Past-paper walk-through

Conservation of Angular Momentum ▪ 6.4

eXercise

Questions in this set

- 2023 Set 1 Q3
- 2019 Set 1 Q3
- 2016 Q3
- 2014 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2023 Set 1 ▪ Q3

[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a $90°$ angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total

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rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(a) Derive an expression for the angular speed of the rod just before striking the sphere in terms of the length ℓ and physical constants as appropriate.

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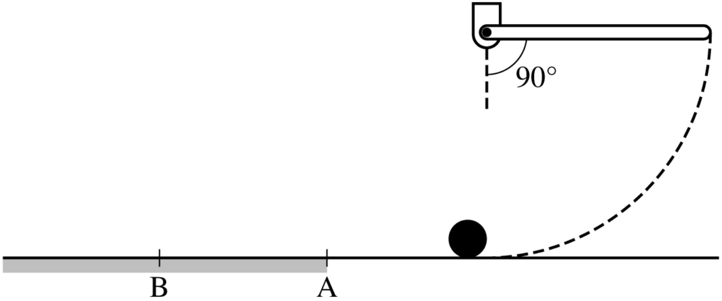

Note: Figure not drawn to scale.

Figure 1

Question 3

2023 Set 1 ▪ Q3

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A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision,

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the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(b) Derive an expression for the linear speed v_0 of the sphere immediately after colliding with the rod in terms of the length ℓ and physical constants as appropriate. After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ, beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

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A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision,

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the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(c) In the following diagram, which represents the sphere while the sphere is traveling between Points A and B, draw and label the forces (not components) that act on the sphere. Each force must be represented by a distinct arrow starting on, and pointing away from, the point of application on the sphere.

[A circle is shown representing the sphere, for the student to draw and label force vectors starting on the circle.]

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the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(d)(i) Derive an expression for each of the following as the sphere is rotating and sliding between points A and B in terms of v_0, μ, R, t, and physical constants as appropriate.

The linear velocity v of the center of mass of the sphere as a function of time t

(d)(ii) The angular velocity ω of the sphere as a function of time t

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Question 3

the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(e)(i) Derive an expression for the time it takes the sphere to travel from Point A to Point B in terms of v_0, μ, and physical constants as appropriate.

(e)(ii) Derive an expression for the linear velocity of the sphere upon reaching Point B in terms of v_0.

Question 3

2019 Set 1 ▪ Q3

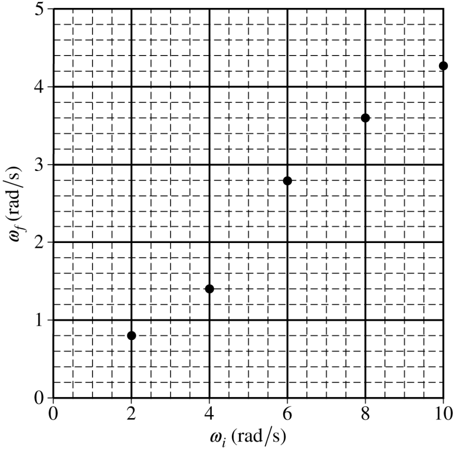
[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(a) Derive an expression for the angular speed ω_P of the platform. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.

Question 3



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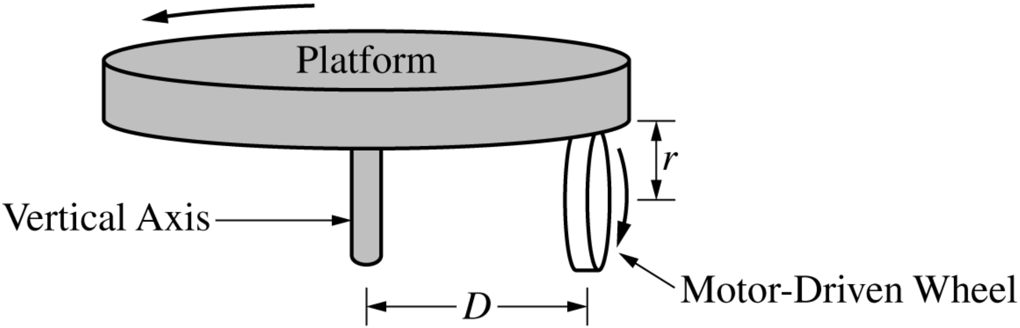

