Past-paper walk-through

Frequency and Period of SHM ▪ 7.2

eXercise

Questions in this set

- 2024 Set 1 Q1
- 2024 Set 2 Q1
- 2023 Set 1 Q2
- 2023 Set 2 Q2
- 2017 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

2024 Set 1 ▪ Q1

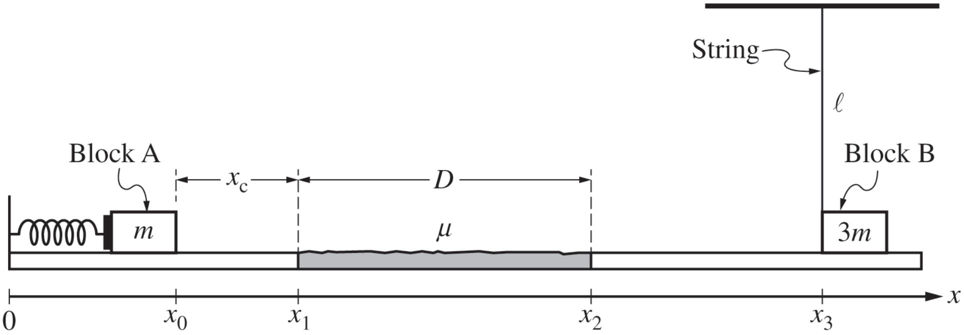
Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

[Figure 1: A horizontal setup on surface with position axis x from 0 to x_3. Block A (mass m) sits at the left against a spring, with a labelled distance x_c from Block A to a dashed reference line. A region of length D with friction coefficient μ (shaded) lies between positions x_1 and x_2. Block B (mass $3m$) hangs from a string of length ℓ attached to the ceiling, positioned at x_3 on the right. Marked positions along the x-axis: $0, x_0, x_1, x_2, x_3$. Note: Figure not drawn to scale.]

Question 1

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.
- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position. - At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction. - At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.
(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.
(a)(i) Derive an expression for the speed v of Block A at time t_1.
(a)(ii) Derive an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3.

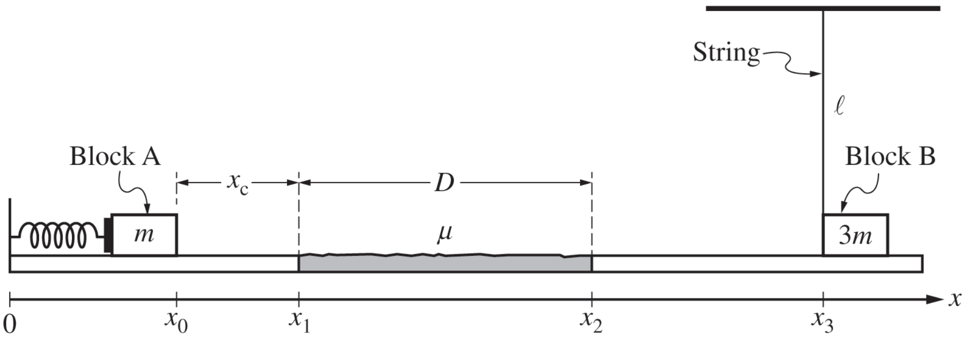
Question 1



Note: Figure not drawn to scale.

Figure 1

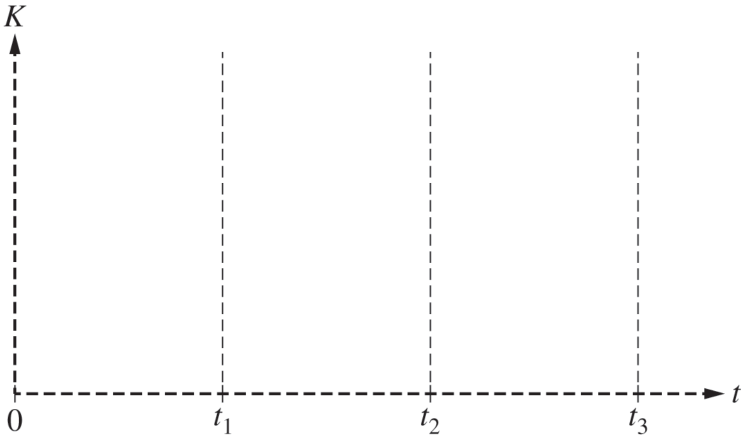
Question 1



Note: Figure not drawn to scale.

Figure 1

Question 1



Question 1

2024 Set 1 ▪ Q1

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[Figure 1: A horizontal setup on surface with position axis x from 0 to x_3. Block A (mass m) sits at the left against a spring, with a labelled distance x_c from Block A to a dashed reference line. A region of length D with friction coefficient μ (shaded) lies between positions x_1 and x_2. Block B (mass $3m$) hangs from a string of length ℓ attached to the ceiling, positioned at x_3 on the right. Marked positions along the x-axis: $0, x_0, x_1, x_2, x_3$. Note: Figure not drawn to scale.]
At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

Question 1

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position. - At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction. - At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

(b)(i) [Figure 1 repeated: same horizontal setup with Block A (mass m) against the spring, distance x_c, friction region of length D with coefficient μ between x_1 and x_2, and Block B (mass $3m$) hanging by a string of length ℓ at x_3. Positions marked $0, x_0, x_1, x_2, x_3$ along axis x. Note: Figure not drawn to scale.]

