Past-paper walk-through

Defining Simple Harmonic Motion (SHM) ▪ 7.1

eXercise

Questions in this set

- 2017 Q2
- 2016 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

2017 ▪ Q2

[Figure: A block of mass m rests at the top of an inclined plane of height h, which descends smoothly onto a horizontal surface. Farther along the horizontal surface, a coiled spring is fixed against a wall. Note: Figure not drawn to scale.]
A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.
(a)(i) Derive an expression for the speed of the block just before it collides with the spring.

Question 2

(a)(ii) Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

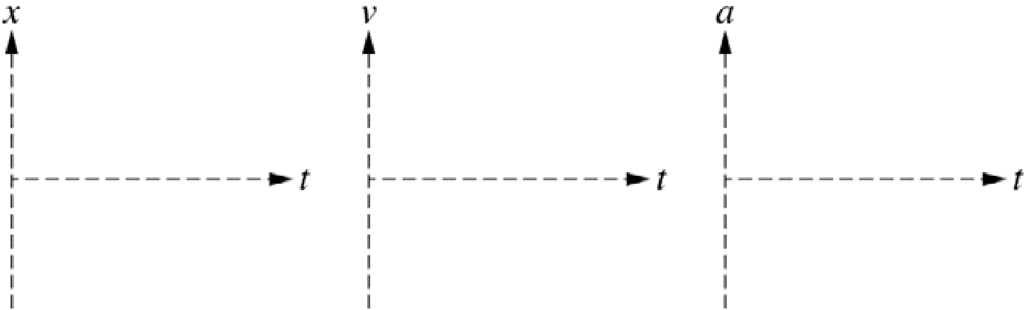
_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Question 2

Note: Figure not drawn to scale.

Question 2



Question 2

2017 ▪ Q2

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(b) Derive an expression for the maximum compression of the spring.

Question 2

2017 ▪ Q2

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(c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

Question 2

The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0. Now suppose the block experiences a resistive force as it slides on the horizontal surface.
The magnitude of the resistive force F is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of kg/m.

Question 2

2017 ▪ Q2

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Question 2

(d)(i) Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m, h, k, β, v, and physical constants, as appropriate.

(d)(ii) Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time t can be written in the form $\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$, where v_0 is the speed at $t = 0$.

Question 2

2017 ▪ Q2

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Question 2

(e) Sketch graphs of position x as a function of time t, velocity v as a function of time t, and acceleration a as a function of time t for the block as it is moving on the horizontal surface before it reaches the spring.
[Three blank axes side by side, each with a vertical axis and horizontal axis labeled t: the first labeled x, the second labeled v, the third labeled a — for sketching the respective graphs.]

Solution 2

(a)(i)

Mark scheme

- Using conservation of energy for the block sliding down the frictionless incline: loss in gravitational PE = gain in KE, $U_g = K$, i.e. $mgh = \frac{1}{2}mv^2$. Solving: $v = \sqrt{2gh}$. 1 point for correctly applying conservation of energy ($U_g = K$), 1 point for the correct final answer $v = \sqrt{2gh}$.

Solution 2

(a)(ii) Mark scheme

- Correct answer: Greater than. Since $v = \sqrt{2gh}$ is proportional to $\sqrt{h}$, reducing the height by a factor of 2 reduces the speed by a factor of $\sqrt{2} \approx 1.41$ (not by a factor of 2). So the speed halfway down the incline is $v/\sqrt{2} \approx 0.71v$, which is greater than half the bottom speed $(0.5v)$. 1 point for correctly selecting 'Greater than' AND giving this correct justification – an incorrect selection cannot earn credit even with valid-looking reasoning.

Solution 2

(b)

Mark scheme

- Using conservation of energy ($U_g = U_s$) for the block compressing the spring, consistent with part (a): $mgh = \frac{1}{2}kx_{max}^2$. Solving for the maximum compression: $x_{max} = \sqrt{\frac{2mgh}{k}}$. 1 point for this correct answer, consistent with the part (a) approach.