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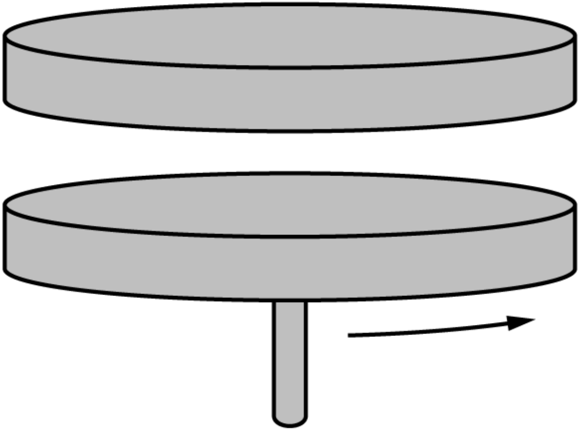

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2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

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Question 3

(b) Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed ω_P. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.

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Question 3

(c) Derive an expression for the angular speed ω_W of the wheel when the platform has reached angular speed ω_P. Express your answer in terms of D, r, ω_P, and physical constants, as appropriate.

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[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

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Question 3

(d) [Figure: the platform shown alone from a 3D perspective, spinning (arrow indicating rotation direction), with the motor-driven wheel removed.]
When the platform is spinning at angular speed ω_P, the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia I_P as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed ω_f.
Derive an expression for ω_f. Express your answer in terms of ω_P, I_P, and physical constants, as appropriate.

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2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(e)(i) A student now uses the rotating platform ($I_P = 3.1$ kg·m^2) to determine the rotational inertia I_U of an unknown object about a vertical axis that passes through the object's center of mass. The platform is rotating at an initial angular speed ω_i when the unknown object is dropped with its center of mass directly above the center of the platform. The platform and object are observed to be rotating together at angular speed ω_f. Trials are repeated for different values of ω_i. A graph of ω_f as a function of ω_i is shown on the axes below.
[Graph: ω_f (rad/s) vs. ω_i (rad/s); y-axis from 0 to 5 in increments of 1, x-axis from 0 to 10 in increments of 2. Data points roughly follow a straight line through the origin, e.g. near (2, 0.9), (4, 1.4), (6, 2.6), (8, 3.6), (10, 4.2).]
On the graph on the previous page, draw a best-fit line for the data.
(e)(ii) Using the straight line, calculate the rotational inertia of the unknown object I_U about a vertical axis passing through its center of mass.

Question 3

2019 Set 1 ▪ Q3

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Question 3

(f) The kinetic energy of the spinning platform before the object is dropped on it is K_i. The total kinetic energy of the platform-object system when it reaches angular speed ω_f is K_f. Which of the following expressions is true?

______ $K_f < K_i$ ______ $K_f = K_i$ ______ $K_f > K_i$

Justify your answer.

Question 3

2019 Set 1 ▪ Q3

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Question 3

(g) One of the students observes that the center of mass of the object is not actually aligned with the axis of the platform. Is the experimental value of I_U obtained in part (e) greater than, less than, or equal to the actual value of the rotational inertia of the unknown object about a vertical axis that passes through its center of mass?

______ Greater than   ______ Less than   ______ Equal to

Justify your answer.

Solution 3

(a)

Mark scheme

- Substitute into the rotational form of Newton's second law (1 pt): $\tau = I\alpha \Rightarrow FD = I_P\alpha \Rightarrow \alpha = \dfrac{FD}{I_P}$. Substitute into a rotational kinematic equation to find the angular speed (1 pt): $\omega_2 = \omega_1 + \alpha\Delta t = 0 + \dfrac{FD}{I_P}\Delta t$, giving $\omega_P = \dfrac{FD\Delta t}{I_P}$.