On the following axes, sketch a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to t_3.

Question 1

[Blank graph: vertical axis K, horizontal axis t with gridlines/tick marks at t_1, t_2, t_3 (origin at 0); axes given with no curve drawn — student is to sketch the curve.]
(b)(ii) Use principles of work and energy to justify the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

Question 1

2024 Set 1 ▪ Q1

Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

[Figure 1: A horizontal setup on surface with position axis x from 0 to x_3. Block A (mass m) sits at the left against a spring, with a labelled distance x_c from Block A to a dashed reference line. A region of length D with friction coefficient μ (shaded) lies between positions x_1 and x_2. Block B (mass $3m$) hangs from a string of length ℓ attached to the ceiling, positioned at x_3 on the right. Marked positions along the x-axis: $0, x_0, x_1, x_2, x_3$. Note: Figure not drawn to scale.]

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

Question 1

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position. - At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction. - At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

(c) [Figure 2: The two-block system (combined mass $m + 3m$) hangs from a string of length ℓ attached to a fixed support at the top. The string makes an angle θ_{max} with the vertical (dashed vertical reference line shown), with the block system displaced to one side at time t_4, having swung from its position at time t_3 (shown hanging straight down). Note: Figure not drawn to scale.]

After the collision, the two-block system instantaneously comes to rest at time t_4, which occurs when the string makes a small angle θ_{max} with the vertical, as shown in

Question 1

Figure 2. For times $t > t_4$, the system oscillates with frequency f_i. The support holding the string is raised, and the procedure is then repeated using a new string of length 2ℓ. Indicate how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length 2ℓ will compare to the frequency of oscillation f_i from the original procedure.

______ $f_{2\ell} > f_i$ ______ $f_{2\ell} < f_i$ ______ $f_{2\ell} = f_i$

Briefly justify your answer.

Solution 1

(a)(i)

Mark scheme

- Apply conservation of mechanical energy: all of the initial energy is stored as spring potential energy U_s, so $E_{initial} = E_{final}$ gives $\frac{1}{2}kx_c^2 = \frac{1}{2}mv^2$. Solving for v: $v = x_c\sqrt{k/m}$. Point 1: multi-step energy derivation that identifies the initial energy as entirely spring PE U_s. Point 2: correct algebraic solution for v.

Solution 1

(a)(ii) Mark scheme

- Step 1 (energy or kinematics to get the speed v_2 at x_2, after friction acts over distance D): Energy method — $K_{x_1} - \Delta E_{friction} = K_{before\ collision}$: $\frac{1}{2}m\left(x_c\sqrt{k/m}\right)^2 - \mu mgD = \frac{1}{2}mv_2^2 \Rightarrow v_2 = \sqrt{\frac{kx_c^2}{m} - 2\mu gD}$. (Equivalently by kinematics: $a = -\mu g$ from $-\mu mg = ma$, then $v_2^2 = v^2 + 2aD = \left(x_c\sqrt{k/m}\right)^2 - 2\mu gD$, same result.) Step 2 (perfectly inelastic collision, conservation of momentum): $\Sigma p_{before} = \Sigma p_{after}$: $mv_2 = (m+3m)v_{A,B} = 4m\,v_{A,B}$. Solving: $v_{A,B} = \dfrac{m\sqrt{\frac{kx_c^2}{m} - 2\mu gD}}{4m} = \dfrac{1}{4}\sqrt{\dfrac{kx_c^2}{m} - 2\mu gD}$. Points: (1) energy or kinematics

Solution 1

application to find speed at x_2; (2) correct expression for friction energy loss or deceleration ($\Delta E_{friction} = \mu mgD$ or $a = -\mu g$); (3) attempt at conservation of momentum $m_{initial}v_{initial} = m_{A,B}v_{A,B}$; (4) correctly substituting the consistent speed and masses into the momentum equation.

Solution 1

(b)(i) Mark scheme

- Sketch: $K = 0$ at $t = 0$; the curve rises nonlinearly and increases throughout $0 \leq t \leq t_1$, reaching a maximum at t_1 (when Block A leaves the spring). From t_1 to t_2 (the friction region) the curve decreases but does NOT reach zero, and is concave up (a constant friction deceleration makes v decrease linearly in time, so $K = \frac{1}{2}mv^2$ decreases at a decreasing rate). From t_2 to t_3 (after leaving the friction region, coasting to the collision) the curve is a horizontal line at a constant value greater than zero, and the whole curve from t_1 to t_3 is continuous (no jump at t_2). Points: (1) nonlinear sketch beginning at zero and increasing throughout $0 \leq t \leq t_1$; (2) decreasing throughout $t_1 \leq t \leq t_2$ without reaching

Solution 1

zero; (3) concave up on $t_1 \leq t \leq t_2$; (4) a continuous function on $t_1 \leq t \leq t_3$ that is a horizontal line greater than zero on $t_2 \leq t \leq t_3$.

