Past-paper walk-through

Rolling ▪ 6.5

eXercise

Questions in this set

- 2026 Q4
- 2025 Q4
- 2023 Set 1 Q3
- 2019 Set 2 Q3
- 2018 Q3
- 2017 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 4

2026 ▪ Q4

A unicycle is placed on a stand such that the wheel does not touch the ground. The rotational inertia of the wheel about the axle is I_W. One end of a crank arm can be attached to the wheel, as shown in the side view.
[Figure: A side-view illustration of a unicycle (seat, seat post, frame, wheel with spokes, and stand legs) standing on the ground with the wheel raised off the ground, labeled “Unicycle.” A dashed-line callout box magnifies the wheel region, labeled “Side View,” showing the wheel’s outer rim labeled “Wheel,” the central hub labeled “Axle,” and a straight bar extending from the axle outward labeled “Crank Arm.”]
In Scenarios 1 and 2, crank arms of different lengths are connected to the wheel. In each scenario, the wheel is initially at rest. A force of magnitude F is then exerted at the end of each crank arm. The direction of the force remains perpendicular to each

Question 4

crank arm at all times. The mass of each crank arm is negligible. The scenarios differ as described.

- Scenario 1: The length of the crank arm is ℓ_1. The angular speed of the wheel immediately after the crank arm has been turned through one rotation is ω_1.
- Scenario 2: The length of the crank arm is ℓ_2, where $\ell_2 > \ell_1$. The angular speed of the wheel immediately after the longer crank arm has been turned through one rotation is ω_2.

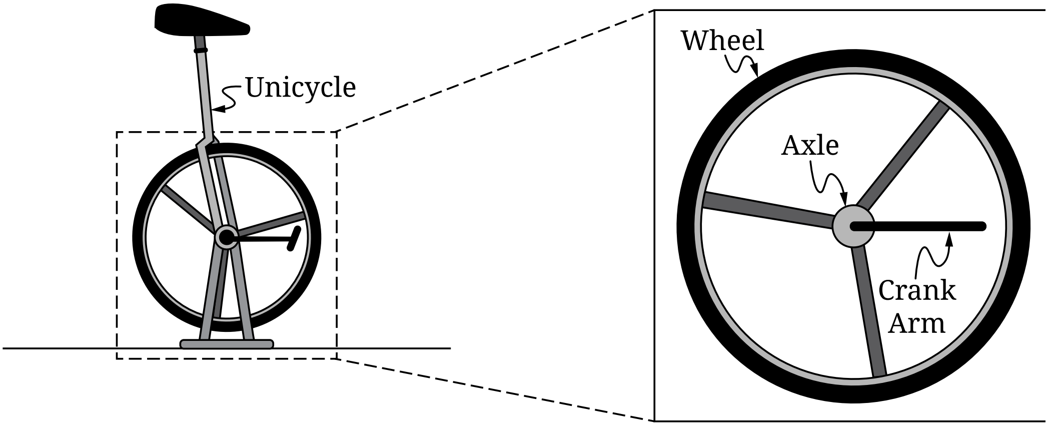
(a) Indicate whether ω_2 is greater than, less than, or equal to ω_1 by writing one of the following.

- $\omega_2 > \omega_1$
- $\omega_2 < \omega_1$
- $\omega_2 = \omega_1$

Question 4

Justify your answer. Your justification may reference equations but must include conceptual reasoning beyond algebraic solutions.

Question 4



Question 4

2026 ▪ Q4

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Question 4

- Scenario 1: The length of the crank arm is ℓ_1. The angular speed of the wheel immediately after the crank arm has been turned through one rotation is ω_1.
- Scenario 2: The length of the crank arm is ℓ_2, where $\ell_2 > \ell_1$. The angular speed of the wheel immediately after the longer crank arm has been turned through one rotation is ω_2.

(b) Derive an expression for the angular speed ω_1 of the wheel in Scenario 1 immediately after the crank arm has been turned through one rotation. Express your answer in terms of I_W, F, ℓ_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 4

2026 ▪ Q4

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In Scenarios 1 and 2, crank arms of different lengths are connected to the wheel. In each scenario, the wheel is initially at rest. A force of magnitude F is then exerted at the end of each crank arm. The direction of the force remains perpendicular to each crank arm at all times. The mass of each crank arm is negligible. The scenarios differ as described.