Solution 3

(b)

Mark scheme

- Use the rotational kinetic energy equation (1 pt): $K = \frac{1}{2}I\omega_P^2 = \frac{1}{2}(I)\left(\frac{FD\Delta t}{I}\right)^2$. Give an answer consistent with part (a) (1 pt): $K = \dfrac{(FD\Delta t)^2}{2I}$.

Solution 3

(c)

Mark scheme

- Indicate that the linear speed of the platform's edge equals the linear speed of the wheel's edge at the point of contact (1 pt): $v_P = v_W$ or $(r\omega)_P = (r\omega)_W$. Correctly relate the linear speeds to the angular speeds (1 pt):
$$D\omega_P = r\omega_W \Rightarrow \omega_W = \frac{D\omega_P}{r}.$$

Solution 3

(d)

Mark scheme

- Use conservation of angular momentum for the disk landing on the platform (1 pt): $L_1 = L_2 \Rightarrow I_1\omega_1 = I_2\omega_2$. Correctly substitute (the disk has the same rotational inertia I_P as the platform, so the combined inertia doubles) (1 pt): $I_P\omega_P = (2I_P)\omega_f \Rightarrow \omega_f = \frac{1}{2}\omega_P$.

Solution 3

(e)(i)

Mark scheme

- Draw a reasonable straight best-fit line through the plotted ω_f vs. ω_i data points on the given graph (1 pt for an appropriate best-fit line).

Solution 3

(e)(ii)

Mark scheme

- Use conservation of angular momentum to derive an expression containing I_U (1 pt): $L_i = L_f \Rightarrow I_P\omega_i = (I_P + I_U)\omega_f \Rightarrow I_U = I_P\left(\frac{\omega_i}{\omega_f}\right) - I_P$. Substitute two points read from the best-fit line (1 pt): $I_U = (3.1\ \mathsf{kg}\cdot\mathsf{m}^2)\left(\frac{4.0 - 1.0\ \mathsf{rad/s}}{9.3 - 2.4\ \mathsf{rad/s}}\right) - (3.1\ \mathsf{kg}\cdot\mathsf{m}^2) = 4.1\ \mathsf{kg}\cdot\mathsf{m}^2$ (the point (0,0) may be used implicitly if the best-fit line passes through the origin).

Solution 3

(f)

Mark scheme

- Select $K_f < K_i$ with an attempt at a relevant justification (1 pt), plus a fully correct justification (1 pt). Example: Because the two disks end up rotating together at the same final angular speed, this is an inelastic collision, so kinetic energy is lost during the collision — therefore $K_f < K_i$.

Solution 3

(g)

Mark scheme

- Select 'Greater than' with an attempt at a relevant justification (1 pt), plus a fully correct justification (1 pt). Example: Because the object's center of mass is off the platform's axis, the parallel-axis theorem $I = I_{CM} + Mh^2$ applies, so the true (actual) rotational inertia about the axis through the center of mass is smaller than what was measured — the experimental value obtained in part (e) is therefore greater than the actual value, increased by Mh^2.

Question 3

2016 ▪ Q3

[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.]
[Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]
A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in

Question 3

frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

	Mass	Rotational Inertia
Block	m	
Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)
Platform	$m_P = 5m$	$\frac{m_P R^2}{2}$ (about the central axis)

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

Question 3

(a) Derive an expression for the spring constant of the spring.