Solution 1

(b)(ii) Mark scheme

- From $0 < t < t_1$, the kinetic energy of Block A increases. The force exerted on the block by the compressed spring transfers the elastic potential energy in the block-spring system to the kinetic energy of the block. Because the force exerted by the spring is not applied at a constant rate (it decreases as the spring relaxes), the kinetic energy of the block does not increase at a constant rate — hence the nonlinear shape. Points: (1) a statement about the change in KE consistent with the graph (KE increasing on this interval); (2) a correct explanation for why KE is increasing (positive work done on the block / mechanical energy conserved as spring PE converts to KE); (3) a correct explanation for why the graph is

Solution 1

nonlinear (the spring force — and hence the rate of work done — changes as the block moves, so KE does not increase at a constant rate).

Solution 1

(c)

Mark scheme

- Select $f_{2\ell} < f_\ell$ (doubling the pendulum length decreases its frequency). Justification: the period of a pendulum is $T = 2\pi\sqrt{\ell/g}$. As the length increases, the period increases. Because frequency and period are inversely related ($f = 1/T$), an increase in period results in a decrease in frequency, so $f_{2\ell} < f_\ell$. Points: (1) selecting $f_{2\ell} < f_\ell$ with an attempt at a relevant justification; (2) correctly applying the pendulum period/frequency-length relationship $T = 2\pi\sqrt{\ell/g}$.

Question 1

2024 Set 2 ▪ Q1

Blocks A and B of masses $2m$ and m, respectively, are arranged in a setup consisting of a ramp that makes an angle θ with a smooth horizontal table and an ideal spring of spring constant k fixed to a wall, as shown. Block A is held at rest a distance D up the ramp, and Block B is at rest on the horizontal table. The coefficient of kinetic friction between Block A and the rough ramp is μ in the region of length D, and there is negligible friction between the blocks and the smooth table.

[Figure: A ramp inclined at angle θ to a horizontal table. Block A (mass $2m$) sits on the ramp at a distance D up the incline, with the region of length D on the ramp labeled with friction coefficient μ. The ramp meets the table at the origin 0. On the horizontal table, Block B (mass m) rests at position x_1. Further right along the table, a spring of constant k is fixed to a wall, with its uncompressed (equilibrium) position at x_3; a distance x_c is marked from x_3 back toward the wall/spring, near the wall. The

Question 1

horizontal axis is labeled with positions 0, x_1, x_3 increasing to the right toward the wall.]

Note: Figure not drawn to scale.

Continue your response to QUESTION 1 on this page.

At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$. - At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B. - At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position. - At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.

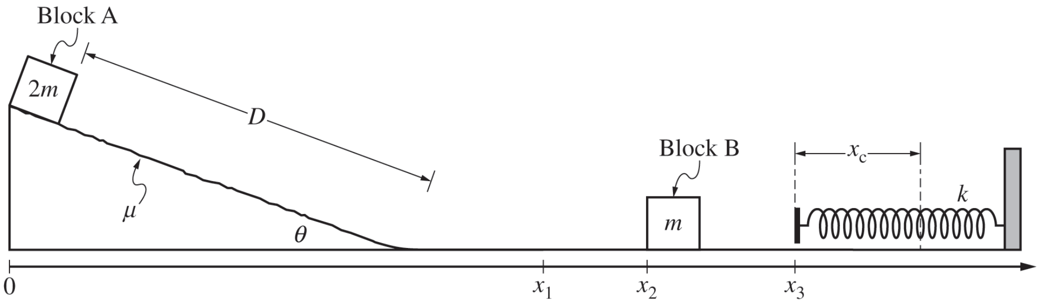
Question 1

For parts (a)(i) and (a)(ii), express your answer in terms of m, θ, D, μ, x_c, and physical constants, as appropriate.

(a)(i) Derive an expression for the speed v of Block A at time t_1.

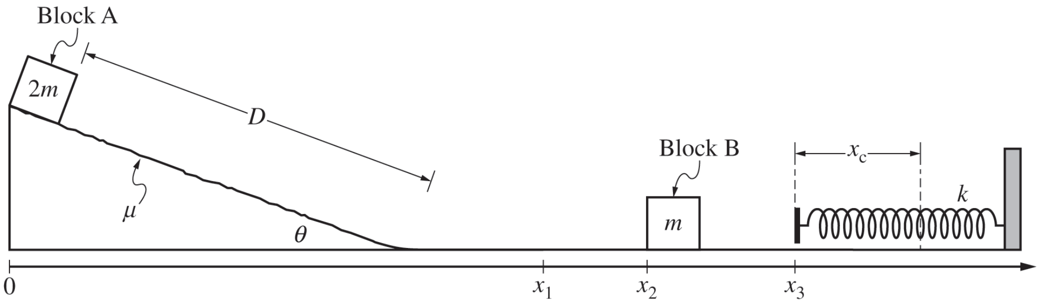
(a)(ii) Derive an expression for the spring constant k of the spring.

