

Exercise walk-through

Representing and Analyzing SHM ■ 7.3

eXercise

You must be able to

- use the position equation $x(t) = A \cos(\omega t + \varphi)$ with **amplitude** 振幅 A and **phase** 相位 φ
- find the maximum speed $v_{max} = A\omega$ (at equilibrium)
- find the maximum acceleration $a_{max} = A\omega^2$ (at the extremes)
- state that displacement and acceleration are out of phase

Method — The position equation

$$x(t) = A \cos(\omega t + \varphi).$$

A is the amplitude (maximum displacement) and φ the phase (starting point).

Method — Maximum speed and acceleration

Differentiating gives

$$v_{max} = A\omega \text{ (at equilibrium),} \quad a_{max} = A\omega^2 \text{ (at the extremes).}$$

Method — Out of step

Speed is greatest at the middle, where the displacement and acceleration are zero; acceleration is greatest at the ends, where the speed is zero.

Practice 2.1 ■ 2 marks

An oscillator has $A = 0.2$ m and $\omega = 4$ rad s⁻¹. Find its maximum speed.

Solution 2.1

Method: Maximum speed and acceleration

Worked steps

$$v_{\max} = A\omega = 0.2 \times 4 = 0.8 \text{ m s}^{-1} \quad \text{M1} \quad \text{A1}.$$

Practice 2.2 ■ 2 marks

For the same oscillator, find the maximum acceleration.

Solution 2.2

Method: Maximum speed and acceleration

Worked steps

$$a_{\max} = A\omega^2 = 0.2 \times 16 = 3.2 \text{ m s}^{-2} \quad \text{M1} \quad \text{A1}.$$

Practice 2.3 ■ 1 mark

Where in the motion is the speed greatest?

Solution 2.3

Method: Out of step

Worked steps

At the equilibrium position **B1**.

Practice 2.4 ■ 1 mark

In $x = A \cos(\omega t + \varphi)$, what does A represent?

Solution 2.4

Method: The position equation

Worked steps

The amplitude (the maximum displacement) **B1**.

Exam-style 3.1 ■ 1 mark

In SHM, the maximum speed occurs at the

- A** extremes
- B** equilibrium position
- C** turning points
- D** it is constant

Solution 3.1

Method: Maximum speed and acceleration

- A extremes
- B equilibrium position 
- C turning points
- D it is constant

Worked steps

B — maximum speed is at equilibrium.

Exam-style 3.2 ■ 1 mark

The maximum acceleration in SHM equals

A $A\omega$

B $A\omega^2$

C A/ω

D ω^2/A

Solution 3.2

Method: Maximum speed and acceleration

A $A\omega$

B $A\omega^2$



C A/ω

D ω^2/A

Worked steps

B — $a_{\max} = A\omega^2$.

Exam-style 3.3 ■ 2 marks

An oscillator has $A = 0.05$ m and $\omega = 10$ rad s⁻¹.

- (a) Find the maximum speed.
- (b) Find the maximum acceleration.

Solution 3.3

Method: Maximum speed and acceleration

Worked steps

(a) $v_{max} = 0.05 \times 10 = 0.5 \text{ m s}^{-1}$ **M1** **A1**. (b) $a_{max} = 0.05 \times 100 = 5 \text{ m s}^{-2}$ **M1** **A1**.

Exam-style 3.4 ■ 1 mark

A mass in SHM starts from rest at its far extreme.

- (a) State whether to model it with cosine or sine (phase).
- (b) State where its acceleration is largest.

Solution 3.4

Method: Out of step

Worked steps

(a) Cosine, with $\varphi = 0$ **B1**. (b) At the extremes **B1**.