Question 3

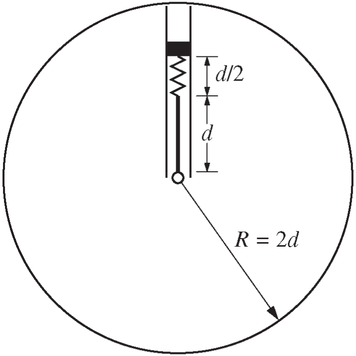

Figure 1

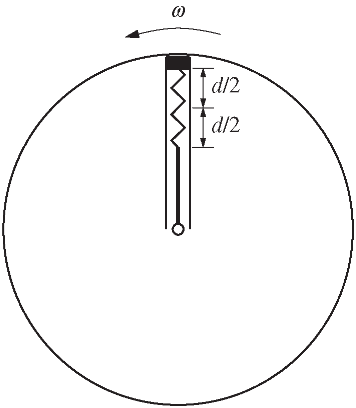

Figure 2

Question 3

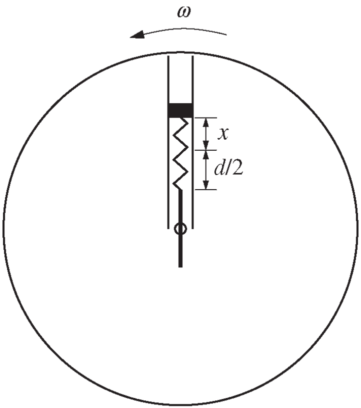

Figure 3

Question 3

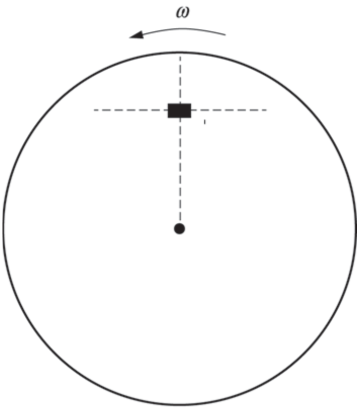

Figure 4

Question 3

2016 ▪ Q3

[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.] [Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium

Question 3

length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

	Mass	Rotational Inertia
Block	m	
Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)
Platform	$m_P = 5m$	$\frac{m_P R^2}{2}$ (about the central axis)

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(b)(i) Determine an expression for the rotational inertia of the block around the axis of the platform.

Question 3

(b)(ii) Derive an expression for the rotational inertia of the entire system about the axis of the platform.

Question 3

2016 ▪ Q3

[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.] [Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]
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Question 3

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	Mass	Rotational Inertia
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A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(c) Determine an expression for the angular momentum of the entire system about the axis of the platform.

Question 3

[Figure 3: The same circular platform, rotating counterclockwise at ω. The rod's block position is shown partway along the rod, with the spring stretched by amount x and a remaining segment labeled $d/2$ below it.]
While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed ω is maintained by the motor driving the platform.

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2016 ▪ Q3

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A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium

Question 3

length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

| | Mass | Rotational Inertia | |—|—|—| | Block | m | | | Rod | $m_R = 3m$ | $\frac{m_R d^2}{3}$ (about the end of the rod) | | Platform | $m_P = 5m$ | $\frac{m_P R^2}{2}$ (about the central axis) |

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(d) Derive an expression for the distance x that the spring is stretched when the rod reaches the position shown in Figure 3 above.

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For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

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[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.] [Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]
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Question 3

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A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

_____ Increasing   _____ Decreasing   _____ Staying the same

Question 3

Justify your answer.

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[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.] [Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]

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Question 3

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A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(f) In order to keep the system rotating with constant angular speed ω, is the motor doing positive work, negative work, or no work on the rotating system?

______ Positive   ______ Negative   ______ No work

Question 3

Justify your answer.

Question 3

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[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.] [Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]

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Question 3

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A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.

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[Figure 4: The circular platform rotating counterclockwise at ω, with the block shown at the center on the axis (a small dark square marker), a dashed horizontal line through it, and a dot below the platform on the vertical rod line, for the student to draw an acceleration vector on the block.]

Solution 3

(a)

Mark scheme

- Radial form of Newton's second law: spring force equals centripetal force, $F_S = F_C = ma_C$, i.e. $kx = mr\omega^2$. With spring stretch $x = d/2$ and orbit radius $r = 2d$ (per the given geometry): $k\left(\frac{d}{2}\right) = m(2d)\omega^2$. Solving: $k = 4m\omega^2$. 1 pt for indicating $F_S = F_C = ma_C$; 1 pt for a correct substitution for x; 1 pt for a correct substitution for r that lets you solve for k. Correct answer: $k = 4m\omega^2$.