Question 1



<u>Note</u>: Figure not drawn to scale.

Question 1



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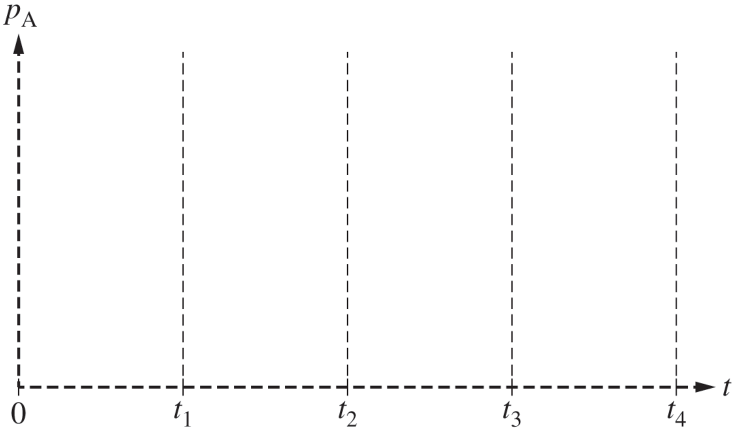
Question 1



Question 1

2024 Set 2 ▪ Q1

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Question 1

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Continue your response to QUESTION 1 on this page.

At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$. - At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B. - At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position. - At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.

For parts (a)(i) and (a)(ii), express your answer in terms of m, θ, D, μ, x_c, and physical constants, as appropriate.

(b)(i) [Figure repeated: the same ramp-table-spring setup with Block A (mass $2m$) on the ramp, Block B (mass m) on the table, and the spring of constant k near the wall;

Question 1

axis positions 0, x_1, x_2, x_3 marked left to right, with x_c marked near the spring. Note: Figure not drawn to scale.]

On the following axes, sketch a graph of the magnitude of the momentum p_A of Block A as a function of time t from $t = 0$ to t_4.

[Graph: vertical axis labeled p_A (unlabeled scale), horizontal axis labeled t with tick marks at 0, t_1, t_2, t_3, t_4 (unlabeled scale/blank grid for the student to draw on).]

(b)(ii) Use principles of forces to justify the graph drawn in part (b)(i) for the time interval $t = t_3$ to $t = t_4$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

Question 1

2024 Set 2 ▪ Q1

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Question 1

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For parts (a)(i) and (a)(ii), express your answer in terms of m, θ, D, μ, x_c, and physical constants, as appropriate.

Question 1

(c) For times $t > t_4$, the two-block-spring system oscillates with period T_O. The procedure is then repeated using a new ramp, where there is negligible friction between Block A and the ramp.

Indicate how the new period of oscillation T_N in the procedure that uses the new ramp compares with the period of oscillation T_O from the original procedure.

_____ $T_N > T_O$ _____ $T_N < T_O$ _____ $T_N = T_O$

Briefly justify your answer.

Solution 1

(a)(i) Mark scheme

- Using energy conservation for Block A sliding down the ramp (mass $2m$, ramp length D, angle θ, coefficient of friction μ): $U_g - \Delta E_{friction} = K$, i.e. $m_A g D \sin\theta - \mu m_A g D \cos\theta = \frac{1}{2} m_A v^2$, giving $v = \sqrt{2gD(\sin\theta - \mu\cos\theta)}$. (Equivalently via kinematics: Newton's second law gives $a = g\sin\theta - \mu g\cos\theta$, then $v^2 = v_0^2 + 2a\Delta x = 2(g\sin\theta - \mu g\cos\theta)D$.) Points: 1 for a correct multi-step derivation start (energy-conservation statement with U_g, or Newton's second law including the signed friction force), 1 for a consistent next step (friction-energy expression with correct sign, or substituting acceleration into a kinematics equation), 1 for the final correct expression $v = \sqrt{2gD(\sin\theta - \mu\cos\theta)}$.

Solution 1

(a)(ii) Mark scheme

- Momentum conservation for the collision between Block A (mass $2m$, speed v from part (a)(i)) and Block B (mass m, at rest): $2mv = (2m + m)v_{A,B}$, so $v_{A,B} = \frac{2}{3}v = \frac{2}{3}\sqrt{2gD(\sin\theta - \mu\cos\theta)}$. Then energy conservation from just after the collision to maximum spring compression: $K_{after\ collision} = U_{s,max}$, i.e. $\frac{1}{2}(3m)v_{A,B}^2 = \frac{1}{2}kx_c^2$ (using that the combined blocks' speed just before contacting the spring equals $v_{A,B}$). Solving: $k = \dfrac{3m}{x_c^2}v_{A,B}^2 = \dfrac{8}{3}\dfrac{mgD(\sin\theta - \mu\cos\theta)}{x_c^2}$. Points: 1 for using momentum conservation to find $v_{A,B}$, 1 for equating post-collision kinetic energy to the maximum elastic potential energy of the spring

Solution 1

$(K_{after\ collision} = U_{s,max})$, 1 for indicating the speed just before the spring contact equals $v_{A,B}$.