Question 4

- Scenario 1: The length of the crank arm is ℓ_1. The angular speed of the wheel immediately after the crank arm has been turned through one rotation is ω_1.
- Scenario 2: The length of the crank arm is ℓ_2, where $\ell_2 > \ell_1$. The angular speed of the wheel immediately after the longer crank arm has been turned through one rotation is ω_2.

(c) In Scenario 3, the unicycle is placed on the ground such that the unicycle can move forward without the wheel slipping on the ground. A force of the same magnitude F is exerted at the end of the crank arm of length ℓ_1, as in Scenario 1. The direction of the force remains perpendicular to the crank arm at all times. The resulting angular speed of the wheel immediately after the crank arm has been turned through one rotation is ω_3.

Indicate whether ω_3 is greater than, less than, or equal to ω_1 by writing one of the following.

Question 4

- $\omega_3 > \omega_1$
- $\omega_3 < \omega_1$
- $\omega_3 = \omega_1$

Briefly **justify** your answer. Your justification may reference equations but must include conceptual reasoning beyond algebraic solutions.

Question 4

2025 ▪ Q4

A uniform disk and ring, each of mass M and radius R, roll without slipping along a horizontal surface, as shown in Figure 1. The outer edges of the disk and ring are made of the same material. The center of mass of the disk and the center of mass of the ring each initially move with the same constant speed v.

The disk and the ring then smoothly transition to a ramp that is inclined at an angle θ above the horizontal. Both the disk and the ring continue to roll without slipping as they move up the ramp, as shown in Figure 2.

The ring travels a greater distance along the ramp than the disk travels before each momentarily comes to rest.

[Figure 1: A horizontal surface with a Disk (solid gray circle, radius R labeled) on the left and a Ring (gray annulus/hoop) to its right, both moving to the right (motion lines shown behind each) toward a ramp inclined at angle θ.]

Question 4

[Figure 2: The disk and ring shown together partway up the ramp inclined at angle θ, moving up-slope (motion lines behind them), with the ring positioned slightly behind/around the disk.]

(a) While the disk and the ring are rolling on the ramp without slipping, the magnitudes of the static frictional force exerted on the disk and on the ring by the ramp are f_D and f_R, respectively.

Indicate whether f_D is greater than, less than, or equal to f_R by writing one of the following.

- $f_D > f_R$
- $f_D < f_R$
- $f_D = f_R$

Justify your answer using qualitative reasoning beyond referencing equations.

Question 4

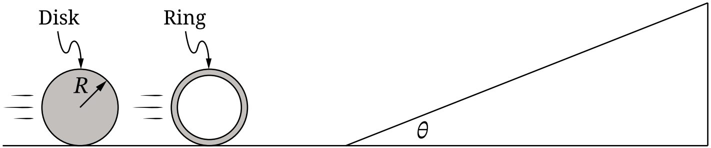
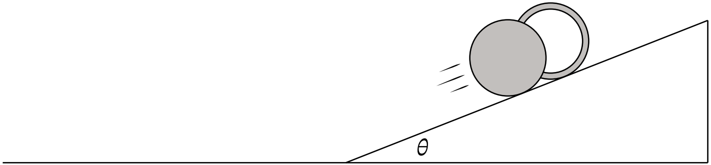


Figure 1



Question 4

2025 ▪ Q4

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The ring travels a greater distance along the ramp than the disk travels before each momentarily comes to rest.

[Figure 1: A horizontal surface with a Disk (solid gray circle, radius R labeled) on the left and a Ring (gray annulus/hoop) to its right, both moving to the right (motion lines shown behind each) toward a ramp inclined at angle θ.]

Question 4

[Figure 2: The disk and ring shown together partway up the ramp inclined at angle θ, moving up-slope (motion lines behind them), with the ring positioned slightly behind/around the disk.]

(b) A cylinder has mass M, radius R, and rotational inertia I about its central axis. The cylinder rolls without slipping up a ramp that is inclined at an angle θ above the horizontal.

Derive an expression for the magnitude of the static frictional force f exerted on the cylinder by the ramp. Express your answer in terms of M, R, I, θ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 4

2025 ▪ Q4

A uniform disk and ring, each of mass M and radius R, roll without slipping along a horizontal surface, as shown in Figure 1. The outer edges of the disk and ring are made of the same material. The center of mass of the disk and the center of mass of the ring each initially move with the same constant speed v.

The disk and the ring then smoothly transition to a ramp that is inclined at an angle θ above the horizontal. Both the disk and the ring continue to roll without slipping as they move up the ramp, as shown in Figure 2.

The ring travels a greater distance along the ramp than the disk travels before each momentarily comes to rest.