Solution 2

(c) Mark scheme

- Recognize that the block's contact with the spring, from first touching it to maximum compression, is one-quarter of a simple-harmonic-motion cycle, with period $T = 2\pi\sqrt{m/k}$. So $t = \frac{1}{4}T = \frac{1}{4}\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{\pi}{2}\sqrt{\frac{m}{k}}$. 1 point for identifying the simple-harmonic-motion (quarter-period) approach, 1 point for the correct final expression.

Solution 2

(d)(i) Mark scheme

- Newton's second law for the block on the horizontal surface, with the resistive force opposing the motion: $F_{net} = ma$, and $F_{net} = -\beta v^2$ (negative because it decelerates the block), giving $-\beta v^2 = ma$. Writing acceleration as dv/dt gives the differential equation $-\beta v^2 = m\frac{dv}{dt}$. 1 point for correctly applying Newton's second law ($F_{net} = ma$), 1 point for correctly indicating that F_{net} is opposite the direction of motion, 1 point for expressing the result as a differential equation in dv/dt.

Solution 2

(d)(ii) Mark scheme

- Separate variables in $-\beta v^2 = m\,dv/dt$ to get $-\dfrac{\beta}{m}dt = \dfrac{1}{v^2}dv$. Integrating both sides: $\int -\dfrac{\beta}{m}dt = \int \dfrac{1}{v^2}dv$ gives $-\dfrac{\beta}{m}[t] = \left[-\dfrac{1}{v}\right]$. Applying the limits $t : 0 \to t$ and $v : v_0 \to v(t)$: $-\dfrac{\beta t}{m} = -\dfrac{1}{v(t)} - \left(-\dfrac{1}{v_0}\right) = -\dfrac{1}{v(t)} + \dfrac{1}{v_0}$. Rearranging gives $\dfrac{1}{v(t)} = \dfrac{1}{v_0} + \dfrac{\beta t}{m}$, as required. 1 point for correctly separating variables, 1 point for correctly integrating, 1 point for correctly applying the limits/constant of integration to reach the given result.

Solution 2

(e) Mark scheme

- Since the resistive force continually decelerates the block (from the $1/v(t) = 1/v_0 + \beta t/m$ result, $v(t)$ decreases toward 0 but never reaches it in finite time, and the deceleration itself decreases), the three sketches are: position $x(t)$ – concave down, increasing and leveling off toward (approaching but not reaching) a horizontal asymptote; velocity $v(t)$ – concave up, decreasing from v_0 toward the horizontal (time) axis as an asymptote; acceleration $a(t)$ – concave down (magnitude decreasing, stays negative), also approaching the horizontal axis as an asymptote. (Full credit is also earned if all three curves are reflected vertically; if a plausible but different nonlinear $v(t)$ shape is drawn, $x(t)$ and $a(t)$ consistent with that shape still earn credit.) 1 point per correct curve (x, v, a).

Question 2

2016 ▪ Q2

[Figure: A horizontal table. On the left, a spring (drawn as a coil) is anchored to a wall and attached to a block labeled "2M" at position marked "0". On the right, separated by open space, a block labeled "3M" moves to the left with initial velocity v_0 (arrow pointing left labeled v_0).]

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(a) On the dots below, which represent the blocks of mass $2M$ and $3M$, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

Question 2

[Two dots are shown side by side, labeled "Block of Mass $2M$" (left dot) and "Block of Mass $3M$" (right dot), for the student to draw force vectors on.]
[Figure: The spring (coil) anchored at the wall on the left is now attached to a combined block labeled "2M" next to "3M" (the two blocks shown adjacent, having moved together). A distance D is marked from the wall/spring's compressed end to a reference position "0".]
The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance D, as shown above. Express your answers to parts (b), (c), and (d) in terms of M, B, v_0, and physical constants, as appropriate.

Question 2

Question 2

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Question 2

Question 2

2016 ▪ Q2

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(b) Derive an expression for the speed of the blocks immediately after the collision.

Question 2

2016 ▪ Q2

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(c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.

Question 2

2016 ▪ Q2

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(d) Derive an expression for the maximum distance D that the spring is compressed.

Question 2

2016 ▪ Q2

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(e)(i) In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

_____ Left   _____ Right   _____ The net force on the block of mass $2M$ is zero.

Question 2

Justify your answer.