Solution 1

(b)(i) Mark scheme

- Sketch of p_A (magnitude of Block A's momentum) vs. t from $t = 0$ to t_4, with four required features: (1) a straight line increasing from 0 at $t = 0$ to a value $p_1 = m_A v$ at $t = t_1$ (Block A accelerates down the ramp under the net gravity-minus-friction force); (2) a horizontal line from t_1 to t_2 at the same level p_1, continuous at t_1 (Block A crosses the frictionless table at constant velocity before the collision); (3) a horizontal line from t_2 to t_3 at a smaller constant level $p_2 = m_A v_{A,B} < p_1$ (the perfectly inelastic collision with Block B at t_2 reduces Block A's speed to $v_{A,B} = \frac{2}{3} v$, then A+B move together at constant velocity toward the spring); (4) a concave-down curve from t_3 to t_4, continuous with level p_2 at t_3, decreasing to 0 at t_4 (the spring's restoring force grows as compression

Solution 1

increases, decelerating the blocks at an increasing rate until they momentarily stop at maximum compression).

Solution 1

(b)(ii) Mark scheme

- For $t_3 \leq t \leq t_4$: the momentum of Block A decreases because the spring exerts a force on the blocks directed opposite to their velocity, decelerating them toward rest. As the spring compresses further, the spring force increases in magnitude, so the rate of decrease of momentum (the magnitude of the slope of the p_A-vs-t graph) increases with time – this is why the curve is concave down, becoming steeper as $t \to t_4$, consistent with the sketch in part (b)(i). Points: 1 for a statement about the change in momentum consistent with the (b)(i) graph, 1 for identifying that a decreasing graph means the net force on Block A is opposite its motion, 1 for explicitly relating the changing steepness of the curve to the increasing magnitude of the spring force.

Solution 1

(c)

Mark scheme

- $T_N = T_O$ (the period is unchanged). Justification: the period of a spring-block oscillator depends only on the oscillating mass and the spring constant ($T = 2\pi\sqrt{m/k}$), not on friction, amplitude, velocity, or compression distance. Removing friction on the new ramp changes only the compression distance/amplitude of the oscillation, not m or k, so the period stays the same.

Question 2

2023 Set 1 ▪ Q2

[Figure 1: A torsional pendulum with a single uniform disk ($N = 1$) suspended from a light wire, showing an angular displacement θ_0 from the untwisted position. Figure 2: The same torsional pendulum with a second identical disk attached below the first ($N = 2$), also displaced by angle θ_0.]

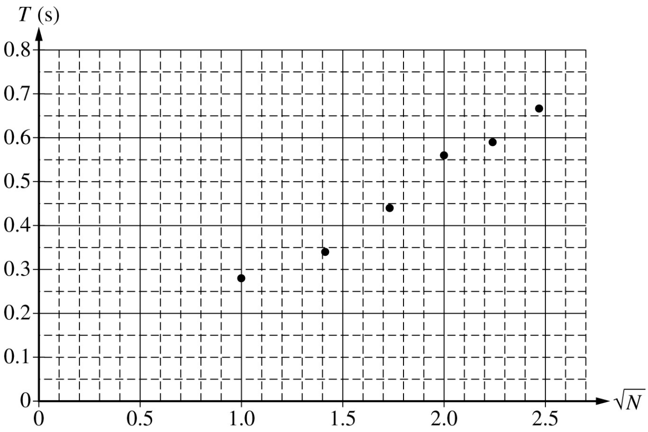
A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

Question 2

The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

(a) Using $T = 2\pi\sqrt{\frac{I}{\kappa}}$, derive an expression for T as a function of N. Express your answer in terms of M, R, κ, N, and physical constants, as appropriate.

Question 2



Question 2

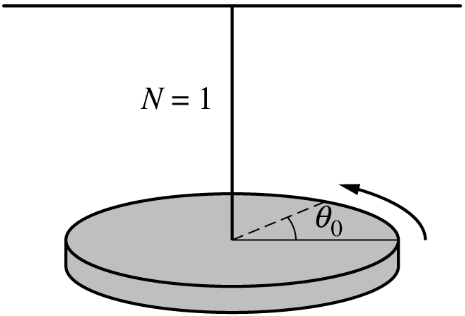


Figure 1

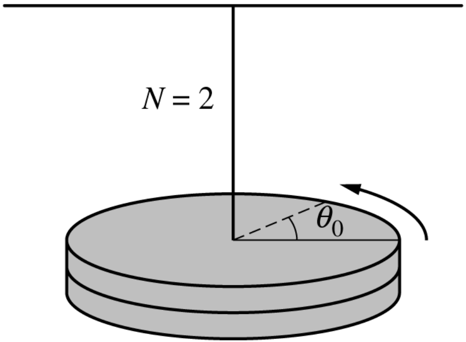


Figure 2

Question 2

Question 2

2023 Set 1 ▪ Q2

[Figure 1: A torsional pendulum with a single uniform disk ($N = 1$) suspended from a light wire, showing an angular displacement θ_0 from the untwisted position. Figure 2: The same torsional pendulum with a second identical disk attached below the first ($N = 2$), also displaced by angle θ_0.]