[Figure 1: A horizontal surface with a Disk (solid gray circle, radius R labeled) on the left and a Ring (gray annulus/hoop) to its right, both moving to the right (motion lines shown behind each) toward a ramp inclined at angle θ.]

Question 4

[Figure 2: The disk and ring shown together partway up the ramp inclined at angle θ, moving up-slope (motion lines behind them), with the ring positioned slightly behind/around the disk.]

(c) In a different scenario, the centers of mass of the original disk and ring each have the same initial speed v as they did in the original scenario. The ramp is replaced by a new ramp on which the disk and the ring initially slip as they roll up the new ramp. **Indicate** whether the magnitude of the kinetic frictional force exerted on the disk by the new ramp is greater than, less than, or equal to the magnitude of the kinetic frictional force exerted on the ring by the new ramp while both are slipping. Briefly **justify** your answer.

Solution 4

(a) Mark scheme

- Compare the magnitudes of static friction f_D (on the disk) and f_R (on the ring) as both roll without slipping on the ramp (same mass, radius, initial conditions). Answer: $f_D < f_R$ [Point A1]. Justification (model, from the scoring guidelines): the ring has a greater rotational inertia because it has more mass distributed toward the edge, so the ring travels farther and has less acceleration down the ramp; because the ring and disk have the same mass, the gravitational force components on them are the same, so from Newton's second law the ring must have less net force down the ramp and more friction (up the ramp) than the disk. This justification compares the disk's and ring's rotational inertias (the ring's is larger, since its mass sits farther from the axis) [Point A2: a justification

Solution 4

comparing translational kinematics, OR rotational kinematics, OR rotational inertias of the disk and ring – any ONE suffices], and it reasons via Newton's second law in translational form (equal gravity component, so the shape with the smaller net accelerating force down the ramp – the ring – must have the larger friction force) [Point A3: justification using N2L in translational form, OR rotational form, OR conservation of energy – any ONE suffices]. Reasoning must go beyond simply citing equations.

Solution 4

(b) Mark scheme

- Cylinder of mass M, radius R, rotational inertia I about its central axis, rolling without slipping UP a ramp inclined at angle θ. Derive an expression for the magnitude of the static friction force f from the ramp, in terms of M, R, I, θ. Rotational form of Newton's second law about the center of mass (friction f is the only force producing a torque about the axis, torque arm R): $\alpha_{sys} = \frac{\sum \tau}{I_{sys}} \Rightarrow \frac{a}{R} = \frac{fR}{I} \Rightarrow a = \frac{fR^2}{I}$ (uses the rolling-without-slipping constraint $a = R\alpha$) [Point B3: uses $a = r\alpha$ in an attempt to solve the system of equations]. Translational form of Newton's second law along the incline (the cylinder decelerates as it rolls up, so net force points down the incline):

Solution 4

$\vec{a}_{sys} = \dfrac{\sum \vec{F}}{m_{sys}} \Rightarrow f - Mg\sin\theta = -Ma$ – friction (up the incline) and the gravity component (down the incline) enter with opposite signs [Point B2: opposite signs for the gravitational and frictional force terms]. Combining Newton's second law in BOTH translational and rotational forms is the required multistep derivation [Point B1]. Substituting:

$f - Mg\sin\theta = -M\left(\dfrac{fR^2}{I}\right) \Rightarrow f + M\dfrac{fR^2}{I} = Mg\sin\theta \Rightarrow f\left(1 + \dfrac{MR^2}{I}\right) = Mg\sin\theta$, giving $f = \dfrac{Mg\sin\theta}{1 + \dfrac{MR^2}{I}}$. (A response using conservation of energy instead may earn full credit per the scoring note.)

Solution 4

(c) Mark scheme

- New scenario: the disk and the ring (same initial center-of-mass speed v as before) roll up a DIFFERENT ramp on which they initially SLIP while rolling. Compare the magnitude of the kinetic friction force on the disk to that on the ring while both are slipping. Answer: 'Equal to' [Point C1]. Justification (model, from the scoring guidelines): the friction here is kinetic (since the objects are slipping), and kinetic friction depends only on the coefficient of kinetic friction between the surfaces in contact and on the normal force ($f_k = \mu_k N$), not on rotational inertia or shape. Since the disk and ring have the same mass and are on the same ramp, their normal forces are equal, and the coefficient of kinetic friction is the same for both (same pair of surfaces), so the kinetic friction forces on the disk and the ring

Solution 4

are equal in magnitude [Point C2: justification includes that kinetic friction depends only on the surfaces/coefficient of kinetic friction and the normal force].

