Exercise walk-through

Energy of Simple Harmonic Oscillators ▪ 7.4

eXercise

You must be able to

- use the total mechanical energy $E = \frac{1}{2}kA^2$
- describe the trade between **potential energy 势能** $\frac{1}{2}kx^2$ and **kinetic energy 动能** $\frac{1}{2}mv^2$
- state that E is all potential at the extremes and all kinetic at equilibrium
- explain that $E \propto A^2$

Method — Total energy

$$E = \tfrac{1}{2} k A^2.$$

A spring with $k = 200$ N m^{-1} and amplitude 0.05 m stores $E = \tfrac{1}{2}(200)(0.05)^2 = 0.25$ J.

Method — The energy trade

At the extremes it is all potential; at equilibrium it is all kinetic. In between it is shared, but the total is constant.

Method — Amplitude squared

Because $E \propto A^2$, doubling the amplitude quadruples the energy.

Practice 2.1 ▪ 2 marks

A spring with $k = 100$ N m^{-1} oscillates with amplitude 0.2 m. Find the total energy.

Solution 2.1

Method: Total energy

Worked steps

$E = \frac{1}{2}kA^2 = \frac{1}{2}(100)(0.2)^2 = 2$ J M1 A1.

Practice 2.2 ▪ 1 mark

At which point is the energy entirely kinetic?

Solution 2.2

Method: The energy trade

Worked steps

At the equilibrium position B1.

Practice 2.3 ▪ 1 mark

If the amplitude doubles, by what factor does the total energy change?

Solution 2.3

Method: Total energy

Worked steps

Four times B1.

Practice 2.4 ▪ 2 marks

A spring with $k = 100$ N m^{-1} is displaced 0.1 m. Find the stored potential energy.

Solution 2.4

Method: Total energy

Worked steps

$U = \frac{1}{2}kx^2 = \frac{1}{2}(100)(0.1)^2 = 0.5$ J M1 A1.

Exam-style 3.1 ▪ 1 mark

The total energy of a simple harmonic oscillator is

A $\frac{1}{2}kA^2$

B $\frac{1}{2}kA$

C kA^2

D $\frac{1}{2}mA^2$

Solution 3.1

A $\frac{1}{2}kA^2$ ✓

B $\frac{1}{2}kA$

C kA^2

D $\frac{1}{2}mA^2$

Worked steps

A — $E = \frac{1}{2}kA^2$.

Exam-style 3.2 ▪ 1 mark

Doubling the amplitude changes the total energy by a factor of

A 2

B 4

C $1/2$

D 1

Solution 3.2

A 2

B 4 ✓

C $1/2$

D 1

Worked steps

B — $E \propto A^2$, so $2^2 = 4$.

Exam-style 3.3 ▪ 2 marks

A spring with $k = 400$ N m^{-1} oscillates with amplitude 0.1 m.
(a) Find the total energy.
(b) State the kinetic energy at the equilibrium position.

Solution 3.3

Method: Total energy

Worked steps

(a) $E = \frac{1}{2}(400)(0.1)^2 = 2$ J M1 A1. (b) 2 J (all kinetic there) B1.

Exam-style 3.4 ▪ 1 mark

A mass oscillates on a spring.
(a) State where the potential energy is greatest.
(b) State whether the kinetic and potential energies can both be maximum at the same instant.

Solution 3.4

Method: The energy trade

Worked steps

(a) At the extremes (maximum x) B1. (b) No — each is greatest where the other is zero B1.