A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical

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disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

(b) The potential energy stored in the torsional pendulum when the disks are displaced is $U = \frac{1}{2}\kappa(\Delta\theta)^2$. On the following axes, sketch a graph of the maximum kinetic energy K_{max} of the torsional pendulum as a function of N for $N \geq 1$.
[Blank axes provided: y-axis labeled K_{max}, x-axis labeled N, for the student to sketch a curve.]

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[Figure 1: A torsional pendulum with a single uniform disk ($N = 1$) suspended from a light wire, showing an angular displacement θ_0 from the untwisted position. Figure 2: The same torsional pendulum with a second identical disk attached below the first ($N = 2$), also displaced by angle θ_0.]

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disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

(c)(i) [Figure: Graph of T (s) on the y-axis from 0 to 0.8 in increments of 0.1, versus $\sqrt{N}$ on the x-axis from 0 to 2.5 in increments of 0.5. Scattered data points are plotted showing an approximately linear increasing trend, roughly from $(\sqrt{N} \approx 0.7, T \approx 0.25)$ up to $(\sqrt{N} \approx 2.3, T \approx 0.65)$, for the student to draw a best-fit line through.]
The student plots the data for T as a function of $\sqrt{N}$, as shown.
Draw the best-fit line for the data.

(c)(ii) The student previously determined that the radius of a disk is $R = 0.2$ m and found that $\kappa = 1.6$ N $\cdot$ m. Using the graph, calculate the mass M of a single disk.

(c)(iii) The student finds that the value given by the manufacturer for the mass of the disk is less than the value determined experimentally in part (c)(ii). Determine a single

Question 2

source of experimental error that could result in the observed difference in the value of M. Justify your answer.

Question 2

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[Figure 1: A torsional pendulum with a single uniform disk ($N = 1$) suspended from a light wire, showing an angular displacement θ_0 from the untwisted position. Figure 2: The same torsional pendulum with a second identical disk attached below the first ($N = 2$), also displaced by angle θ_0.]

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The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical

Question 2

disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

(d)(i) The student repeats the experiment, but now the disks have a density that varies as a function of the radius of the disk according to $\rho = 0.3r$. Would the slope of the best-fit line for this new data be greater than, less than, or the same as the slope of the best-fit line in part (c)(i)?

_______ greater than _______ less than _______ the same as

Justify your answer.

(d)(ii) When $N = 1$, the maximum angular speed of the torsional pendulum with a uniform disk is found to be ω_{U}. When $N = 1$, the maximum angular speed of the torsional pendulum with a nonuniform disk is $\omega_{\text{non-U}}$. Which of the following correctly compares ω_{U} and $\omega_{\text{non-U}}$?

Question 2

$\omega_{\text{U}} > \omega_{\text{non-U}}$ $\omega_{\text{U}} < \omega_{\text{non-U}}$ $\omega_{\text{U}} = \omega_{\text{non-U}}$
Briefly justify your answer.

Question 2

2023 Set 2 ▪ Q2

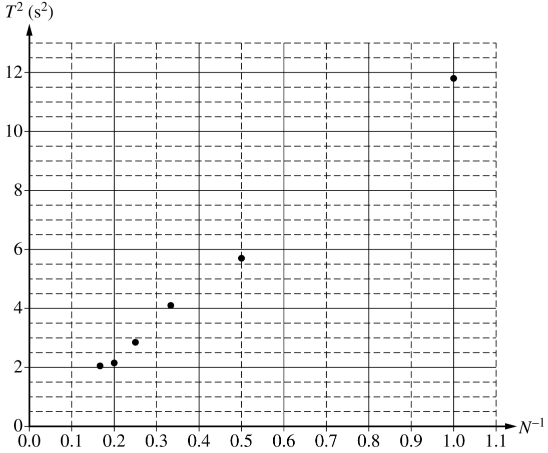
A student conducts an investigation to determine the relationship between the period of oscillation T of a system consisting of a block and N attached springs. The student starts with a block of mass m attached to a single ideal spring of spring constant k, as shown in Figure 1. The student holds the block so that the spring is neither stretched nor compressed at a vertical height 1.00 m above a motion detector. The student releases the block from rest and records the period of oscillation for the system consisting of the single spring and block. An additional identical spring is attached in parallel, as shown in Figure 2, and the procedure is repeated for $N = 2$ springs. This procedure is repeated through $N = 10$ springs.
[Figure 1: A spring of spring constant k (labeled "$N = 1$") hangs from a fixed support, with a block of mass m attached below it, positioned 1.00 m above a motion detector on the ground.]

Question 2

[Figure 2: Two identical springs, each of spring constant k, attached in parallel (labeled "$N = 2$") hang from a fixed support, with a block of mass m attached below them, positioned 1.00 m above a motion detector on the ground.]

