adding the starting value. If $a = 4$ (constant) and $v_0 = 2$, then

$$v = \int a\,dt = 4t + 2.$$

■ Slopes and areas on graphs

- The **slope** of an x-t graph is the velocity.
- The **area** under a v-t graph is the displacement.

A v-t graph holds a steady 5 m s^{-1} for 4 s: displacement $= 5 \times 4 = 20$ m (the area).

2 Practice

Now apply the methods above.

2.1 A particle has $x(t) = 2t^3$. Find $v(t)$. [2]

2.2 A particle has $v(t) = 6t$. Find $a(t)$. [1]

2.3 A constant acceleration $a = 3\ \text{m s}^{-2}$ acts, with $v_0 = 5\ \text{m s}^{-1}$. Write $v(t)$. [2]

2.4 A v-t graph is a constant $8\ \text{m s}^{-1}$ for 3 s. Find the displacement. [2]

3 Exam-style questions

3.1 The velocity of an object is the _____ of its position with respect to time. [1]

- **A** integral
- **B** derivative
- **C** reciprocal
- **D** square

3.2 On a position-time graph, the velocity at a point is given by the [1]

- **A** area under the curve
- **B** slope of the tangent
- **C** height of the curve
- **D** curvature

3.3 A particle moves with $x(t) = t^3 - 6t$.

(a) Find $v(t)$. [2]

(b) Find the acceleration $a(t)$. [1]

3.4 A particle starts from rest with acceleration $a(t) = 6t$.

(a) Find $v(t)$. [2]

(b) Find the velocity at $t = 2$ s. [1]

4 Go further

You are ready for more on this subtopic:

- work through the interactive **1.2 Displacement, Velocity, and Acceleration** lesson on the **Learn** page;
- read the **Kinematics** section of the AP Physics C: Mechanics handout on the **Know** page for the full explanation and worked diagrams.

Solutions

2.1 $v = \dfrac{dx}{dt} = 6t^2$.

2.2 $a = \dfrac{dv}{dt} = 6 \text{ m s}^{-2}$.

2.3 $v = \int a\,dt = 3t + 5$.

2.4 Displacement = area $= 8 \times 3 = 24$ m.

3.1 **B** —velocity is the derivative of position.

3.2 **B** —the slope of the tangent gives the velocity.

3.3 (a) $v = 3t^2 - 6$. (b) $a = 6t$.

3.4 (a) $v = \int 6t\,dt = 3t^2$. (b) $v(2) = 3(2)^2 = 12 \text{ m s}^{-1}$.