Solution 3

(b)(i)

Mark scheme

- Treating the block as a point mass at radius $r = 2d$:
  $I = \sum mr^2 = m(2d)^2 = 4md^2$. 1 pt for a correct answer, or an answer consistent with the radius used in part (a).

Solution 3

(b)(ii) Mark scheme

- Total rotational inertia is the sum of the platform's, rod's, and block's: $I = I_P + I_R + I_O = \frac{m_P R^2}{2} + \frac{m_R d^2}{3} + mr^2$. Substituting $m_P = 5m$, $R = 2d$ (platform as a disk), $m_R = 3m$ (rod), and $mr^2 = 4md^2$ from (b)i: $I = \frac{5m(2d)^2}{2} + \frac{3md^2}{3} + 4md^2 = 10md^2 + md^2 + 4md^2 = 15md^2$. 1 pt for indicating the system's rotational inertia must include the platform, rod, and object; 1 pt for correctly substituting each piece consistent with (b)i. Correct answer: $I = 15md^2$.

Solution 3

(c)

Mark scheme

- $L = I\omega$. Using $I = 15md^2$ from part (b)ii: $L = 15md^2\omega$. 1 pt for an answer consistent with the answer from part (b)ii.

Solution 3

(d)

Mark scheme

- Same radial equation as part (a): $F_S = F_C = ma_C \Rightarrow kx = mr\omega^2$. Using $k = 4m\omega^2$ (from part (a), or the student's own consistent value) and the new radius $r = d + x$ (rod's center now at the axis, spring stretched by x): $(4m\omega^2)x = m(d+x)\omega^2 \Rightarrow 4x = d + x \Rightarrow 3x = d \Rightarrow x = d/3$. 1 pt for indicating the spring force equals mass times centripetal acceleration; 1 pt for a correct substitution for k (or one consistent with part (a)); 1 pt for a correct substitution for r that lets you solve for x. Correct answer: $x = d/3$.

Solution 3

(e)

Mark scheme

- Answer: Decreasing. As the block moves inward toward the axis, the system's rotational inertia I decreases while ω is held constant; since $L = I\omega$, a decreasing I with constant ω means L decreases. 1 pt for selecting 'Decreasing' (no credit for the justification if the wrong selection is made); 1 pt for a correct justification (I decreases while ω stays the same, so $L = I\omega$ decreases).

Solution 3

(f)

Mark scheme

- Answer: Negative. With ω held constant while I decreases, the rotational kinetic energy $K = \frac{1}{2}I\omega^2$ decreases; by the work-energy theorem the net work on the rotating system is negative, so the motor is doing negative work (removing energy to keep ω constant as I shrinks). 1 pt for selecting 'Negative' (no credit for the justification if the wrong selection is made); 1 pt for a correct justification (ω is constant, I has decreased, so rotational KE has decreased, so the work done must be negative).

Solution 3

(g)

Mark scheme

- The scoring guidelines show the correct acceleration direction as a single arrow starting on the block and pointing down and to the right — i.e. combining a centripetal (inward) component with a tangential component consistent with the block's inward radial motion while the platform rotates counterclockwise. 1 pt for an arrow starting on the box and pointing down and to the right.