(a) Derive an expression for T as a function of N. Express your answer in terms of m, k, N, and physical constants as appropriate.

Question 2



Question 2

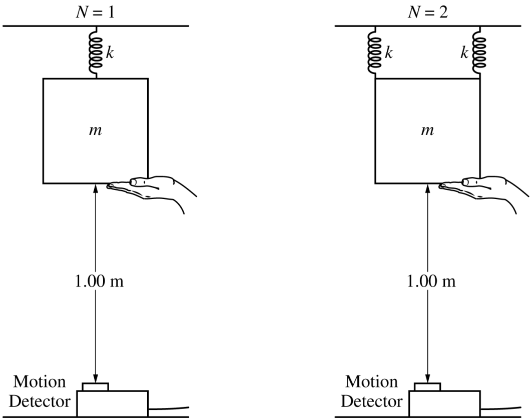


Figure 1

Figure 2

Question 2

Question 2

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[Figure 1: A spring of spring constant k (labeled "$N = 1$") hangs from a fixed support, with a block of mass m attached below it, positioned 1.00 m above a motion detector on the ground.]

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Question 2

(b) On the following axes, sketch a graph of T as a function of N for $N \geq 1$.
[Grid: axes labeled T (vertical) vs. N (horizontal), both unlabeled with values, blank for the student to sketch on.]

Question 2

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[Figure 1: A spring of spring constant k (labeled "$N = 1$") hangs from a fixed support, with a block of mass m attached below it, positioned 1.00 m above a motion detector on the ground.]

[Figure 2: Two identical springs, each of spring constant k, attached in parallel (labeled "$N = 2$") hang from a fixed support, with a block of mass m attached below them, positioned 1.00 m above a motion detector on the ground.]

Question 2

(c)(i) The student plots the data for T^2 as a function of N^{-1}, as shown.
[Graph: T^2 (s^2) vs. N^{-1}; y-axis from 0 to 12 in increments of 2, x-axis from 0.0 to 1.1 in increments of 0.1; scattered data points roughly along an increasing linear trend, approximately at $(0.1, 1)$, $(0.15, 1.5)$, $(0.2, 2)$, $(0.33, 4)$, $(0.5, 6)$, $(1.0, 12)$.]
Draw the best-fit line for the data.
(c)(ii) The mass of the block is measured to be $m = 1.5$ kg. Using the graph, calculate an experimental value for the spring constant k for a single spring.
(c)(iii) The student finds that the value given by the manufacturer for the spring constant is larger than the value determined experimentally in part (c)(ii). Determine a single source of experimental error that could result in the observed difference in the value for k. Briefly justify your answer.

Question 2

2023 Set 2 ▪ Q2

A student conducts an investigation to determine the relationship between the period of oscillation T of a system consisting of a block and N attached springs. The student starts with a block of mass m attached to a single ideal spring of spring constant k, as shown in Figure 1. The student holds the block so that the spring is neither stretched nor compressed at a vertical height 1.00 m above a motion detector. The student releases the block from rest and records the period of oscillation for the system consisting of the single spring and block. An additional identical spring is attached in parallel, as shown in Figure 2, and the procedure is repeated for $N = 2$ springs. This procedure is repeated through $N = 10$ springs.

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Question 2

(d)(i) The student conducts a similar investigation to determine the relationship between the period of oscillation T of a block-spring system and a number N of identical springs horizontally on a table, as shown in Figure 3. Frictional forces between the table and the block are negligible. In each trial, the block is displaced the same horizontal distance from equilibrium and released from rest.
[Figure 3: A block of mass m attached to $N = 2$ springs on a horizontal table, springs shown attached to a wall on the left and the block on the right.]
The student plots T^2 as a function of N^{-1} for this new investigation. Would the slope of the best-fit line from this new investigation be greater than, less than, or the same as the slope of the best-fit line in part (c)(i)?

______ greater than ______ less than ______ the same as

Briefly justify your answer.

Question 2

(d)(ii) When $N = 1$, the maximum speed of the block is found to be v_{max}. When N increases, will v_{max} increase, decrease, or stay the same?

_____ increase _____ decrease _____ stay the same

Justify your answer.

Question 2

2017 ▪ Q2

[Figure: A block of mass m rests at the top of an inclined plane of height h, which descends smoothly onto a horizontal surface. Farther along the horizontal surface, a coiled spring is fixed against a wall. Note: Figure not drawn to scale.]
A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.
(a)(i) Derive an expression for the speed of the block just before it collides with the spring.

Question 2

(a)(ii) Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

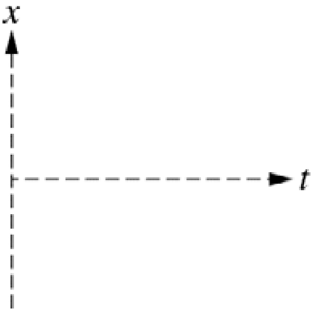
_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Question 2

Note: Figure not drawn to scale.