(e)(ii) Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

_____ The magnitude of the net force is greater on the block of mass $2M$. _____ The magnitude of the net force is greater on the block of mass $3M$. _____ The magnitude of the net force on each block has the same nonzero value. _____ The magnitude of the net force on each block is zero.

Justify your answer.

Question 2

2016 ▪ Q2

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A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

______ Yes   ______ No

Justify your answer.

Solution 2

(a)

Mark scheme

- Draw two force diagrams (blocks not yet touching the spring, frictionless surface): on the $2M$ block, normal force F_{N1} up and weight $2Mg$ down; on the $3M$ block, normal force F_{N2} up and weight $3Mg$ down (distinct symbols since the two blocks have different masses/normal forces). 1 pt for correctly drawing/labeling the vectors on the $2M$ block; 1 pt for correctly drawing/labeling the vectors on the $3M$ block, using different, physically correct symbols. Note: a maximum of 1 of the 2 points can be earned if any extraneous vectors are drawn.

Solution 2

(b)

Mark scheme

- The $3M$ block moving at v_0 collides and sticks to the stationary $2M$ block (perfectly inelastic). Conservation of momentum: $p_1 = p_2 \Rightarrow 3Mv_0 = (3M + 2M)v_f$. 1 pt for correctly substituting into the momentum equation; 1 pt for the correct final answer $v_f = \frac{3}{5}v_0$.

Solution 2

(c)

Mark scheme

- Kinetic energy of the combined system: $K = \frac{1}{2}mv^2 = \frac{1}{2}(5M)\left(\frac{3}{5}v_0\right)^2 = \frac{9}{10}Mv_0^2$. 1 pt for an answer consistent with the speed found in part (b).

Solution 2

(d) Mark scheme

- By conservation of energy, $\Delta K_{\text{system}} + \Delta U_{\text{system}} = 0$, so $K_0 = U_{\text{final}}$: the kinetic energy right after the collision converts entirely to spring potential energy at maximum compression. With nonlinear spring force $F = Bx^3$: $\frac{9}{10}Mv_0^2 = \int_0^D Bx^3\,dx = \left[\frac{Bx^4}{4}\right]_0^D = \frac{BD^4}{4}$. Solving: $D = \sqrt[4]{\frac{18Mv_0^2}{5B}}$. 1 pt for a correct conservation-of-energy expression ($\Delta K + \Delta U = 0$, $K_0 = U_{\text{final}}$); 1 pt for attempting to integrate the spring-force equation; 1 pt for using correct limits of integration (0 to D) or an appropriate constant of integration; 1 pt for a final answer consistent with the speed from (b) or the kinetic energy from (c). Correct answer: $D = \sqrt[4]{18Mv_0^2/(5B)}$.

Solution 2

(e)(i)

Mark scheme

- Answer: Right. At maximum compression the two-block system is instantaneously at rest, but the (still-compressed) spring continues to exert a rightward force on the system, so the block of mass $2M$ has a net force/acceleration to the right at that instant. 1 pt for selecting 'Right' with a reasonable justification attempt (no credit for the justification if the wrong choice is made); 1 pt for indicating that at maximum compression the $2M$ block has a rightward acceleration, due either to the forces acting on it directly or to the external spring force on the system.

Solution 2

(e)(ii)

Mark scheme

- Answer: The magnitude of the net force is greater on the block of mass $3M$. Because the blocks are stuck together they share the same acceleration; since $F = ma$, the block with more mass ($3M$) experiences the greater net force at that same acceleration. 1 pt for indicating both blocks have the same acceleration; 1 pt for a correct justification that the net force is greater on the $3M$ block (no credit if the wrong block is selected).

Solution 2

(f) Mark scheme

- Answer: No. Although the spring exerts a restoring force on the stuck-together blocks, the spring is nonlinear ($F = -Bx^3$, not $F = -kx$), so the force is not proportional to displacement from equilibrium. SHM requires a linear restoring force ($a = -\frac{k}{m}\Delta x$), so the blocks do not exhibit simple harmonic motion. 1 pt for selecting 'No' with a reasonable justification attempt (the selection point is not earned if 'No' is chosen with no justification; no credit for the justification if the wrong answer is selected); 1 pt for indicating the spring's force is not linear and that SHM specifically requires a linear restoring force.