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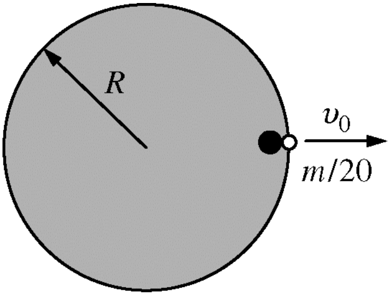
[Figure, Top View: A large filled circle of radius R (labelled with an arrow from center to edge) representing a disk, with a small stone of mass $m/20$ shown just outside the edge of the disk moving away with velocity v_0 (arrow labelled v_0, $m/20$).]
[Figure, Side View: A person holding a stone stands atop a raised platform of height h above the ground, throwing the stone horizontally with initial speed v_0 (arrow labelled v_0); h is marked as the vertical distance from the platform surface to the ground.]
Mech. 3.
A large circular disk of mass m and radius R is initially stationary on a horizontal icy surface. A person of mass $m/2$ stands on the edge of the disk. Without slipping on the disk, the person throws a large stone of mass $m/20$ horizontally at initial speed v_0 from a height h above the ice in a radial direction, as shown in the figures above. The coefficient of friction between the disk and the ice is μ. All velocities are measured

Question 3

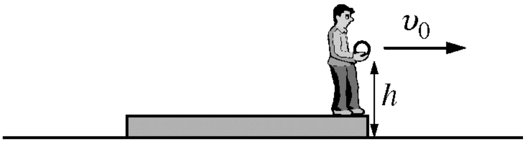
relative to the ground. The time it takes to throw the stone is negligible. Express all algebraic answers in terms of m, R, v_0, h, μ, and fundamental constants, as appropriate.

(a) Derive an expression for the length of time it will take the stone to strike the ice.

Question 3



Top View



Side View

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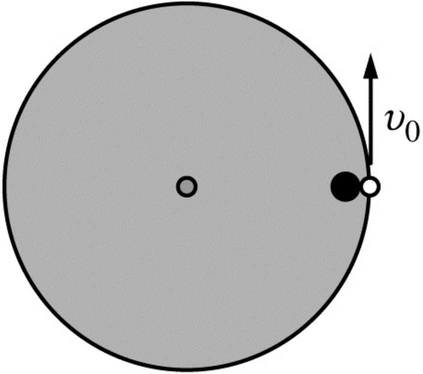

Top View

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[Figure, Top View: A large filled circle of radius R (labelled with an arrow from center to edge) representing a disk, with a small stone of mass $m/20$ shown just outside the edge of the disk moving away with velocity v_0 (arrow labelled v_0, $m/20$).]
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Question 3

the stone is negligible. Express all algebraic answers in terms of m, R, v_0, h, μ, and fundamental constants, as appropriate.

(b) Assuming that the disk is free to slide on the ice, derive an expression for the speed of the disk and person immediately after the stone is thrown.

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Question 3

the stone is negligible. Express all algebraic answers in terms of m, R, v_0, h, μ, and fundamental constants, as appropriate.

(c) Derive an expression for the time it will take the disk to stop sliding.

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Question 3

the stone is negligible. Express all algebraic answers in terms of m, R, v_0, h, μ, and fundamental constants, as appropriate.

(d) [Figure, Top View: A large filled circle (disk) with a small open circle marking its center, and a small stone (filled circle) at the edge of the disk moving tangentially with velocity v_0 (arrow labelled v_0 pointing tangent to the circle at the edge point).]

The person now stands on a similar disk of mass m and radius R that has a fixed pole through its center so that it can only rotate on the ice. The person throws the same stone horizontally in a tangential direction at initial speed v_0, as shown in the figure above. The rotational inertia of the disk is $mR^2/2$.

Derive an expression for the angular speed ω of the disk immediately after the stone is thrown.

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[Figure, Top View: A large filled circle of radius R (labelled with an arrow from center to edge) representing a disk, with a small stone of mass $m/20$ shown just outside the edge of the disk moving away with velocity v_0 (arrow labelled v_0, $m/20$).]
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Question 3

the stone is negligible. Express all algebraic answers in terms of m, R, v_0, h, μ, and fundamental constants, as appropriate.

(e) The person now stands on the disk at rest $R/2$ from the center of the disk. The person now throws the stone horizontally with a speed v_0 in the same direction as in part (d). Is the angular speed of the disk immediately after throwing the stone from this new position greater than, less than, or equal to the angular speed found in part (d)?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.