Question 2



Question 2

2017 ▪ Q2

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A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

(b) Derive an expression for the maximum compression of the spring.

Question 2

2017 ▪ Q2

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(c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

Question 2

The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0. Now suppose the block experiences a resistive force as it slides on the horizontal surface.
The magnitude of the resistive force F is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of kg/m.

Question 2

2017 ▪ Q2

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Question 2

(d)(i) Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m, h, k, β, v, and physical constants, as appropriate.

(d)(ii) Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time t can be written in the form $\dfrac{1}{v(t)} = \dfrac{1}{v_0} + \dfrac{\beta t}{m}$, where v_0 is the speed at $t = 0$.

Question 2

2017 ▪ Q2

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Question 2

(e) Sketch graphs of position x as a function of time t, velocity v as a function of time t, and acceleration a as a function of time t for the block as it is moving on the horizontal surface before it reaches the spring.
[Three blank axes side by side, each with a vertical axis and horizontal axis labeled t: the first labeled x, the second labeled v, the third labeled a — for sketching the respective graphs.]

Solution 2

(a)(i)

Mark scheme

- Using conservation of energy for the block sliding down the frictionless incline: loss in gravitational PE = gain in KE, $U_g = K$, i.e. $mgh = \frac{1}{2}mv^2$. Solving: $v = \sqrt{2gh}$. 1 point for correctly applying conservation of energy ($U_g = K$), 1 point for the correct final answer $v = \sqrt{2gh}$.

Solution 2

(a)(ii) Mark scheme

- Correct answer: Greater than. Since $v = \sqrt{2gh}$ is proportional to $\sqrt{h}$, reducing the height by a factor of 2 reduces the speed by a factor of $\sqrt{2} \approx 1.41$ (not by a factor of 2). So the speed halfway down the incline is $v/\sqrt{2} \approx 0.71v$, which is greater than half the bottom speed $(0.5v)$. 1 point for correctly selecting 'Greater than' AND giving this correct justification – an incorrect selection cannot earn credit even with valid-looking reasoning.

Solution 2

(b)

Mark scheme

- Using conservation of energy ($U_g = U_s$) for the block compressing the spring, consistent with part (a): $mgh = \frac{1}{2}kx_{max}^2$. Solving for the maximum compression: $x_{max} = \sqrt{\frac{2mgh}{k}}$. 1 point for this correct answer, consistent with the part (a) approach.

Solution 2

(c) Mark scheme

- Recognize that the block's contact with the spring, from first touching it to maximum compression, is one-quarter of a simple-harmonic-motion cycle, with period $T = 2\pi\sqrt{m/k}$. So $t = \frac{1}{4}T = \frac{1}{4}\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{\pi}{2}\sqrt{\frac{m}{k}}$. 1 point for identifying the simple-harmonic-motion (quarter-period) approach, 1 point for the correct final expression.

Solution 2

(d)(i) Mark scheme

- Newton's second law for the block on the horizontal surface, with the resistive force opposing the motion: $F_{net} = ma$, and $F_{net} = -\beta v^2$ (negative because it decelerates the block), giving $-\beta v^2 = ma$. Writing acceleration as dv/dt gives the differential equation $-\beta v^2 = m\dfrac{dv}{dt}$. 1 point for correctly applying Newton's second law ($F_{net} = ma$), 1 point for correctly indicating that F_{net} is opposite the direction of motion, 1 point for expressing the result as a differential equation in dv/dt.

Solution 2

(d)(ii) Mark scheme

- Separate variables in $-\beta v^2 = m\,dv/dt$ to get $-\frac{\beta}{m}dt = \frac{1}{v^2}dv$. Integrating both sides: $\int -\frac{\beta}{m}dt = \int \frac{1}{v^2}dv$ gives $-\frac{\beta}{m}[t] = \left[-\frac{1}{v}\right]$. Applying the limits $t : 0 \to t$ and $v : v_0 \to v(t)$: $-\frac{\beta t}{m} = -\frac{1}{v(t)} - \left(-\frac{1}{v_0}\right) = -\frac{1}{v(t)} + \frac{1}{v_0}$. Rearranging gives $\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$, as required. 1 point for correctly separating variables, 1 point for correctly integrating, 1 point for correctly applying the limits/constant of integration to reach the given result.

Solution 2

(e) Mark scheme

- Since the resistive force continually decelerates the block (from the $1/v(t) = 1/v_0 + \beta t/m$ result, $v(t)$ decreases toward 0 but never reaches it in finite time, and the deceleration itself decreases), the three sketches are: position $x(t)$ – concave down, increasing and leveling off toward (approaching but not reaching) a horizontal asymptote; velocity $v(t)$ – concave up, decreasing from v_0 toward the horizontal (time) axis as an asymptote; acceleration $a(t)$ – concave down (magnitude decreasing, stays negative), also approaching the horizontal axis as an asymptote. (Full credit is also earned if all three curves are reflected vertically; if a plausible but different nonlinear $v(t)$ shape is drawn, $x(t)$ and $a(t)$ consistent with that shape still earn credit.) 1 point per correct curve (x, v, a).