Question 3

2023 Set 1 ▪ Q3

[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a 90° angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

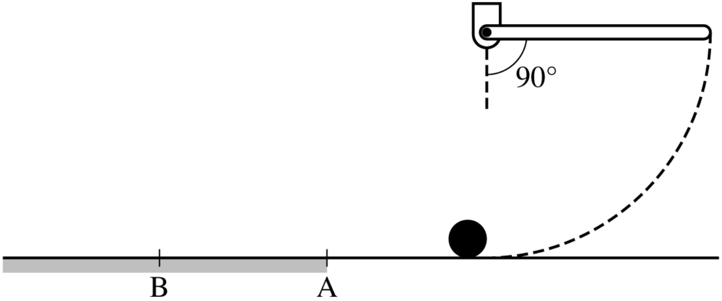
A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total

Question 3

rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(a) Derive an expression for the angular speed of the rod just before striking the sphere in terms of the length ℓ and physical constants as appropriate.

Question 3



Note: Figure not drawn to scale.

Figure 1

Question 3

2023 Set 1 ▪ Q3

[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a $90°$ angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

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Question 3

the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(b) Derive an expression for the linear speed v_0 of the sphere immediately after colliding with the rod in terms of the length ℓ and physical constants as appropriate. After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ, beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

Question 3

2023 Set 1 ▪ Q3

[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a $90°$ angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision,

Question 3

the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(c) In the following diagram, which represents the sphere while the sphere is traveling between Points A and B, draw and label the forces (not components) that act on the sphere. Each force must be represented by a distinct arrow starting on, and pointing away from, the point of application on the sphere.

[A circle is shown representing the sphere, for the student to draw and label force vectors starting on the circle.]

Question 3

2023 Set 1 ▪ Q3

[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a $90°$ angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

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Question 3

the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(d)(i) Derive an expression for each of the following as the sphere is rotating and sliding between points A and B in terms of v_0, μ, R, t, and physical constants as appropriate.

The linear velocity v of the center of mass of the sphere as a function of time t

(d)(ii) The angular velocity ω of the sphere as a function of time t

Question 3

2023 Set 1 ▪ Q3

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Question 3

the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(e)(i) Derive an expression for the time it takes the sphere to travel from Point A to Point B in terms of v_0, μ, and physical constants as appropriate.

(e)(ii) Derive an expression for the linear velocity of the sphere upon reaching Point B in terms of v_0.

Question 3

2019 Set 2 ▪ Q3

[Figure: A point P at height h above the floor, at the top of a straight incline going down and to the right, with a small circle labeled m, r at P representing the rolling sphere. The incline descends to a circular vertical loop of radius R (labeled with a radius line from the center to the edge of the loop), with point A marked at the top of the loop. After the loop, the track continues to the right and up as a straight incline. A vertical dashed line with arrows shows the height h measured from point P down to the level of the bottom of the loop/floor. Note: Figure not drawn to scale.]
The rotational inertia of a rolling object may be written in terms of its mass m and radius r as $I = bmr^2$, where b is a numerical value based on the distribution of mass within the rolling object. Students wish to conduct an experiment to determine the value of b for a partially hollowed sphere. The students use a looped track of radius

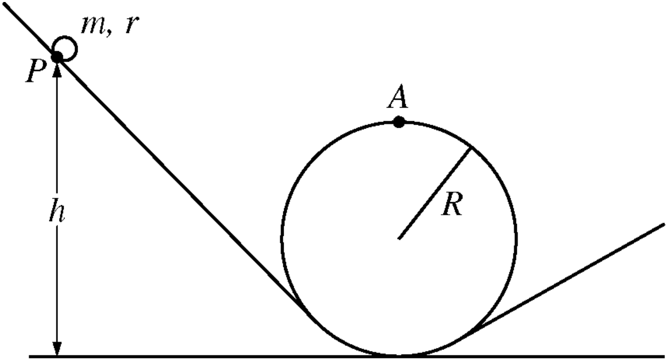
Question 3

$R \gg r$, as shown in the figure above. The sphere is released from rest a height h above the floor and rolls around the loop.

(a) Derive an expression for the minimum speed of the sphere's center of mass that will allow the sphere to just pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

Question 3

Question 3



Note: Figure not drawn to scale.

Question 3

Object	b	h (m)
Solid sphere	0.40	1.08
Hollow sphere	0.67	1.13
Solid cylinder	0.50	1.10
Hollow cylinder	1.0	1.20